Experimental determination of friction factor

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Introduction

To partially fulfill our course requirements, we planned and conducted an experiment to determine the friction factor of a straight PVC pipe. The apparatus is an Armfield flow bench instrumented for measuring pressure and flow rate. The experimental findings are compared to published values in a Moody chart. Results within 30% of expected values are considered successful.

Apparatus

The apparatus is an Armfield flow bench instrumented as illustrated in Figure 1. The flow bench has an adjustable flow pump that cycles water from a reservoir through the straight pipe and back to the reservoir. The pump speed is adjusted each trial to obtain a desired flow rate measured by a flow meter.

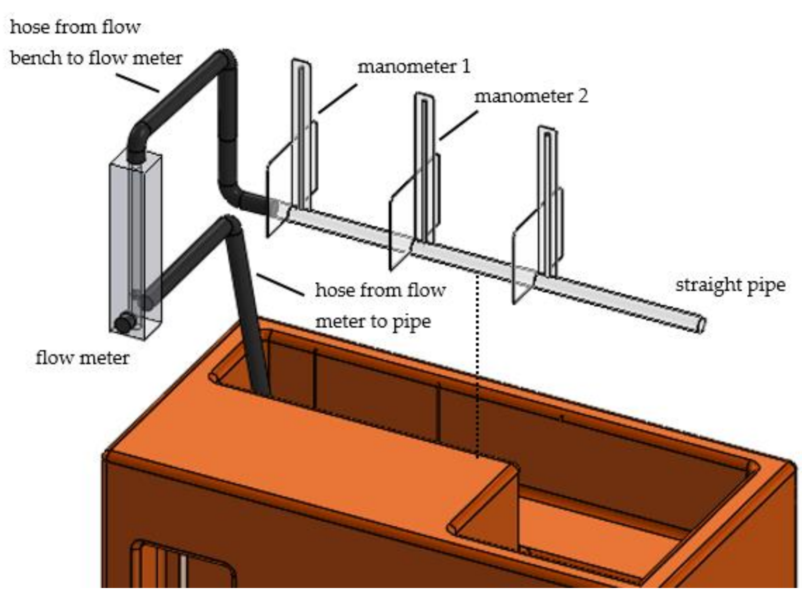

Figure 1. Configuration of the flow bench

Attached to the pipe are three manometers distanced at 20 cm apart for measuring fluid pressure at that point. Only two of the manometers are used.

Modeling

For an incompressible fluid (in this experiment, water at room temperature) the pressure difference Δp between two points along a straight pipe is modeled by the Darcy-Weisbach equation (Gerhart, 2013),

$$\Delta p = f \frac{L}{D} \frac{1}{2} \rho v^2, \tag{1}$$

where L is the pipe length of the pipe, D is the pipe inner diameter, v is the fluid velocity, and f is the friction factor, as illustrated in Figure 2. Determining the friction factor is the goal of the experiment.

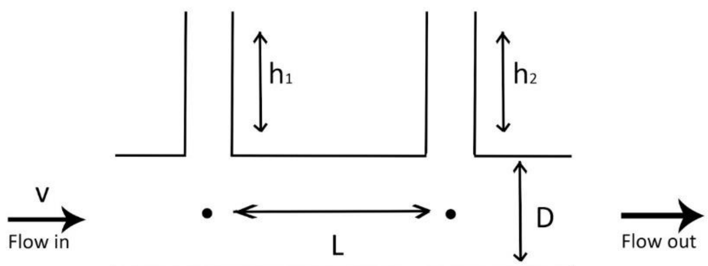

Figure 2. Illustrating the variables that appear in the data reduction equation

From hydrostatics, pressure at the base of each manometer is given by

$$p = \rho g h, \tag{2}$$

where ρ is the fluid density and h is the measured manometer fluid height. The pressure difference, therefore, is given by

$$\Delta p = \rho g (h_1 - h_2). \tag{3}$$

Substituting (3) into (1) and rearranging, we obtain an expression for friction factor,

