Optimal Overtaking on a Two-Lane Highway

Peter Schleede, Andrew Shoats, and Elliot Weiss

***Abstract*—This paper covers the design and implementation of a model predictive control (MPC) scheme to safely navigate a vehicle through a highway overtaking scenario. The MPC algorithm controls the lateral motion of the vehicle, enforcing discrete-time linear dynamics at each time-step. Our controller minimizes deviation from the path and actuator effort while enforcing constraints that keep the vehicle operating within actuator limits, a stable handling region, and a collision-free envelope. In simulation of a simple overtaking scenario, the controller was found to be robust to many potential parameter mismatch errors. With the addition of an oncoming vehicle, a hybrid longitudinal control scheme was introduced to ensure persistent feasibility along collision-free trajectories. Future work will aim to combine longitudinal and lateral dynamics in a nonlinear MPC framework.**

I. Introduction

The capability of a vehicle to autonomously overtake a lead car in real-world settings will consist of many improvements in existing control strategies. Current automated overtaking solutions require regulated conditions and precise knowledge of not only the lead and following vehicle states, but also the state of the surrounding environment and, therefore, may not be suited for all real-world driving scenarios. Additionally, they often contain of a set of heuristic maneuvers, which can be inflexible to unforeseen situations. An optimal control approach that incorporates knowledge of the vehicles dynamics and environment can provide adaptability [1].

We propose the design of an optimal control strategy for a simple model of a two car system on a two-lane (opposing traffic direction) highway, at constant speed, with the high-level goal of maximizing both d_a (the distance between lead and follower before passing) and d_c (the distance between lead and follower after passing), while minimizing d_b (the distance traveled in the opposing traffic lane). We simultaneously aim to minimize uncomfortably quick input sequences while ensuring that the following vehicle remains within a stable handling envelope and is collision free at all times. Going beyond this initial problem, we implement a hybrid control strategy to handle the more complicated scenario involving an oncoming vehicle by discretely adjusting desired longitudinal velocity. For all constant velocity simulations in this paper, the following vehicle travels at 25 m/s, and the lead vehicle travels at 17.5 m/s in the same lane, having started 37.5 m in front. Both vehicles have the same physical body and wheel properties, matching that of a typical passenger car.

II. Problem Formulation

A. MPC Framework

The following vehicle's lateral dynamics are controlled by a model predictive control (MPC) scheme as described in [2]. Longitudinal speed for the following vehicle is either kept constant or set by a separate control strategy based on maximum front-wheel acceleration due to tire friction limits. The MPC problem is as follows:

$$\begin{aligned}
\text{minimize} \quad & J = \sum_{k=1}^{N} x_k^T Q x_k + u_k^T R u_k + W_{\text{veh}}\sigma_{\text{veh}} + W_{\text{env}}\sigma_{\text{env}} \\
\text{subject to} \quad & x_{k+1} = Ax_k + B_1 F_k + B_2 K + B_3 \\
& v_k = F_{k+1} - F_k \\
& F_{\min} \preceq F \preceq F_{\max} \\
& v_{\min} \preceq v \preceq v_{\max} \\
& G_{\text{env}} x \preceq h_{\text{env}} + \sigma_{\text{env}} \\
& G_{\text{veh}} x \preceq h_{\text{veh}} + \sigma_{\text{veh}} \\
& 0 \preceq \sigma_{\text{env}} \preceq e_{\text{buff}} \\
& 0 \preceq \sigma_{\text{veh}}.
\end{aligned}$$

In the above optimization problem, σ_{veh} and σ_{env} are slack variables designed to allow slight constraint violations to ensure persistent feasibility. In plain English, we seek to minimize lateral and heading error, along with slew rate, or rate of change of steering inputs. The constraints correspond to:

1) Discrete lateral vehicle dynamics are upheld from one time-step to the next.
2) Commanded lateral force and slew rate values are within actuator limits.
3) The vehicle stays within a stable handling region throughout its entire planned trajectory.
4) The vehicle stays within a collision-free envelope throughout its entire planned trajectory.

