

## Supplementary data for

### *Determinants of Mahler measures and special values of L-functions*

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In this data file, we compile a complete list of  $t \in \mathbb{T}_{P,4}$  or  $\mathbb{T}_{Q,4}$ , together with the associated formulas that express  $\mu(t)$  or  $\nu(t)$  as  $L$ -values of cusp forms. We use the following labeling system:

- the 3 **rational**  $t \in \mathbb{T}_{P,4}$  are labeled from [μ-R-1](#) to [μ-R-3](#);
- the 12 pairs of **real quadratic**  $t \in \mathbb{T}_{P,4}$  are labeled from [μ-R2-1-x](#) to [μ-R2-12-x](#);
- the 4 pairs of **imaginary quadratic**  $t \in \mathbb{T}_{P,4}$  are labeled from [μ-I2-1-x](#) to [μ-I2-4-x](#);
- the 24 quartets of **totally real quartic**  $t \in \mathbb{T}_{P,4}$  are labeled from [μ-T4-1-x](#) to [μ-T4-24-x](#);
- the 24 quartets of **non-totally real quartic**  $t \in \mathbb{T}_{P,4}$  are labeled from [μ-N4-1-x](#) to [μ-N4-24-x](#);
- the 3 **rational**  $t \in \mathbb{T}_{Q,4}$  are labeled from [ν-R-1](#) to [ν-R-3](#);
- the 15 pairs of **real quadratic**  $t \in \mathbb{T}_{Q,4}$  are labeled from [ν-R2-1-x](#) to [ν-R2-15-x](#);
- the 2 pairs of **imaginary quadratic**  $t \in \mathbb{T}_{Q,4}$  are labeled from [ν-I2-1-x](#) to [ν-I2-2-x](#);
- the 4 trios of **(non-totally real) cubic**  $t \in \mathbb{T}_{Q,4}$  are labeled from [ν-C-1-x](#) to [ν-C-4-x](#);
- the 24 quartets of **totally real quartic**  $t \in \mathbb{T}_{Q,4}$  are labeled from [ν-T4-1-x](#) to [ν-T4-24-x](#);
- the 25 quartets of **non-totally real quartic**  $t \in \mathbb{T}_{Q,4}$  are labeled from [ν-N4-1-x](#) to [ν-N4-25-x](#).

#### [μ-R-1](#)

Let  $\tau_0 = [2, 2, 1] = -\frac{1}{2} + \frac{i}{2}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = -16$$

has minimal polynomial  $T + 16$ .

We have  $\mu(t) = \frac{4}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (4m + 2n) q^{8m^2 + 4mn + n^2} \\ &= 4q - 8q^5 - 12q^9 + 24q^{13} + 8q^{17} - 4q^{25} - 40q^{29} - 8q^{37} + 40q^{41} + 24q^{45} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(32), (\frac{64}{\cdot}))$ .

#### [μ-R-2](#)

Let  $\tau_0 = [8, 4, 1] = -\frac{1}{4} + \frac{i}{4}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 8$$

has minimal polynomial  $T - 8$ .

We have  $\mu(t) = \frac{1}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (8m + 8n) q^{2m^2 + 2mn + n^2} \\ &= 16q - 32q^5 - 48q^9 + 96q^{13} + 32q^{17} - 16q^{25} - 160q^{29} - 32q^{37} + 160q^{41} + 96q^{45} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(32), (\frac{16}{\cdot}))$ .

#### [μ-R-3](#)

Let  $\tau_0 = [4, 0, 1] = \frac{i}{2}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 32$$

has minimal polynomial  $T - 32$ .

We have  $\mu(t) = \frac{2}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 4n q^{4m^2 + n^2} \\ &= 8q + 16q^5 - 24q^9 - 48q^{13} + 16q^{17} - 8q^{25} + 80q^{29} + 16q^{37} + 80q^{41} - 48q^{45} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(64), (\frac{64}{\cdot}))$ .

#### [μ-R2-1-1 \(#1 in the paper\)](#)

Let  $\tau_0 = [16, 0, 1] = \frac{i}{4}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 8 + 6\sqrt{2}$$

has minimal polynomial  $T^2 - 16T - 8$ .

We have  $\mu(t) = \frac{1}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 16n q^{m^2 + n^2} \\ &= 32q + 64q^2 + 64q^5 - 96q^9 - 128q^{10} - 192q^{13} + 64q^{17} - 192q^{18} - 32q^{25} + 384q^{26} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(64), (\frac{16}{\cdot}))$ .

#### [μ-R2-1-2 \(#1 in the paper\)](#)

Let  $\tau_0 = [16, 16, 5] = -\frac{1}{2} + \frac{i}{4}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 8 - 6\sqrt{2}$$

has minimal polynomial  $T^2 - 16T - 8$ .

We have  $\mu(t) = \frac{1}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(32m+16n)q^{5m^2+4mn+n^2} \\ = 32q - 64q^2 + 64q^5 - 96q^9 + 128q^{10} - 192q^{13} + 64q^{17} + 192q^{18} - 32q^{25} - 384q^{26} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(64), (\frac{16}{\cdot}))$ .

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**$\mu$ -R2-2-1 (#2 in the paper)**

Let  $\tau_0 = [8, 0, 1] = \frac{i}{2\sqrt{2}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 8 + 8\sqrt{2}$$

has minimal polynomial  $T^2 - 16T - 64$ .

We have  $\mu(t) = \frac{1}{\sqrt{2}\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)8nq^{2m^2+n^2} \\ = 16q + 32q^3 - 16q^9 - 96q^{11} - 96q^{17} + 32q^{19} + 80q^{25} + 64q^{27} + 192q^{33} - 96q^{41} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(64), (\frac{32}{\cdot}))$ .

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**$\mu$ -R2-2-2 (#2 in the paper)**

Let  $\tau_0 = [8, 8, 3] = -\frac{1}{2} + \frac{i}{2\sqrt{2}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 8 - 8\sqrt{2}$$

has minimal polynomial  $T^2 - 16T - 64$ .

We have  $\mu(t) = \frac{1}{\sqrt{2}\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(16m+8n)q^{6m^2+4mn+n^2} \\ = 16q - 32q^3 - 16q^9 + 96q^{11} - 96q^{17} - 32q^{19} + 80q^{25} - 64q^{27} + 192q^{33} - 96q^{41} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(64), (\frac{32}{\cdot}))$ .

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**$\mu$ -R2-3-1 (#3 in the paper)**

Let  $\tau_0 = [12, 4, 1] = -\frac{1}{6} + \frac{i}{3\sqrt{2}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = -32 + 32\sqrt{2}$$

has minimal polynomial  $T^2 + 64T - 1024$ .

We have  $\mu(t) = \frac{2\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m+12n)q^{4m^2+4mn+3n^2} \\ = 32q^3 + 32q^{11} - 96q^{19} - 64q^{27} - 96q^{43} + 192q^{51} + 160q^{59} - 96q^{67} + 160q^{75} + 32q^{83} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(128), (\frac{128}{\cdot}))$ .

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**$\mu$ -R2-3-2 (#3 in the paper)**

Let  $\tau_0 = [4, -4, 3] = \frac{1}{2} + \frac{i}{\sqrt{2}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = -32 - 32\sqrt{2}$$

has minimal polynomial  $T^2 + 64T - 1024$ .

We have  $\mu(t) = \frac{2\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-8m+4n)q^{12m^2-4mn+n^2} \\ = 8q - 40q^9 + 48q^{17} + 40q^{25} - 64q^{33} - 48q^{41} - 56q^{49} + 192q^{57} - 16q^{73} + 8q^{81} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(128), (\frac{128}{\cdot}))$ .

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**$\mu$ -R2-4-1 (#4 in the paper)**

Let  $\tau_0 = [2, 0, 1] = \frac{i}{\sqrt{2}}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 48 + 32\sqrt{2}$$

has minimal polynomial  $T^2 - 96T + 256$ .

We have  $\mu(t) = \frac{4\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)2nq^{8m^2+n^2} \\ = 4q - 4q^9 - 24q^{17} + 20q^{25} + 48q^{33} - 24q^{41} - 28q^{49} - 16q^{57} + 8q^{73} - 44q^{81} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(64), (\frac{128}{\cdot}))$ .

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**$\mu$ -R2-4-2 (#4 in the paper)**

Let  $\tau_0 = [6, -4, 1] = \frac{1}{3} + \frac{i}{3\sqrt{2}}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 48 - 32\sqrt{2}$$

has minimal polynomial  $T^2 - 96T + 256$ .