$$f = \frac{2Dg(h_1 - h_2)}{Lv^2}. \tag{4}$$

Flow rate is the product of fluid velocity and pipe cross-sectional area A, thus, velocity can be written,

$$v = \frac{\dot{V}}{A} \tag{5}$$

where $\dot{V}$ is measured flow rate and area A of a circular pipe is $\pi D^2/4$. Substituting (5) and the area expression into (4) yields the data reduction equation,

$$f = \frac{\pi^2 D^5 g(h_1 - h_2)}{8L\dot{V}^2}. \tag{6}$$

To compare our experimental results to the published values in a Moody chart, we compute the Reynolds number for each operating condition, given by,

$$Re = \frac{\rho v D}{\upsilon}, \tag{7}$$

where υ is the dynamic viscosity of the fluid, a table look-up value for water at room temperature. Substituting for velocity, we obtain Reynolds number in terms of the measurands,

$$Re = \frac{4\rho\dot{V}}{\pi\upsilon D}. \tag{8}$$

Procedure

Several one-time measurements and table look-ups are made. Measurements are made of the D (using calipers) and L (using a tape measure). Reference look-up values are obtained for PVC pipe roughness and water density and viscosity at room temperature. (citation needed)

The test sequence is based on ten trials of three operating conditions. The operating conditions are a low, medium, and high flow rate (determined by the range of pump speeds available that produce steady flow rates). In this case, the three flow rates are approximately 1.5, 2.5, and 3.5 gallons per minute. Ten observations are planned for each flow rate and the test sequence is randomized.

As shown in Figure 3, for each observation, the motor speed is adjusted until the flow rate measurement indicates steady flow at the desired flow rate. Manometer readings for that observation are recorded.

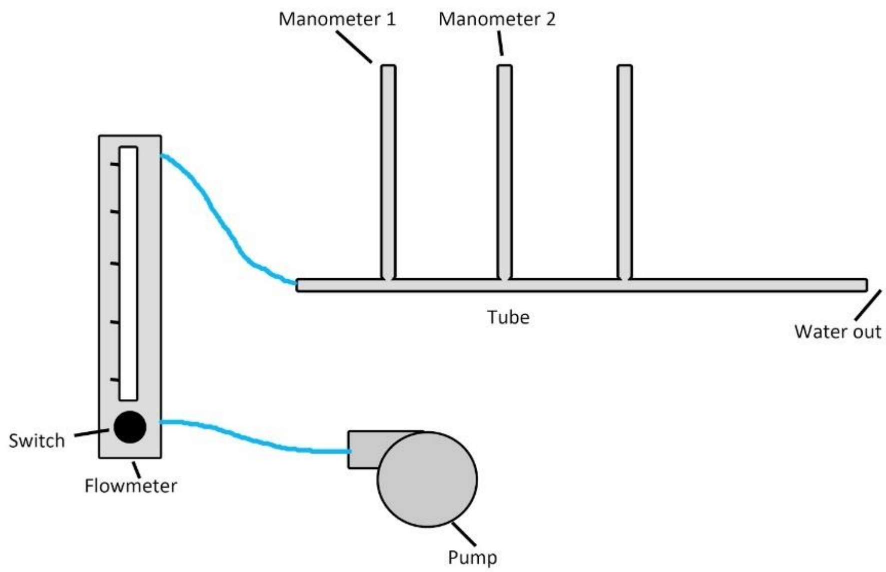

Figure 3. Elements of the apparatus important for the procedure.

Data

The thirty observation are shown in Table 2. The data have been organized by flow rate. The test sequence in random order is given in an appendix.