For an explanation of the dynamics and implementation details, see [2] and our GitHub repository, linked below.[1]

This cost function and constraint formulation comprise a quadratic program (QP), making this a convex optimization problem that can be efficiently solved with the **CVX** package [3] in Matlab. We use variable length time-steps in the MPC horizon to enable more precise optimization in the near future, while ensuring tractability by more coarsely discretizing in the farther future. There are 9 short time-steps of 0.05 sec each, one intermediate time-step of adaptive length, and 20 long time-steps of 0.20 sec each. There was no constraint on the final state to ensure persistent feasibility. This was intentional because a failure to find a solution meant that the vehicle was violating the constraint on σ_{env} somewhere in the horizon

[1] https://github.com/petershlady/linear-mpc-overtaking

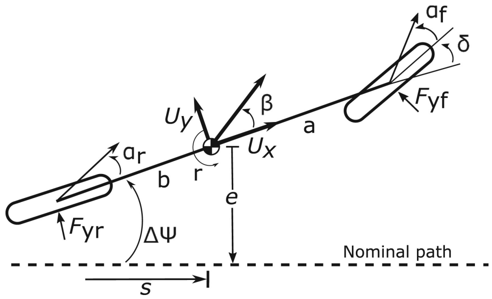

Fig. 1. Dynamic bicycle model, figure from [2]

(corresponding to a crash), which is knowledge that can be used to modify the problem.

B. Vehicle Dynamics

A single-track bicycle model, shown in Figure 1, was utilized to simulate lateral vehicle dynamics. In this model, the left and right tires on each axle are combined together, and force and moment balances are taken around the center of gravity of the vehicle. It is shown that this single-track model is sufficiently accurate (compared to a model that individually computes forces on each of the four tires) for a vehicle that is not making very tight turns at low speeds in which left-right tire angle differences may be large.

These continuous vehicle dynamics were implemented computationally by linearizing and discretizing the dynamics. Nonlinearities in system dynamics – most notably the nonlinear relationship between lateral tire forces and slip angles for higher acceleration turns – were accounted for with local linearization along the tire curve. To improve computational efficiency, many large sets of computations were combined together into matrix and vector operations. This is particularly important given the large number of matrix operations necessary to propagate linearized vehicle dynamics forward with time. Quick computational updates at each time-step at a rate of 10-100 Hz are necessary to control the motion of a real vehicle with suitable fidelity. In Matlab, we achieved a speed of roughly 1.9 Hz. This is encouraging, as compiled code will run far faster, likely achieving the desired speed.

III. Results

A. Initial Results

We initially computed optimal trajectories for the case where the following vehicle travels at a constant velocity (25 m/s) that is greater than the constant velocity (15 m/s) of the lead vehicle. Once cost function weights and slack variable values were appropriately tuned, the following vehicle was able to safely and smoothly pass the lead vehicle. Even though the nominal trajectory that the following vehicle aims to follow is the centerline of the right lane of the road, it temporarily incurs path tracking cost to avoiding a costly collision with the other vehicle. This trade-off between various cost function components can be seen in Figure 2. The final trajectory of the following vehicle overlaid on the optimal open-loop