We have  $\mu(t) = \frac{4\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-8m+6n)q^{8m^2-8mn+3n^2} \\ = 8q^3 - 24q^{11} + 8q^{19} + 16q^{27} + 40q^{43} - 48q^{51} - 24q^{59} - 56q^{67} + 40q^{75} + 72q^{83} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(64), (\frac{128}{\cdot}))$ .

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**$\mu$ -R2-5-1 (#5 in the paper)**

Let  $\tau_0 = [20, 4, 1] = -\frac{1}{10} + \frac{i}{5}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = -256 + 192\sqrt{2}$$

has minimal polynomial  $T^2 + 512T - 8192$ .

We have  $\mu(t) = \frac{4}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m+20n)q^{4m^2+4mn+5n^2} \\ = 64q^5 + 64q^{13} + 64q^{29} - 192q^{37} - 192q^{45} + 64q^{53} - 192q^{61} - 128q^{85} + 320q^{101} + 320q^{109} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(256), (\frac{256}{\cdot}))$ .

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**$\mu$ -R2-5-2 (#5 in the paper)**

Let  $\tau_0 = [4, -4, 5] = \frac{1}{2} + i$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = -256 - 192\sqrt{2}$$

has minimal polynomial  $T^2 + 512T - 8192$ .

We have  $\mu(t) = \frac{4}{\pi^2} L(\Theta_{P, \tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-4}(n) (-8m + 4n) q^{20m^2 - 4mn + n^2} \\ &= 8q - 24q^9 - 16q^{17} + 88q^{25} - 80q^{41} - 56q^{49} + 128q^{65} - 48q^{73} + 72q^{81} + 80q^{89} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(256), (\frac{256}{\cdot}))$ .

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#### [μ-R2-6-1 \(#6 in the paper\)](#)

Let  $\tau_0 = [1, 0, 1] = i$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 272 + 192\sqrt{2}$$

has minimal polynomial  $T^2 - 544T + 256$ .

We have  $\mu(t) = \frac{16}{\pi^2} L(\Theta_{P, \tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-4}(n) n q^{16m^2 + n^2} \\ &= 2q - 6q^9 + 4q^{17} - 2q^{25} + 20q^{41} - 14q^{49} - 24q^{65} - 12q^{73} + 18q^{81} + 20q^{89} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(64), (\frac{256}{\cdot}))$ .

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#### [μ-R2-6-2 \(#6 in the paper\)](#)

Let  $\tau_0 = [5, 4, 1] = -\frac{2}{5} + \frac{i}{5}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 272 - 192\sqrt{2}$$

has minimal polynomial  $T^2 - 544T + 256$ .

We have  $\mu(t) = \frac{16}{\pi^2} L(\Theta_{P, \tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-4}(n) (8m + 5n) q^{16m^2 + 16mn + 5n^2} \\ &= 4q^5 - 12q^{13} + 20q^{29} + 4q^{37} - 12q^{45} - 28q^{53} + 20q^{61} + 8q^{85} + 4q^{101} - 12q^{109} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(64), (\frac{256}{\cdot}))$ .

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#### [μ-R2-7-1 \(#7 in the paper\)](#)

Let  $\tau_0 = [16, 4, 1] = -\frac{1}{8} + \frac{i\sqrt{3}}{8}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 8 + 4\sqrt{3}$$

has minimal polynomial  $T^2 - 16T + 16$ .

We have  $\mu(t) = \frac{\sqrt{3}}{8\pi^2} L(\Theta_{P, \tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-4}(n) (8m + 16n) q^{m^2 + mn + n^2} \\ &= 48q + 48q^3 - 96q^7 - 144q^9 - 96q^{13} + 96q^{19} + 288q^{21} + 240q^{25} - 144q^{27} + 288q^{31} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(48), (\frac{12}{\cdot}))$ .

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#### [μ-R2-7-2 \(#7 in the paper\)](#)

Let  $\tau_0 = [16, 12, 3] = -\frac{3}{8} + \frac{i\sqrt{3}}{8}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 8 - 4\sqrt{3}$$

has minimal polynomial  $T^2 - 16T + 16$ .

We have  $\mu(t) = \frac{\sqrt{3}}{8\pi^2} L(\Theta_{P, \tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-4}(n) (24m + 16n) q^{3m^2 + 3mn + n^2} \\ &= 16q - 48q^3 + 96q^7 - 48q^9 - 32q^{13} - 96q^{19} + 96q^{21} + 80q^{25} + 144q^{27} - 288q^{31} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(48), (\frac{12}{\cdot}))$ .

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#### [μ-R2-8-1 \(#8 in the paper\)](#)

Let  $\tau_0 = [3, 3, 1] = -\frac{1}{2} + \frac{i}{2\sqrt{3}}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = -112 + 64\sqrt{3}$$

has minimal polynomial  $T^2 + 224T + 256$ .

We have  $\mu(t) = \frac{8\sqrt{3}}{\pi^2} L(\Theta_{P, \tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-4}(n) (6m + 3n) q^{16m^2 + 12mn + 3n^2} \\ &= 6q^3 - 12q^7 + 12q^{19} - 18q^{27} + 36q^{31} - 12q^{39} - 36q^{43} + 36q^{63} + 12q^{67} + 30q^{75} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(48), (\frac{192}{\cdot}))$ .

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#### [μ-R2-8-2 \(#8 in the paper\)](#)

Let  $\tau_0 = [1, -1, 1] = \frac{1}{2} + \frac{i\sqrt{3}}{2}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = -112 - 64\sqrt{3}$$

has minimal polynomial  $T^2 + 224T + 256$ .

We have  $\mu(t) = \frac{8\sqrt{3}}{\pi^2} L(\Theta_{P, \tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-4}(n) (-2m + n) q^{16m^2 - 4mn + n^2} \\ &= 2q - 6q^9 - 4q^{13} + 12q^{21} + 10q^{25} - 20q^{37} - 10q^{49} - 12q^{57} + 28q^{61} + 20q^{73} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(48), (\frac{192}{\cdot}))$ .

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#### [μ-R2-9-1 \(#9 in the paper\)](#)

Let  $\tau_0 = [4, 0, 3] = \frac{i\sqrt{3}}{2}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 128 + 64\sqrt{3}$$

has minimal polynomial  $T^2 - 256T + 4096$ .

We have  $\mu(t) = \frac{2\sqrt{3}}{\pi^2} L(\Theta_{P, \tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-4}(n) 4n q^{12m^2 + n^2} \\ &= 8q - 24q^9 + 16q^{13} - 48q^{21} + 40q^{25} + 80q^{37} - 40q^{49} - 48q^{57} - 112q^{61} + 80q^{73} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(192), (\frac{192}{\cdot}))$ .

[μ-R2-9-2 \(#9 in the paper\)](#)

Let  $\tau_0 = [12, 0, 1] = \frac{i}{2\sqrt{3}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 128 - 64\sqrt{3}$$

has minimal polynomial  $T^2 - 256T + 4096$ .

We have  $\mu(t) = \frac{2\sqrt{3}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 12n q^{4m^2+3n^2} \\ &= 24q^3 + 48q^7 + 48q^{19} - 72q^{27} - 144q^{31} + 48q^{39} - 144q^{43} - 144q^{63} + 48q^{67} + 120q^{75} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(192), (\frac{192}{\cdot}))$ .

[μ-R2-10-1 \(#10 in the paper\)](#)

Let  $\tau_0 = [32, 4, 1] = -\frac{1}{16} + \frac{i\sqrt{7}}{16}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 8 + 3\sqrt{7}$$

has minimal polynomial  $T^2 - 16T + 1$ .

We have  $\mu(t) = \frac{\sqrt{7}}{8\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (8m + 32n) q^{m^2+mn+2n^2} \\ &= 112q^2 + 112q^4 + 112q^8 + 112q^{14} - 336q^{16} - 336q^{18} - 224q^{22} - 336q^{28} + 112q^{32} - 336q^{36} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(112), (\frac{28}{\cdot}))$ .

[μ-R2-10-2 \(#10 in the paper\)](#)

Let  $\tau_0 = [32, 28, 7] = -\frac{7}{16} + \frac{i\sqrt{7}}{16}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 8 - 3\sqrt{7}$$

has minimal polynomial  $T^2 - 16T + 1$ .

We have  $\mu(t) = \frac{\sqrt{7}}{8\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (56m + 32n) q^{7m^2+7mn+2n^2} \\ &= 16q^2 - 48q^4 + 80q^8 - 112q^{14} + 16q^{16} - 48q^{18} + 224q^{22} - 112q^{28} - 176q^{32} + 144q^{36} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(112), (\frac{28}{\cdot}))$ .

