Problem 1

S=2

```
ρ[w_] := w^2 - w
σ[w_] := 3 w/2 - 1/2
z[θ_] := ρ[Exp[I θ]]/σ[Exp[I θ]]

(*Determine the order of the method*)
Series[ρ[Exp[w]] - w σ[Exp[w]], {w, 0, 5}]
```

$$\frac{5\,w^3}{12}+\frac{3\,w^4}{8}+\frac{47\,w^5}{240}+O[w]^6$$

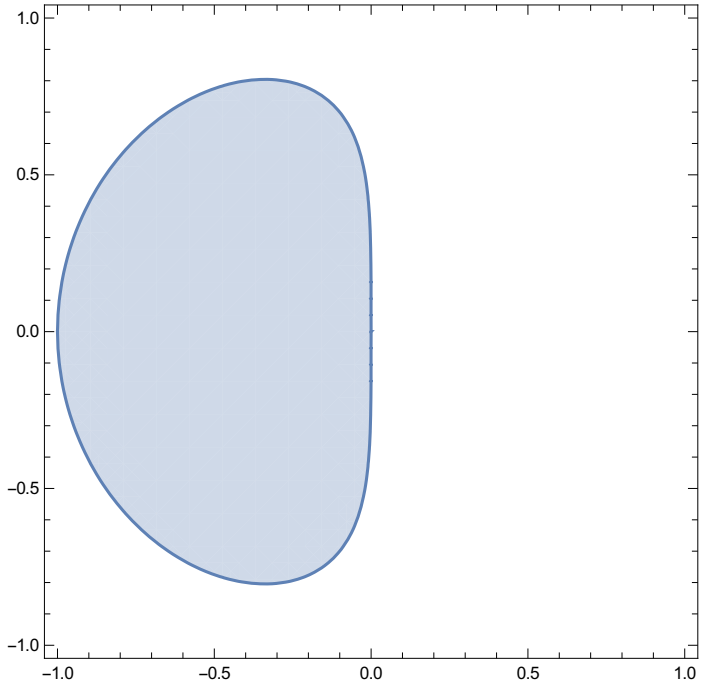
```
(*Determine its region of stability*)

p[w_, k_] := ρ[w] - k σ[w]
Solve[p[w, k] == 0, w]
```

$$\left\{\left\{w\to\frac{1}{4}\left(2+3\,k-\sqrt{4+4\,k+9\,k^2}\right)\right\},\left\{w\to\frac{1}{4}\left(2+3\,k+\sqrt{4+4\,k+9\,k^2}\right)\right\}\right\}$$

```
root[k_] := Max[Abs[1/4 (2 + 3 k - Sqrt[4 + 4 k + 9 k^2])], Abs[1/4 (2 + 3 k + Sqrt[4 + 4 k + 9 k^2])]]

RegionPlot[root[x + I y] ≤ 1, {x, -1, 1}, {y, -1, 1}]
```



S=3

```
ρ[w_] := w^3 - w^2
σ[w_] := 23 w^2/12 - 4 w/3 + 5/12
z[θ_] := ρ[Exp[I θ]]/σ[Exp[I θ]]

(*Determine the order of the method*)
Series[ρ[Exp[w]] - w σ[Exp[w]], {w, 0, 5}]
```

