Minimal surfaces exercises

Lecture 2

- Really do exercise 4 from last week if you haven't.
- Let x(u,v) be a chart of a surface on a region $\Omega \in \mathbb{R}^2$, and let N(x) be a unit normal.
  - Show that for ϵ sufficiently small, the map X(u,v,t) = x(u,v) + tN(x(u,v)) is a diffeomorphism on the region $U \times (-\epsilon, \epsilon) \subset \mathbb{R}^3$. (This is pretty hard.)
  - Assuming the first part, show that the area of $x(\Omega)$ is given by

$$\lim_{\epsilon \to 0} \frac{1}{2\epsilon} \mathrm{Vol}(X(\Omega \times (-\epsilon, \epsilon)))$$

- A helicoid is the surface swept out by rotating a horizontal line around the vertical axis at a constant speed as you translate it upwards at constant speed. Find which speeds of translating and rotating make the helicoid a minimal surface.

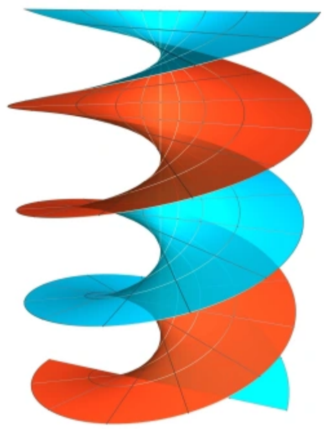

Figure 1: Helicoid