[μ-R2-11-1 \(#11 in the paper\)](#)

Let  $\tau_0 = [7, 7, 2] = -\frac{1}{2} + \frac{i}{2\sqrt{7}}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = -2032 + 768\sqrt{7}$$

has minimal polynomial  $T^2 + 4064T + 256$ .

We have  $\mu(t) = \frac{8\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (14m + 7n) q^{32m^2+28mn+7n^2} \\ &= 14q^7 - 28q^{11} + 28q^{23} - 28q^{43} - 42q^{63} + 84q^{67} + 28q^{71} - 84q^{79} + 84q^{99} - 28q^{107} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(112), (\frac{448}{\cdot}))$ .

[μ-R2-11-2 \(#11 in the paper\)](#)

Let  $\tau_0 = [1, -1, 2] = \frac{1}{2} + \frac{i\sqrt{7}}{2}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = -2032 - 768\sqrt{7}$$

has minimal polynomial  $T^2 + 4064T + 256$ .

We have  $\mu(t) = \frac{8\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-2m + n) q^{32m^2-4mn+n^2} \\ &= 2q - 6q^9 + 10q^{25} - 4q^{29} + 12q^{37} - 14q^{49} - 20q^{53} + 28q^{77} + 18q^{81} - 36q^{109} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(112), (\frac{448}{\cdot}))$ .

[μ-R2-12-1 \(#12 in the paper\)](#)

Let  $\tau_0 = [4, 0, 7] = \frac{i\sqrt{7}}{2}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 2048 + 768\sqrt{7}$$

has minimal polynomial  $T^2 - 4096T + 65536$ .

We have  $\mu(t) = \frac{2\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 4n q^{28m^2+n^2} \\ &= 8q - 24q^9 + 40q^{25} + 16q^{29} - 48q^{37} - 56q^{49} + 80q^{53} - 112q^{77} + 72q^{81} + 144q^{109} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(448), (\frac{448}{\cdot}))$ .

[μ-R2-12-2 \(#12 in the paper\)](#)

Let  $\tau_0 = [28, 0, 1] = \frac{i}{2\sqrt{7}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 2048 - 768\sqrt{7}$$

has minimal polynomial  $T^2 - 4096T + 65536$ .

We have  $\mu(t) = \frac{2\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 28n q^{4m^2+7n^2} \\ &= 56q^7 + 112q^{11} + 112q^{23} + 112q^{43} - 168q^{63} - 336q^{67} + 112q^{71} - 336q^{79} - 336q^{99} + 112q^{107} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(448), (\frac{448}{\cdot}))$ .

[μ-I2-1-1](#)

Let  $\tau_0 = [4, 2, 1] = -\frac{1}{4} + \frac{i\sqrt{3}}{4}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 8 + 8i\sqrt{3}$$

has minimal polynomial  $T^2 - 16T + 256$ .

We have  $\mu(t) = \frac{\sqrt{3}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(4m+4n)q^{4m^2+2mn+n^2} \\ &= 8q - 24q^9 - 16q^{13} + 48q^{21} + 40q^{25} - 80q^{37} - 40q^{49} - 48q^{57} + 112q^{61} + 80q^{73} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(48), \left(\frac{48}{\cdot}\right))$ .

---

#### $\mu$ -I2-1-2

Let  $\tau_0 = [4, -2, 1] = \frac{1}{4} + \frac{i\sqrt{3}}{4}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 8 - 8i\sqrt{3}$$

has minimal polynomial  $T^2 - 16T + 256$ .

We have  $\mu(t) = \frac{\sqrt{3}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-4m+4n)q^{4m^2-2mn+n^2} \\ &= 8q - 24q^9 - 16q^{13} + 48q^{21} + 40q^{25} - 80q^{37} - 40q^{49} - 48q^{57} + 112q^{61} + 80q^{73} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(48), \left(\frac{48}{\cdot}\right))$ .

---

#### $\mu$ -I2-2-1

Let  $\tau_0 = [2, 1, 1] = -\frac{1}{4} + \frac{i\sqrt{7}}{4}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 8 + 24i\sqrt{7}$$

has minimal polynomial  $T^2 - 16T + 4096$ .

We have  $\mu(t) = \frac{2\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(2m+2n)q^{8m^2+2mn+n^2} \\ &= 4q - 12q^9 + 20q^{25} - 8q^{29} + 24q^{37} - 28q^{49} - 40q^{53} + 56q^{77} + 36q^{81} - 72q^{109} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(56), \left(\frac{112}{\cdot}\right))$ .

---

#### $\mu$ -I2-2-2

Let  $\tau_0 = [2, -1, 1] = \frac{1}{4} + \frac{i\sqrt{7}}{4}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 8 - 24i\sqrt{7}$$

has minimal polynomial  $T^2 - 16T + 4096$ .

We have  $\mu(t) = \frac{2\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-2m+2n)q^{8m^2-2mn+n^2} \\ &= 4q - 12q^9 + 20q^{25} - 8q^{29} + 24q^{37} - 28q^{49} - 40q^{53} + 56q^{77} + 36q^{81} - 72q^{109} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(56), \left(\frac{112}{\cdot}\right))$ .

---

#### $\mu$ -I2-3-1

Let  $\tau_0 = [4, 3, 1] = -\frac{3}{8} + \frac{i\sqrt{7}}{8}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = \frac{1}{2} + \frac{3i\sqrt{7}}{2}$$

has minimal polynomial  $T^2 - T + 16$ .

We have  $\mu(t) = \frac{\sqrt{7}}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(6m+4n)q^{4m^2+3mn+n^2} \\ &= 8q - 4q^2 - 12q^4 + 20q^8 - 24q^9 + 28q^{14} + 4q^{16} + 12q^{18} - 56q^{22} + 40q^{25} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(28), \left(\frac{28}{\cdot}\right))$ .

---

#### $\mu$ -I2-3-2

Let  $\tau_0 = [4, -3, 1] = \frac{3}{8} + \frac{i\sqrt{7}}{8}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = \frac{1}{2} - \frac{3i\sqrt{7}}{2}$$

has minimal polynomial  $T^2 - T + 16$ .

We have  $\mu(t) = \frac{\sqrt{7}}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-6m+4n)q^{4m^2-3mn+n^2} \\ &= 8q - 4q^2 - 12q^4 + 20q^8 - 24q^9 + 28q^{14} + 4q^{16} + 12q^{18} - 56q^{22} + 40q^{25} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(28), \left(\frac{28}{\cdot}\right))$ .

---

#### $\mu$ -I2-4-1

Let  $\tau_0 = [8, 2, 1] = -\frac{1}{8} + \frac{i\sqrt{7}}{8}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = \frac{31}{2} + \frac{3i\sqrt{7}}{2}$$

has minimal polynomial  $T^2 - 31T + 256$ .

We have  $\mu(t) = \frac{\sqrt{7}}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(4m+8n)q^{2m^2+mn+n^2} \\ &= 16q + 8q^2 + 24q^4 - 40q^8 - 48q^9 - 56q^{14} - 8q^{16} - 24q^{18} + 112q^{22} + 80q^{25} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(56), \left(\frac{28}{\cdot}\right))$ .

---

#### $\mu$ -I2-4-2

Let  $\tau_0 = [8, -2, 1] = \frac{1}{8} + \frac{i\sqrt{7}}{8}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = \frac{31}{2} - \frac{3i\sqrt{7}}{2}$$

has minimal polynomial  $T^2 - 31T + 256$ .

We have  $\mu(t) = \frac{\sqrt{7}}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-4m+8n)q^{2m^2-mn+n^2} \\ &= 16q + 8q^2 + 24q^4 - 40q^8 - 48q^9 - 56q^{14} - 8q^{16} - 24q^{18} + 112q^{22} + 80q^{25} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(56), \left(\frac{28}{\cdot}\right))$ .

---

[μ-T4-1-1 \(#28 in the paper\)](#)

Let  $\tau_0 = [16, 16, 7] = -\frac{1}{2} + \frac{i\sqrt{3}}{4}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - 3\sqrt{2} - 5\sqrt{6}$$

has minimal polynomial  $T^4 - 32T^3 + 48T^2 + 3328T + 16$ .

We have  $\mu(t) = \frac{\sqrt{3}}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(32m + 16n)q^{7m^2 + 4mn + n^2} \\ = 32q - 64q^4 - 96q^9 + 192q^{12} + 64q^{13} - 192q^{21} + 160q^{25} - 384q^{28} + 192q^{36} + 320q^{37} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(192), (\frac{48}{\cdot}))$ .

---

[μ-T4-1-2 \(#28 in the paper\)](#)

Let  $\tau_0 = [48, 48, 13] = -\frac{1}{2} + \frac{i}{4\sqrt{3}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + 3\sqrt{2} - 5\sqrt{6}$$

has minimal polynomial  $T^4 - 32T^3 + 48T^2 + 3328T + 16$ .

