Peter Sheridan Dodds

Principles of Complex Systems
Volume I: What's the John Dory?

June 7, 2017

☐ Write a real acknowledgments section.

Inspiration: On Growth and Form: http://www.nature.com/news/the-100-year-old-challenge-to-darwin-that-is-still-making-waves-in-research-1.21806

"And what's more, the Dundee Advertiser loved the book and recommended it to readers. When the author, it noted, wrote of maths, "he never fails to translate his mathematics into English; and he is one of the relatively few men of science who can write in flawless English and who never grudge the effort to make every sentence balanced and good.""

The Unfollower Monks of the Infinitely Many Wrong Paths. Our Un-guides.

Monk life spans

Monks does suggest men in cowls. I would hope you will be able to imagine monks of all kinds and that the cowls and the suffering are the key features. Octopus monks for example.

Finders and Unfollowers.

Workmonks.

Monkhandling

We will have a lot of fun but do not eat the ... Charlie and the Chocolate Factory

Part A:

The first 40 minutes of the second lecture of PoCS this Fall covers 1. The definition of Complex Systems (not "A", "The") 2. Reductionism, the Golden Age thereof, 3. The Manifesto,

The video is housed within the course's episodes page: `http://www.uvm.edu/~pdodds/teaching/courses/2015-08UVM-300/content/lectures.html#episodes-E25` and on youtube: `https://www.youtube.com/watch?v=Q69DXymX9OE`

Manifesto slides are here: `http://www.uvm.edu/~pdodds/teaching/courses/2015-08UVM-300/content/lectures.html#slides-manifesto`

Part B:

I came up with the manifesto exactly for the reasons you mentioned today. People kept coming up saying complex systems aren't defined and I would say no, they are, here it is. And then I was compelled to lay out the bigger framing of how science simply must unfold (whether we're octopuses or people).

Part C:

The rest of the lecture above and then the next one are on scaling, and travels across all kinds of systems. Includes the awesome Buckingham π Theorem. Everything from physical to biological to sociotechnical. This is a new structuring of the course I introduced this year and it feels very good. Power law size distributions follow—description then a laying out of major mechanisms.

acknowledgement

The hummingbird.

Cite the tweet.

Why did I write this book?

1. First: Complex Systems absolutely are a thing and can be defined well enough. We'll get to this soon.
2. We now understand that there are universal features and properties of systems—stories—that at least touch all fields dealing with complex systems.
3. I want to share what I've gleaned over 20+ years of becoming a scientist wanting to understand the story of everything.

The Schelling model remains our archetypal example. Simple to describe micro dynamics leading to interesting/meaningful macro phenomena which gives us a new narrative of collective behaviour (which we are ill placed to intuit).

Religion:

Successful religions have, ironically, been in part a result of natural selection of stories.

Not believing in evolution is an evolutionary outcome.

Soon after arriving at the University of Vermont, I began constructing my two courses on Complex Systems in the 2007:

- Principles of Complex Systems. □ Include logo
- Complex Networks.

The two, three, no four!, main goals of this book:

What this book won't do for you:

-

Success Jk Rowling writing with a pen name, did okay. Sales exploded when revealed.

`http://www.theguardian.com/books/2014/jul/19/jk-rowling-crime-thriller-series-longer-harry-potter`
Can you see the story?

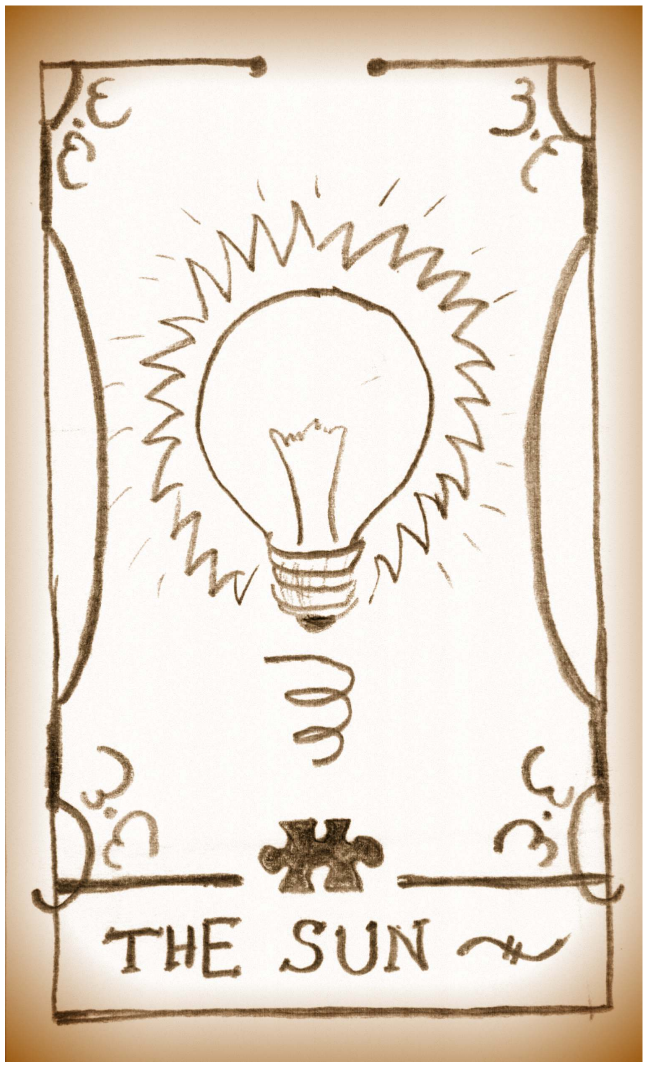
Diseases, ebola spreading.

Easiest stories to follow.

Braiding conspiracy theories.

SimulPast @SimulPast Why do we need the science of complexity to tackle the most difficult questions? #ComplexSystems #ComplexityScience vimeo.com/125538277

We only need a Science of Complexity if such a thing exists. Step 1: What we need to understand is complexity. Honey and water may have followed different equations.


THE SUN

Story No. 1

Introduction

1.1 Orientation

Course Website:
http://www.uvm.edu/~pdodds/teaching/courses/2014-08UVM-300

Course Twitter handle: @pocsvox

Graduate Certificate:

Principles of Complex Systems is one of two core requirements for UVM's five course Certificate of Graduate Study in Complex Systems.

Other required course: Prof. Maggie Eppstein's "Modelling Complex Systems" (CSYS/CS 302).

The Sequel to PoCS: "Complex Networks" (CSYS/MATH 303).

Exciting details regarding these slides:

Web links look like this and are eminently clickable.

References in slides link to full citation at end. [2]

Citations contain links to pdfs for papers (if available).

Some books will be linked to on amazon.

Brought to you by a frightening melange of LaTeX, Beamer, perl, PerlTeX, fevered command-line madness, and an almost fanatical devotion to the indomitable emacs.
#superpowers

More super exciting details:

This is Season 8 of Principles of Complex Systems.

Lectures will be called Episodes.

All lectures are bottle episodes.

Other tropes will be involved.

Last season's Episodes are here.

Grading breakdown:

Projects/talks (36%)—Students will work on semester-long projects. Students will develop a proposal in the first few weeks of the course which will be discussed with the instructor for approval.

Details: 12% for the first talk, 12% for the final talk, and 12% for the written project.

Assignments (60%)—All assignments will be of equal weight and there will be nine or ten of them.

General attendance/Class participation (4%)

How grading works:

Questions are worth 3 points according to the following scale:

3 = correct or very nearly so.

2 = acceptable but needs some revisions.

1 = needs major revisions.

0 = way off.

Important things:

1. Classes run from Tuesday, August 26 to Tuesday, December 2.
2. Add/Drop, Audit, Pass/No Pass deadline—Monday, September 8.
3. Last day to withdraw—Monday, October 27 (Sadness!).
4. Reading and Exam period—Thursday, December 4 to Friday, December 12.

Do check your zoo account for updates regarding the course.

Academic assistance: Anyone who requires assistance in any way (as per the ACCESS program or due to athletic endeavors), please see or contact me as soon as possible.

1.1.1 Topics

The nature of PoCS:

Transitional from standard coursework to research-focused work. #alittlescary

Major themes:

The Complexity Manifesto;

Complex Systems ≡ Modern, Normal Science;

Roles and limits of Data, Theory, and Experiment;

Emergence;

Universality and Accidents of History;

Structure and Stories: Micro-to-macro Mechanisms;

Elements: Scaling, Surprise, Networks, Robustness, Failure, and Spreading.

The Theory of Anything: Why Complexify?

Topics:

Scaling phenomena:

Power law size distributions and non-Gaussian statistics and

Zipf's law

Key mechanisms for generating power law size distributions

Allometry

Scaling of social phenomena: crime, creativity, and consumption.

Scaling in biology (elephants and platypuses).

Renormalization techniques

Topics:

Complex networks:

Structure and Dynamics;

Statistical Mechanics;

Phase transitions;

Random Networks;

Scale-free Networks;

Small-world Networks.

Multiscale complex systems:

Hierarchies and Scaling;

Modularity;

Topics:

Integrity of complex systems:

Generic failure mechanisms

Network robustness

Highly Optimized Tolerance (HOT): Robustness and fragility

Predictablity

Information and Language:

Search in networked systems
(e.g., the web, social systems)

Search on scale-free networks

Knowledge trees, metadata and tagging

Evolution and structure of natural languages

Topics:

Sociotechnical Systems:

Biological and social spreading models;

Schelling's model of segregation; [43]

Granovetter's model of imitation; [23]

Collective behavior and Synchrony;

Global cooperation from bad actors;

Global conflicts from good actors;

Stories (Homo Narrativus);

The Sociotechnocene.

Topics:

Large-scale social patterns:

Movement of individuals;

Cities;

Happiness;

Twitter.

Collective decision making:

Wisdom and madness of crowds;

Systems of voting;

The role of randomness and chance;

Success inequality: superstardom;

Season's Narrative Arc (or Places We Will Go):

Overview of Complexity with bonus Manifesto.

Size distributions of system elements:

Power-law size distributions.

Description and Mechanisms of Becoming.

Robustness of Complex Systems.

Complex networks—how system elements are connected:

Structure, Growth Mechanisms, Processes on Networks.

Social Contagion, Voting, Fame and Fate, Stories.

Allometric scaling in complex systems.

Happiness.

Complexification: The Theory of Anything.

Schedule in detail:

Richard Feynmann on the Social Sciences:

Sheldon Cooper on the Social Sciences:

1.1.2 Projects

Projects

Semester-long projects, teams of 2 or 3.

Develop proposal in first few weeks.

May range from novel research to investigation of an established area of complex systems.

Two talks + written piece.

Usage of the VACC is encouraged (ability to code well = super powers).

Massive data sets available, including Twitter.

Academic output (journal papers) resulting from Principles of Complex Systems and Complex Networks can be found here. Add more!

We'll go through a list of possible projects soon.

Projects

The narrative hierarchy—explaining things on all scales:

1 to 3 word encapsulation = a soundbite = a buzzframe,

1 sentence, title,

The Nutshell (or Log line), Also: the Nut Graph

Log line,

few sentences,

a paragraph, abstract,

short paper, essay,

long paper,

chapter,

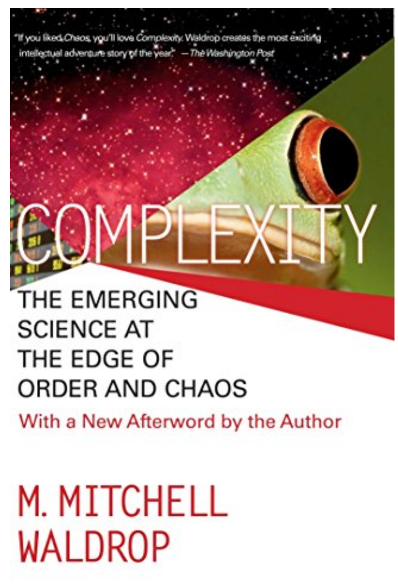
book,

…

1.1.3 Centers, Books, Resources

Popular Science Books:

Historical artifact:



"Complexity: The Emerging Science at the Edge of Order and Chaos"
by M. Mitchell Waldrop (1993). [53]

Shout-out: Dr. Andrew P. Morokoff,
MBBS PhD FRACS D.Thau (Bug)

Popular Science Books:

"Simply Complexity: A Clear Guide to Complexity Theory"
by Neil F. Johnson (2009). [27]

"Complexity: A Guided Tour"
by Melanie Mitchell (2009). [39]

"The Information: A History, A Theory, A Flood"
by James Gleick (2011). [22]

On complex sociotechnical systems:

"Human Behaviour and the Principle of Least-Effort"
by G. K. Zipf (1949). [56]

"Micromotives and Macrobehavior"
by Thomas C. Schelling (1978). [45]

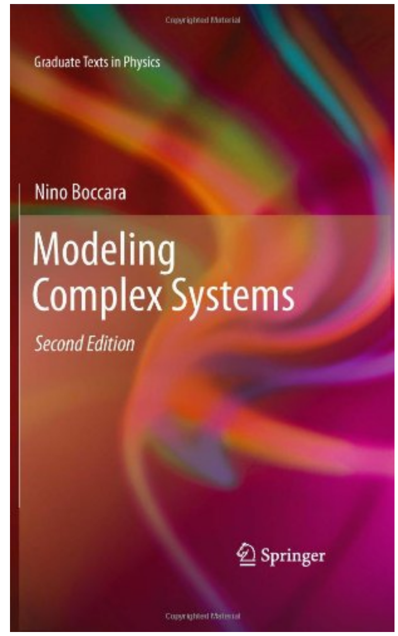
"Critical Mass: How One Thing Leads to Another"
by Philip Ball (2004). [4]

A few textbooky books:

"Complex Adaptive Systems: An introduction to computational models of social life"
by John H. Miller and Scott E. Page and (2007). [38]

"Critical Phenomena in Natural Sciences"
by Didier Sornette (2003). [49]

"Modeling Complex Systems"
by Nino Boccara (2004). [9]



Relevant online courses:

Melanie Mitchell (Santa Fe Institute):
Introduction to Complexity

Lada Adamic (Michigan):
Social Network Analysis

Centers:

Santa Fe Institute (SFI)

New England Complex Systems Institute (NECSI)

Michigan's Center for the Study of Complex Systems (CSCS)

Northwestern Institute on Complex Systems (NICO)

Also: Indiana, Davis, Brandeis, University of Illinois, Duke, Warsaw, Melbourne, ...,

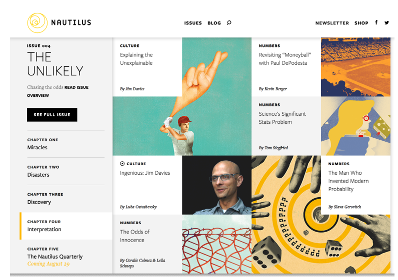
Vermont Complex Systems Center

Other inputs:

Complexity Digest:

http://www.comdig.org
https://twitter.com/@cxdig

Nautilus Magazine: http://nautil.us/



1.2 Fundamentals

1.2.1 Complexity

Definitions

Complex: (Latin = with + fold/weave (com + plex))

Adjective:

Made up of multiple parts; intricate or detailed. Not simple or straightforward.

