Mathematical Induction Proof assignment

Assignment objective: The objective of this assignment is to give students a beginning experience at writing a logically sound, well written, well presented proof by mathematical induction. The assignment will also provide an opportunity to complete a solution from exploration of a problem to mathematical proof of its validity.

The Problem: The problem is given at the end of the instructions.

Due date: Friday, Dec 1, 2017 *by midnight*.

Deliverables: The assignment should be uploaded to Blackboard as a single document. It must be typed and formatted correctly. The illustrations must be included in the document. A solution will include at least all of the following parts.

- A description of the problem with illustrations of at least 4 stages for the problem.
- A table of data showing the correct numbers for the first 6 stages of the problem and a recursive formula, including initial conditions that would generate the values in the table.
- Arithmetical evidence that the recursive formula correctly generates the data obtained from the physical model.
- An algebraic derivation of the closed form formula that would generate the data in the table. This would include the system of equations that you solved as well as documentation clearly stating what technology you used to solve the system. A screen shot would be valuable documentation if an online calculator is used.
- Arithmetical evidence that the proposed closed form formula generates the data.
- The mathematical proof of the truth of the closed form formula. Proofs must be typed and **all mathematical notation should be typeset correctly**. Use either LaTex or the equation editor in Word.

Grading: The project will be graded according to the attached rubric. All of the deliverables for the bullet points above must be turned in for the assignment to be considered for grading. If an assignment is missing any

one of the deliverables, it will be handed back and a 0 will be recorded in the grade book. The assignment may be turned in again once all parts are included, but a 25% penalty will be assessed. That is, 75% will be the highest grade possible.

Proof: Each student is expected to write their own proof, although students may share ideas about the problem. All sources must be adequately cited. Suspected plagiarism will result in loss of points for the assignment. If you work with another student and use their ideas, they should be listed as a coauthor on your assignment.

Your proof should be self-contained. You must identify each variable used and state any characteristics the variable has. Be sure you don't use the same variable to represent different quantities in your proof.

Don't forget to start with a base case.

Be careful that when you present the induction step that you proceed logically from the k case to the $k+1$ case. You need to start by assuming the k state is true and show how that logically to claim being true for the $k+1$ case. This step will involve algebra and justifications should be given for each step in the algebra.

Err on the side of too much detail. While the details of the proof may include algebraic notation, sentences written in English will be necessary to adequately explain the proof.

Notation: The proof must be typed and all mathematical notations must be typeset correctly and presented in its proper form. If you use Word, use the equation editor any time you are presenting mathematical symbols. For example x^2 is typeset correctly, but x^2 is not.

Presentation: Ideally, the presentation of the proof should enhance the content of the proof. Typesetting the algebraic steps can be challenging, but take the time to do it right. The equal signs should line up vertically. You can us a table in Word to get them to line up. Mathematical symbols should be presented in a different typeface than the English text. (For example, italics is a different typeface. The equation editor and LaTex change the typeset automatically.)

Citing: Plagiarism is act of presenting someone else's thoughts, ideas, or expressions as your own. If you use other people's work, be certain to give them credit by properly citing your source. This includes any work found electronically or ideas obtained from other people. If someone else that you know works with you and contributes ideas, you can list them as a coauthor on the proof. It is a violation of honesty policy to have another student, tutor, or math website do any substantial part of the proof without citation.

If you include other people's work in whole or in part, you must give them credit for the work by giving a clear citation. **If it seems that sources were used and not cited, there will be a 100% deduction from the points earned on the proof.**

Grading Rubric	Point Values		
	50	30	10
Objective			
Use correct logical structure for a proof by mathematical induction when proving a mathematical statement.	The proof correctly shows the basis step, and logically proceeds from the induction assumption to the conclusion. The proof finishes by stating the use of mathematical induction.	The proof was missing some of the components of a proof by mathematical induction, but the essential logic for the induction stage of the proof was included.	Essential parts of the proof by mathematical induction were missing and the logic for the induction step was missing or incorrect.
	20	10	0
Give numeric evidence that the statement you are proving is in fact true for some small natural numbers. Provide illustrations that enhance the understanding of the problem.	**The proof was accompanied by well-organized and correctly computed evidence that adequately documents the truthfulness of the statement,** by showing both recursive and closed form computations. Illustrations were valuable.	The numeric evidence was computed correctly, but was not well-organized. Illustrations were too messy to be of much value.	The evidence was missing or incorrectly computed. The student did not show both the closed form computation and the recursive computation.
	10 each	5 each	0
Correctly write a recurrence equation.	The equation was expressed correctly, including initial values, and generates the data in the table.	The equation was not expressed correctly	The equation did not generate the data.
Correctly derive a closed from formula that generates the numerical evidence in a problem.	The algebraic derivation included the system of equation, its solution and also contained necessary documentation on technology used.	The algebraic derivation was correct, but documentation was not given.	There were errors in the derivation and documentation was missing.
Use mathematical notation correctly and present work in a manner that adds value to the content.	The proof was presented in a professional looking document. The presentation made the content appear to be worth reading. All mathematical notations were correctly defined and presented. The work was turned in as specified.	The proof was presented in an acceptable style. The presentation did not distract from the content. Most mathematical notations were correctly defined and presented.	The document was presented in such a way that it distracted from the content. The presentation made the content look unreliable. Mathematical notations were seldom defined and presented correctly.

The Problem: The problem consists of deriving a recursive and closed form solution that will count the total number of disks need to create the following structure for the nth stage.

Stage 1 will have a stack of 7 disks.

Stage 2 is created by laying disks around the center stack, and adding one disk to the center stack. There will be a 3x3 square of stacks made up of 8 short stacks of 1 disk and 1 tall stack in the center.

Stage 3 is created by laying disks all around the existing square, and then adding one disk to every existing stack. This will create a square with single stacks of 1 disk on the edge and 8 stacks of 2 disks and 1 tall stack in the center.

Stage n continues in the same fashion by surrounding the current square with single disks and adding one disk to all of the existing stacks.

(If you were in class, you saw this demonstrated. If you missed class you will probably need someone to demonstrate the construction to you.)

Do all of the following.

- Provide illustrations (photos or computer generated sketches) for at least 4 stages of the problem.
- Create a table of data showing the correct number of disks needed for the first 6 arrangements, and generate a recursive formula for the values in the table.
- Provide arithmetical evidence that the recursive formula correctly generates the data obtained from the physical model.
- Produce an algebraic derivation of the closed form formula using differencing technique that would generate the data in the table. This will include the system of equations that can be solved to find the parameters of the equation, and either a hand written solution or documentation clearly stating what technology you used to solve the system.
- Provide arithmetical evidence that the closed form formula correctly generates the data obtained from the physical model.

- Write a well written and well-presented induction proof showing that the closed form formula will compute the correct value for the number of disks in the arrangement for all $n \in \mathbb{N}$. Proofs must be typed and all mathematical notation should be typeset correctly. Use either LaTex or something similar to the equation editor in Word.

All parts of the assignment must be turned in to receive credit for any part of the assignment.