We have  $\mu(t) = \frac{\sqrt{3}}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(96m + 48n)q^{13m^2 + 12mn + 3n^2} \\ = 96q^3 - 192q^4 + 192q^7 - 192q^{12} + 192q^{19} - 288q^{27} + 384q^{28} - 576q^{31} + 576q^{36} + 192q^{39} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(192), (\frac{48}{\cdot}))$ .

---

[μ-T4-1-3 \(#28 in the paper\)](#)

Let  $\tau_0 = [48, 0, 1] = \frac{i}{4\sqrt{3}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - 3\sqrt{2} + 5\sqrt{6}$$

has minimal polynomial  $T^4 - 32T^3 + 48T^2 + 3328T + 16$ .

We have  $\mu(t) = \frac{\sqrt{3}}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)48nq^{m^2 + 3n^2} \\ = 96q^3 + 192q^4 + 192q^7 + 192q^{12} + 192q^{19} - 288q^{27} - 384q^{28} - 576q^{31} - 576q^{36} + 192q^{39} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(192), (\frac{48}{\cdot}))$ .

---

[μ-T4-1-4 \(#28 in the paper\)](#)

Let  $\tau_0 = [16, 0, 3] = \frac{i\sqrt{3}}{4}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + 3\sqrt{2} + 5\sqrt{6}$$

has minimal polynomial  $T^4 - 32T^3 + 48T^2 + 3328T + 16$ .

We have  $\mu(t) = \frac{\sqrt{3}}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)16nq^{3m^2 + n^2} \\ = 32q + 64q^4 - 96q^9 - 192q^{12} + 64q^{13} - 192q^{21} + 160q^{25} + 384q^{28} - 192q^{36} + 320q^{37} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(192), (\frac{48}{\cdot}))$ .

---

[μ-T4-2-1 \(#29 in the paper\)](#)

Let  $\tau_0 = [8, -8, 5] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{3}{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - 16\sqrt{3} - 8\sqrt{6}$$

has minimal polynomial  $T^4 - 32T^3 - 1920T^2 + 34816T + 4096$ .

We have  $\mu(t) = \frac{\sqrt{\frac{3}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-16m + 8n)q^{10m^2 - 4mn + n^2} \\ = 16q - 32q^7 - 48q^9 + 96q^{15} + 112q^{25} - 160q^{31} - 96q^{33} + 48q^{49} + 192q^{55} + 96q^{63} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(192), (\frac{96}{\cdot}))$ .

---

[μ-T4-2-2 \(#29 in the paper\)](#)

Let  $\tau_0 = [24, 24, 7] = -\frac{1}{2} + \frac{i}{2\sqrt{6}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - 16\sqrt{3} + 8\sqrt{6}$$

has minimal polynomial  $T^4 - 32T^3 - 1920T^2 + 34816T + 4096$ .

We have  $\mu(t) = \frac{\sqrt{\frac{3}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(48m + 24n)q^{14m^2 + 12mn + 3n^2} \\ = 48q^3 - 96q^5 + 96q^{11} - 96q^{21} - 144q^{27} + 288q^{29} - 192q^{35} + 288q^{45} - 96q^{53} - 288q^{59} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(192), (\frac{96}{\cdot}))$ .

---

[μ-T4-2-3 \(#29 in the paper\)](#)

Let  $\tau_0 = [24, 0, 1] = \frac{i}{2\sqrt{6}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + 16\sqrt{3} - 8\sqrt{6}$$

has minimal polynomial  $T^4 - 32T^3 - 1920T^2 + 34816T + 4096$ .

We have  $\mu(t) = \frac{\sqrt{\frac{3}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)24nq^{2m^2 + 3n^2} \\ = 48q^3 + 96q^5 + 96q^{11} + 96q^{21} - 144q^{27} - 288q^{29} - 192q^{35} - 288q^{45} + 96q^{53} - 288q^{59} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(192), (\frac{96}{\cdot}))$ .

---

[μ-T4-2-4 \(#29 in the paper\)](#)

Let  $\tau_0 = [8, 0, 3] = \frac{1}{2}i\sqrt{\frac{3}{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + 16\sqrt{3} + 8\sqrt{6}$$

has minimal polynomial  $T^4 - 32T^3 - 1920T^2 + 34816T + 4096$ .

We have  $\mu(t) = \frac{\sqrt{\frac{3}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 8nq^{6m^2+n^2} \\ = 16q + 32q^7 - 48q^9 - 96q^{15} + 112q^{25} + 160q^{31} - 96q^{33} + 48q^{49} - 192q^{55} - 96q^{63} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(192), \left(\frac{96}{\cdot}\right))$ .

#### [μ-T4-3-1 \(#30 in the paper\)](#)

Let  $\tau_0 = [8, -8, 11] = \frac{1}{2} + \frac{3i}{2\sqrt{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - 280\sqrt{2} - 224\sqrt{3}$$

has minimal polynomial  $T^4 - 32T^3 - 614272T^2 + 9832448T + 4096$ .

We have  $\mu(t) = \frac{3}{\sqrt{2}\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-16m + 8n) q^{22m^2 - 4mn + n^2} \\ = 16q - 48q^9 - 32q^{19} + 80q^{25} + 96q^{27} - 160q^{43} - 112q^{49} + 224q^{67} + 32q^{73} + 48q^{81} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(576), \left(\frac{288}{\cdot}\right))$ .

#### [μ-T4-3-2 \(#30 in the paper\)](#)

Let  $\tau_0 = [72, 72, 19] = -\frac{1}{2} + \frac{i}{6\sqrt{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - 280\sqrt{2} + 224\sqrt{3}$$

has minimal polynomial  $T^4 - 32T^3 - 614272T^2 + 9832448T + 4096$ .

We have  $\mu(t) = \frac{3}{\sqrt{2}\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (144m + 72n) q^{38m^2 + 36mn + 9n^2} \\ = 144q^9 - 288q^{11} + 288q^{17} - 288q^{27} + 288q^{41} - 288q^{59} - 144q^{81} + 864q^{83} - 864q^{89} + 864q^{99} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(576), \left(\frac{288}{\cdot}\right))$ .

#### [μ-T4-3-3 \(#30 in the paper\)](#)

Let  $\tau_0 = [72, 0, 1] = \frac{i}{6\sqrt{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + 280\sqrt{2} - 224\sqrt{3}$$

has minimal polynomial  $T^4 - 32T^3 - 614272T^2 + 9832448T + 4096$ .

We have  $\mu(t) = \frac{3}{\sqrt{2}\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 72nq^{2m^2 + 9n^2} \\ = 144q^9 + 288q^{11} + 288q^{17} + 288q^{27} + 288q^{41} + 288q^{59} - 144q^{81} - 864q^{83} - 864q^{89} - 864q^{99} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(576), \left(\frac{288}{\cdot}\right))$ .

#### [μ-T4-3-4 \(#30 in the paper\)](#)

Let  $\tau_0 = [8, 0, 9] = \frac{3i}{2\sqrt{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + 280\sqrt{2} + 224\sqrt{3}$$

has minimal polynomial  $T^4 - 32T^3 - 614272T^2 + 9832448T + 4096$ .

We have  $\mu(t) = \frac{3}{\sqrt{2}\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 8nq^{18m^2 + n^2} \\ = 16q - 48q^9 + 32q^{19} + 80q^{25} - 96q^{27} + 160q^{43} - 112q^{49} - 224q^{67} + 32q^{73} + 48q^{81} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(576), \left(\frac{288}{\cdot}\right))$ .

#### [μ-T4-4-1 \(#31 in the paper\)](#)

Let  $\tau_0 = [4, -4, 7] = \frac{1}{2} + i\sqrt{\frac{3}{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -544 - 384\sqrt{2} - 320\sqrt{3} - 224\sqrt{6}$$

has minimal polynomial  $T^4 + 2176T^3 - 30720T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{6}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-8m + 4n) q^{28m^2 - 4mn + n^2} \\ = 8q - 24q^9 + 24q^{25} + 48q^{33} - 136q^{49} + 112q^{73} + 72q^{81} + 16q^{97} - 192q^{105} - 8q^{121} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(384), \left(\frac{384}{\cdot}\right))$ .