Definitions

Complicated versus Complex:

Complicated: Mechanical watches, airplanes, ... Engineered systems can be made to be highly robust but not adaptable. But engineered systems can become complex (power grid, planes). They can also fail spectacularly. Explicit distinction: Complex Adaptive Systems.

Definitions

The Wikipedia on Complex Systems:

"Complexity science is not a single theory: it encompasses more than one theoretical framework and is highly interdisciplinary, seeking the answers to some fundamental questions about living, adaptable, changeable systems."

Nino Boccara in *Modeling Complex Systems*: [10] "... there is no universally accepted definition of a complex system ... most researchers would describe a system of connected agents that exhibits an emergent global behavior not imposed by a central controller, but resulting from the interactions between the agents."

Definitions

Philip Ball in *Critical Mass*: [4] "...complexity theory seeks to understand how order and stability arise from the interactions of many components according to a few simple rules."

Definitions

A meaningful definition of a Complex System:

Distributed system of many interrelated (possibly networked) parts with no centralized control exhibiting emergent behavior—'More is Different' [2]

A few optional features:

Explicit nonlinear relationships

Presence of feedback loops

Being open or driven, opaque boundaries

Presence of memory

Modular (nested)/multiscale structure

Examples of Complex Systems:

human societies financial systems cells ant colonies weather systems ecosystems

animal societies disease ecologies brains social insects geophysical systems the world wide web

i.e., everything that's interesting...

Relevant fields:

Physics

Economics

Sociology

Psychology

Information Sciences

Cognitive Sciences

Biology

Ecology

Geociences

Geography

Medical Sciences

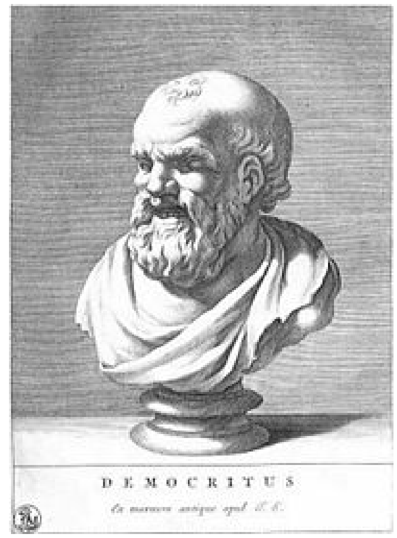
Systems Engineering

Computer Science

...

i.e., everything that's interesting...

Reductionism:



Democritus
(ca. 460 BC – ca. 370 BC)

Atomic hypothesis

Atom ∼ a (not) – temnein (to cut)

Plato allegedly wanted his books burned.

John Dalton
1766–1844

Chemist, Scientist

Developed atomic theory

First estimates of atomic weights

Reductionism:

Ludwig Boltzmann**, 1844–1906.**
Atomic Theory.

"Boltzmann's kinetic theory of gases seemed to presuppose the reality of atoms and molecules, but almost all German philosophers and many scientists like Ernst Mach and the physical chemist Wilhelm Ostwald disbelieved their existence."

"In 1904 at a physics conference in St. Louis most physicists seemed to reject atoms and he was not even invited to the physics section. Rather, he was stuck in a section called "applied mathematics," he violently attacked philosophy, especially on allegedly Darwinian grounds but actually in terms of Lamarck's theory of the inheritance of acquired characteristics that people inherited bad philosophy from the past and that it was hard for scientists to overcome such inheritance."

Reductionism:

Albert Einstein 1879–1955

Annus Mirabilis paper: "the Motion of Small Particles Suspended in a Stationary Liquid, as Required by the Molecular Kinetic Theory of Heat" [18, 19]

Showed Brownian motion followed from an atomic model giving rise to diffusion.

Jean Perrin 1870–1942

1908: Experimentally verified Einstein's work and Atomic Theory.

Feynmann: If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generation of creatures, what statement would contain the most information in the fewest words? I believe it is the atomic hypothesis that all things are made of atoms—little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another. In that one sentence, you will see, there is an enormous amount of information about the world, if just a little imagination and thinking are applied.

Snared from brainpickings.org

Complexity Manifesto:

Systems are ubiquitous and systems matter.

Consequently, much of science is about understanding how pieces dynamically fit together.

1700 to 2000 = Golden Age of Reductionism.

Atoms!, sub-atomic particles, DNA, genes, people, ...

Understanding and creating systems (including new 'atoms') is the greater part of science and engineering.

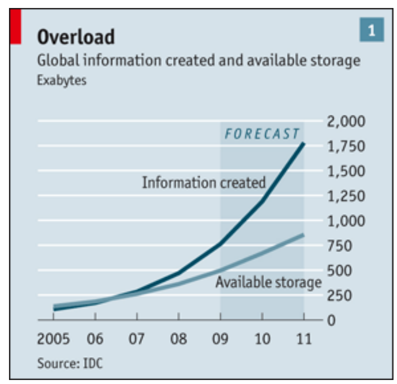
Universality: systems with quantitatively different micro details exhibit qualitatively similar macro behavior.

Computing advances make the Science of Complexity possible:

We can measure and record enormous amounts of data, research areas continue to transition from data scarce to data rich.

We can simulate, model, and create complex systems in extraordinary detail.

Data, Data, Everywhere—the Economist, Feb 25, 2010



Exponential growth: $\sim 60\%$ per year.

Big Data Science:

2013: year traffic on Internet estimate to reach 2/3 Zettabytes
($1\text{ZB} = 10^3\text{EB} = 10^6\text{PB} = 10^9\text{TB}$)

Large Hadron Collider: 40 TB/second.

2016—Large Synoptic Survey Telescope:
140 TB every 5 days.

Data inflation 2

Unit	Size	What it means
Bit (b)	1 or 0	Short for "binary digit", after the binary code (1 or 0) computers use to store and process data
Byte (B)	8 bits	Enough information to create an English letter or number in computer code. It is the basic unit of computing
Kilobyte (KB)	1,000, or 2^{10}, bytes	From "thousand" in Greek. One page of typed text is 2KB
Megabyte (MB)	1,000KB; 2^{20} bytes	From "large" in Greek. The complete works of Shakespeare total 5MB. A typical pop song is about 4MB
Gigabyte (GB)	1,000MB; 2^{30} bytes	From "giant" in Greek. A two-hour film can be compressed into 1-2GB
Terabyte (TB)	1,000GB; 2^{40} bytes	From "monster" in Greek. All the catalogued books in America's Library of Congress total 15TB
Petabyte (PB)	1,000TB; 2^{50} bytes	All letters delivered by America's postal service this year will amount to around 5PB. Google processes around 1PB every hour
Exabyte (EB)	1,000PB; 2^{60} bytes	Equivalent to 10 billion copies of *The Economist*
Zettabyte (ZB)	1,000EB; 2^{70} bytes	The total amount of information in existence this year is forecast to be around 1.2ZB
Yottabyte (YB)	1,000ZB; 2^{80} bytes	Currently too big to imagine

Source: *The Economist*

The prefixes are set by an intergovernmental group, the International Bureau of Weights and Measures. Yotta and Zetta were added in 1991; terms for larger amounts have yet to be established.

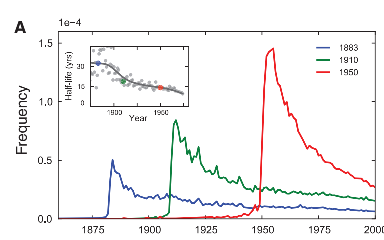
Facebook: ~ 250 billion photos (mid 2013)

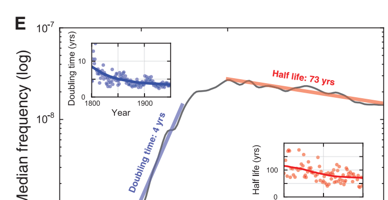
Twitter: ~ 500 billion tweets (mid 2013)

No really, that's a lot of data

Big Data—Culturomics:

"Quantitative analysis of culture using millions of digitized books" by Michel et al., Science, 2011 [37]





http://www.culturomics.org/
Google Books ngram viewer

Basic Science $\simeq$ Describe + Explain:

Lord Kelvin (possibly):

"To measure is to know."

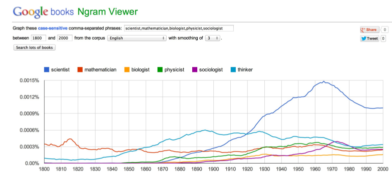
"If you cannot measure it, you cannot improve it."

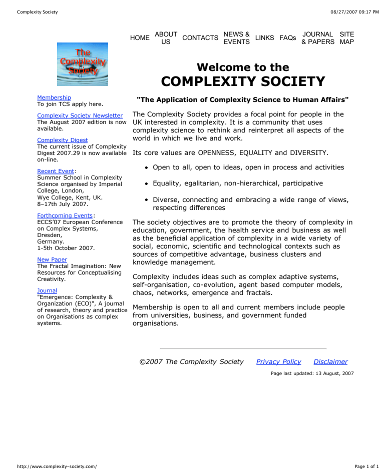
Bonus:

"X-rays will prove to be a hoax."

"There is nothing new to be discovered in physics now, All that remains is more and more precise measurement."

The Newness of being a Scientist:





Complexity Society 08/27/2007 09:17 PM

HOME | ABOUT US | CONTACTS | NEWS & EVENTS | LINKS | FAQs | JOURNAL & PAPERS | SITE MAP

The Complexity Society

Welcome to the
COMPLEXITY SOCIETY

"The Application of Complexity Science to Human Affairs"

The Complexity Society provides a focal point for people in the UK interested in complexity. It is a community that uses complexity science to rethink and reinterpret all aspects of the world in which we live and work.

Its core values are OPENNESS, EQUALITY and DIVERSITY.

- Open to all, open to ideas, open in process and activities
- Equality, egalitarian, non-hierarchical, participative
- Diverse, connecting and embracing a wide range of views, respecting differences

The society objectives are to promote the theory of complexity in education, government, the health service and business as well as the beneficial application of complexity in a wide variety of social, economic, scientific and technological contexts such as sources of competitive advantage, business clusters and knowledge management.

Complexity includes ideas such as complex adaptive systems, self-organisation, co-evolution, agent based computer models, chaos, networks, emergence and fractals.

Membership is open to all and current members include people from universities, business, and government funded organisations.

Membership
To join TCS apply here.

Complexity Society Newsletter
The August 2007 edition is now available.

Complexity Digest
The current issue of Complexity Digest 2007.29 is now available on-line.

Recent Event:
Summer School in Complexity Science organised by Imperial College, London,
Wye College, Kent, UK.
8–17th July 2007.

Forthcoming Events:
ECCS'07 European Conference on Complex Systems,
Dresden,
Germany.
1-5th October 2007.

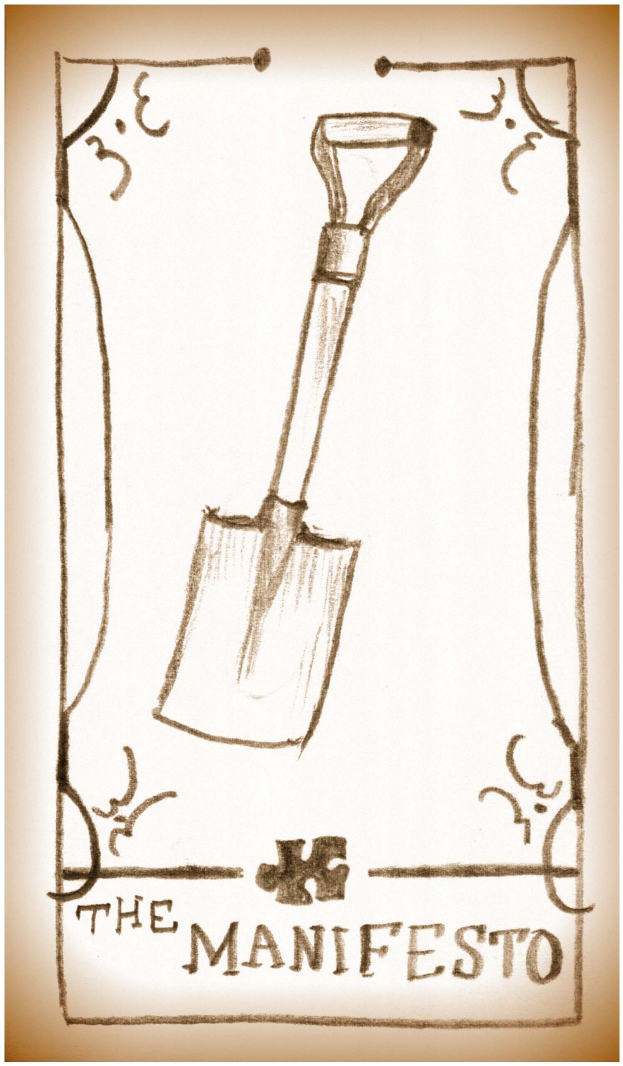
New Paper
The Fractal Imagination: New Resources for Conceptualising Creativity.

Journal
"Emergence: Complexity & Organization (ECO)", A journal of research, theory and practice on Organisations as complex systems.

©2007 The Complexity Society *Privacy Policy* *Disclaimer*

Page last updated: 13 August, 2007

http://www.complexity-society.com/ Page 1 of 1


THE MANIFESTO

Story No. 2

Free Accompanying Manifesto

2.1 Defining Complexity

Definitions

Complex: (Latin = with + fold/weave (com + plex))

Adjective:

Made up of multiple parts; intricate or detailed. Not simple or straightforward.
Definitions

Complicated versus Complex: Complicated: Mechanical watches, airplanes, ...

Engineered systems can be made to be highly robust but not adaptable.

But engineered systems can become complex (power grid, planes).

They can also fail spectacularly.

Explicit distinction: Complex Adaptive Systems.