Table 1. Complete experimental data

Flow gpm	h_1 mm	h_2 mm	Flow gpm	h_1 mm	h_2 mm	Flow gpm	h_1 mm	h_2 mm
1.5	57	32	2.5	155	94	3.5	278	170
1.5	54	30	2.5	152	92	3.5	274	168
1.5	57	30	2.5	153	94	3.5	280	171
1.5	59	36	2.5	155	93	3.5	279	170
1.5	57	32	2.5	154	94	3.5	280	172
1.5	54	30	2.5	157	95	3.5	280	171
1.5	59	35	2.5	157	97	3.5	283	173
1.5	60	33	2.5	154	94	3.5	275	168
1.5	58	33	2.5	158	96	3.5	277	170
1.5	59	32	2.5	153	92	3.5	274	168

Table 2 lists the physical constants and one-time measurands. Values that may be uncertain are the pipe roughness and dynamic viscosity.

Table 2: Constants and one-time measurands

quantity	value	
D	12.1	mm
L	20.0	cm
ρ	1000	kg/m^3
ε	0.0015	mm
ν	1.3	mPa-sec

The range for the surface roughness is 0.0015 to 0.007 mm, therefore we did not have an exact roughness value for our specific experimental PVC Pipe.

Analysis

For a given flow rate, a Thompson tau test (citation needed) is performed to identify portential outliers. The test is repeated for each flow rate.

For every trial, friction factor (6) and Reynolds number (8) are computed. At each of the three operating conditions (low, medium, and higjh flow rate) a mean friction factor and mean Reynolds number are determined.

These three mean value are graphed on a conventional Moody chart for comparison to expected values and a percent difference from expected values is determined.

Total uncertainty of the friction factor is given by

$$u_f^2 = u_{f,sys}^2 + u_{f,rand}^2, \tag{9}$$

comprised of systematic and random components. . Because each operating condition is assumed to be at steady state, random uncertainty is computed in ethe resultant. Systematic uncertainty is estimated by applying uncertainty propagation to the data reduction equation (6), yielding,

$$u_{f,sys}^2 = a_{h_1}^2 u_{h_1,sys}^2 + a_{h_2}^2 u_{h_2,sys}^2 + a_D^2 u_{D,sys}^2 + a_L^2 u_{L,sys}^2 + a_V^2 u_{V,sys}^2, \tag{10}$$

where a is the sensitivity coefficient and u_{sys} is the systematic uncertainty in each measurand due to sensor accuracy and readability

Total uncertainty in Reynolds number is estimated in a similar manner. Additional details of the uncertainty analysis with numerical values is provided in an appendix.

Results

The outlier tests indicate that no data were potential outlier, therefore, all data in Table 1 were used in the analysis.

Average results for friction factor and Reynolds number at each operating condition are shown in Table 3.

Table 3. Mean friction factor and Reynolds number at three operating conditions.

	Flow rate (gpm)	Mean friction factor	Reynolds number
Low	1.5	0.0438	7.67×10^3
Medium	2.5	0.0381	1.28×10^4
High	3.5	0.0347	1.79×10^4

We see that the lower flow rates yield lower Reynolds numbers and higher friction factors, consistent with expectations from the Moody chart.

These values are graphed on the Moody chart in Figure 4. To assist in the visual comparison, the experimental data are drawn with circles and the expected values are shown as triangle data markers.

Overall, the experimental friction factors are higher than expected. The percent difference is shown in Table 4. All values are within 30% of the expected values.

Table 4 friction factor comparison.

	Reference value f	Percent difference
Low	0.0341	22.1%
Medium	0.0305	19.9%
High	0.0280	19.3%

