<u>Felix Baumgartner's Supersonic Freefall</u>

Peter Spencer-Smith

Introduction:

In this exercise the free fall of an object under various conditions, is to be modelled using variations of the Euler method.

In section a), the Euler method is used to solve the differential equation which describes the motion of an object in free fall. The Euler method works by taking a differential and replacing it with a ratio of finite differences. We can then evaluate the differential to a chosen degree of accuracy.

Equations (1), (2) and (3) summarise the Euler method and provide the basis for the development of the code which models the free fall of the object throughout the exercise.

$$\frac{dy}{dt} = f(y,t) \qquad (1)$$

$$t_{n+1} = t_n + \Delta t \qquad (2)$$

$$y_{n+1} = y_n + \Delta t \,.\, f(y,t) \qquad (3)$$

In section b), an analytical solution (4) & (5), to the problem described in part a), is to be compared with the Euler method.

$$y = y_0 - \frac{m}{2k} log_e \left[cosh^2 \left(\sqrt{\frac{kgt}{m}} \right) \right] \qquad (4)$$

$$v_y = -\sqrt{\frac{mg}{k}} tanh \left(\sqrt{\frac{kgt}{m}} \right) \qquad (5)$$

In section c), a modified version of the Euler method is coded and the results compared to those presented in part a). The modified Euler method functions by taking the derivative between time steps which reduces the overshoot.

Finally in section d), the air density is introduced as a function of position in order to more accurately re-create the motion of a body in free fall within the earths atmosphere.

Part a: The Euler method

The program:

In order to model the velocity and position of the object in free fall using the Euler method, a 'while' loop was employed to iterate through each time step.

Initially, three empty lists were created to be filled with the output values for time, velocity and position. The .append function was utilised in order to fill the empty lists with values taken from each iteration of the while loop.

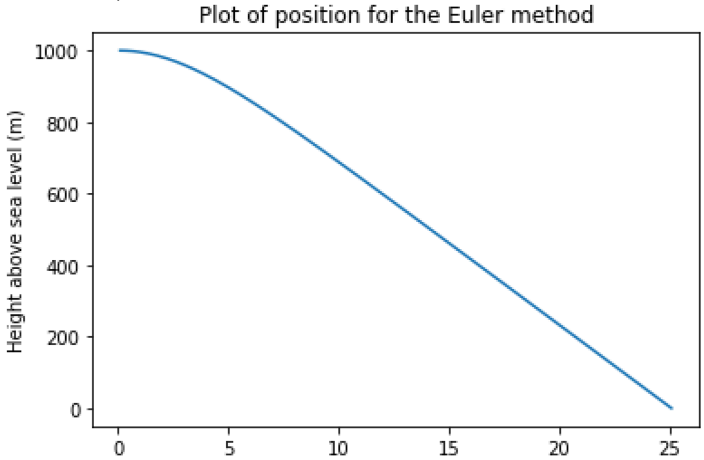
The termination condition for the 'while' loop was set to be when the y-position of the object was zero.

Finally the populated lists were then plotted, with the time list mapped to the x-axis in each case.

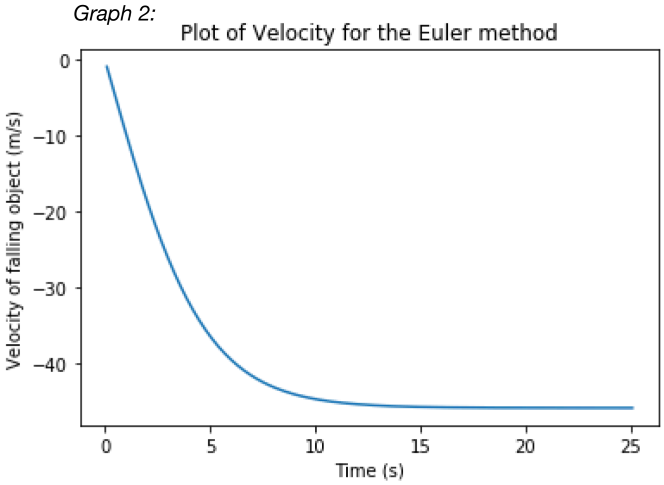
Results & Discussion:

Graphs 1 & 2 display the position and velocity respectively of the output values from the Euler method. They clearly show the object reaching a terminal velocity which remains constant and independent of the height of the object. The gradient of graph 1 remains unchanged after approximately 10 seconds which corresponds to the exponentially decreasing value of velocity displayed in graph 2. For both graphs the time step, h = 0.1 s.

Graph 1:



Graph 2:



Part b: The analytical solution

Part b) provided results consistent with those measured in part a). Differences, as expected, were observed between the two methods as the time step was increased .