A while ago: **The Wikipedia on Complex Systems:** "Complexity science is not a single theory: it encompasses more than one theoretical framework and is highly interdisciplinary, seeking the answers to some fundamental questions about living, adaptable, changeable systems." **Now:**

"Complex systems present problems both in mathematical modelling and philosophical foundations. The study of complex systems represents a new approach to science that investigates how relationships between parts give rise to the collective behaviors of a system and how the system interacts and forms relationships with its environment." **Definitions**

Nino Boccara in *Modeling Complex Systems*: [10] "... there is no universally accepted definition of a complex system ... most researchers would describe a system of connected agents that exhibits an emergent global behavior not imposed by a central controller, but resulting from the interactions between the agents." **Philip Ball in *Critical Mass*:** [4] "...complexity theory seeks to understand how order and stability arise from the interactions of many components according to a few simple rules."

Definitions

A working definition of a Complex System: Distributed system of many interrelated (possibly networked) parts with no centralized control exhibiting emergent behavior—'More is Different' [2] **Other features/aspects:** Explicit nonlinear relationships.

Presence of feedback loops.

Being open or driven, opaque boundaries.

Memory.

Modular (nested)/multiscale structure.

Mechanisms range from being purely physical to purely algorithmic in nature.
Examples of Complex Systems:

human societies

financial systems

cells

ant colonies

weather systems

ecosystems

power grids

animal societies

disease ecologies

brains

social insects

geophysical systems

Internet + Web

i.e., everything that's interesting …**Relevant fields:**

Physics

Economics

Sociology

Psychology

Information Sciences

Cognitive Sciences

Biology

Ecology

Geociences

Geography

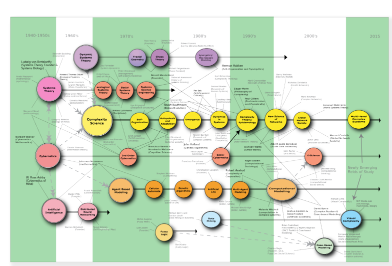
Medical Sciences

Systems Engineering

Computer Science

…

i.e., everything that's interesting …**A visualized history of Complex Systemsish fields:**



"Complexity Map" by Brian Castellani/Wiki

Online here: https://en.wikipedia.org/wiki/Complex_systems#History **The Golden Age of Reductionism:**

Reductionism:

Democritus
(ca. 460 BC – ca. 370 BC) Atomic hypothesis

Atom ∼ a (not) – temnein (to cut)

Plato allegedly wanted his books burned.

John Dalton
1766–1844 Chemist, Scientist

Developed atomic theory

First estimates of atomic weights

Ludwig Boltzmann, 1844–1906. Atomic Theory.

"Boltzmann's kinetic theory of gases seemed to presuppose the reality of atoms and molecules, but almost all German philosophers and many scientists like Ernst Mach and the physical chemist Wilhelm Ostwald disbelieved their existence."

"In 1904 at a physics conference in St. Louis most physicists seemed to reject atoms and he was not even invited to the physics section. Rather, he was stuck in a section called "applied mathematics," he violently attacked philosophy, especially on allegedly Darwinian grounds but actually in terms of Lamarck's theory of the inheritance of acquired characteristics that people inherited bad philosophy from the past and that it was hard for scientists to overcome such inheritance."

See: epigenetics.

Albert Einstein 1879–1955 Annus Mirabilis paper: "the Motion of Small Particles Suspended in a Stationary Liquid, as Required by the Molecular Kinetic Theory of Heat" [18, 19]

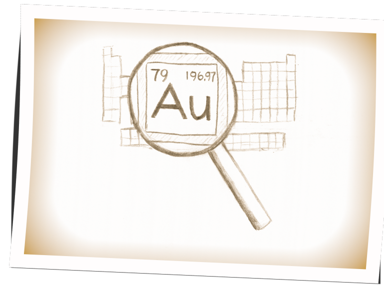
Showed Brownian motion followed from an atomic model giving rise to diffusion.

Jean Perrin 1870–1942 1908: Experimentally verified Einstein's work and Atomic Theory.

Feynmann:

"If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generation of creatures, what statement would contain the most information in the fewest words?

"I believe it is the atomic hypothesis that all things are made of atoms—little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another. "In that one sentence, you will see, there is an enormous amount of information about the world, if just a little imagination and thinking are applied." Snared from brainpickings.org



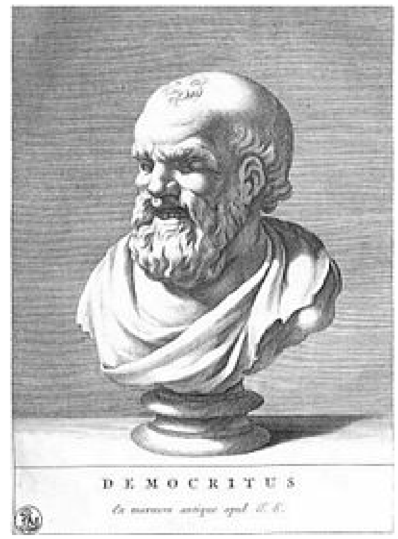


Figure 2.1: Burn all of this things.

2.2 A Manifesto

The Science of Complex Systems Manifesto: Systems are ubiquitous and systems matter.

Consequently, much of science is about understanding how pieces dynamically fit together.

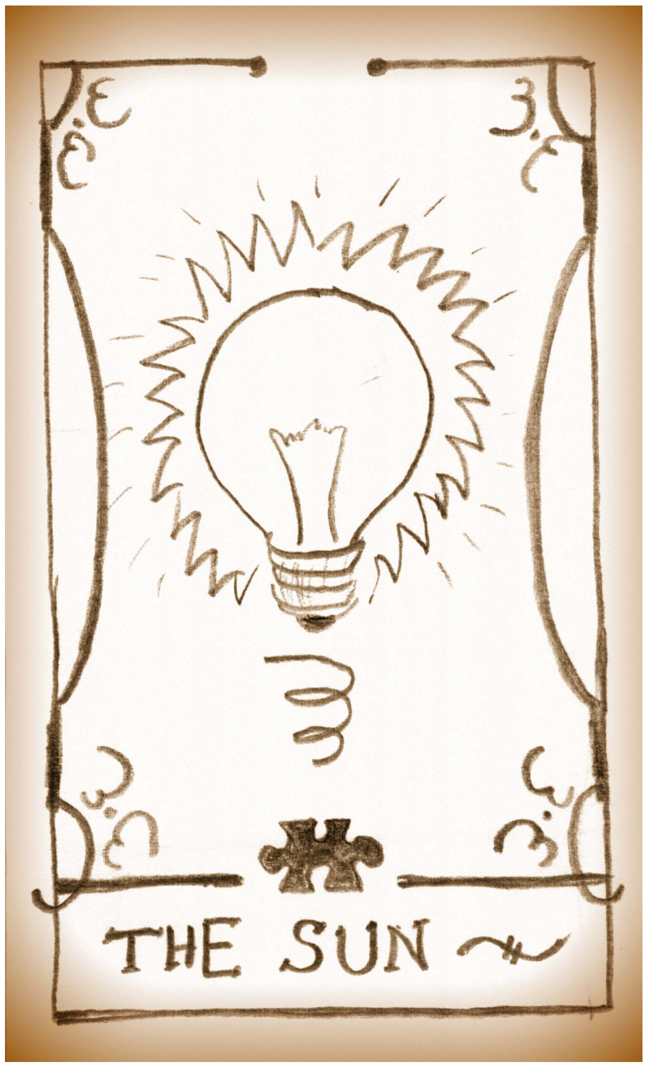
1700 to 2000 = Golden Age of Reductionism:
Atoms!, sub-atomic particles, DNA, genes, people, ...

Understanding and creating systems (including new 'atoms') is the greater part of science and engineering.

Universality: systems with quantitatively different micro details exhibit qualitatively similar macro behavior.

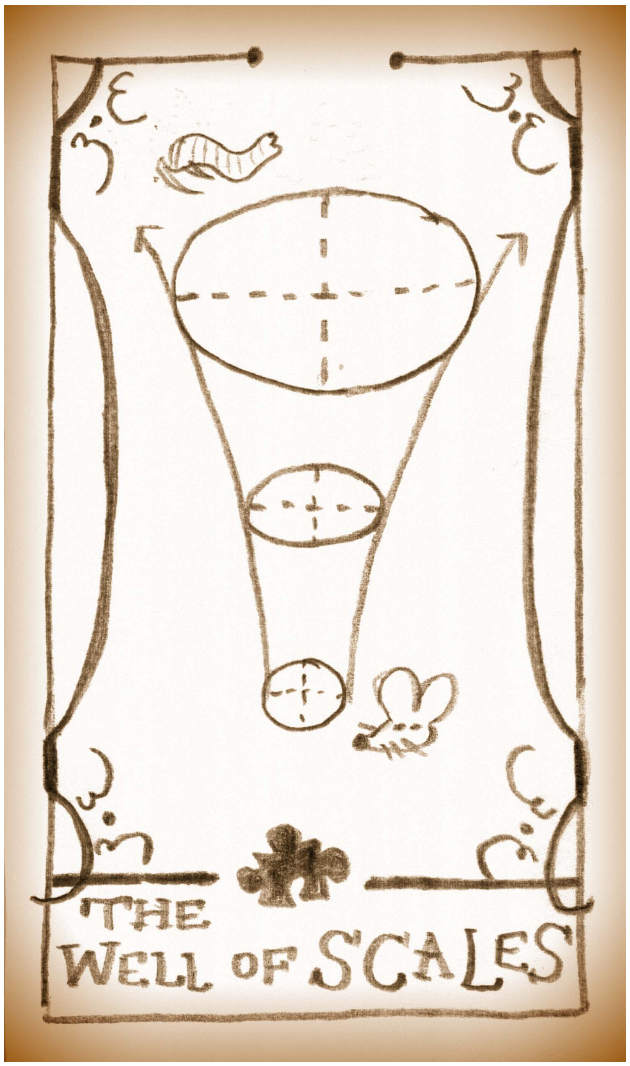
Computing advances make the Science of Complex Systems possible: We can measure and record enormous amounts of data, research areas continue to transition from data scarce to data rich.

We can simulate, model, and create complex systems in extraordinary detail.


THE SUN

Story No. 3

Fundamentals


THE
WELL OF SCALES

Story No. 4

Scaling

4.1 Introduction to Scalingology

Systems whose components and properties cross many spatial and temporal scales often exhibit some form of **scaling**.

Examples

We'll find that some scaling laws can be uncovered with an ancient technology: the back of envelope.[1] But we'll also see that describing and explaining scaling can both be fraught scientific activities.[2]

[1] The best magic rectangle is a blank piece of paper.

[2] In a "Behind the Science" kind of way.

While we'll lay out general principles for thinking about scaling, some derivations of scaling laws will have a magical feel.

We're going to take a tour of the vast world of scaling, starting with physical systems and moving up through biological to social and informational realms. Some scalings are

The systems may be

The surface areas Economic productivity as a function of country or city population, the expected number of heartbeats in an organism's lifespan as a function of body mass;

In this chapter, we'll move through

- some basic definitions,
- take in a host of examples,
- How to measure your power-law relationship.

A simple power law[3] relates two variables x and y as:

$$y = cx^{\alpha}. \tag{4.1}$$

We say α is the law's scaling exponent or just its exponent.[4] The possible values of the exponent α will find various restrictions, depending on context.

Unexcitingly, we call the constant c the prefactor. The prefactor is often overlooked in more cavalier scaling calculations[5] but we want to make sure to mark it here as being potentially important.

A super simple physical example is the scaling of a sphere's surface area S as a function of its radius r: $S = 4\phi r^2$ where $c = 4\pi$ and $\alpha = 2$.

[3] The widespread use of 'law' is often generous and should not be interpreted as meaning every power law is set in stone. In many cases, lore would be the better word to hear.

[4] In some scientific victories, expwnent may be appropriate.

[5] Theoretical physicists capable of feeling shame begin to inspect their feet.

For clarity of presentation and to prevent back of the envelope misadventures, the Unfollower Monks tell us that if we know a scaling exponent is negative, we should express that scaling explicitly:

$$y = cx^{-\alpha}. \tag{4.2}$$

Definitions

The prefactor c must balance dimensions.

Imagine the height ℓ and volume v of a family of shapes are related as:

$$\ell = cv^{1/4}$$

Using $[\cdot]$ to indicate dimension, then

$$[c] = [l]/[V^{1/4}] = L/L^{3/4} = L^{1/4}.$$

Looking at data

Power-law relationships are linear in log-log space:

$$y = cx^{\alpha}$$

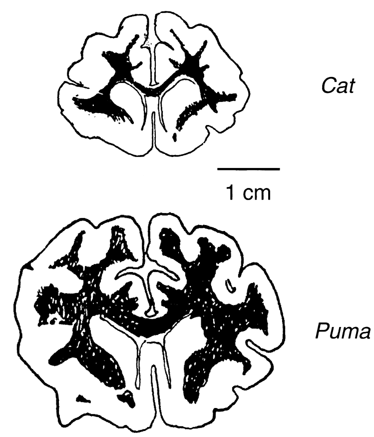
$$\Rightarrow \log_b y = \alpha \log_b x + \log_b c$$

with slope equal to α, the scaling exponent.

Much searching for straight lines on log-log or double-logarithmic plots.

Good practice: Always, always, always use base 10.

Talk only about orders of magnitude (powers of 10). **A beautiful, heart-warming example:**

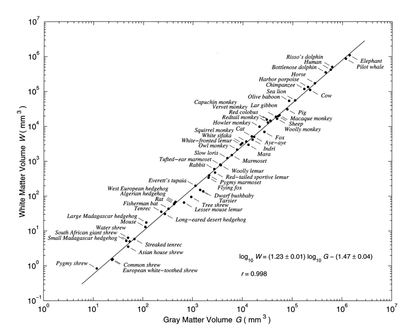


G = volume of gray matter:
'computing elements'

W = volume of white matter:
'wiring'

$W \sim cG^{1.23}$

from Zhang & Sejnowski, PNAS (2000) [55]



Why is $\alpha \simeq 1.23$?

Quantities (following Zhang and Sejnowski):

G = Volume of gray matter (cortex/processors) W = Volume of white matter (wiring) T = Cortical thickness (wiring) S = Cortical surface area L = Average length of white matter fibers p = density of axons on white matter/cortex interface **A rough understanding:**

$G \sim ST$ (convolutions are okay) $W \sim \frac{1}{2}pSL$ $G \sim L^3$ ← this is a little sketchy... Eliminate S and L to find $W \propto G^{4/3}/T$

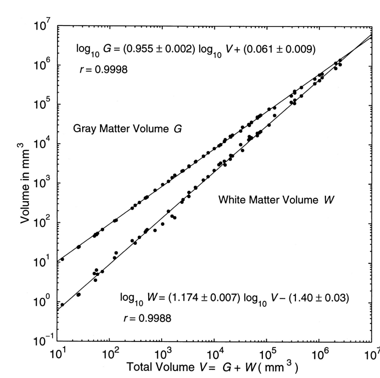
Why is $\alpha \simeq 1.23$?