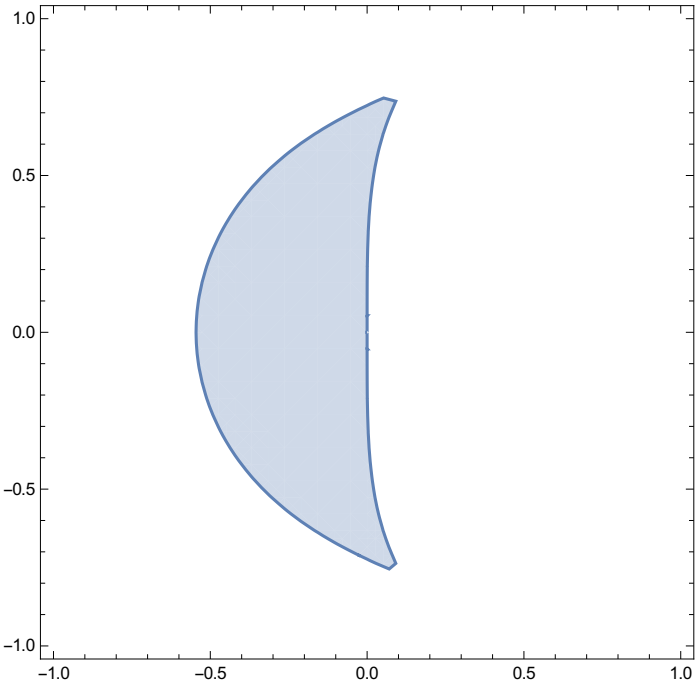
$$\frac{3\,w^4}{8} + \frac{193\,w^5}{360} + O[w]^6$$

```
(*Determine its region of stability*)

p[w_, k_] := ρ[w] - k σ[w];
roots[k_] := w /. Solve[p[w, k] == 0, w]

root[k_] := Max[Abs[roots[k]]]

RegionPlot[root[x + I y] ≤ 1, {x, -1, 1}, {y, -1, 1}]
```



Adams-Moulton

```
ρ[w_] := w^2 - w
σ[w_] := 5 w^2/12 + 2 w/3 - 1/12
z[θ_] := ρ[Exp[I θ]]/σ[Exp[I θ]]
```

```
(*Determine the order of the method*)
Series[ρ[Exp[w]] - w σ[Exp[w]], {w, 0, 5}]
```

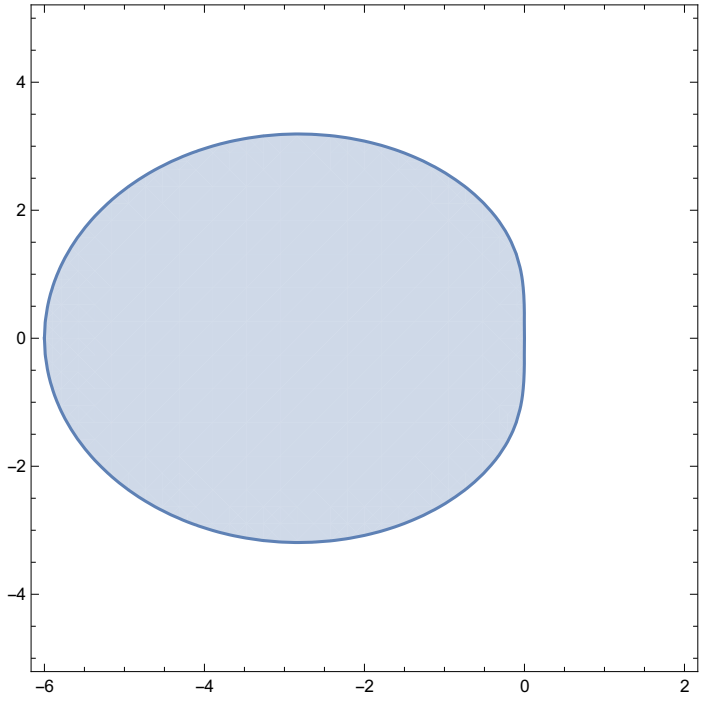
$$-\frac{w^4}{24}-\frac{17\,w^5}{360}+O[w]^6$$

```
(*Determine its region of stability*)

p[w_, k_] := ρ[w] - k σ[w];
roots[k_] := w /. Solve[p[w, k] == 0, w]

root[k_] := Max[Abs[roots[k]]]

RegionPlot[root[x + I y] ≤ 1, {x, -6, 2}, {y, -5, 5}]
```



