# Indexed Containers

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**Abstract**

Blah

## 1 Introduction

## 2 Background

### 2.1 Type Theory

```
_⇾̇_ : {I : Set} → (A B : I → Set) → Set
_⇾̇_ {I} A B = (i : I) → A i → B i
```

### 2.2 Containers

## 3 Indexed Functors

```
record IFunc (I : Set) : Set where
  field
    obj : (A : I → Set) → Set
    mor : ∀ {A B} → (A ⇾̇ B) → obj A → obj B
```

```
η^F : ∀ {I} → I → IFunc I
η^F i = record {obj = λ A → A i; mor = λ f → f i}
_ ≫=^F _ : ∀ {I J} → (I → IFunc J) → IFunc I → IFunc J
H ≫=^F F =
  record {obj = λ A → F (λ i → (H i) A)
         ; mor = λ f → F (λ i → (H i) f )}
```

```
IFunc* : (I J : Set) → Set
IFunc* I J = J → IFunc I
obj* : ∀ {I J} → IFunc* I J → (I → Set) → J → Set
obj* F A j = (F j) A
```

```
mor⋆ : ∀ {I J A B} (F : IFunc⋆ I J) → A ∴→ B → obj⋆ F A ∴→ obj⋆ F B
mor⋆ F m j = (F j) m


Σ^F : ∀ {I J K} → (J → K) → IFunc⋆ I J → IFunc⋆ I K
Σ^F {J = J} f F k =
  record {obj = λ A → Σ J λ j → f j ≡ k × F A j
          ; mor = λ m → < π₀ , < π₀ ∘ π₁ , F m _ ∘ π₁ ∘ π₁ > >}
Π^F : ∀ {I J K} → (J → K) → IFunc⋆ I J → IFunc⋆ I K
Π^F {J = J} f F k =
  record {obj = λ A → (j : J) → f j ≡ k → F A j
          ; mor = λ m f j p → F m j (f j p)}


_[_]^F : ∀ {I J} → IFunc (I ⊎ J) → IFunc⋆ I J → IFunc I
F [ G ]^F =
  record {obj = λ A → F [A , G A]
          ; mor = λ f → F [f , G f ]}
```

# 4 Indexed containers

# 5 Initial Algebras of Indexed Containers

# 6 Strictly Positive Types

# 7 Conclusions