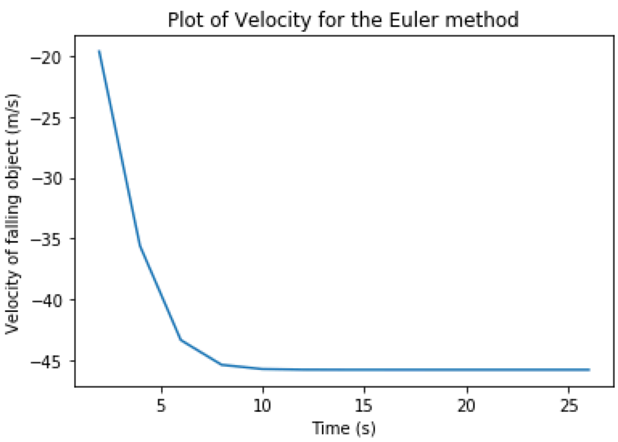
The program:

A list of time values was generated and used as the argument for equations (4) & (5). Equations (4) & (5) were programmed as mathematical objects and then plotted as variables against the list of time values.

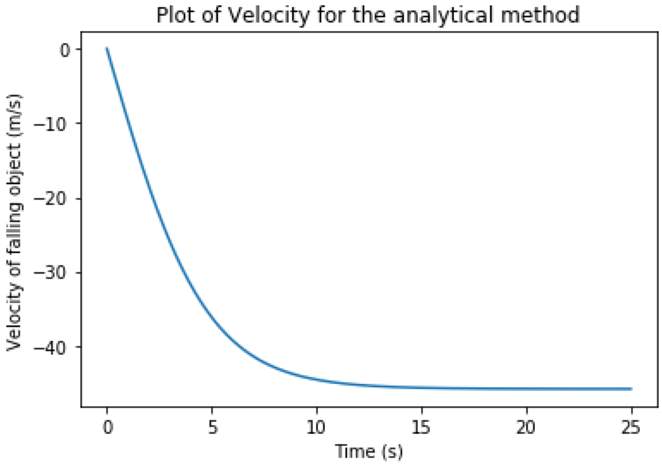
Results & Discussion:

Graph 4 displays the velocity output for the analytical method. It clearly matches to a high degree of accuracy the corresponding output for the Euler method when the time step is small (h = 0.1's). Graph 3 displays the velocity output of the Euler method when the time step is made much larger, h = 2's. We can clearly see how the Euler method no longer accurately reproduces the analytical method at large time steps.

Graph 3:

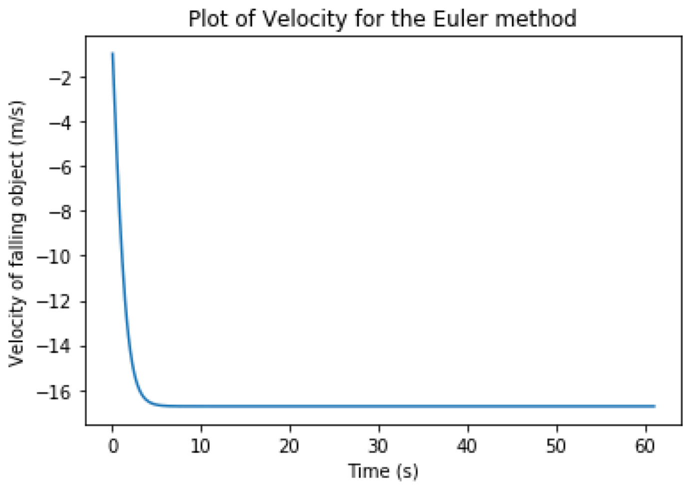


Graph 4:



The effect of varying the ratio k/m can be seen displayed in graph 5, where k/m has been increased by a factor of 10 as compared to the ratio in graph 2. We can see that the effect of decreasing the mass of the object or increasing its surface area has the result of decreasing both the time taken to reach terminal velocity and the final value itself.

Graph 5



Part c): The modified Euler method

Part c) showed the effect of improving the accuracy of the Euler method at larger time intervals.

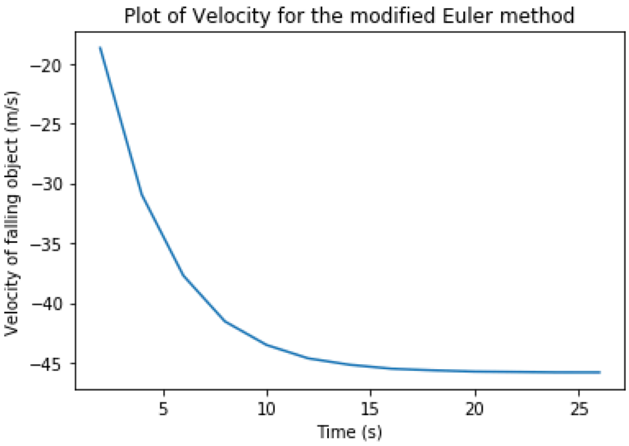
The program:

The code used for part c) required only a minor alteration to the coding nested within the 'for' loop for that used in part a). The basic structure and architecture remained unchanged.