#### [μ-T4-4-2 \(#31 in the paper\)](#)

Let  $\tau_0 = [12, 12, 5] = -\frac{1}{2} + \frac{i}{\sqrt{6}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -544 + 384\sqrt{2} - 320\sqrt{3} + 224\sqrt{6}$$

has minimal polynomial  $T^4 + 2176T^3 - 30720T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{6}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (24m + 12n) q^{20m^2 + 12mn + 3n^2} \\ = 24q^3 - 48q^{11} - 72q^{27} + 192q^{35} - 144q^{59} + 72q^{75} - 240q^{83} + 144q^{99} + 240q^{107} + 48q^{131} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(384), \left(\frac{384}{\cdot}\right))$ .

#### [μ-T4-4-3 \(#31 in the paper\)](#)

Let  $\tau_0 = [20, 12, 3] = -\frac{3}{10} + \frac{1}{5}i\sqrt{\frac{3}{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -544 + 384\sqrt{2} + 320\sqrt{3} - 224\sqrt{6}$$

has minimal polynomial  $T^4 + 2176T^3 - 30720T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{6}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (24m + 20n) q^{12m^2 + 12mn + 5n^2} \\ = 32q^5 - 96q^{21} + 32q^{29} - 96q^{45} + 160q^{53} + 192q^{77} - 96q^{93} - 224q^{101} - 64q^{125} + 32q^{149} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(384), \left(\frac{384}{\cdot}\right))$ .

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[μ-T4-4-4 \(#31 in the paper\)](#)

Let  $\tau_0 = [28, 4, 1] = -\frac{1}{14} + \frac{1}{7}i\sqrt{\frac{3}{2}}$  with  $h(D) = 4$ . Then

$t = t_P(\tau_0) = -544 - 384\sqrt{2} + 320\sqrt{3} + 224\sqrt{6}$   
has minimal polynomial  $T^4 + 2176T^3 - 30720T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{6}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 28n)q^{4m^2 + 4mn + 7n^2} \\ &= 96q^7 + 96q^{15} + 96q^{31} - 192q^{55} - 288q^{63} - 288q^{79} + 96q^{87} - 288q^{103} + 96q^{127} - 288q^{135} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(384), \left(\frac{384}{\cdot}\right))$ .

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[μ-T4-5-1 \(#32 in the paper\)](#)

Let  $\tau_0 = [14, 12, 3] = -\frac{3}{7} + \frac{1}{7}i\sqrt{\frac{3}{2}}$  with  $h(D) = 2$ . Then

$t = t_P(\tau_0) = 560 + 384\sqrt{2} - 320\sqrt{3} - 224\sqrt{6}$   
has minimal polynomial  $T^4 - 2240T^3 + 75264T^2 - 573440T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{6}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(24m + 14n)q^{24m^2 + 24mn + 7n^2} \\ &= 8q^7 - 24q^{15} + 40q^{31} - 48q^{55} - 24q^{63} + 40q^{79} + 72q^{87} - 56q^{103} - 88q^{127} + 72q^{135} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(192), \left(\frac{384}{\cdot}\right))$ .

---

[μ-T4-5-2 \(#32 in the paper\)](#)

Let  $\tau_0 = [10, 4, 1] = -\frac{1}{5} + \frac{1}{5}i\sqrt{\frac{3}{2}}$  with  $h(D) = 2$ . Then

$t = t_P(\tau_0) = 560 - 384\sqrt{2} - 320\sqrt{3} + 224\sqrt{6}$   
has minimal polynomial  $T^4 - 2240T^3 + 75264T^2 - 573440T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{6}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 10n)q^{8m^2 + 8mn + 5n^2} \\ &= 24q^5 + 24q^{21} - 72q^{29} - 72q^{45} + 24q^{53} + 48q^{77} + 120q^{93} + 24q^{101} + 48q^{125} - 168q^{149} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(192), \left(\frac{384}{\cdot}\right))$ .

---

[μ-T4-5-3 \(#32 in the paper\)](#)

Let  $\tau_0 = [6, 0, 1] = \frac{i}{\sqrt{6}}$  with  $h(D) = 2$ . Then

$t = t_P(\tau_0) = 560 - 384\sqrt{2} + 320\sqrt{3} - 224\sqrt{6}$   
has minimal polynomial  $T^4 - 2240T^3 + 75264T^2 - 573440T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{6}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)6nq^{8m^2 + 3n^2} \\ &= 12q^3 + 24q^{11} - 36q^{27} - 48q^{35} - 72q^{59} + 84q^{75} + 120q^{83} - 72q^{99} + 120q^{107} + 24q^{131} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(192), \left(\frac{384}{\cdot}\right))$ .

---

[μ-T4-5-4 \(#32 in the paper\)](#)

Let  $\tau_0 = [2, 0, 3] = i\sqrt{\frac{3}{2}}$  with  $h(D) = 2$ . Then

$t = t_P(\tau_0) = 560 + 384\sqrt{2} + 320\sqrt{3} + 224\sqrt{6}$   
has minimal polynomial  $T^4 - 2240T^3 + 75264T^2 - 573440T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{6}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)2nq^{24m^2 + n^2} \\ &= 4q - 12q^9 + 28q^{25} - 24q^{33} + 12q^{49} - 56q^{73} + 36q^{81} + 8q^{97} + 48q^{105} - 4q^{121} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(192), \left(\frac{384}{\cdot}\right))$ .

---

[μ-T4-6-1 \(#33 in the paper\)](#)

Let  $\tau_0 = [4, -4, 13] = \frac{1}{2} + i\sqrt{3}$  with  $h(D) = 4$ . Then

$t = t_P(\tau_0) = -13312 - 9408\sqrt{2} - 7680\sqrt{3} - 5440\sqrt{6}$   
has minimal polynomial  $T^4 + 53248T^3 + 196608T^2 - 33554432T + 268435456$ .

We have  $\mu(t) = \frac{4\sqrt{3}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-8m + 4n)q^{52m^2 - 4mn + n^2} \\ &= 8q - 24q^9 + 40q^{25} - 72q^{49} + 48q^{57} - 80q^{73} + 72q^{81} + 112q^{97} - 88q^{121} - 144q^{129} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(768), \left(\frac{768}{\cdot}\right))$ .

---

[μ-T4-6-2 \(#33 in the paper\)](#)

Let  $\tau_0 = [12, -12, 7] = \frac{1}{2} + \frac{i}{\sqrt{3}}$  with  $h(D) = 4$ . Then

$t = t_P(\tau_0) = -13312 + 9408\sqrt{2} + 7680\sqrt{3} - 5440\sqrt{6}$   
has minimal polynomial  $T^4 + 53248T^3 + 196608T^2 - 33554432T + 268435456$ .

We have  $\mu(t) = \frac{4\sqrt{3}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-24m + 12n)q^{28m^2 - 12mn + 3n^2} \\ &= 24q^3 - 48q^{19} - 72q^{27} + 144q^{43} + 48q^{67} + 120q^{75} - 384q^{91} + 240q^{139} - 216q^{147} + 336q^{163} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(768), \left(\frac{768}{\cdot}\right))$ .

---

[μ-T4-6-3 \(#33 in the paper\)](#)

Let  $\tau_0 = [28, 12, 3] = -\frac{3}{14} + \frac{i\sqrt{3}}{7}$  with  $h(D) = 4$ . Then

$t = t_P(\tau_0) = -13312 - 9408\sqrt{2} + 7680\sqrt{3} + 5440\sqrt{6}$



has minimal polynomial  $T^4 + 53248T^3 + 196608T^2 - 33554432T + 268435456$ .

We have  $\mu(t) = \frac{4\sqrt{3}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(24m + 28n)q^{12m^2 + 12mn + 7n^2} \\ &= 64q^7 + 64q^{31} - 192q^{39} - 192q^{63} + 64q^{79} + 320q^{103} - 192q^{111} + 320q^{127} + 64q^{151} + 320q^{175} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(768), (\frac{768}{\cdot}))$ .

---

#### [μ-T4-6-4 \(#33 in the paper\)](#)

Let  $\tau_0 = [52, 4, 1] = -\frac{1}{26} + \frac{i\sqrt{3}}{13}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -13312 + 9408\sqrt{2} - 7680\sqrt{3} + 5440\sqrt{6}$$

has minimal polynomial  $T^4 + 53248T^3 + 196608T^2 - 33554432T + 268435456$ .

We have  $\mu(t) = \frac{4\sqrt{3}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 52n)q^{4m^2 + 4mn + 13n^2} \\ &= 192q^{13} + 192q^{21} + 192q^{37} + 192q^{61} + 192q^{93} - 576q^{109} - 576q^{117} - 384q^{133} - 576q^{157} + 192q^{181} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(768), (\frac{768}{\cdot}))$ .

---

#### [μ-T4-7-1 \(#34 in the paper\)](#)

Let  $\tau_0 = [13, 12, 3] = -\frac{6}{13} + \frac{i\sqrt{3}}{13}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 13328 - 9408\sqrt{2} + 7680\sqrt{3} - 5440\sqrt{6}$$

has minimal polynomial  $T^4 - 53312T^3 + 2754048T^2 - 13647872T + 65536$ .