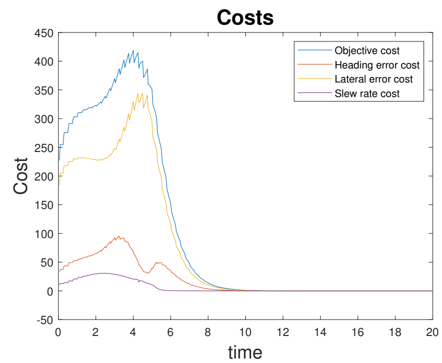

Fig. 2. Costs with no oncoming vehicle

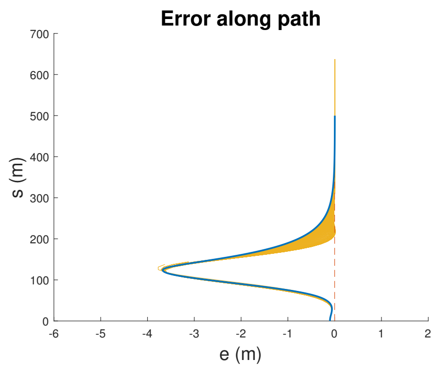

Fig. 3. Trajectories with no oncoming vehicle

trajectories computed at each time-step is show in Figure 3. A plot of the vehicle states, showing stability across the maneuver, is shown in Figure 4.

B. Sensitivity Analysis

To measure robustness of this MPC algorithm to various forms of model-mismatch error, several simulation variables were individually changed over a wide range of values. This model mismatch was tested through repeated simulation against a controller with a fixed parameter set. We varied several vehicle parameters in this way, including mass, tire cornering stiffnesses, mass moment of inertia, location of

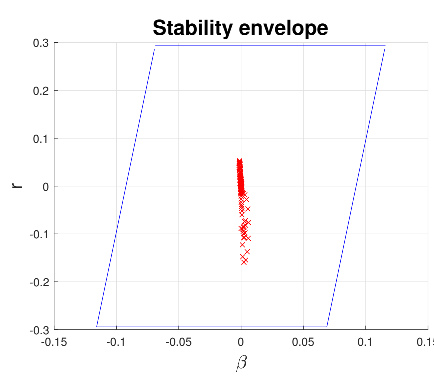

Fig. 4. Stability with no oncoming vehicle

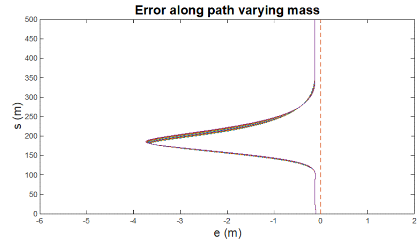

Fig. 5. Sensitivity of the trajectory to mass disturbance

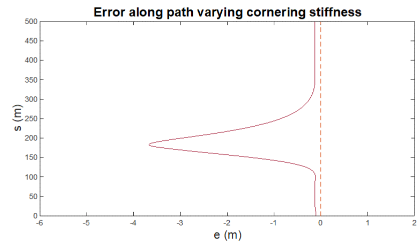

Fig. 6. Sensitivity of the trajectory to cornering disturbance

the center of gravity, and friction coefficient. Figures 5 and 6 demonstrate the variation in optimal trajectories for mass deviations of $\pm$ 300 kgs and cornering stiffness deviations of $\pm$ 40,000 N/rad. These are relatively large mismatches for what would be expected in realistic scenarios, and thus, we are confident that when changing our vehicle set-up by e.g. adding or removing passengers or changing tires, we will still have safe, feasible trajectories for our overtaking maneuvers. For all of these physical vehicle properties, variations in parameter value can be robustly handled by this control scheme without major changes to trajectory, stability, or safety. When longitudinal velocity mismatches occur, however, this controller computes very different optimal trajectories from the nominal trajectory. Figure 7 shows these deviations. Clearly, deviations in longitudinal velocity need to be carefully handled in this linear MPC framework, and this mismatch can cause issues when accelerating (i.e., waiting for the dynamics to catch up).

C. Oncoming Vehicle

To test the effectiveness of our MPC control scheme in realistic overtaking scenarios, an oncoming vehicle was introduced in the other lane of the road. This oncoming vehicle travels in the opposite direction of the lead and following vehicles at a constant speed of 15 m/s and begins travel at a point 200 m in front of the following vehicle. The oncoming vehicle is identical in geometry and physical properties to the other two vehicles, representing another typical passenger car. This combination of parameters results in a quickly closing gap between the oncoming and lead vehicles through which the following vehicle must drive to avoid a collision with either vehicle. If the following vehicle keeps a constant speed of 25 m/s, it cannot find a feasible trajectory that safely overtakes the lead vehicle. Therefore, the following vehicle must pre-plan for the situation in which a safe trajectory cannot be computed by varying its longitudinal speed and lateral position in some way, either by speeding up to pass or by slowing down and waiting for a safe path.