Results & Discussion:

By increasing the time step to h = 2's, we see displayed in graph 6 and by comparison to graph 3, that the modified Euler method retains a higher degree of accuracy than the un-modified method, at higher time step values.

Graph 6:



Part d: Modelling Felix Baumgartner's jump with varying drag

The effect of varying drag was modelled and investigated using a positional dependence for air density.

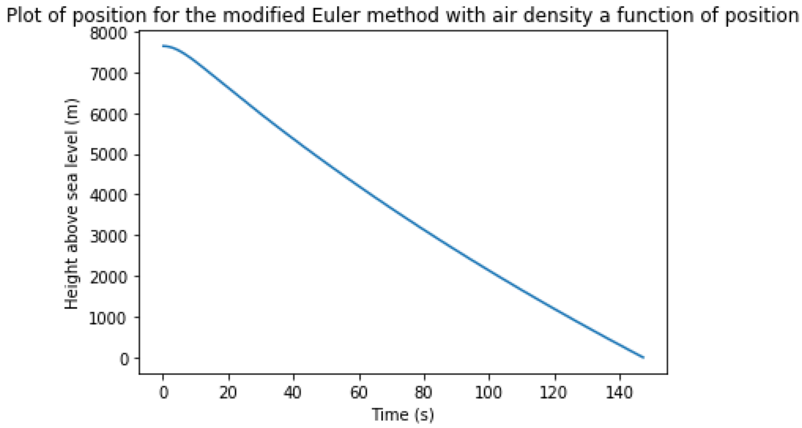
The program:

The code used for part c) was reused in part d) with the addition of an air density equation nested within the 'while' loop.

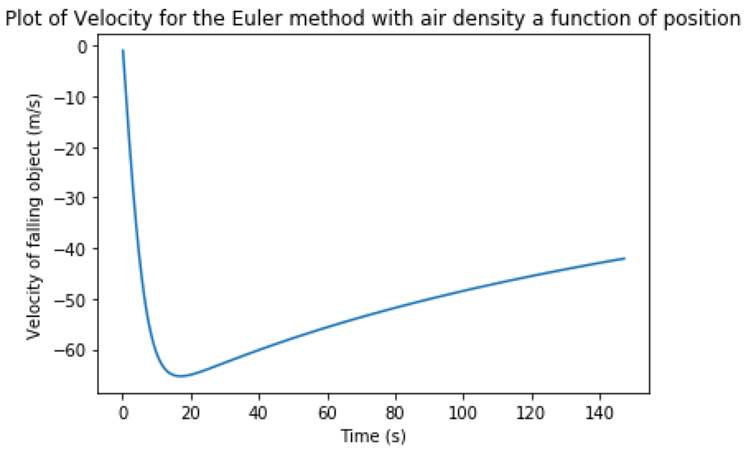
Results & Discussion:

Graphs 7 & 8 show the position and velocity respectively, of Felix as he fell through the imperfect cyberspace of my virtual skydive simulation. We clearly see his velocity reach a maximum and then gradually reduce as the air density increases. Based on the values of surface area A = 0.7m^2, drag coefficient C = 1 and mass = 75 kg, we see that Felix did not break the sound barrier, only reaching approximately 65m/s.

Graph 7:



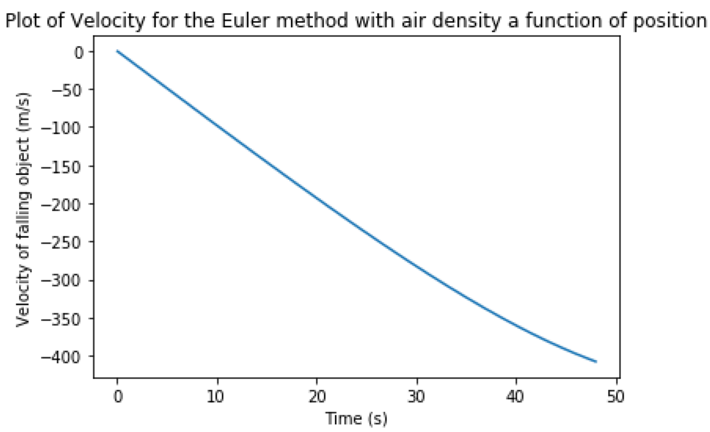
Graph 8



Part e: How fast can we make him go?

By telling Felix to dive head first to reduce his surface area to only A = 0.1 m^2 and wear some super slippery space age low drag suit, so that his drag coefficient C = 0.1. His maximum speed dramatically increased. He also took on some extra weight so that he now weighs 100kg and jumped from an even grater height of 10,000m.

Graph 9:



Graph 9 displays the slightly dangerous decent and gives us a value for his velocity of around 400m/s before he smashed into the ground leaving behind a broken sound barrier.

Conclusion:

The Euler method was successfully modelled and an improved version demonstrated by comparison to an analytical solution. A real world situation was re-created by the introduction of a varying drag force and the effect of various parameters on maximum velocity explored.