We have  $\mu(t) = \frac{16\sqrt{3}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(24m + 13n)q^{48m^2 + 48mn + 13n^2} \\ &= 4q^{13} - 12q^{21} + 20q^{37} - 28q^{61} + 36q^{93} + 4q^{109} - 12q^{117} - 24q^{133} - 28q^{157} + 52q^{181} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(192), (\frac{768}{\cdot}))$ .

---

#### [μ-T4-7-2 \(#34 in the paper\)](#)

Let  $\tau_0 = [7, 4, 1] = -\frac{2}{7} + \frac{i\sqrt{3}}{7}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 13328 - 9408\sqrt{2} - 7680\sqrt{3} - 5440\sqrt{6}$$

has minimal polynomial  $T^4 - 53312T^3 + 2754048T^2 - 13647872T + 65536$ .

We have  $\mu(t) = \frac{16\sqrt{3}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 7n)q^{16m^2 + 16mn + 7n^2} \\ &= 12q^7 - 36q^{31} + 12q^{39} - 36q^{63} + 60q^{79} + 12q^{103} + 60q^{111} - 36q^{127} - 84q^{151} + 60q^{175} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(192), (\frac{768}{\cdot}))$ .

---

#### [μ-T4-7-3 \(#34 in the paper\)](#)

Let  $\tau_0 = [3, 0, 1] = \frac{i}{\sqrt{3}}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 13328 - 9408\sqrt{2} - 7680\sqrt{3} + 5440\sqrt{6}$$

has minimal polynomial  $T^4 - 53312T^3 + 2754048T^2 - 13647872T + 65536$ .

We have  $\mu(t) = \frac{16\sqrt{3}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)3nq^{16m^2 + 3n^2} \\ &= 6q^3 + 12q^{19} - 18q^{27} - 36q^{43} + 12q^{67} + 30q^{75} + 24q^{91} + 60q^{139} - 30q^{147} - 84q^{163} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(192), (\frac{768}{\cdot}))$ .

---

#### [μ-T4-7-4 \(#34 in the paper\)](#)

Let  $\tau_0 = [1, 0, 3] = i\sqrt{3}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 13328 + 9408\sqrt{2} + 7680\sqrt{3} + 5440\sqrt{6}$$

has minimal polynomial  $T^4 - 53312T^3 + 2754048T^2 - 13647872T + 65536$ .

We have  $\mu(t) = \frac{16\sqrt{3}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)nq^{48m^2 + n^2} \\ &= 2q - 6q^9 + 10q^{25} - 10q^{49} - 12q^{57} + 20q^{73} + 18q^{81} - 28q^{97} - 22q^{121} + 36q^{129} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(192), (\frac{768}{\cdot}))$ .

---

#### [μ-T4-8-1 \(#35 in the paper\)](#)

Let  $\tau_0 = [4, -4, 19] = \frac{1}{2} + \frac{3i}{\sqrt{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -153632 - 108640\sqrt{2} - 88704\sqrt{3} - 62720\sqrt{6}$$

has minimal polynomial  $T^4 + 614528T^3 - 9828352T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{6\sqrt{2}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-8m + 4n)q^{76m^2 - 4mn + n^2} \\ &= 8q - 24q^9 + 40q^{25} - 56q^{49} - 16q^{73} + 120q^{81} - 80q^{97} + 24q^{121} - 144q^{153} + 104q^{169} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1152), (\frac{1152}{\cdot}))$ .

---

#### [μ-T4-8-2 \(#35 in the paper\)](#)

Let  $\tau_0 = [36, 36, 11] = -\frac{1}{2} + \frac{i}{3\sqrt{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -153632 - 108640\sqrt{2} + 88704\sqrt{3} + 62720\sqrt{6}$$

has minimal polynomial  $T^4 + 614528T^3 - 9828352T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{6\sqrt{2}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(72m + 36n)q^{44m^2 + 36mn + 9n^2} \\ &= 72q^9 - 144q^{17} + 144q^{41} - 360q^{81} + 432q^{89} - 432q^{113} + 144q^{137} + 432q^{153} - 576q^{209} + 360q^{225} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1152), (\frac{1152}{\cdot}))$ .

---

 **$\mu$ -T4-8-3 (#35 in the paper)**

Let  $\tau_0 = [44, 36, 9] = -\frac{9}{22} + \frac{3i}{11\sqrt{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -153632 + 108640\sqrt{2} - 88704\sqrt{3} + 62720\sqrt{6}$$

has minimal polynomial  $T^4 + 614528T^3 - 9828352T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{6\sqrt{2}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(72m + 44n)q^{36m^2 + 36mn + 11n^2} \\ = 32q^{11} - 96q^{27} + 160q^{59} + 32q^{83} - 96q^{99} - 224q^{107} + 160q^{131} + 288q^{171} - 224q^{179} + 32q^{227} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1152), (\frac{1152}{\cdot}))$ .

---

 **$\mu$ -T4-8-4 (#35 in the paper)**

Let  $\tau_0 = [76, 4, 1] = -\frac{1}{38} + \frac{3i}{19\sqrt{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -153632 + 108640\sqrt{2} + 88704\sqrt{3} - 62720\sqrt{6}$$

has minimal polynomial  $T^4 + 614528T^3 - 9828352T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{6\sqrt{2}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 76n)q^{4m^2 + 4mn + 19n^2} \\ = 288q^{19} + 288q^{27} + 288q^{43} + 288q^{67} + 288q^{99} + 288q^{139} - 864q^{163} - 864q^{171} - 576q^{187} - 864q^{211} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1152), (\frac{1152}{\cdot}))$ .

---

 **$\mu$ -T4-9-1 (#36 in the paper)**

Let  $\tau_0 = [38, 36, 9] = -\frac{9}{19} + \frac{3i}{19\sqrt{2}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 153648 - 108640\sqrt{2} - 88704\sqrt{3} + 62720\sqrt{6}$$

has minimal polynomial  $T^4 - 614592T^3 + 19670528T^2 - 157335552T + 65536$ .

We have  $\mu(t) = \frac{12\sqrt{2}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(72m + 38n)q^{72m^2 + 72mn + 19n^2} \\ = 8q^{19} - 24q^{27} + 40q^{43} - 56q^{67} + 72q^{99} - 88q^{139} + 8q^{163} - 24q^{171} + 144q^{187} - 56q^{211} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(576), (\frac{1152}{\cdot}))$ .

---

 **$\mu$ -T4-9-2 (#36 in the paper)**

Let  $\tau_0 = [22, 4, 1] = -\frac{1}{11} + \frac{3i}{11\sqrt{2}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 153648 - 108640\sqrt{2} + 88704\sqrt{3} - 62720\sqrt{6}$$

has minimal polynomial  $T^4 - 614592T^3 + 19670528T^2 - 157335552T + 65536$ .

We have  $\mu(t) = \frac{12\sqrt{2}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 22n)q^{8m^2 + 8mn + 11n^2} \\ = 72q^{11} + 72q^{27} + 72q^{59} - 216q^{83} - 216q^{99} + 72q^{107} - 216q^{131} + 72q^{171} - 216q^{179} + 360q^{227} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(576), (\frac{1152}{\cdot}))$ .

---

 **$\mu$ -T4-9-3 (#36 in the paper)**

Let  $\tau_0 = [18, 0, 1] = \frac{i}{3\sqrt{2}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 153648 + 108640\sqrt{2} - 88704\sqrt{3} - 62720\sqrt{6}$$

has minimal polynomial  $T^4 - 614592T^3 + 19670528T^2 - 157335552T + 65536$ .

We have  $\mu(t) = \frac{12\sqrt{2}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)18nq^{8m^2 + 9n^2} \\ = 36q^9 + 72q^{17} + 72q^{41} - 36q^{81} - 216q^{89} - 216q^{113} + 72q^{137} - 216q^{153} - 144q^{209} + 180q^{225} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(576), (\frac{1152}{\cdot}))$ .

---

 **$\mu$ -T4-9-4 (#36 in the paper)**

Let  $\tau_0 = [2, 0, 9] = \frac{3i}{\sqrt{2}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 153648 + 108640\sqrt{2} + 88704\sqrt{3} + 62720\sqrt{6}$$

has minimal polynomial  $T^4 - 614592T^3 + 19670528T^2 - 157335552T + 65536$ .