A rough understanding: We are here: $W \propto G^{4/3}/T$

Observe weak scaling $T \propto G^{0.10 \pm 0.02}$.

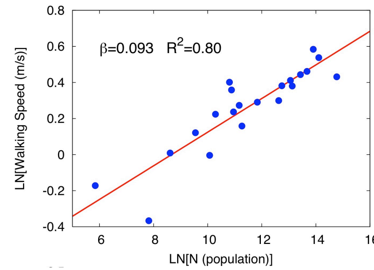
(Implies $S \propto G^{0.9} \rightarrow$ convolutions fill space.)

$\Rightarrow W \propto G^{4/3}/T \propto G^{1.23 \pm 0.02}$ **Trickiness:**



With $V = G + W$, some power laws must be approximations. Measuring exponents is a hairy business... **Good scaling:**

General rules of thumb: *High quality:* scaling persists over
three or more orders of magnitude
for each variable. *Medium quality:* scaling persists over
three or more orders of magnitude
for only one variable and at least one for the other. *Very dubious:* scaling 'persists' over
less than an order of magnitude
for both variables. **Unconvincing scaling:**



Average walking speed as a function of city population:

Two problems: use of natural log, and

minute varation in dependent variable.

from Bettencourt et al. (2007) [7]; otherwise totally great—see later. **Definitions**

Power laws are the signature of scale invariance:

Scale invariant 'objects'
look the 'same'
when they are appropriately rescaled.

Objects = geometric shapes, time series, functions, relationships, distributions,...

'Same' might be 'statistically the same'

To rescale means to change the units of measurement for the relevant variables
Scale invariance

Our friend $y = cx^\alpha$: If we rescale x as $x = rx'$ and y as $y = r^\alpha y'$,

then

$$r^\alpha y' = c(rx')^\alpha$$

$$\Rightarrow y' = cr^\alpha x'^\alpha r^{-\alpha}$$

$$\Rightarrow y' = cx'^\alpha$$

Scale invariance

Compare with $y = ce^{-\lambda x}$: If we rescale x as $x = rx'$, then

$$y = ce^{-\lambda r x'}$$

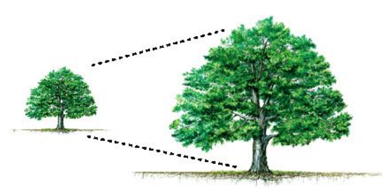
Original form cannot be recovered.

Scale matters for the exponential. **More on $y = ce^{-\lambda x}$:** Say $x_0 = 1/\lambda$ is the characteristic scale.

For $x \gg x_0$, y is small,
while for $x \ll x_0$, y is large.

4.2 Allometry

Isometry:



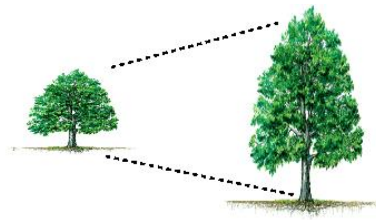
Dimensions scale linearly with each other.

Allometry:

Dimensions scale nonlinearly.

Allometry: Refers to differential growth rates of the parts of a living organism's body part or process.

First proposed by Huxley and Teissier, Nature, 1936
"Terminology of relative growth" [26, 47] **Definitions**



Isometry versus Allometry: Iso-metry = 'same measure'

Allo-metry = 'other measure'

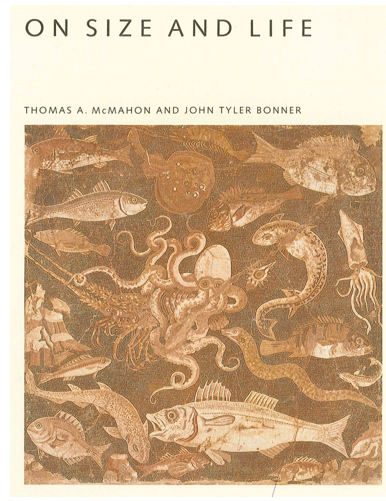
Confusingly, we use allometric scaling to refer to both: Nonlinear scaling of a dependent variable on an independent one (e.g., $y \propto x^{1/3}$)

The relative scaling of correlated measures
(e.g., white and gray matter).

4.3 Examples in Biology

An interesting, earlier treatise on scaling:

McMahon and Bonner, 1983 [35]



The many scales of life:

p. 2, McMahon and Bonner [35]

The biggest living things (*left*). All the organisms are drawn to the same scale. *1*, The largest flying bird (albatross); *2*, the largest known animal (the blue whale), *3*, the largest extinct land mammal (*Baluchitherium*) with a human figure shown for scale; *4*, the tallest living land animal (giraffe); *5*, *Tyrannosaurus*; *6*, *Diplodocus*; *7*, one of the largest flying reptiles (*Pteranodon*); *8*, the largest extinct snake; *9*, the length of the largest tapeworm found in man; *10*, the largest living reptile (West African crocodile); *11*, the largest extinct lizard; *12*, the largest extinct bird (*Aepyornis*); *13*, the largest jellyfish (*Cyanea*); *14*, the largest living lizard (Komodo dragon); *15*, sheep; *16*, the largest bivalve mollusc (*Tridacna*); *17*, the largest fish (whale shark); *18*, horse; *19*, the largest crustacean (Japanese spider crab); *20*, the largest sea scorpion (Eurypterid); *21*, large tarpon; *22*, the largest lobster; *23*, the largest mollusc (deep-water squid, *Architeuthis*); *24*, ostrich; *25*, the lower 105 feet of the largest organism (giant sequoia), with a 100-foot larch superposed.

The many scales of life:

p. 3, McMahon and Bonner [35]
More on the Elephant Bird here.

Medium-sized creatures (*above*). *1*, Dog; *2*, common herring; *3*, the largest egg (*Aepyornis*); *4*, song thrush with egg; *5*, the smallest bird (hummingbird) with egg; *6*, queen bee; *7*, common cockroach; *8*, the largest stick insect; *9*, the largest polyp (*Branchiocerianthus*); *10*, the smallest mammal (flying shrew); *11*, the smallest vertebrate (a tropical frog); *12*, the largest frog (goliath frog); *13*, common grass frog; *14*, house mouse; *15*, the largest land snail (*Achatina*) with egg; *16*, common snail; *17*, the largest beetle (goliath beetle); *18*, human hand; *19*, the largest starfish (*Luidia*); *20*, the largest free-moving protozoan (an extinct nummulite).

The many scales of life:

p. 3, McMahon and Bonner [35]

Small, "naked-eye" creatures (*lower left*).

Size range (in grams) and cell differentiation:

10^{-13} to 10^{8}, p. 3, McMahon and Bonner [35]

Non-uniform growth:

p. 32, McMahon and Bonner [35] **Non-uniform growth—arm length versus height:**

Good example of a break in scaling:

A crossover in scaling occurs around a height of 1 metre. p. 32, McMahon and Bonner [35]

Weightlifting: $M_{\text{world record}} \propto M_{\text{lifter}}^{2/3}$

Idea: Power $\sim$ cross-sectional area of isometric lifters. p. 53, McMahon and Bonner [35]

Titanothere horns: $L_{\text{horn}} \sim L_{\text{skull}^4}$

p. 36, McMahon and Bonner [35]; a bit dubious. **Animal power**

Fundamental biological and ecological constraint:

$$\boxed{P = c\,M^{\alpha}}$$

P = basal metabolic rate

$$M = \text{organismal body mass}$$

Stories—The Fraction Assassin:

Ecology—Species-area law:

Allegedly (data is messy): [30, 29]

Display paper: macarthur1963a, 2

$N_{\text{species}} \propto A^{\beta}$

According to physicists—on islands: $\beta \approx 1/4$.

Also—on continuous land: $\beta \approx 1/8$.

Cancer:

Display paper: tomasetti2015a, 2

Roughly: $p \sim r^{2/3}$ where p = life time probability and r = rate of stem cell replication.

Display paper: meyer-vernet2015a, 4

Insert question from assignment 1

4.4 Physics

Engines:

BHP = brake horse power **The allometry of nails:**

Observed: Diameter $\propto$ Length$^{2/3}$ or $d \propto \ell^{2/3}$.

Since $\ell d^2 \propto$ Volume v: Diameter $\propto$ Mass$^{2/7}$ or $d \propto v^{2/7}$.

Length $\propto$ Mass$^{3/7}$ or $\ell \propto v^{3/7}$.

Nails lengthen faster than they broaden (c.f. trees). p. 58–59, McMahon and Bonner [35] **The allometry of nails:**

A buckling instability?: Physics/Engineering result: Columns buckle under a load which depends on d^4/ℓ^2.

To drive nails in, posit resistive force $\propto$ nail circumference $= \pi d$.

Match forces independent of nail size: $d^4/\ell^2 \propto d$.

Leads to $d \propto \ell^{2/3}$.

Argument made by Galileo [21] in 1638 in "Discourses on Two New Sciences." Also, see here.

Another smart person's contribution: Euler, 1757

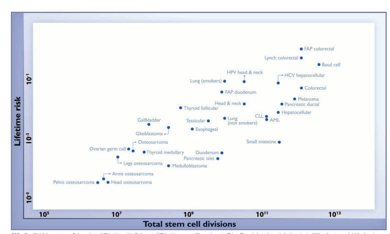
Also see McMahon, "Size and Shape in Biology," Science, 1973. [34]

Rowing: Speed $\propto$ (number of rowers)$^{1/9}$

Physics:

Scaling in elementary laws of physics: Inverse-square law of gravity and Coulomb's law:

$$F \propto \frac{m_1 m_2}{r^2} \quad \text{and} \quad F \propto \frac{q_1 q_2}{r^2}.$$



FAP = Familial Adenomatous Polyposis ◆ HCV = Hepatitis C virus ◆ HPV = Human papillomavirus ◆ CLL = Chronic lymphocytic leukemia ◆ AML = Acute myeloid leukemia

Fig. 1. The relationship between the number of stem cell divisions in the lifetime of a given tissue and the lifetime risk of cancer in that tissue. Values are from table S1, the derivation of which is discussed in the supplementary materials.

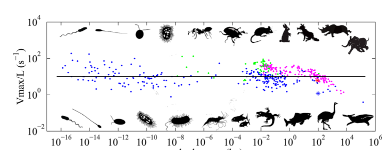
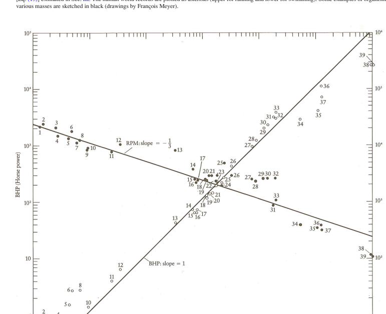


Fig. 1. Maximum relative speed versus body mass for 202 running species (157 mammals plotted in magenta and 45 non-mammals plotted in green), 127 swimming species and 91 micro-organisms (plotted in blue). The sources of the data are given in Ref. 16. The solid line is the maximum relative speed [Eq. (13)] estimated in Sec. III. The human world records are plotted as asterisks (upper for running and lower for swimming). Some examples of organisms of various masses are sketched in black (drawings by François Meyer).



Force is diminished by expansion of space away from source.

The square is $d - 1 = 3 - 1 = 2$, the dimension of a sphere's surface. **Dimensional Analysis:**

The Buckingham π theorem:[1]

Display paper: buckingham1914a, 1

As captured in the 1990s in the MIT physics library:

Dimensional Analysis:[2]

Fundamental equations cannot depend on units: System involves n related quantities with some unknown equation $f(q_1, q_2, \ldots, q_n) = 0$.

Geometric ex.: area of a square, side length ℓ:
$A = \ell^2$ where $[A] = L^2$ and $[\ell] = L$.

Rewrite as a relation of $p \leq n$ independent dimensionless parameters where p is the number of independent dimensions (mass, length, time, luminous intensity ...):

$$F(\pi_1, \pi_2, \ldots, \pi_p) = 0$$

e.g., $A/\ell^2 - 1 = 0$ where $\pi_1 = A/\ell^2$.

Another example: $F = ma \Rightarrow F/ma - 1 = 0$.

Plan: solve problems using only backs of envelopes.

Example:

Simple pendulum:

Idealized mass/platypus swinging forever.

Four quantities: Length ℓ,

mass m,

gravitational acceleration g, and

pendulum's period τ.

Variable dimensions: $[\ell] = L$, $[m] = M$, $[g] = LT^{-2}$, and $[\tau] = T$.

Turn over your envelopes and find some π's.

A little formalism: Game: find all possible independent combinations of the $\{q_1, q_2, \ldots, q_n\}$, that form dimensionless quantities $\{\pi_1, \pi_2, \ldots, \pi_p\}$, where we need to figure out $p \leq n$.

Consider $\pi_i = q_1^{x_1} q_2^{x_2} \cdots q_n^{x_n}$.

We (desperately) want to find all sets of powers x_j that create dimensionless quantities.

[1] Stigler's Law of Eponymy applies. See here.
[2] Length is a dimension, furlongs and smoots are units

Dimensions: want $[\pi_i] = [q_1]^{x_1}[q_2]^{x_2}\cdots[q_n]^{x_n} = 1$.

For the platypus pendulum we have
$[q_1] = L$, $[q_2] = M$, $[q_3] = LT^{-2}$, and $[q_4] = T$,

with dimensions $d_1 = L$, $d_2 = M$, and $d_3 = T$.

So: $[\pi_i] = L^{x_1}M^{x_2}(LT^{-2})^{x_3}T^{x_4}$.

We regroup: $[\pi_i] = L^{x_1+x_3}M^{x_2}T^{-2x_3+x_4}$.

We now need: $x_1 + x_3 = 0$, $x_2 = 0$, and $-2x_3 + x_4$.

Time for matrixology … **Well, of course there are matrices:**

Thrillingly, we have:

$$\mathbf{A}\vec{x} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A nullspace equation: $\mathbf{A}\vec{x} = \vec{0}$.

Number of dimensionless parameters = Dimension of null space = $n - r$ where n is the number of columns of $\mathbf{A}$ and r is the rank of $\mathbf{A}$.

