Section 12.4 Solutions

12.4.1 $\vec{a} = \langle 2,3,0 \rangle$, $\vec{b} = \langle 1,0,5 \rangle$

Find $\vec{a} \times \vec{b}$:

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & 3 & 0 \\ 1 & 0 & 5 \end{vmatrix}$$

$$= (3\cdot5 - 0\cdot0)\vec{i} - (2\cdot5 - 1\cdot0)\vec{j} + (2\cdot0 - 3\cdot1)\vec{k}$$

$$= \boxed{15\vec{i} - 10\vec{j} - 3\vec{k}}$$

12.4.4 $\vec{a} = \langle 3, 3, -3 \rangle$, $\vec{b} = \langle 3, -3, 3 \rangle$

find $\vec{a} \times \vec{b}$:

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 3 & 3 & -3 \\ 3 & -3 & 3 \end{vmatrix}$$

$$= (3\cdot 3 - (-3)\cdot(-3))\vec{i} - (3\cdot 3 - (-3)\cdot 3)\vec{j} + (3\cdot(-3) - 3\cdot 3)\vec{k}$$

$$= (9-9)\vec{i} - (9+9)\vec{j} + (-9-9)\vec{k}$$

$$= \boxed{0\vec{i} - 18\vec{j} - 18\vec{k}}$$

12.4.20 $\vec{u} = \vec{j} - \vec{k}$, $\vec{v} = \vec{i} + \vec{j}$.

find two vectors perpendicular to both $\vec{u}$ and $\vec{v}$.

just find $\vec{u} \times \vec{v}$ and $\vec{v} \times \vec{u} = -(\vec{u} \times \vec{v})$

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= (1\cdot 0 - (-1)\cdot 1)\vec{i} - (0\cdot 0 - (-1)\cdot 1)\vec{j} + (0\cdot 1 - 1\cdot 1)\vec{k}$$

$$= \vec{i} - \vec{j} - \vec{k} = \langle 1, -1, -1 \rangle$$

two vectors: $\boxed{\langle 1,-1,-1\rangle, \langle -1,1,1\rangle}$

$\boxed{12.4.30}$ $P(0,0,-3)$, $Q(4,2,0)$, $R(3,3,1)$

Let $\vec{u} = \overrightarrow{PQ} = \langle 4, 2, 3\rangle$

$\vec{v} = \overrightarrow{PR} = \langle 3,3,4\rangle$

Then, find $\vec{u} \times \vec{v}$:

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 4 & 2 & 3 \\ 3 & 3 & 4 \end{vmatrix}$$

$$= (2\cdot 4 - 3\cdot 3)\hat{i} - (4\cdot 4 - 3\cdot 3)\hat{j} + (4\cdot 3 - 3\cdot 2)\vec{k}$$

$$= (8-9)\hat{i} - (16-9)\hat{j} + (12-6)\vec{k}$$

$$= \boxed{-\hat{i} - 7\hat{j} + 6\vec{k}}$$