We have  $\mu(t) = \frac{12\sqrt{2}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)2nq^{72m^2 + n^2} \\ = 4q - 12q^9 + 20q^{25} - 28q^{49} + 8q^{73} + 12q^{81} + 40q^{97} - 100q^{121} + 72q^{153} + 52q^{169} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(576), (\frac{1152}{\cdot}))$ .

---

 **$\mu$ -T4-10-1 (#37 in the paper)**

Let  $\tau_0 = [8, -8, 7] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{5}{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - 48\sqrt{2} - 24\sqrt{10}$$

has minimal polynomial  $T^4 - 32T^3 - 20352T^2 + 329728T + 4096$ .

We have  $\mu(t) = \frac{\sqrt{5}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-16m + 8n)q^{14m^2 - 4mn + n^2} \\ = 16q - 48q^9 - 32q^{11} + 96q^{19} + 80q^{25} - 160q^{35} + 32q^{41} - 208q^{49} + 224q^{59} + 160q^{65} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(320), (\frac{160}{\cdot}))$ .

---

 **$\mu$ -T4-10-2 (#37 in the paper)**

Let  $\tau_0 = [40, 40, 11] = -\frac{1}{2} + \frac{i}{2\sqrt{10}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + 48\sqrt{2} - 24\sqrt{10}$$

has minimal polynomial  $T^4 - 32T^3 - 20352T^2 + 329728T + 4096$ .

We have  $\mu(t) = \frac{\sqrt{\frac{5}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(80m+40n)q^{22m^2+20mn+5n^2} \\ &= 80q^5 - 160q^7 + 160q^{13} - 160q^{23} + 160q^{37} - 240q^{45} + 480q^{47} - 480q^{53} - 160q^{55} + 480q^{63} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(320), (\frac{160}{\cdot}))$ .

---

#### [μ-T4-10-3 \(#37 in the paper\)](#)

Let  $\tau_0 = [40, 0, 1] = \frac{i}{2\sqrt{10}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - 48\sqrt{2} + 24\sqrt{10}$$

has minimal polynomial  $T^4 - 32T^3 - 20352T^2 + 329728T + 4096$ .

We have  $\mu(t) = \frac{\sqrt{\frac{5}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)40nq^{2m^2+5n^2} \\ &= 80q^5 + 160q^7 + 160q^{13} + 160q^{23} + 160q^{37} - 240q^{45} - 480q^{47} - 480q^{53} + 160q^{55} - 480q^{63} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(320), (\frac{160}{\cdot}))$ .

---

#### [μ-T4-10-4 \(#37 in the paper\)](#)

Let  $\tau_0 = [8, 0, 5] = \frac{1}{2}i\sqrt{\frac{5}{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + 48\sqrt{2} + 24\sqrt{10}$$

has minimal polynomial  $T^4 - 32T^3 - 20352T^2 + 329728T + 4096$ .

We have  $\mu(t) = \frac{\sqrt{\frac{5}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)8nq^{10m^2+n^2} \\ &= 16q - 48q^9 + 32q^{11} - 96q^{19} + 80q^{25} + 160q^{35} + 32q^{41} - 208q^{49} - 224q^{59} + 160q^{65} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(320), (\frac{160}{\cdot}))$ .

---

#### [μ-T4-11-1 \(#38 in the paper\)](#)

Let  $\tau_0 = [4, -4, 11] = \frac{1}{2} + i\sqrt{\frac{5}{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -5152 - 3648\sqrt{2} - 2304\sqrt{5} - 1632\sqrt{10}$$

has minimal polynomial  $T^4 + 20608T^3 - 325632T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{10}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-8m+4n)q^{44m^2-4mn+n^2} \\ &= 8q - 24q^9 + 40q^{25} - 16q^{41} - 8q^{49} - 80q^{65} + 72q^{81} + 112q^{89} - 232q^{121} + 192q^{161} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(640), (\frac{640}{\cdot}))$ .

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#### [μ-T4-11-2 \(#38 in the paper\)](#)

Let  $\tau_0 = [20, 20, 7] = -\frac{1}{2} + \frac{i}{\sqrt{10}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -5152 + 3648\sqrt{2} + 2304\sqrt{5} - 1632\sqrt{10}$$

has minimal polynomial  $T^4 + 20608T^3 - 325632T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{10}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(40m+20n)q^{28m^2+20mn+5n^2} \\ &= 40q^5 - 80q^{13} + 80q^{37} - 120q^{45} + 240q^{53} - 320q^{77} + 240q^{117} + 200q^{125} - 320q^{133} + 400q^{157} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(640), (\frac{640}{\cdot}))$ .

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#### [μ-T4-11-3 \(#38 in the paper\)](#)

Let  $\tau_0 = [28, 20, 5] = -\frac{5}{14} + \frac{1}{7}i\sqrt{\frac{5}{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -5152 - 3648\sqrt{2} + 2304\sqrt{5} + 1632\sqrt{10}$$

has minimal polynomial  $T^4 + 20608T^3 - 325632T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{10}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(40m+28n)q^{20m^2+20mn+7n^2} \\ &= 32q^7 - 96q^{23} + 32q^{47} + 160q^{55} - 96q^{63} + 160q^{95} - 224q^{103} + 32q^{127} - 320q^{143} + 288q^{167} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(640), (\frac{640}{\cdot}))$ .

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#### [μ-T4-11-4 \(#38 in the paper\)](#)

Let  $\tau_0 = [44, 4, 1] = -\frac{1}{22} + \frac{1}{11}i\sqrt{\frac{5}{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -5152 + 3648\sqrt{2} - 2304\sqrt{5} + 1632\sqrt{10}$$

has minimal polynomial  $T^4 + 20608T^3 - 325632T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{10}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m+44n)q^{4m^2+4mn+11n^2} \\ &= 160q^{11} + 160q^{19} + 160q^{35} + 160q^{59} - 320q^{91} - 480q^{99} - 480q^{115} + 160q^{131} - 480q^{139} - 480q^{171} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(640), (\frac{640}{\cdot}))$ .

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#### [μ-T4-12-1 \(#39 in the paper\)](#)

Let  $\tau_0 = [22, 20, 5] = -\frac{5}{11} + \frac{1}{11}i\sqrt{\frac{5}{2}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 5168 - 3648\sqrt{2} + 2304\sqrt{5} - 1632\sqrt{10}$$

has minimal polynomial  $T^4 - 20672T^3 + 665088T^2 - 5292032T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{10}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(40m+22n)q^{40m^2+40mn+11n^2}$$

$$= 8q^{11} - 24q^{19} + 40q^{35} - 56q^{59} + 80q^{91} - 24q^{99} + 40q^{115} - 88q^{131} - 56q^{139} + 72q^{171} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(320), \left(\frac{640}{\cdot}\right))$ .

[μ-T4-12-2 \(#39 in the paper\)](#)

Let  $\tau_0 = [14, 4, 1] = -\frac{1}{7} + \frac{1}{7}i\sqrt{\frac{5}{2}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 5168 + 3648\sqrt{2} - 2304\sqrt{5} - 1632\sqrt{10}$$

has minimal polynomial  $T^4 - 20672T^3 + 665088T^2 - 5292032T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{10}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 14n)q^{8m^2 + 8mn + 7n^2}$$

$$= 40q^7 + 40q^{23} - 120q^{47} + 40q^{55} - 120q^{63} - 120q^{95} + 40q^{103} + 200q^{127} + 80q^{143} + 40q^{167} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(320), \left(\frac{640}{\cdot}\right))$ .

[μ-T4-12-3 \(#39 in the paper\)](#)

Let  $\tau_0 = [10, 0, 1] = \frac{i}{\sqrt{10}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 5168 - 3648\sqrt{2} - 2304\sqrt{5} + 1632\sqrt{10}$$

has minimal polynomial  $T^4 - 20672T^3 + 665088T^2 - 5292032T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{10}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)10nq^{8m^2 + 5n^2}$$

$$= 20q^5 + 40q^{13} + 40q^{37} - 60q^{45} - 120q^{53} - 80q^{77} - 120q^{117} + 100q^{125} + 240q^{133} + 200q^{157} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(320), \left(\frac{640}{\cdot}\right))$ .

[μ-T4-12-4 \(#39 in the paper\)](#)

Let  $\tau_0 = [2, 0, 5] = i\sqrt{\frac{5}{2}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 5168 + 3648\sqrt{2} + 2304\sqrt{5} + 1632\sqrt{10}$$

has minimal polynomial  $T^4 - 20672T^3 + 665088T^2 - 5292032T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{10}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)2nq^{40m^2 + n^2}$$

$$= 4q - 12q^9 + 20q^{25} + 8q^{41} - 52q^{49} + 40q^{65} + 36q^{81} - 56q^{89} + 28q^{121} - 80q^{161} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(320), \left(\frac{640}{\cdot}\right))$ .