Problem 2

S=2

```
ρ[w_] := w^2 - 4 w/3 + 1/3
σ[w_] := 2 w^2/3
z[θ_] := ρ[Exp[I θ]]/σ[Exp[I θ]]
```

```
(*Determine the order of the method*)
Series[ρ[Exp[w]] - w σ[Exp[w]], {w, 0, 5}]
```

$$-\frac{2\,w^3}{9}-\frac{5\,w^4}{18}-\frac{17\,w^5}{90}+O[w]^6$$

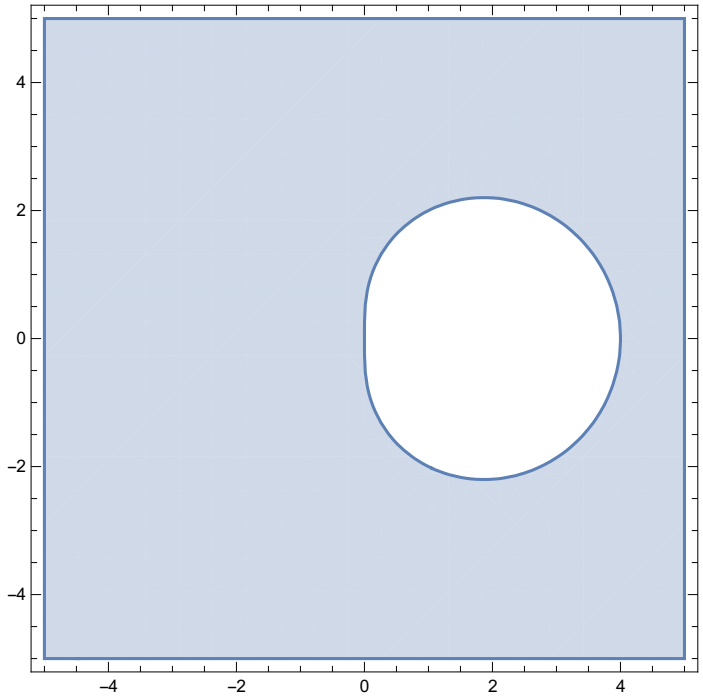
```
(*Determine its region of stability*)

p[w_, k_] := ρ[w] - k σ[w]
Solve[p[w, k] == 0, w]
```

$$\left\{\left\{w\to\frac{-2-\sqrt{1+2\,k}}{-3+2\,k}\right\},\left\{w\to\frac{-2+\sqrt{1+2\,k}}{-3+2\,k}\right\}\right\}$$

```
p[w_, k_] := ρ[w] - k σ[w];
roots[k_] := w /. Solve[p[w, k] == 0, w]
root[k_] := Max[Abs[roots[k]]]

RegionPlot[root[x + I y] ≤ 1, {x, -5, 5}, {y, -5, 5}]
```



S=3

```
ρ[w_] := w^3 - 18 w^2/11 + 9 w/11 - 2/11
σ[w_] := 6 w^3/11
z[θ_] := ρ[Exp[I θ]]/σ[Exp[I θ]]
```

```
(*Determine the order of the method*)
Series[ρ[Exp[w]] - w σ[Exp[w]], {w, 0, 5}]
```

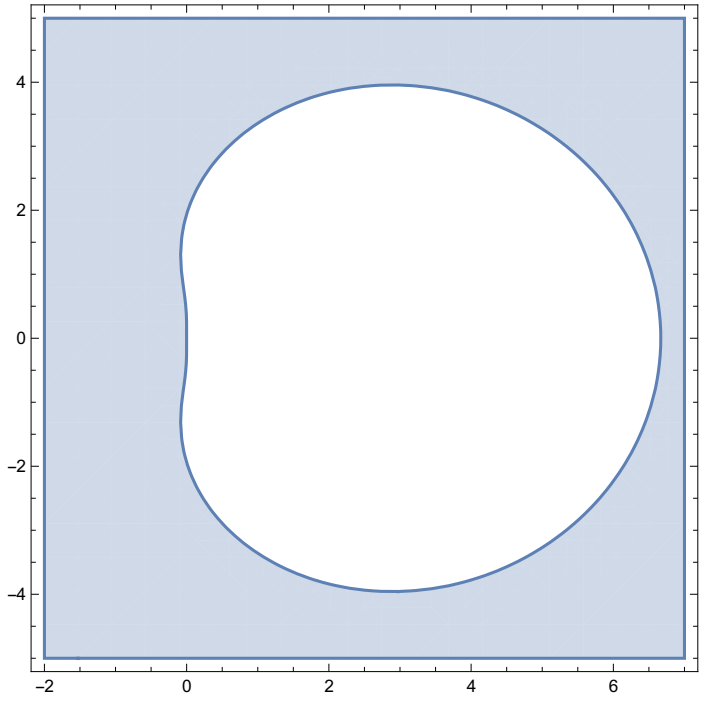
$$-\frac{3\,w^4}{22}-\frac{27\,w^5}{110}+O[w]^6$$

```
(*Determine its region of stability*)

p[w_, k_] := ρ[w] - k σ[w];
roots[k_] := w /. Solve[p[w, k] == 0, w]

root[k_] := Max[Abs[roots[k]]]

RegionPlot[root[x + I y] ≤ 1, {x, -2, 7}, {y, -5, 5}]
```