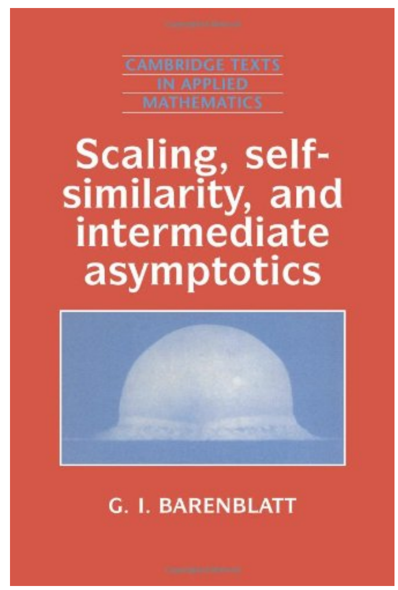
Here: $n = 4$ and $r = 3 \rightarrow F(\pi_1) = 0 \rightarrow \pi_1 = \text{const}$.

In general: Create a matrix $\mathbf{A}$ where ijth entry is the power of dimension i in the jth variable, and solve by row reduction to find basis null vectors.

We (you) find: $\pi_1 = \ell/g\tau^2 = \text{const.}$

Upshot: $\tau \propto \sqrt{\ell}$.

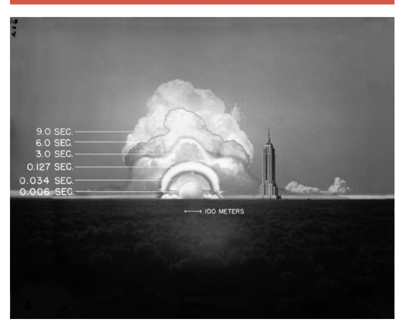
Insert question from assignment 1



"Scaling, self-similarity, and intermediate asymptotics"
by G. I. Barenblatt (1996).[5]

G. I. Taylor, magazines, and classified secrets:

1945
New Mexico
Trinity test:



Self-similar blast wave: Radius: $[R] = L$,
Time: $[t] = T$,
Density of air: $[\rho] = M/L^3$,
Energy: $[E] = ML^2/T^2$.

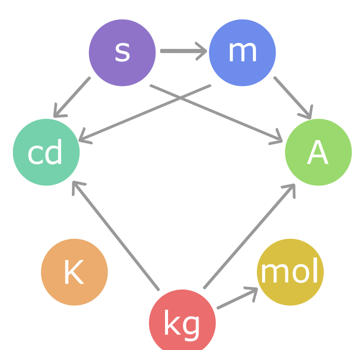
Four variables, three dimensions.

One dimensionless variable:
$E = \text{constant} \times \rho R^5/t^2$.

Scaling: Speed decays as $1/R^{3/2}$.

Related: Radiolab's Elements on the Cold War, the Bomb Pulse, and the dating of cell age (33:30). **We're still sorting out units:**

Proposed 2018 revision of SI base units:



by Dono/Wikipedia

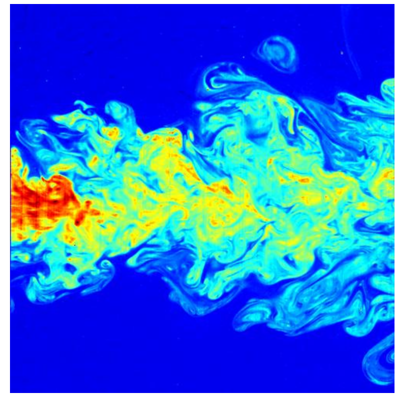
by Wikipetzi/Wikipedia

Now: kilogram is an artifact in Sèvres, France.

Future: Defined by fixing Planck's constant as $6.62606X \times 10^{-34}\ \mathrm{s}^{-1}\cdot\mathrm{m}^2\cdot\mathrm{kg}$.[3]

Metre chosen to fix speed of light at 299792458 $\mathrm{m}\cdot\mathrm{s}^{-1}$.

Radiolab piece: $\leq$ kg



Turbulence:

Big whirls have little whirls
That heed on their velocity,
And little whirls have littler whirls
And so on to viscosity.

— Lewis Fry Richardson

Image from here.

Jonathan Swift (1733): "Big fleas have little fleas upon their backs to bite 'em, And little fleas have lesser fleas, and so, ad infinitum." The Siphonaptera.
Display paper: aragon2008a, 4

Examined the probability pixels a distance R apart share the same luminance.

"Van Gogh painted perfect turbulence" by Phillip Ball, July 2006.

Apparently not observed in other famous painter's works or when van Gogh was settled.

Oops: Small ranges and natural log used. **Advances in turbulence:**

Kolmogorov, armed only with dimensional analysis and an envelope figures this out in 1941:

$$E(k) = C\epsilon^{2/3}k^{-5/3}$$

$E(k)$ = energy spectrum function.

ϵ = rate of energy dissipation.

$k = 2\pi/\lambda$ = wavenumber.

Energy is distributed across all modes, decaying with wave number.

No internal characteristic scale to turbulence.

Stands up well experimentally and there has been no other advance of similar magnitude.

"The Geometry of Nature": Fractals

[4]

"Anomalous" scaling of lengths, areas, volumes relative to each other.

The enduring question: how do self-similar geometries form?

Robert E. Horton: Self-similarity of river (branching) networks (1945). [24]

Harold Hurst—Roughness of time series (1951). [25]

Lewis Fry Richardson—Coastlines (1961).

Benoît Mandelbrot—Introduced the term "Fractals" and explored them everywhere, 1960s on. [31, 32, 33]

[3] X = still arguing …
[4] Note to self: Make millions with the "Fractal Diet"

4.5 Cities

Scaling in Cities: Display paper: bettencourt2007a, 2

Quantified levels of

Infrastructure Wealth Crime levels Disease Energy consumption

as a function of city size N (population).

Scaling in Cities:

Scaling in Cities:

Scaling in Cities:

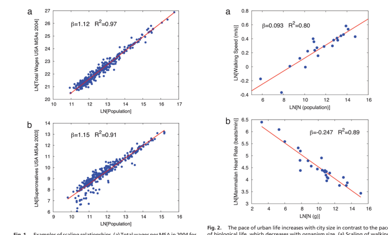


Fig. 1. Examples of scaling relationships. (*a*) Total wages per MSA in 2004 for the U.S. (blue points) vs. metropolitan population. (*b*) Supercreative employment per MSA in 2003, for the U.S. (blue points) vs. metropolitan population. Best-fit scaling relations are shown as solid lines.

Fig. 2. The pace of urban life increases with city size in contrast to the pace of biological life, which decreases with organism size. (*a*) Scaling of walking speed vs. population for cities around the world. (*b*) Heart rate vs. the size (mass) of organisms.

Table 1. Scaling exponents for urban indicators vs. city size

Y	β	95% CI	Adj-R^2	Observations	Country–year
New patents	1.27	[1.25,1.29]	0.72	331	U.S. 2001
Inventors	1.25	[1.22,1.27]	0.76	331	U.S. 2001
Private R&D employment	1.34	[1.29,1.39]	0.92	266	U.S. 2002
"Supercreative" employment	1.15	[1.11,1.18]	0.89	287	U.S. 2003
R&D establishments	1.19	[1.14,1.22]	0.77	287	U.S. 1997
R&D employment	1.26	[1.18,1.43]	0.93	295	China 2002
Total wages	1.12	[1.09,1.13]	0.96	361	U.S. 2002
Total bank deposits	1.08	[1.03,1.11]	0.91	267	U.S. 1996
GDP	1.15	[1.06,1.23]	0.96	295	China 2002
GDP	1.26	[1.09,1.46]	0.64	196	EU 1999–2003
GDP	1.13	[1.03,1.23]	0.94	37	Germany 2003
Total electrical consumption	1.07	[1.03,1.11]	0.88	392	Germany 2002
New AIDS cases	1.23	[1.18,1.29]	0.76	93	U.S. 2002–2003
Serious crimes	1.16	[1.11, 1.18]	0.89	287	U.S. 2003
Total housing	1.00	[0.99,1.01]	0.99	316	U.S. 1990
Total employment	1.01	[0.99,1.02]	0.98	331	U.S. 2001
Household electrical consumption	1.00	[0.94,1.06]	0.88	377	Germany 2002
Household electrical consumption	1.05	[0.89,1.22]	0.91	295	China 2002
Household water consumption	1.01	[0.89,1.11]	0.96	295	China 2002
Gasoline stations	0.77	[0.74,0.81]	0.93	318	U.S. 2001
Gasoline sales	0.79	[0.73,0.80]	0.94	318	U.S. 2001
Length of electrical cables	0.87	[0.82,0.92]	0.75	380	Germany 2002
Road surface	0.83	[0.74,0.92]	0.87	29	Germany 2002

Data sources are shown in *SI Text*. CI, confidence interval; Adj-R^2, adjusted R^2; GDP, gross domestic product.

Intriguing findings: Global supply costs scale sublinearly with N ($\beta < 1$).

Returns to scale for infrastructure. Total individual costs scale linearly with N ($\beta = 1$)

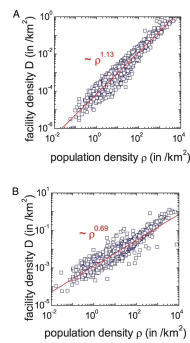
Individuals consume similar amounts independent of city size. Social quantities scale superlinearly with N ($\beta > 1$)

Creativity (# patents), wealth, disease, crime, ...

Density doesn't seem to matter...

Surprising given that across the world, we observe two orders of magnitude variation in area covered by agglomerations of fixed populations. **A possible theoretical explanation?** Display paper: bettencourt2013a, 4 #sixthology

Density of public and private facilities:

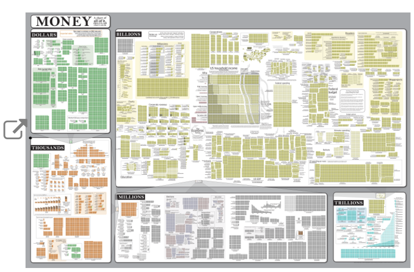


$$\rho_{\text{fac}} \propto \rho_{\text{pop}}^{\alpha}$$

Left plot: ambulatory hospitals in the U.S.

Right plot: public schools in the U.S.

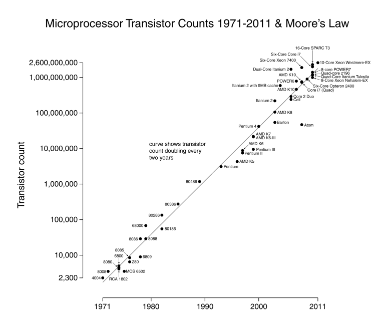
4.6 Money



`http://xkcd.com/980/`

Explore the original zoomable and interactive version here: http://xkcd.com/980/

Moore's Law:



4.7 Technology

Scaling laws for technology production: "Statistical Basis for Predicting Technological Progress [41]" Nagy et al., PLoS ONE, 2013.

y_t = stuff unit cost; x_t = total amount of stuff made.

Wright's Law, cost decreases as a power of total stuff made: [54]

$$y_t \propto x_t^{-w}.$$

Moore's Law, framed as cost decrease connected with doubling of transistor density every two years: [40]

$$y_t \propto e^{-mt}.$$

Sahal's observation that Moore's law gives rise to Wright's law if stuff production grows exponentially: [42]

$$x_t \propto e^{gt}.$$

Sahal + Moore gives Wright with $w = m/g$.

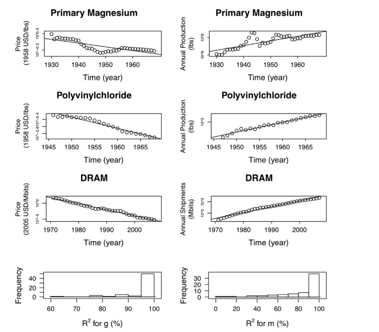


Figure 3. Three examples showing the logarithm of price as a function of time in the left column and the logarithm of production as a function of time in the right column, based on industry-wide data. We have chosen these examples to be representative: The top row contains an example with one of the worst fits, the second row an example with an intermediate goodness of fit, and the third row one of the best examples. The fourth row of the figure shows histograms of R^2 values for fitting g and m for the 62 datasets.
doi:10.1371/journal.pone.0052669.g003

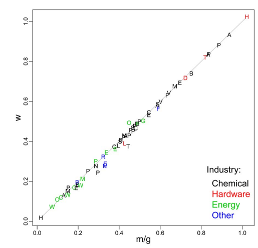


Figure 4. An illustration that the combination of exponentially increasing production and exponentially decreasing cost are equivalent to Wright's law. The value of the Wright parameter w is plotted against the prediction m/g based on the Sahal formula, where m is the exponent of cost reduction and g the exponent of the increase in cumulative production.
doi:10.1371/journal.pone.0052669.g004

4.8 Specialization

Scaling of Specialization: "Scaling of Differentiation in Networks: Nervous Systems, Organisms, Ant Colonies, Ecosystems, Businesses, Universities, Cities, Electronic Circuits, and Legos"
M. A. Changizi, M. A. McDannald and D. Widders [15]
J. Theor. Biol., 2002.

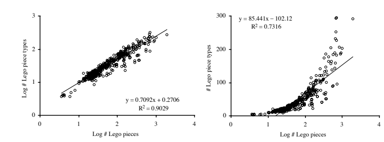


FIG. 3. Log–log (base 10) (left) and semi-log (right) plots of the number of Lego piece types vs. the total number of parts in Lego structures ($n = 391$). To help to distinguish the data points, logarithmic values were perturbed by adding a random number in the interval [−0.05, 0.05], and non-logarithmic values were perturbed by adding a random number in the interval [−1, 1].