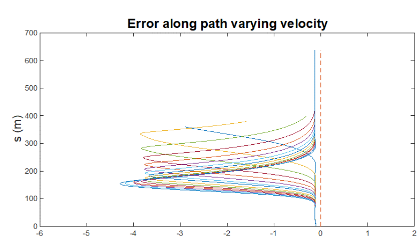

Fig. 7. Sensitivity of the trajectory to velocity disturbance

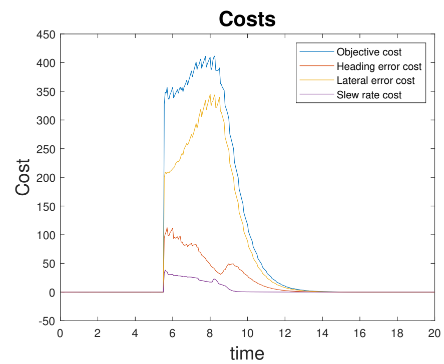

Fig. 8. Costs with oncoming vehicle

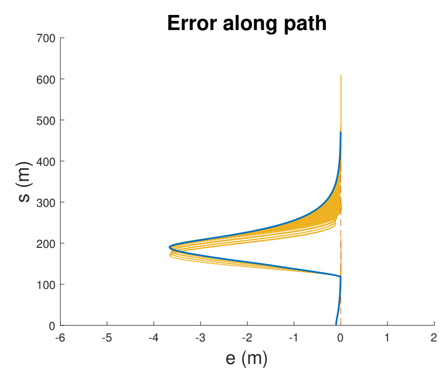

Fig. 9. Trajectories with oncoming vehicle. Notice that the overtaking vehicle waits to pass until it is safe to do so.

D. Longitudinal Speed Control

A longitudinal speed controller was implemented – entirely decoupled from the lateral dynamics controller – to ensure safe trajectories are always possible and to handle potentially tight emergency turning maneuvers at the vehicle's handling limits. Several different longitudinal speed control schemes were explored, which are presented in more detail in the discussion section below. The controller that worked best with our current lateral MPC framework was a hybrid scheme in which the following vehicle's desired longitudinal speed is set according to two different system states. The following vehicle starts in the overtaking state in which it tries to pass the lead vehicle at its nominal speed of 25 m/s. Once the MPC optimization problem cannot compute a feasible solution due to the narrowing gap between the two other vehicles, the following vehicle enters a waiting state. In this state, the following vehicle slows down to 15 m/s, which keeps it at a safe following distance from the lead vehicle. At this decreased speed, the following vehicle tends to wait in the right lane to avoid a collision with the oncoming vehicle. Once the oncoming vehicle has passed, the lead vehicle returns to the overtaking state and speeds up to 25 m/s. It can now reliably compute a safe trajectory for passing the lead vehicle since the oncoming vehicle no longer restricts the left bound of the safe driving envelope. The final trajectory (plus the open-loop trajectories computed by MPC) and cost distribution over time are shown in Figures 8 and 9. An animation of this overtaking maneuver is linked below. [2]

IV. Discussion

For scenarios in which a feasible, safe trajectory always exists, our lateral motion MPC performs quite well at overtaking another vehicle. Lateral error from the nominal vehicle path (the right lane of the road) is only incurred so that the following vehicle can safely continue at its current speed without colliding with the lead vehicle. These overtaking maneuvers also minimize slew rate (rate of change of steering input), so the left and right steering commands are comfortable for any vehicle occupants. This controller has limitations, however, when confronted with scenarios in which persistent feasibility is not guaranteed. This occurs when an oncoming vehicle is introduced, which crosses the longitudinal position of the lead vehicle around the same time that the following vehicle would need to be in the left lane to safely overtake the lead vehicle. In these situations, it is not clear how to optimally control the following vehicle while ensuring that safety and stability constraints are always met, without allowing for the overtaking vehicle to accelerate, which does not work well in the current control design because the dynamics are nonlinear in longitudinal velocity.

