Homework - 01

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Instructions: Completed assignments are due on Monday, October 30. They are to be uploaded to the Virtual Campus.

Solutions can be submitted by groups of two or three students. In this case only one copy of each group's work should be uploaded (by any member), clearly stating the names of all contributors.

Format: Exercises 2 and 3 should be submitted either as a pair (text document + code) or as a Jupyter notebook. Exercise 1 just as a document or notebook. Documents should be .pdf files *[never as* `.doc/.docx/.odt`*]*. Code as ASCII `.r` text files. When more than one file has to be entered, prepare a single compressed file.

Exercise 1. *Consider the following experiment:*

1. *We toss n fair coins. We know the number X_1 of heads is a binomial r.v.* $\sim \mathrm{B}(n, \frac{1}{2})$.
2. *We toss again the remaining $R_1 = n - X_1$ coins and denote by X_2 the number of heads obtained this second time.*
3. *We toss again the remaining $R_2 = n - X_1 - X_2$ coins and denote by X_3 the number of heads obtained this second time.*
4. *The process continues in the same manner, stopping when all n coins show heads.*

Questions:

1. *Find the pmf of X_2 conditional to a given value of X_1, the joint pmf of (X_1, X_2), the joint pmf of (X_1, X_2, R_2), the marginal pmf of X_2 and the marginal pmf of R_2.*
2. *For $k \geq 2$, generalize the above computations as far as you can. Meaning: some extensions are feasible, even easy, but others give unwieldy expressions as far as I know. Hence start with $k = 3$, observe what you get and follow any practicable way you find. Do not worry if none appears.*
3. *Define the r.v.*

   $$Y = \text{“Total number of tosses. ”}$$

   Obtain the cdf of Y.

Exercise 2. *Write code to generate random numbers following a* $\mathrm{Beta}(3, 1)$ *distribution, (a) using the inverse transformation and (b) the accept-reject method. With both (a) and (b) generators write and apply code to demonstrate graphically the goodness-of-fit of the output to the target distribution. Comment about the efficiency of both methods.*

Exercise 3. *See the Notebook in the Binomial model chapter.*