Spatio-temporal dynamics of the cystic fibrosis airway microbiome

**Abstract**

**Introduction**

**Methodology**

**Model equations**

Our model consists of three coupled partial differential equations with state variables corresponding to aerobic bacteria , anaerobic bacteria and oxygen concentration . Each variable is a function of time and location, where we will consider locations in both one- and two-spatial domains. Local dynamics for the bacterial communities are governed by logistic growth with oxygen-dependent growth rates, background death rates for each community, and oxygen toxicity for the anaerobic community (1-4). We incorporate spatial dynamics into the model by adding diffusion terms for each variable.

We use Michaelis-Menten kinetics to model the oxygen-dependent growth rates of the two communities, wherein each community’s growth is determined by a maximum growth rate and half-saturation concentration , and slope factor (5, 6). We assume that the aerobic community grows faster as the oxygen concentration increases and take as the aerobic growth function. The anaerobic community should grow slower as oxygen increases, hence we take the anaerobic growth function as ).

The full model can be written as

This system can be nondimensionalized by introducing the scaled quantities , , , , and and scaled parameters , , , , , and (7, 8). Dropping the \*’s, the scaled system is

where and represent the derivate and Laplacian with respect to the scaled time and space variables, respectively.

**Results**

**Critical domain**

**Analytical**

**1D**

**2D**

**Traveling wave solution**

A traveling wave solution is a function that satisfies a PDE while maintaining its shape in time (9-11).

**Analytical**

**1D**

**Discussion**

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