TABLE 1
*Summary of results**

Network	Node	No. data points	Range of log N	Log–log R^2	Semi-log R^2	p_{power}/p_{log}	Relationship between C and N	Comb. degree	Exponent v for type-net scaling	Figure in text
Selected networks										
Electronic circuits	Component	373	2.12	0.747	0.602	0.05/4e−5	Power law	2.29	0.92	2
Legos™	Piece	391	2.65	0.903	0.732	0.09/1e−7	Power law	1.41	—	3
Businesses										
military vessels	Employee	13	1.88	0.971	0.832	0.05/3e−3	Power law	1.60	—	4
military offices	Employee	8	1.59	0.964	0.789	0.16/0.16	Increasing	1.13	—	4
universities	Employee	9	1.55	0.786	0.749	0.27/0.27	Increasing	1.37	—	4
insurance co.	Employee	52	2.30	0.748	0.685	0.11/0.10	Increasing	3.04	—	4
Universities										
across schools	Faculty	112	2.72	0.695	0.549	0.09/0.01	Power law	1.81	—	5
history of Duke	Faculty	46	0.94	0.921	0.892	0.09/0.05	Increasing	2.07	—	5
Ant colonies										
caste = type	Ant	46	6.00	0.481	0.454	0.11/0.04	Power law	8.16	—	6
size range = type	Ant	22	5.24	0.658	0.548	0.17/0.04	Power law	8.00	—	6
Organisms	Cell	134	12.40	0.249	0.165	0.08/0.02	Power law	17.73	—	7
Neocortex	Neuron	10	0.85	0.520	0.584	0.16/0.16	Increasing	4.56	—	9
Competitive networks										
Biotas	Organism	—	—	—	—	—	Power law	≈3	0.3 to 1.0	—
Cities	Business	82	2.44	0.985	0.832	0.08/8e-8	Power law	1.56	—	10

*(1) The kind of network, (2) what the nodes are within that kind of network, (3) the number of data points, (4) the logarithmic range of network sizes N (i.e. $\log(N_{max}/N_{min})$), (5) the log–log correlation, (6) the semi-log correlation, (7) the serial-dependence probabilities under, respectively, power-law and logarithmic models, (8) the empirically determined best-fit relationship between differentiation C and organization size N (if one of the two models can be refuted with $p<0.05$; otherwise we just write "increasing" to denote that neither model can be rejected), (9) the combinatorial degree (i.e. the inverse of the best-fit slope of a log–log plot of C versus N), (10) the scaling exponent for how quickly the edge-degree δ scales with type-network size C (in those places for which data exist), (11) figure in this text where the plots are presented. Values for biotas represent the broad trend from the literature.

Nice 2012 wired.com write-up $C \sim N^{1/d}$, $d \geq 1$: C = network differentiation = # node types.

N = network size = # nodes.

d = combinatorial degree.

Low d: strongly specialized parts.

High d: strongly combinatorial in nature, parts are reused.

Claim: Natural selection produces high d systems.

Claim: Engineering/brains produces low d systems.

Shell of the nut: Scaling is a fundamental feature of complex systems.

Basic distinction between isometric and allometric scaling.

Powerful envelope-based approach: Dimensional analysis.

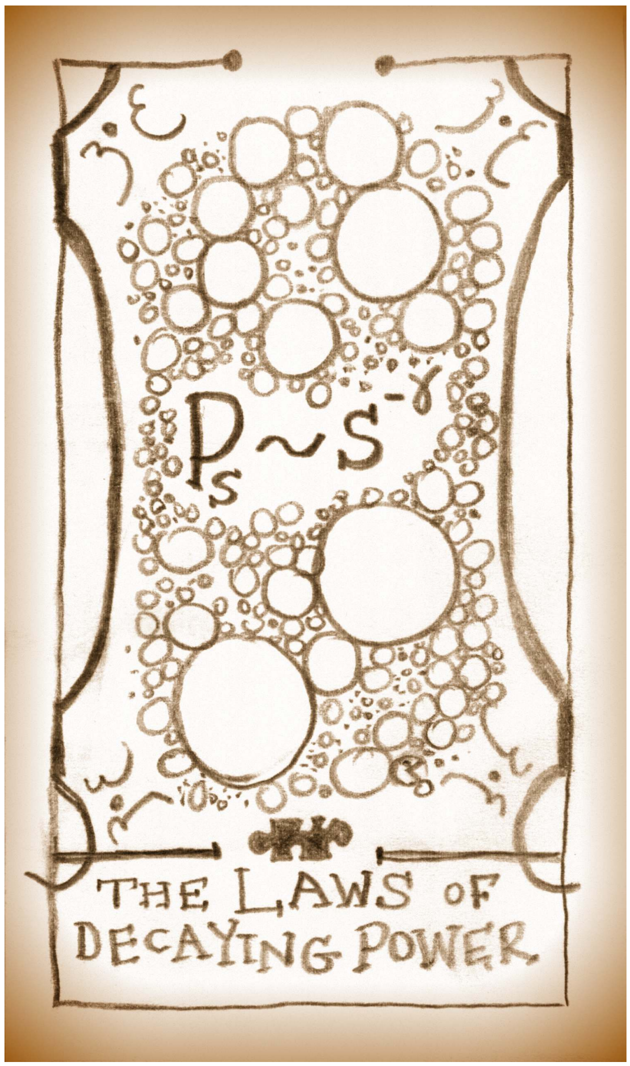
"Oh yeah, well that's just dimensional analysis" said the [insert your own adjective] physicist.

Tricksiness: A wide variety of mechanisms give rise to scalings, both normal and unusual.

To Add?

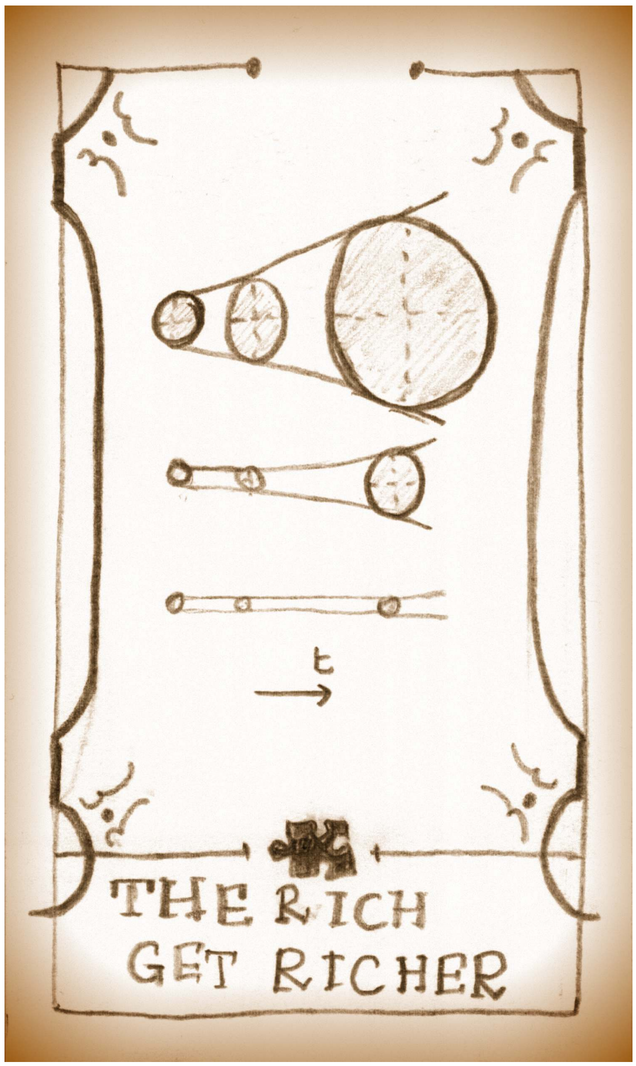
Scaling in blood and river networks.

The Unsolved Allometry Theoricides.