The open-loop trajectories computed with MPC determine a path for the vehicle about 4-5 sec into the future. If the vehicle is accelerating or decelerating according to a control scheme that is decoupled from the MPC optimization, future longitudinal speed information isn't properly accounted for. This is particularly important when predicting whether the following vehicle may collide with one of the other two vehicles based on a forward projection of possible future longitudinal positions. Working within this framework, we implemented a heuristic that encodes either typical overtaking behavior or conservative waiting behavior that slows down the following vehicle until the oncoming vehicle has passed. Additional heuristic longitudinal controllers that enforced accelerating and decelerating modes of operation were also implemented but found to be less effective. In future iterations, we would like to implement coupled longitudinal and lateral vehicle control, so that the balance between choosing to slow down to wait for a safe opportunity to pass and speeding up to overtake before a feasible gap is closed can be optimized. Linking longitudinal and lateral control in this way will will enable the vehicle to determine – for a wide range of scenarios – how best to steer and speed up or slow down without necessitating hard-coded logic to handle interactions with each specific vehicle on the road.

V. Conclusion and Future Work

In this paper, we've detailed a lateral MPC controller to control the motion of a vehicle during a highway overtaking scenario. With our implementation, the following vehicle is always able to overtake a slower moving vehicle with no other vehicles on the road. The MPC controller additionally has proven robustness to model mismatch as caused by parameter error. Introducing an oncoming vehicle presents situations in which the longitudinal speed of the following vehicle must be controlled to ensure persistently feasible trajectories are computed. We've suggested one potential hybrid control scheme that accelerates and decelerates the vehicle according to either waiting or overtaking behaviors. Future work on this project will entail several steps aimed to extend this research to a racing context. Implementing a nonlinear MPC control scheme to couple longitudinal and lateral vehicle dynamics will be critical to optimizing both the lateral motion and speed profile of the following vehicle through highly dynamic maneuvers near the vehicle's friction limits. Introducing a curved racing path and a lead vehicle that, itself, is optimally controlled to follow a racing line will enable a study of interactions between two autonomously controlled racing vehicles. Additionally, current work done in the Dynamic Design Lab regarding so-called contingency MPC and model cascaded trajectory planning (using one higher fidelity model (e.g., bicycle model) for short range predictions, and a lower fidelity model (e.g., point mass model) for extending the prediction horizon) could lead to improvements in our work. Higher level decision making through partially observable Markov decision processes (POMDPs) may represent a more intelligent way to model uncertainty in the lead vehicle's dynamics and future behavior. The ultimate goal of this work is to explore interactions between vehicles operating at the limits of their handling capabilities, each with a set of objectives that are incompatible with one another.

References

[1] S. Dixit, S. Fallah, U. Montanaro, M. Dianati, A. Stevens, F. Mccullough, and A. Mouzakitis, "Trajectory planning and tracking for autonomous overtaking: State-of-the-art and future prospects," *Annual Reviews in Control*, vol. 45, pp. 76–86, 2018.

[2] M. Brown, J. Funke, S. Erlien, and J. C. Gerdes, "Safe driving envelopes for path tracking in autonomous vehicles," *Control Engineering Practice*, vol. 61, pp. 307–316, 2017.

[3] M. Grant, S. Boyd, and Y. Ye, "Cvx: Matlab software for disciplined convex programming," 2008.

[2] https://www.youtube.com/watch?v=yEQfoADEBPs