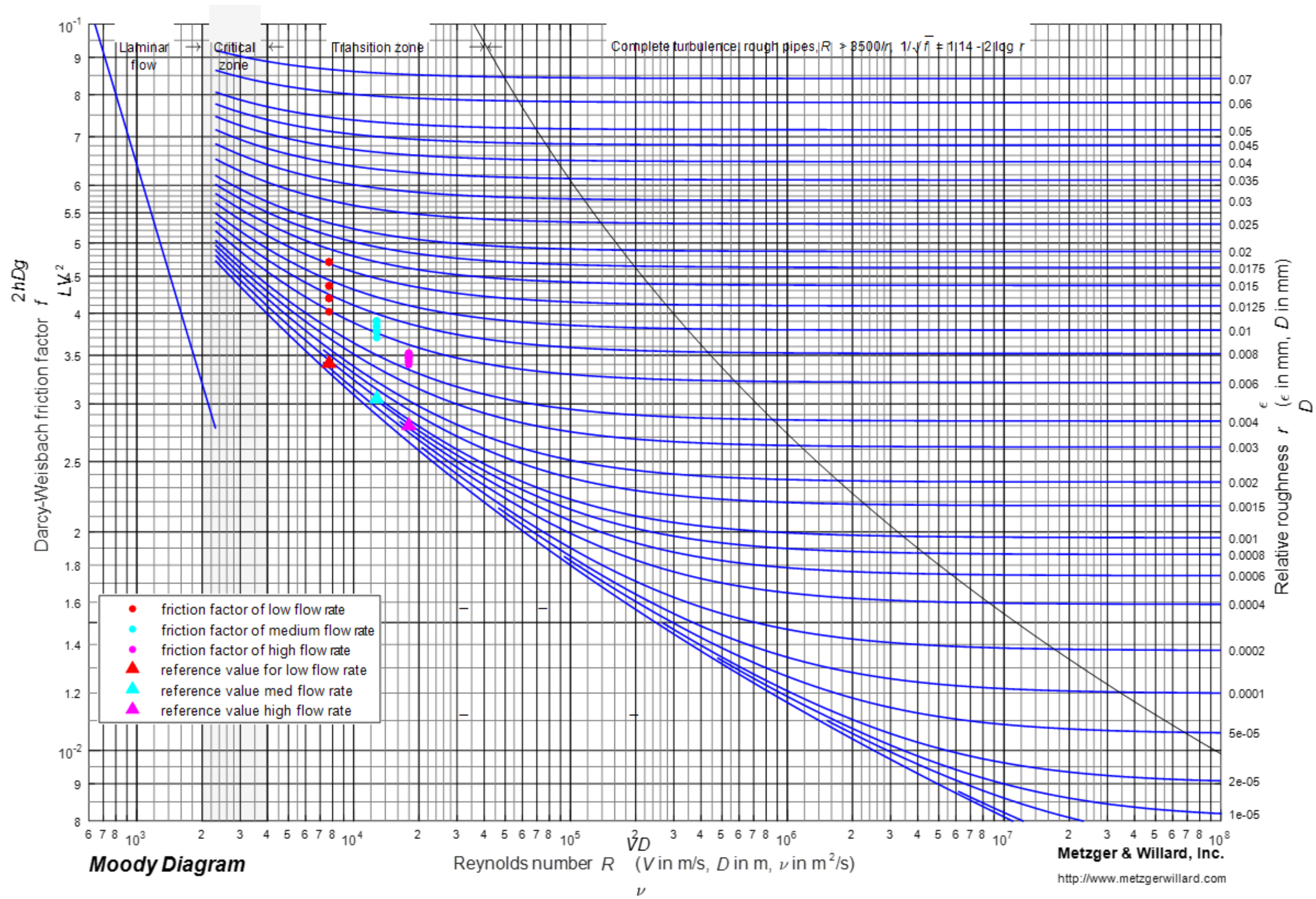

Figure 4: Resultants for "low," "medium," and "high" flow rates graphed on a Moody diagram. (Davis)

Relative uncertainties are shown in Table 5. For fluid system results of this type, relative uncertainties of 25% are common. (citation needed)

Table 5: Summary of relative uncertainties for "low," "medium," and "high" flow rates.

	Relative uncertainty
Low	29.44%
Medium	24.22%
High	22.68%

Conclusion

The goal of this experiment was to determine friction factor of a straight pipe. That goal was achieved for three flow rates. Percent differences to expected values are within 30% and relative uncertainties are within 30%. Both are acceptable norms for experiments of this type.

The discrepancies in our results could have been caused by unstable water flow from the flow bench. Unstable flow will make the height of the manometer unstable and hard to read an accurate number.