$P_s \sim s^{-\gamma}$
THE LAWS OF
DECAYING POWER

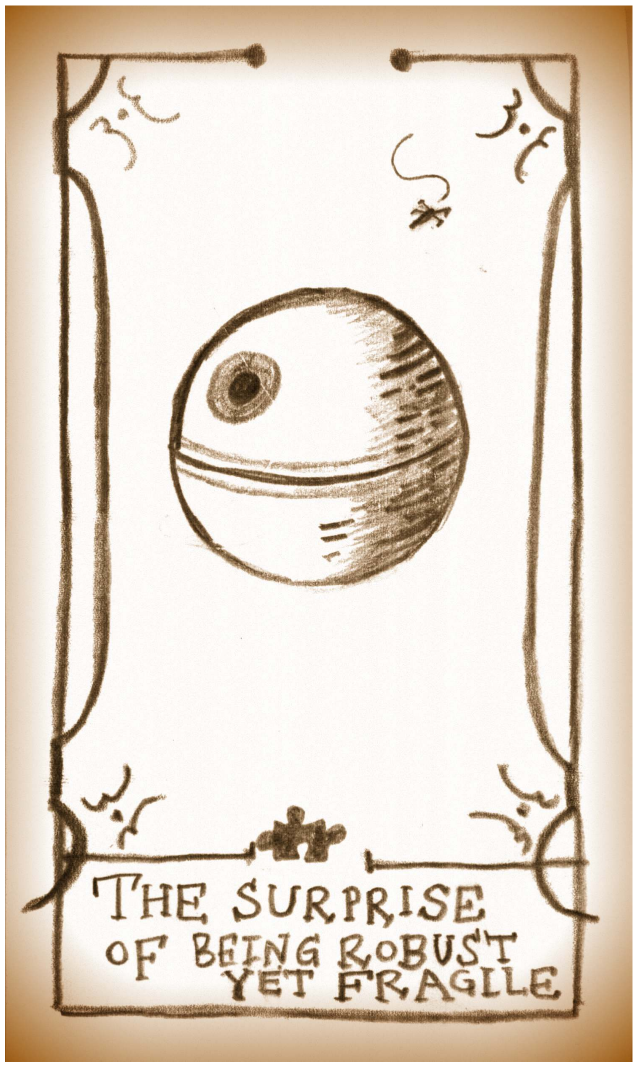
Story No. 5

Power-Law Size Distributions


t
THE RICH
GET RICHER

Story No. 6

Mechanisms for Generating Power-Law Size Distributions

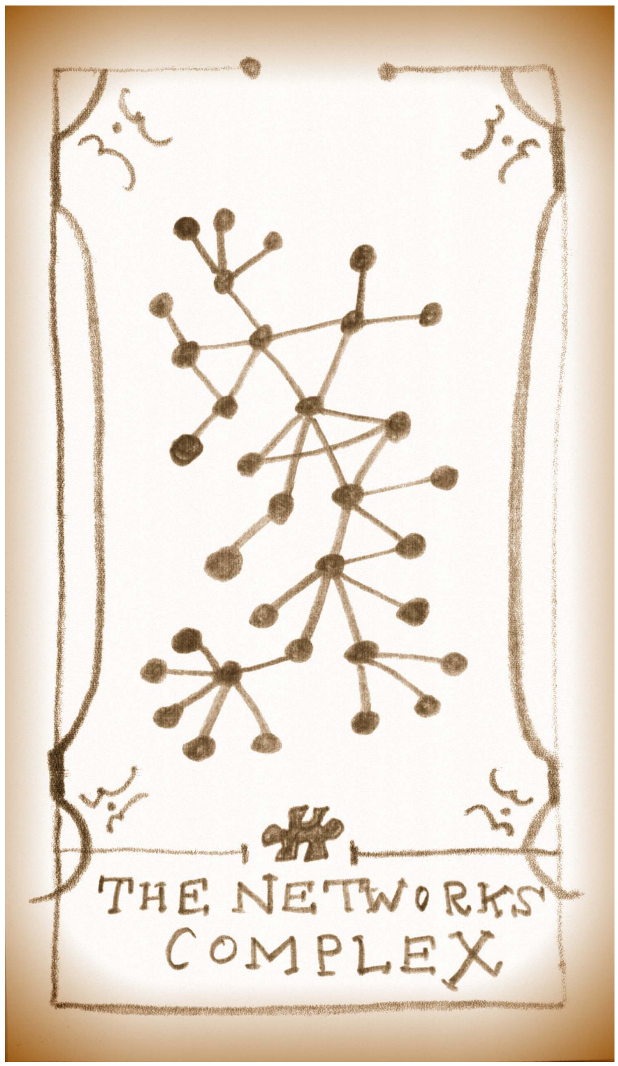

THE SURPRISE
OF BEING ROBUST
YET FRAGILE

Story No. 7

Robustness

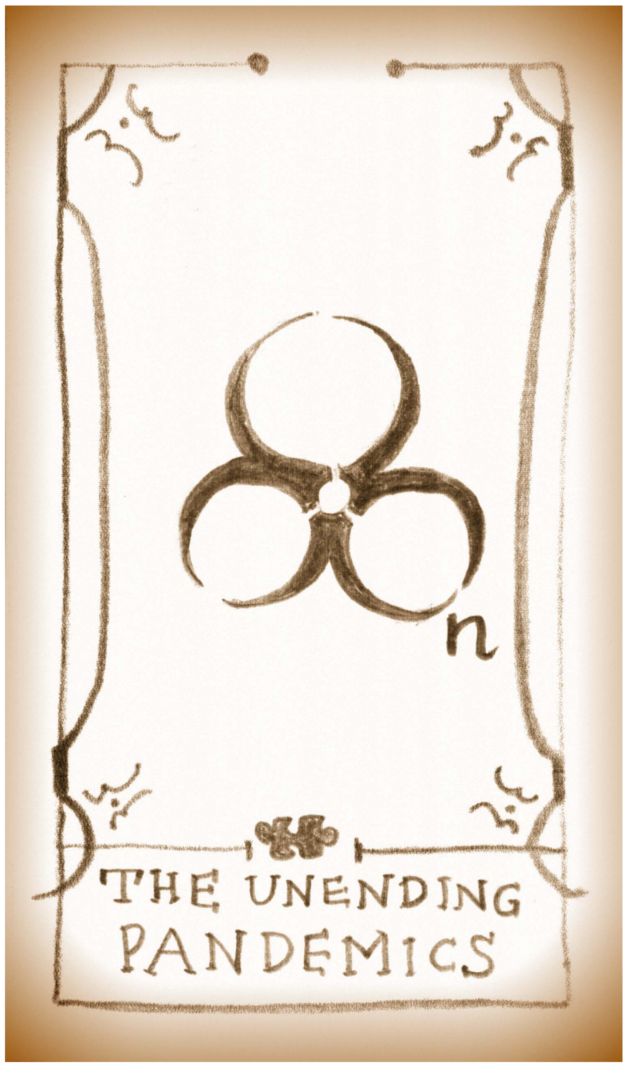
Story No. 8

Lognormals and other disappointments


THE NETWORKS
COMPLEX

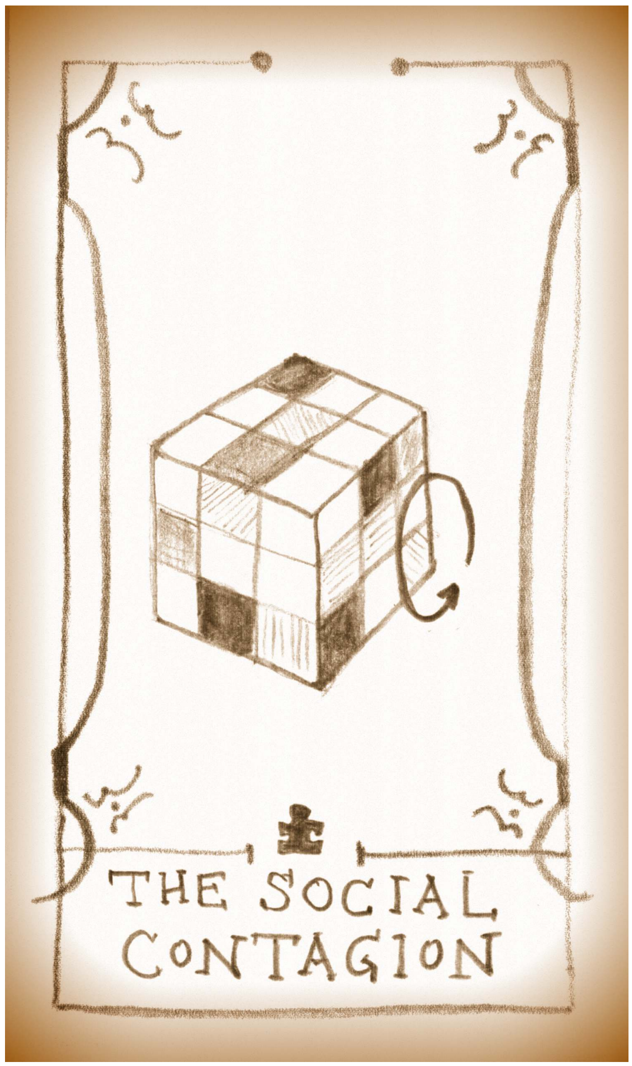
Story No. 9

Complex Networks


n
THE UNENDING
PANDEMICS

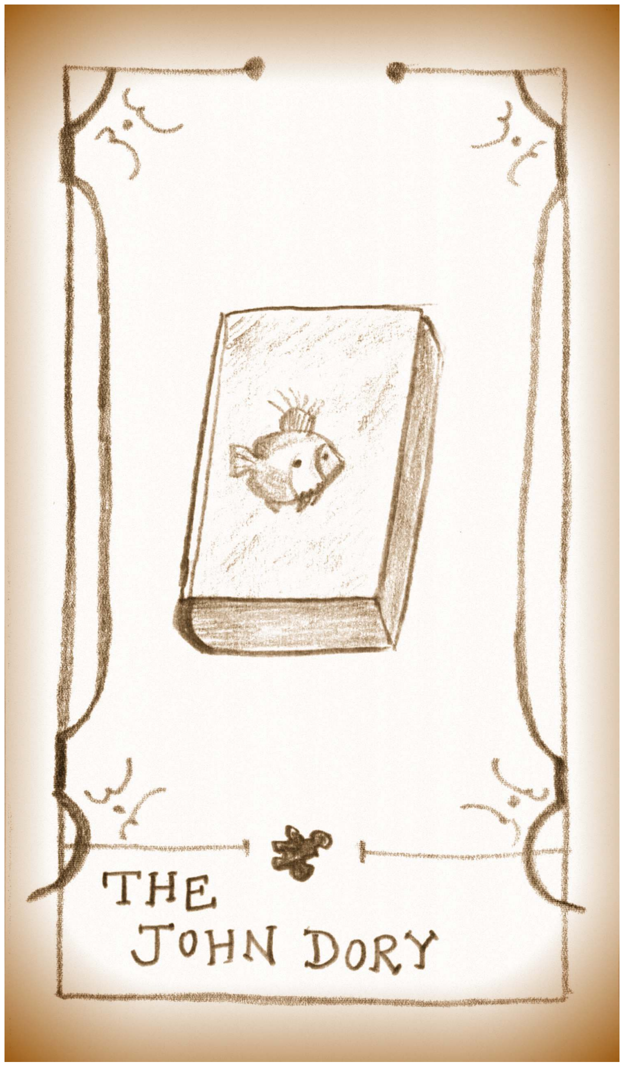
Story No. 10

Biological Contagion


THE SOCIAL
CONTAGION

Story No. 11

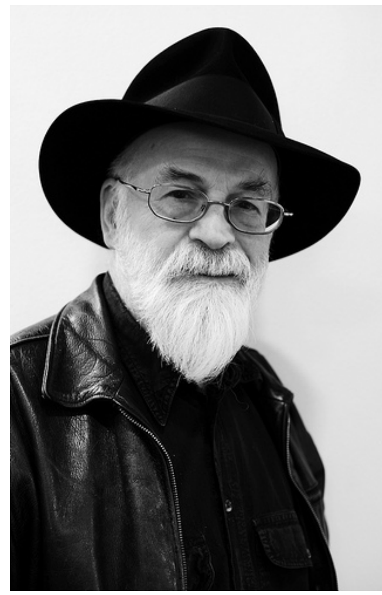
Social Contagion


THE
JOHN DORY

Story No. 12

Differential Equations, Algorithms, and Stories

12.1 Narrativium

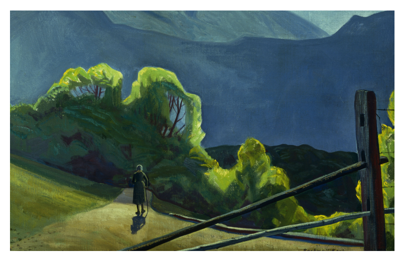


(Sir Terry) Pratchett's Narrativium:

"The most common element on the disc, although not included in the list of the standard five: earth, fire, air, water and surprise. It ensures that everything runs properly as a story."

"A little narrativium goes a long way: the simpler the story, the better you understand it. Storytelling is the opposite of reductionism: 26 letters and some rules of grammar are no story at all."

"Heroes only win when outnumbered, and things which have a one-in-a-million chance of succeeding often do so."



The story trap by Philip Ball, 2015-11-12

"We use neat stories to explain everything from sports matches to symphonies. Is it time to leave the nursery of the mind?"

"…we might wonder if the ultimate intelligibility of the universe will be determined not so much by the capacity of our minds to formulate the appropriate concepts and equations, but by whether we can find a meaningful story to tell about it."

Competing storytelling organizations: News.

Art.

Music industry.

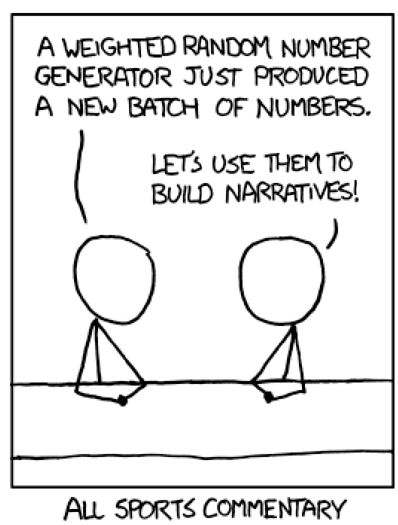
Books, magazines.

Movie studios, Netflix, HBO, Disney.

Video Games.

Social media: Facebook, Medium, Tumblr, blogs. **Cultural products from Pantheon:** Writers, artists, movie directors, video game directors.

Understanding the Sociotechnocene—Stories:



xkcd.com/904/

Perhaps: A true science of stories.

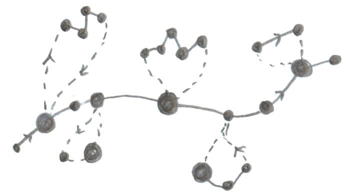
Claim: Homo narrativus—we run on stories.

Claim: The narrative hierarchy and the Scalability of stories.

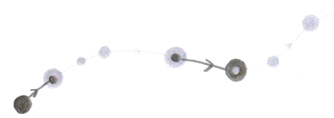
Research: Extraction of metaphors, frames, narratives, and stories from large-scale text.

Research: The taxonomy of human stories.

Harness: Sociotechnical algorithms for measuring/predicting decisions, contagion, demographics, weather, …



Adjacent narratives—why mistruths and conspiracy theories exist and flourish:

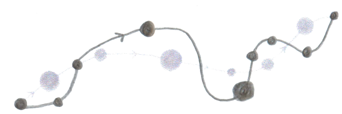


1999 Gallup poll: 6% of Americans said the lunar landings were fake.

5% were undecided. **Video replay:**

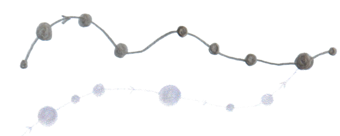
12.2 Power

Story Wars:



Nicholas Hénin, French Journalist, held captive for 10 months.

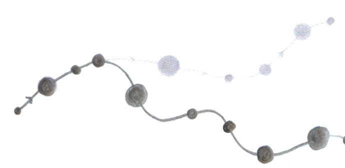
From the end of the interview: NICOLAS HENIN: No, it was just like in a movie. And, by the way, even the people going to Syria, joining ISIS in Syria to fight, even these people see himself as movie characters. They play their own movie. This is why I think that the most powerful way to fight ISIS are not bombs. It is to kill the narrative. We have to write another movie. We have to build other heroes. And this is why I believe that the French are making big mistakes in the ways they, they fight ISIS.



We created, for instance, accounts on the social media named "Stop Jihadism," and this is [BLEEP], like they did not understand anything. And I did understand why we are so bad. It's just because in France we don't know how to write TV series properly.

[BROOKE LAUGHS]

Just because we have no imagination, we cannot just tell beautiful stories, create beautiful characters, beautiful heroes.

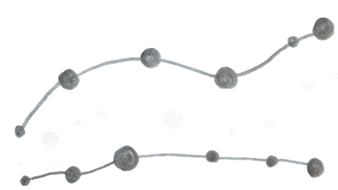


And this is what we have to do because in our world, in our societies what do people want? They want to become heroes. They want to be famous. They want to be, to be recognized. **The American Dream = Rags to Riches**

The story that anyone can become King or Queen.

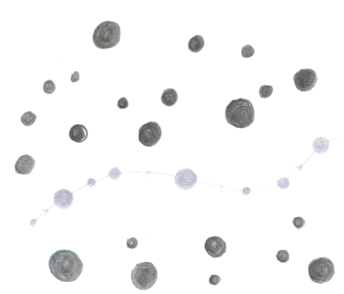
Story of individual, not the collective.

But we know about fame and success:
The presence of outsized fame in a social system means social imitation is a driver of value.



Stories of societies can only hold if they have been and remain believable.

If not Trump, what?, David Brooks, New York Times: "We'll probably need a new national story. Up until now, America's story has been some version of the rags-to-riches story, the lone individual who rises from the bottom through pluck and work. But that story isn't working for people anymore, especially for people who think the system is rigged."



"I don't know what the new national story will be, but maybe it will be less individualistic and more redemptive. Maybe it will be a story about communities that heal those who suffer from addiction, broken homes, trauma, prison and loss, a story of those who triumph over the isolation, social instability and dislocation so common today." **Claim: Stories must have real substance to endure** Enormous disasters: Fabrications of real experiences. Plain old making stuff up: A million little pieces ... Oprah will get you.

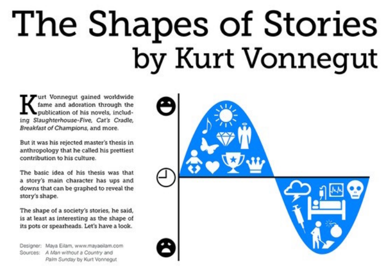
Wikipedia's has a list of famous fake memoirs.

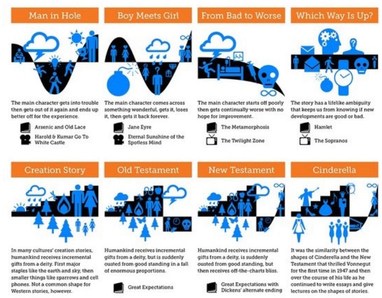
Expansive plagiarism: How Opal Mehta Got Kissed, Got Wild, and Got a Life.
#kudos

Self-plagiarism and more standard badness: Jonah Lehrer.
Amazingly: Made up Bob Dylan quotes.

Lance Armstrong. Also got to meet Oprah. Enormous power: Fiction that speaks to real experiences.

12.3 Shapes





Kurt Vonnegut on the shapes of stories: **Kurt Vonnegut on the shapes of stories:**
The New Yorker, December 16, 2013, p. 56. **Ron Swanson:**

"I hate metaphors. That's why my favorite book is Moby Dick. No frou-frou symbolism. Just a good, simple tale about a man who hates an animal."

The emotional shapes of stories—Moby Dick:

Partly inspired by Vonnegut's Shapes of Stories.

Online, interactive Emotional Shapes of Stories for 10,000+ books:

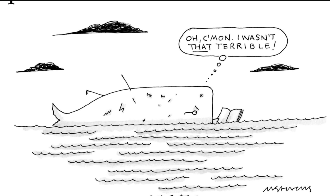
Online, interactive Emotional Shapes of Stories for 10,000+ books:

Online, interactive Emotional Shapes of Stories for 1,000+ movie scripts:

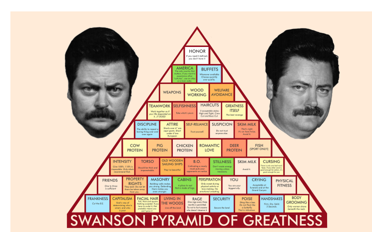
http://whyfiles.org/2015/in-10-languages-happy-words-beat-sad-ones/

Reagan et al. in preparation, 2016.

Harry Potter and the Chamber of Plot Devices: 2016-06-22andy-reagan-hedonometer.mp4
Display paper: kiley2016a, 1



12.4 Taxonomy



The "I wonder who wrote this?" Great Man Theory:

"The Power of Myth"
by Campbell and (1991). [14]

"The Hero with a Thousand Faces"
by Joseph Campbell (2008). [13]

Highly influential but it's a trap! **How to write a screenplay:**

"Save the Cat!"
by Blake Snyder (2005). [48] 9 acts.

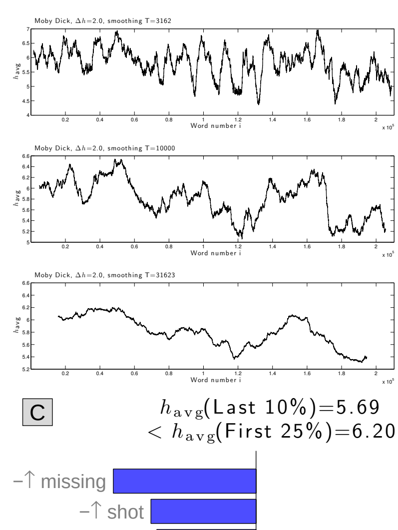
Someone important to the main characters gets toasted in the second act, blah, blah.