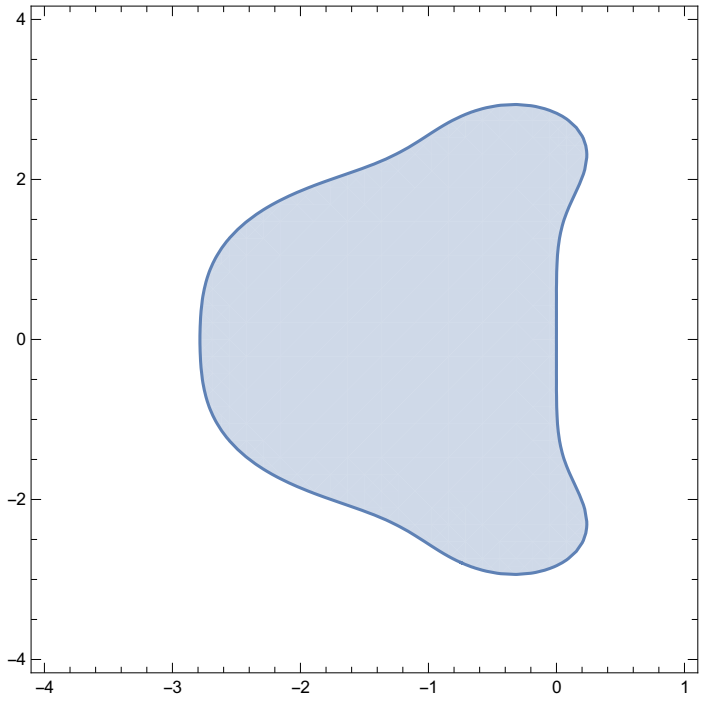


Problem 3

```
k1 = y l;
k2 = Expand[y l (1 + 1/2 l)];
k3 = Expand[y l (1 + 1/2 l (1 + 1/2 l))];
k4 = Expand[y l (1 + l (1 + 1/2 l (1 + 1/2 l)))];

factor = Simplify[y + 1/6 (k1 + 2 k2 + 2 k3 + k4)] / y;
```

```
R[w_] = factor /. {l → w};
RegionPlot[Abs[R[x + I y]] ≤ 1, {x, -4, 1}, {y, -4, 4}]
```




```
(*Where does this region intersect the real axis?*)
N[Solve[R[w] == 1, w][[1]]]
N[Solve[R[w] == 1, w][[4]]]
```

$\{w \to 0.\}$

$\{w \to -2.78529\}$

```
(*Where does this region intersect the imaginary axis?*)
Expand[R[w I] * R[-w I]]
Solve[Expand[R[w I] * R[-w I]] == 1, w]
```

$1 - \frac{w^6}{72} + \frac{w^8}{576}$

$\left\{\{w \to 0\}, \{w \to 0\}, \{w \to 0\}, \{w \to 0\}, \{w \to 0\}, \{w \to 0\}, \left\{w \to -2\sqrt{2}\right\}, \left\{w \to 2\sqrt{2}\right\}\right\}$

Problem 5

In[1]:=
```
Solve[{a0 m a + a0 c - a1 m == g1, b0 m a + b0c - b1 m == g2}, {m, c}]
```

Out[1]=
$$\left\{\left\{m \to -\frac{b0c - g2}{a\,b0 - b1},\ c \to -\frac{-a\,a0\,b0c + a1\,b0c - a\,b0\,g1 + b1\,g1 + a\,a0\,g2 - a1\,g2}{a0\,(a\,b0 - b1)}\right\}\right\}$$