# SLAM Homework 3 - Calibration

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## 1. Task1

Define the projection matrix $P = K[R| - RC] = [KR| - KRC] = [M| - MC]$, where $M$ is a 3x3 matrix

$$M = KR = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$$

According to the procedure, because the $K$ is an upper triangular matrix. Given an $R_x$, indicate $c = -\frac{m_{33}}{\sqrt{m_{32}^2+m_{33}^2}}, s = \frac{m_{32}}{\sqrt{m_{32}^2+m_{33}^2}}$

$$MR_x = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & -s \\ 0 & s & c \end{bmatrix} = \begin{bmatrix} m_{11}^{'} & m_{12}^{'} & m_{13}^{'} \\ m_{21}^{'} & m_{22}^{'} & m_{23}^{'} \\ m_{31}^{'} & m_{32}^{'} & m_{33}^{'} \end{bmatrix}$$

$$MR_xR_y = \begin{bmatrix} m_{11}^{'} & m_{12}^{'} & m_{13}^{'} \\ m_{21}^{'} & m_{22}^{'} & m_{23}^{'} \\ m_{31}^{'} & m_{32}^{'} & m_{33}^{'} \end{bmatrix} * \begin{bmatrix} c^{'} & 0 & s^{'} \\ 0 & 1 & 0 \\ -s^{'} & 0 & c^{'} \end{bmatrix} = \begin{bmatrix} m_{11}^{''} & m_{12}^{''} & m_{13}^{''} \\ m_{21}^{''} & m_{22}^{''} & m_{23}^{''} \\ m_{31}^{''} & m_{32}^{''} & m_{33}^{''} \end{bmatrix}$$

$$MR_xR_yR_z = \begin{bmatrix} m_{11}^{''} & m_{12}^{''} & m_{13}^{''} \\ m_{21}^{''} & m_{22}^{''} & m_{23}^{''} \\ m_{31}^{''} & m_{32}^{''} & m_{33}^{''} \end{bmatrix} * \begin{bmatrix} c^{''} & -s^{''} & 0 \\ s^{''} & c^{''} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{matrix} m_{31}^{''} = m_{31}^{'} * c^{'} - m_{33}^{'} * s' = 0 \\ c^{'2} + s^{'2} = 1 \end{matrix} \implies \begin{matrix} c^{'} = \frac{m_{33}^{'}}{\sqrt{m_{31}^{'2}+m_{33}^{'2}}} = \frac{\sqrt{m_{32}^2+m_{33}^2}}{\sqrt{m_{31}^2+m_{32}^2+m_{33}^2}} \\ s^{'} = \frac{m_{31}^{'}}{\sqrt{m_{31}^{'2}+m_{33}^{'2}}} = \frac{m_{31}}{\sqrt{m_{31}^2+m_{32}^2+m_{33}^2}} \end{matrix}$$

$$\begin{matrix} m_{21}^{''} * c^{''} + m_{22}^{''} * s'' = 0 \\ c^{''2} + s^{''2} = 1 \end{matrix} \implies \begin{matrix} c^{''} = \frac{m_{22}^{''}}{\sqrt{m_{21}^{''2}+m_{22}^{''2}}} = \frac{cm_{22}+sm_{23}}{\sqrt{(c^{'}m_{21}+ss^{'}m_{22}-cs^{'}m_{23})^2+(cm_{22}+sm_{23})^2}} \\ s^{''} = \frac{-m_{21}^{''}}{\sqrt{m_{21}^{''2}+m_{22}^{''2}}} = -\frac{c^{'}m_{21}+ss^{'}m_{22}-cs^{'}m_{23}}{\sqrt{(c^{'}m_{21}+ss^{'}m_{22}-cs^{'}m_{23})^2+(cm_{22}+sm_{23})^2}} \end{matrix}$$

## 2. Usage

1. Dependency:
   (a) Eigen (for computation)
2. Compile:
   Create a directory named **build** under the root. Then run **cmake .. -DCMAKE_BUILD_TYPE=Release**
3. Run: run ./calibration

## 3. Task2

In this task, given 3 pictures, we first create a world frame at the corner as shown in the picture. And choose the 10 corners in the picture as our observations. Then the calibration process is shown as below

Figure 1: coordinate system and features

1. By calculating the 3D coordinates and observing the 2D coordinates, we create correspondence between 3D and 2D.
2. Normalize the 2d and 3d points.
3. Using DLT algorithm to compute the projection matrix.
4. Denormalize the projection matrix.
5. Decompose the projection matrix using RQ-decomposition. Then we can get the intrinsic matrix and the absolute pose.