References

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Appendices

Test sequence

Table A1. Randomized test sequence

Trial	Flow rate
1	low
2	med
3	high
4	med
4b	low
5	med
6	low
7	high
8	med
8b	high
9	med
10	high
10b	low
11	high
11b	med
12	high
13	med
14	low
14b	med
15	low
16	med
17	high
18	low
19	high
20	low
21	high
22	med
23	low
23b	high
24	low

Uncertainty analysis

The systematic uncertainty for each measurand (h_2, D, L, and flow rate) is similar to that for h_1, incorporating readability and accuracy,

$$W_{h_1 sys}^2 = W_{h_1,acc}^2 + W_{h_1,read}^2 \tag{A3}$$

The sensitivity coefficients are computed with,

$$a_{h_1} = \frac{\partial f}{\partial h_1} = \frac{1}{8} \cdot \frac{D^5 \pi^2 g}{\dot{V}^2 L}, \tag{A8}$$

$$a_{h_2} = \frac{\partial f}{\partial h_2} = \frac{-1}{8} \cdot \frac{D^5 \pi^2 g}{\dot{V}^2 L}, \tag{A9}$$

$$a_D = \frac{\partial f}{\partial D} = \frac{5}{8} \cdot \frac{(h_1 - h_2) D^4 \pi^2 g}{\dot{V}^2 L}, \tag{A10}$$

$$a_L = \frac{\partial f}{\partial L} = \frac{-1}{8} \cdot \frac{(h_1 - h_2) D^5 \pi^2 g}{\dot{V}^2 L^2}, \tag{A11}$$

$$a_{\dot{V}} = \frac{\partial f}{\partial \dot{V}} = \frac{-1}{4} \cdot \frac{(h_1 - h_2) D^5 \pi^2 g}{\dot{V}^3 L}, \tag{A12}$$

Random uncertainty in the resultant is given by

$$W_{f,rand} = \frac{t s_f}{\sqrt{n}}, \tag{A14}$$

where $W_{f,rand}$ is the random uncertainty of the friction factor, t is the t-statistic with a two-sided confidence interval of 95%, S_f is the standard deviation of the friction factor, and n is the number of trials. From our experiment,

The calculations of the uncertainties are summarized below in Table A3.

Table A2: Summary of uncertainty analysis (for the low flow rate condition only)

Domain	Component	Value	Units
manometer 1, h_1	average reading, low	0.0574	m
	accuracy	0.0005	m
	read uncertainty	0.0005	m
	syst uncertainty	70.711×10^{3}	m
	sensitivity coeff	0.3206	m^{-1}
manometer 2, h_2	average reading, low	0.0323	m
	accuracy	0.0005	m
	read uncertainty	0.0005	m
	syst uncertainty	70.711×10^{3}	m
	sensitivity coeff	-0.3206	m^{-1}
volumetric flow rate, $\dot{V}$	reading, low	94.64×10^{-6}	m^3s^{-1}
	accuracy	8.8326×10^{-6}	m^3s^{-1}
	read uncertainty	3.1545×10^{-6}	m^3s^{-1}
	syst uncertainty	9.379×10^{-6}	m^3s^{-1}
	sensitivity coeff	-345.5711	sm^{-3}
diameter, D	reading	0.01209	m
	accuracy	0.00002	m
	read uncertainty	0.0005	m
	syst uncertainty	50.04×10^{-3}	m
	sensitivity coeff	15.7786	m^{-1}
length, L	reading	0.20	m
	accuracy	0.0005	m
	read uncertainty	0.0005	m
	syst uncertainty	7.0711×10^{-3}	m
	sensitivity coeff	-0.19077	m^{-1}
friction factor, f	result, low	0.0438	
	syst uncert due to h_1	0.2267×10^{-3}	
	syst uncert due to h_2	0.2267×10^{-3}	
	syst uncert due to flow rate	3.2411×10^{-3}	
	syst uncert due to diameter	7.8956×10^{-3}	
	syst uncert due to length	1.3489×10^{-3}	
	random uncertainty, low	0.0018	
	overall uncertainty, low	0.0129	
	relative uncertainty, low	29%	