Believes irony is key.

Logline = one or two sentence summary.

Logline fails to be a summary of logline. **Seven "good" stories?:**

"The Seven Basic Plots: Why We Tell Stories"
by Christopher Booker (2005). [11] Seven Gateways to the Underworld (?)



Overcoming the Monster ×2 and the Thrilling escape from Death (plot).

Rags to Riches (plot).

The Quest (plot).

Voyage and Return (plot).

Comedy ×2 (plot but really structure).

Tragedy ×3 (plot).

Rebirth (plot).

The Dark Power: From Shadow into Light (master structure).

The taxonomy of stories:

Folkloristics: Academic area formally started around 1900.

Aarne–Thompson classification systems

- ANIMAL TALES 1-299
  - Wild Animals 1-99
    - The Clever Fox (Other Animal) 1-69
    - Other Wild Animals 70-99
  - Wild Animals and Domestic Animals 100-149
  - Wild Animals and Humans 150-199
  - Domestic Animals 200-219
  - Other Animals and Objects 220-299
- TALES OF MAGIC 300-749
  - Supernatural Adversaries 300-399
  - Supernatural or Enchanted Wife (Husband) or Other Relative 400-459
    - Wife 400-424
    - Husband 425-449
    - Brother or Sister 450-459
  - Supernatural Tasks 460-499
  - Supernatural Helpers 500-559
  - Magic Objects 560-649
  - Supernatural Power or Knowledge 650-699
  - Other Tales of the Supernatural 700-749
- RELIGIOUS TALES 750-849
  - God Rewards and Punishes 750-779
  - The Truth Comes to Light 780-799
  - Heaven 800-809
  - The Devil 810-826
  - Other Religious Tales 827-849
- REALISTIC TALES 850-999
  - The Man Marries the Princess 850-869
  - The Woman Marries the Prince 870-879
  - Proofs of FidelitY and Innocence 880-899
  - The Obstinate Wife Learns to Obey 900-909
  - Good Precepts 910-919
  - Clever Acts and Words 920-929

Motif-based taxonomy.

Online classification database

Display paper: abello2012a, 1

Motivation: "As a simple, historical example from the Danish materials, no one has yet classified (according to the ATU index) the several thousand fairy tales in the collections of the Danish Folklore Archive (http://www.dafos.dk), nor does it seem anyone ever will."

'Imagine a system in which the complexities of a folklore corpus can be explored at different levels of resolution, from the broad perspective of "distant reading" down to the narrow perspective of traditional "close reading."'

red riding hood analysis [50]

Famous folklore scholar:

Comic Book Guy (CBG).

Real name: Jeffrey "Jeff" Albertson.

Master's degree in Folklore and Mythology.

Thesis: translated Lord of the Rings into Klingon.

The taxonomy of stories:

Fundamental arcs: Kill the Monster.

Rags to Riches (and Riches to Rags—*Metamophosis*).

The Journey: a Search or a Quest.

Romance.

Narratives in Left Nullspace: All Stories of The Many. **What about comedies?** Comedies are not in themselves a story, but a way of telling stories.

12.5 Essence

Stories are algorithms for life:

Homo narrativus: Provide dynamic paths and trajectories.

If this, then that.

Convey and reinforce how to behave, how not to behave.

Full ecology of stories =
Competing, self-defending operating system for people's minds. **Aphorisms as algorithms:** Pride cometh before the fall.

A stitch in time saves nine.

Look before you leap.

Anti-aphorism: The one who hesitates is lost.

The unifying theme of existence is existence:

The three fundamental events of (non-clone) life:

Hatchings.

Matchings.

Dispatchings.

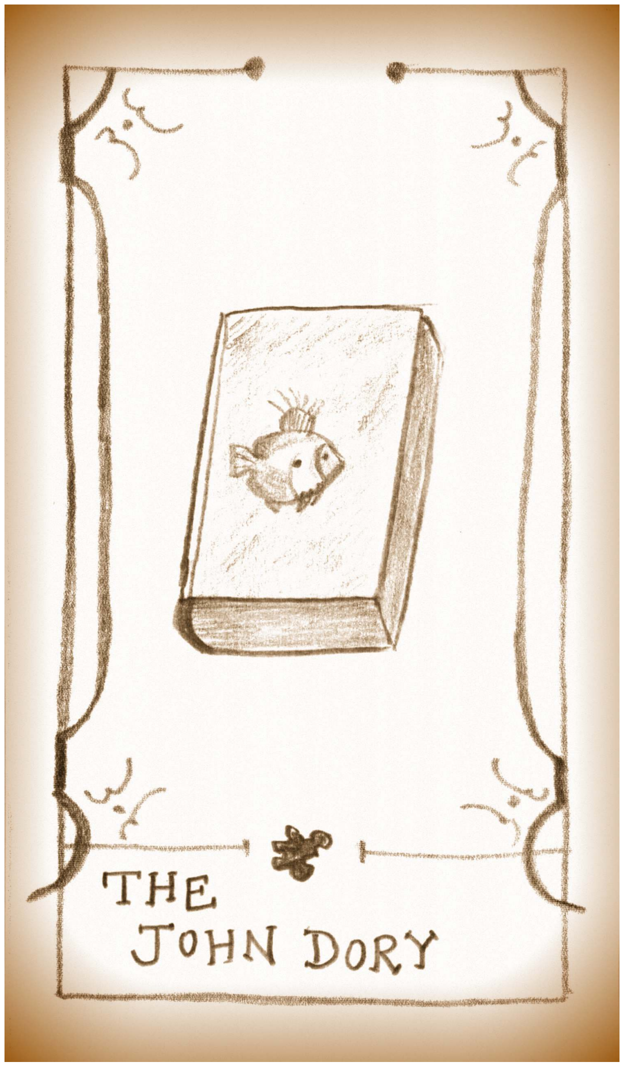
The essence of all stories?

It's survival—Life and Death: Kill the Monster: Bare survival.

Rags to Riches: Flourishing.

Romance: Matchings and Hatchings.

Journey/Odyssey: Search for a salvation, a "Holy Grail".


THE
JOHN DORY

Story No. 13

Complexification

Bibliography

[1] James Abello, Peter Broadwell, and Timothy R. Tangherlini. Computational folkloristics. Communications of the ACM, 55:60–70, 2012.

[2] P. W. Anderson. More is different. Science, 177(4047):393–396, 1972.

[3] J. L. Aragón, Gerardo G. Naumis, M Bai, M. Torres, and P. K. Maini. Turbulent luminance in impassioned van Gogh paintings. J. Math. Imaging Vis., 30:275–283, 2008.

[4] Philip Ball. Critical Mass: How One Thing Leads to Another. Farra, Straus, and Giroux, New York, 2004.

[5] G. I. Barenblatt. Scaling, self-similarity, and intermediate asymptotics, volume 14 of Cambridge Texts in Applied Mathematics. Cambridge University Press, 1996.

[6] Mark A. Bedau. Weak emergence. In J. Tomberlin, editor, Philosophical Perspectives: Mind, Causation, and World, volume 11, pages 375–399. Blackwell, Malden, MA, 1997.

[7] Luís M. A. Bettencourt, José Lobo, Dirk Helbing, Kühnhert, and Geoffrey. B. West. Growth, innovation, scaling, and the pace of life in cities. Proc. Natl. Acad. Sci., 104(17):7301–7306, 2007.

[8] Luís M. A. Bettencourt. The origins of scaling in cities. Science, 340:1438–1441, 2013.

[9] Nino Boccara. Modeling Complex Systems. Springer-Verlag, New York, 2nd edition, 2004.

[10] Nino Boccara. Modeling Complex Systems. Springer-Verlag, New York, 2004.

[11] Christopher Booker. The Seven Basic Plots: Why We Tell Stories. Continuum, 2005.

[12] E. Buckingham. On physically similar systems: Illustrations of the use of dimensional equations. Phys. Rev., 4:345–376, 1914.

[13] Joseph Campbell. The Hero with a Thousand Faces. New World Lbirary, third edition, 2008.

[14] Joseph Campbell and Bill Moyers. The Power of Myth. Anchor, 1991.

[15] M. A. Changizi, M. A. McDannald, and D. Widders. Scaling of differentiation in networks: Nervous systems, organisms, ant colonies, ecosystems, businesses, universities, cities, electronic circuits, and Legos. J. Theor. Biol, 218:215–237, 2002.

[16] James S. Coleman. Foundations of Social Theory. Belknap Press, Cambridge, MA, 1994.

[17] Peter Sheridan Dodds, D. J. Watts, and C. F. Sabel. Information exchange and the robustness of organizational networks. Proc. Natl. Acad. Sci., 100(21):12516–12521, 2003.

[18] Albert Einstein. Über die von der molekularkinetischen theorie der wärme geforderte bewegung von in ruhenden flüssigkeiten suspendierten teilchen. Annalen der Physik, 322:549–560, 1905.

[19] Albert Einstein. On the movement of small particles suspended in a stationary liquid demanded by the molecular-kinetic theory of heat. In R. Fürth, editor, Investigations on the theory of the Brownian motion. Dover Publications, 1956.

[20] Richard Foote. Mathematics and complex systems. Science, 318:410–412, 2007.

[21] Galileo Galilei. Dialogues Concerning Two New Sciences. Kessinger Publishing, 2010. Translated by Henry Crew and Alfonso De Salvio.

[22] James Gleick. The Information: A History, A Theory, A Flood. Pantheon, 2011.

[23] M. Granovetter. Threshold models of collective behavior. Am. J. Sociol., 83(6):1420–1443, 1978.

[24] Robert E. Horton. Erosional development of streams and their drainage basins; hydrophysical approach to quatitative morphology. Bulletin of the Geological Society of America, 56(3):275–370, 1945.

[25] H. E. Hurst. Long term storage capacity of reservoirs. Transactions of the American Society of Civil Engineers, 116:770–808, 1951.

[26] J. S. Huxley and G. Teissier. Terminology of relative growth. Nature, 137:780–781, 1936.

[27] Neil F. Johnson. Simply Complexity: A Clear Guide to Complexity Theory. Oneworld Publications, London, UK, 2009.

[28] Dilan Patrick Kiley, Andrew J. Reagan, Lewis Mitchell, Christopher M. Danforth, and Peter Sheridan Dodds. The game story space of professional sports: Australian Rules Football. Physical Review E, 93, 2016. Available online at `http://arxiv.org/abs/1507.03886`.

[29] S.A. Levin. The problem of pattern and scale in ecology. Ecology, 73(6):1943–1967, 1992. .

[30] Robert H. MacArthur and Edward O. Wilson. An equilibrium theory of insular zoogeography. Evolution, 17:373–387, 1963.

[31] Benoît B. Mandelbrot. How long is the coast of britain? statistical self-similarity and fractional dimension. Science, 156(3775):636–638, 1967.

[32] Benoît B. Mandelbrot. Fractals: Form, Chance, and Dimension. Freeman, San Francisco, 1977.

[33] Benoît B. Mandelbrot. The Fractal Geometry of Nature. Freeman, San Francisco, 1983.

[34] T. McMahon. Size and shape in biology. Science, 179:1201–1204, 1973.

[35] Thomas A. McMahon and John Tyler Bonner. On Size and Life. Scientific American Library, New York, 1983.

[36] Nicole Meyer-Vernet and Jean-Pierre Rospars. How fast do living organisms move: Maximum speeds from bacteria to elephants and whales. American Journal of Physics, pages 719–722, 2015.

[37] Jean-Baptiste Michel, Yuan Kui Shen, Aviva P. Aiden, Adrian Veres, Matthew K. Gray, The Google Books Team, Joseph P. Pickett, Dale Hoiberg, Dan Clancy, Peter Norvig, Jon Orwant, Steven Pinker, Martin A. Nowak, and Erez Aiden Lieberman. Quantitative analysis of culture using millions of digitized books. Science Magazine, 331:176–182, 2011.

[38] John H. Miller and Scott E. Page. Complex Adaptive Systems: An introduction to computational models of social life. Princeton University Press, Princeton, NJ, 2007.

[39] Melanie Mitchell. Complexity: A Guided Tour. Oxford University Press, New York, NY, 2009.

[40] G. E. Moore. Cramming more components onto integrated circuits. Electronics Magazine, 38:114–117, 1965.

[41] Béla Nagy, J. Doyne Farmer, Quan M. Bui, and Jessika E. Trancik. Statistical basis for predicting technological progress. PLoS ONE, 8:352669, 2013.

[42] D. Sahal. A theory of progress functions. AIIE Transactions, 11:23–29, 1979.

[43] Thomas C. Schelling. Dynamic models of segregation. J. Math. Sociol., 1:143–186, 1971.

[44] Thomas C. Schelling. Hockey helmets, concealed weapons, and daylight saving: A study of binary choices with externalities. J. Conflict Resolut., 17:381–428, 1973.

[45] Thomas C. Schelling. Micromotives and Macrobehavior. Norton, New York, 1978.

[46] Thomas C. Schelling. Some fun, thirty-five years ago. In L. Tesfatsion and K. L. Judd, editors, Handbook of Computational Economics, volume 2, pages 1639–1644. Elsevier, 2006.

[47] A. Shingleton. Allometry: The study of biological scaling. Nature Education Knowledge, 1:2, 2010.

[48] Blake Snyder. Save the Cat! Michael Wiese Productions, 2005.

[49] Didier Sornette. Critical Phenomena in Natural Sciences. Springer-Verlag, Berlin, 2nd edition, 2003.

[50] Jamshid J. Tehrani. The phylogeny of Little Red Riding Hood. PLoS ONE, 8:e78871, 2013.

[51] Cristian Tomasetti and Bert Vogelstein. Variation in cancer risk among tissues can be explained by the number of stem cell divisions. Science, pages 78–81, 2015.

[52] Paul B. Umbanhowar, Francisco Melo, and Harry L. Swinney. Localized excitations in a vertically vibrated granular layer. Nature, 382:793–6, 1996.

[53] M. Mitchell Waldrop. Complexity: The Emerging Science at the Edge of Order and Chaos. Simon & Schuster, New York, NY, 1993.

[54] T. P. Wright. Factors affecting the costs of airplanes. Journal of Aeronautical Sciences, 10:302–328, 1936.

[55] K. Zhang and T. J. Sejnowski. A universal scaling law between gray matter and white matter of cerebral cortex. Proceedings of the National Academy of Sciences, 97:5621–5626, 2000.

[56] G. K. Zipf. Human Behaviour and the Principle of Least-Effort. Addison-Wesley, Cambridge, MA, 1949.