The result is shown as below

Table 1: result table

| | projection matrix | intrinsic matrix | absolute pose |
|---|---|---|---|
| img1 | $\begin{bmatrix} -83.38 & 818.0 & 215.4 & 219.08 \\ -12.16 & -26.06 & 754.3 & 204.8 \\ 0.1666 & 0.121 & 0.0991 & 0.2477 \end{bmatrix}$ | $\begin{bmatrix} 3107.64 & 2.2404 & 2038.58 \\ 0 & 3020.75 & 1330 \\ 0 & 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} -0.5951 & -0.3385 & 0.7288 & -1.090 \\ 0.8033 & -0.2711 & 0.5301 & -0.3000 \\ 0.01816 & 0.9010 & 0.4333 & -0.2995 \\ -0 & 0 & -0 & 1 \end{bmatrix}$ |
| img2 | $\begin{bmatrix} 471.9 & 911.4 & 225.5 & 18.47 \\ 45.46 & 5.510 & 955.1 & 417.8 \\ 0.2617 & 0.0184 & 0.108 & 0.4039 \end{bmatrix}$ | $\begin{bmatrix} 3086.36 & 24.2891 & 2044.14 \\ 0 & 3048.86 & 1432.19 \\ 0 & 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} -0.0691 & -0.3806 & 0.9221 & -1.44 \\ 0.9976 & -0.02408 & 0.0648 & 0.8209 \\ -0.00246 & 0.9244 & 0.3813 & -0.3733 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |
| img3 | $\begin{bmatrix} -675.3 & -266.7 & -143.8 & -346.5 \\ -42.11 & 32.43 & -661.96 & -413.6 \\ -0.1506 & 0.1125 & -0.06373 & -0.5049 \end{bmatrix}$ | $\begin{bmatrix} -3111.58 & 65.7217 & 2051.18 \\ 0 & 3071.12 & 1323.25 \\ 0 & 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 0.5982 & 0.2578 & -0.7586 & -1.374 \\ 0.8013 & -0.1910 & 0.5669 & 2.407 \\ 0.00119 & -0.9471 & -0.3209 & -0.4194 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |

## 4. Task3

In this section, we use the 2D correspondence to generate essential matrix and decompose it to get a relative pose between each two frames. Then compare it with the relative pose we generate using the absolute pose in task 2.

The algorithm is shown as below

1. In each two frames, we match the points we selected in 2D image.
2. Using the intrinsic matrix we get from the last task, we transform the 2D points into the normalized 3D plane.
3. Using 8-points algorithm, we can generate the essential matrix, between the two frames, which describe the relative motion of the two frames.
4. Decompose the essential matrix, we can get the relative rotation and translation between the two frames.

The result is shown as below

Table 2: result table

| | relative_pose_from_calibration | relative_pose_from_essential | error(rx, ry, rz, t) |
|---|---|---|---|
| img1 | $\begin{bmatrix} 0.8425 & -0.2493 & 0.4774 & -1.143 \\ 0.2239 & 0.9683 & 0.1103 & -0.0404 \\ -0.4898 & 0.01393 & 0.8717 & 0.2839 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 0.9577 & 0.0832 & -0.275 & 0.156 \\ -0.051 & 0.991 & 0.121 & 0.243 \\ 0.2831 & -0.102 & 0.953 & -0.957 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 0.7682 \\ -2.893 \\ 0.3066 \\ 0.001171 \end{bmatrix}$ |
| img2 | $\begin{bmatrix} 0.7580 & -0.2459 & 0.6040 & -1.314 \\ -0.2061 & -0.9690 & -0.1358 & 0.2409 \\ 0.6187 & -0.02153 & -0.7852 & -0.8599 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 0.98 & -0.0092 & 0.170 & -0.978 \\ 0.00016 & 0.998 & 0.0532 & 0.102 \\ -0.170 & -0.0524 & 0.983 & -0.180 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 2.891 \\ -2.645 \\ 0.07195 \\ 0.6716 \end{bmatrix}$ |
| img3 | $\begin{bmatrix} 0.2877 & -0.3241 & 0.9011 & 2.34 \\ -0.4186 & -0.8888 & -0.1860 & -0.7459 \\ 0.8613 & -0.3237 & -0.3914 & 1.176 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 0.8454 & 0.01359 & 0.5339 & 0.9325 \\ 0.09941 & 0.9782 & -0.1823 & -0.2531 \\ -0.5247 & 0.2072 & 0.8256 & 0.2573 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 2.485 \\ -2.856 \\ 0.1313 \\ 1.643 \end{bmatrix}$ |

The error is large. The reason can be the unmatched correspondence.