[μ-T4-13-1 \(#40 in the paper\)](#)

Let  $\tau_0 = [64, 60, 15] = -\frac{15}{32} + \frac{i\sqrt{15}}{32}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - \frac{7\sqrt{3}}{2} - \frac{\sqrt{15}}{2}$$

has minimal polynomial  $T^4 - 32T^3 + 303T^2 - 752T + 1$ .

We have  $\mu(t) = \frac{\sqrt{15}}{8\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(120m + 64n)q^{15m^2 + 15mn + 4n^2}$$

$$= 16q^4 - 48q^6 + 80q^{10} - 112q^{16} + 144q^{24} - 160q^{34} - 48q^{36} + 80q^{40} + 96q^{46} + 144q^{54} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(240), \left(\frac{60}{\cdot}\right))$ .

[μ-T4-13-2 \(#40 in the paper\)](#)

Let  $\tau_0 = [32, 20, 5] = -\frac{5}{16} + \frac{i\sqrt{15}}{16}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - \frac{7\sqrt{3}}{2} + \frac{\sqrt{15}}{2}$$

has minimal polynomial  $T^4 - 32T^3 + 303T^2 - 752T + 1$ .

We have  $\mu(t) = \frac{\sqrt{15}}{8\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(40m + 32n)q^{5m^2 + 5mn + 2n^2}$$

$$= 48q^2 - 144q^8 + 48q^{12} - 144q^{18} + 240q^{20} + 240q^{30} + 48q^{32} - 480q^{38} - 336q^{48} + 240q^{50} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(240), \left(\frac{60}{\cdot}\right))$ .

[μ-T4-13-3 \(#40 in the paper\)](#)

Let  $\tau_0 = [32, 12, 3] = -\frac{3}{16} + \frac{i\sqrt{15}}{16}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + \frac{7\sqrt{3}}{2} - \frac{\sqrt{15}}{2}$$

has minimal polynomial  $T^4 - 32T^3 + 303T^2 - 752T + 1$ .

We have  $\mu(t) = \frac{\sqrt{15}}{8\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(24m + 32n)q^{3m^2 + 3mn + 2n^2}$$

$$= 80q^2 + 80q^8 - 240q^{12} - 240q^{18} + 80q^{20} - 240q^{30} + 400q^{32} + 480q^{38} - 240q^{48} + 400q^{50} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(240), \left(\frac{60}{\cdot}\right))$ .

[μ-T4-13-4 \(#40 in the paper\)](#)

Let  $\tau_0 = [64, 4, 1] = -\frac{1}{32} + \frac{i\sqrt{15}}{32}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + \frac{7\sqrt{3}}{2} + \frac{\sqrt{15}}{2}$$

has minimal polynomial  $T^4 - 32T^3 + 303T^2 - 752T + 1$ .

We have  $\mu(t) = \frac{\sqrt{15}}{8\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 64n)q^{m^2 + mn + 4n^2}$$

$$= 240q^4 + 240q^6 + 240q^{10} + 240q^{16} + 240q^{24} - 480q^{34} - 720q^{36} - 720q^{40} - 480q^{46} - 720q^{54} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(240), \left(\frac{60}{\cdot}\right))$ .

[μ-T4-14-1 \(#41 in the paper\)](#)

Let  $\tau_0 = [60, 0, 1] = \frac{i}{2\sqrt{15}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 48128 - 27776\sqrt{3} + 21504\sqrt{5} - 12416\sqrt{15}$$

has minimal polynomial  $T^4 - 192512T^3 + 19857408T^2 - 536870912T + 4294967296$ .

We have  $\mu(t) = \frac{2\sqrt{15}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 60nq^{4m^2+15n^2} \\ &= 120q^{15} + 240q^{19} + 240q^{31} + 240q^{51} + 240q^{79} + 240q^{115} - 360q^{135} - 720q^{139} - 720q^{151} + 240q^{159} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(960), \left(\frac{960}{\cdot}\right))$ .

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#### $\mu$ -T4-14-2 (#41 in the paper)

Let  $\tau_0 = [20, 0, 3] = \frac{1}{2}i\sqrt{\frac{3}{5}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 48128 - 27776\sqrt{3} - 21504\sqrt{5} + 12416\sqrt{15}$$

has minimal polynomial  $T^4 - 192512T^3 + 19857408T^2 - 536870912T + 4294967296$ .

We have  $\mu(t) = \frac{2\sqrt{15}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 20nq^{12m^2+5n^2} \\ &= 40q^5 + 80q^{17} - 120q^{45} + 80q^{53} - 240q^{57} - 240q^{93} + 80q^{113} + 200q^{125} + 400q^{137} - 240q^{153} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(960), \left(\frac{960}{\cdot}\right))$ .

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#### $\mu$ -T4-14-3 (#41 in the paper)

Let  $\tau_0 = [12, 0, 5] = \frac{1}{2}i\sqrt{\frac{5}{3}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 48128 + 27776\sqrt{3} - 21504\sqrt{5} - 12416\sqrt{15}$$

has minimal polynomial  $T^4 - 192512T^3 + 19857408T^2 - 536870912T + 4294967296$ .

We have  $\mu(t) = \frac{2\sqrt{15}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 12nq^{20m^2+3n^2} \\ &= 24q^3 + 48q^{23} - 72q^{27} - 144q^{47} + 120q^{75} + 48q^{83} + 240q^{95} - 144q^{107} - 168q^{147} + 240q^{155} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(960), \left(\frac{960}{\cdot}\right))$ .

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#### $\mu$ -T4-14-4 (#41 in the paper)

Let  $\tau_0 = [4, 0, 15] = \frac{i\sqrt{15}}{2}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 48128 + 27776\sqrt{3} + 21504\sqrt{5} + 12416\sqrt{15}$$

has minimal polynomial  $T^4 - 192512T^3 + 19857408T^2 - 536870912T + 4294967296$ .

We have  $\mu(t) = \frac{2\sqrt{15}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 4nq^{60m^2+n^2} \\ &= 8q - 24q^9 + 40q^{25} - 56q^{49} + 16q^{61} - 48q^{69} + 72q^{81} + 80q^{85} - 112q^{109} - 88q^{121} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(960), \left(\frac{960}{\cdot}\right))$ .

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#### $\mu$ -T4-15-1 (#42 in the paper)

Let  $\tau_0 = [1, -1, 4] = \frac{1}{2} + \frac{i\sqrt{15}}{2}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = -48112 - 27776\sqrt{3} - 21504\sqrt{5} - 12416\sqrt{15}$$

has minimal polynomial  $T^4 + 192448T^3 + 10618368T^2 + 49266688T + 65536$ .

We have  $\mu(t) = \frac{8\sqrt{15}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-2m+n)q^{64m^2-4mn+n^2} \\ &= 2q - 6q^9 + 10q^{25} - 14q^{49} - 4q^{61} + 12q^{69} + 18q^{81} - 20q^{85} + 28q^{109} - 22q^{121} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(240), \left(\frac{960}{\cdot}\right))$ .

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#### $\mu$ -T4-15-2 (#42 in the paper)

Let  $\tau_0 = [3, -3, 2] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{5}{3}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = -48112 - 27776\sqrt{3} + 21504\sqrt{5} + 12416\sqrt{15}$$

has minimal polynomial  $T^4 + 192448T^3 + 10618368T^2 + 49266688T + 65536$ .

We have  $\mu(t) = \frac{8\sqrt{15}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-6m+3n)q^{32m^2-12mn+3n^2} \\ &= 6q^3 - 12q^{23} - 18q^{27} + 36q^{47} + 30q^{75} + 12q^{83} - 60q^{95} - 36q^{107} - 42q^{147} + 60q^{155} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(240), \left(\frac{960}{\cdot}\right))$ .

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#### $\mu$ -T4-15-3 (#42 in the paper)

Let  $\tau_0 = [5, 5, 2] = -\frac{1}{2} + \frac{1}{2}i\sqrt{\frac{3}{5}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = -48112 + 27776\sqrt{3} + 21504\sqrt{5} - 12416\sqrt{15}$$

has minimal polynomial  $T^4 + 192448T^3 + 10618368T^2 + 49266688T + 65536$ .

We have  $\mu(t) = \frac{8\sqrt{15}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(10m+5n)q^{32m^2+20mn+5n^2} \\ &= 10q^5 - 20q^{17} - 30q^{45} + 20q^{53} + 60q^{57} - 60q^{93} - 20q^{113} + 50q^{125} - 100q^{137} + 60q^{153} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(240), \left(\frac{960}{\cdot}\right))$ .

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#### $\mu$ -T4-15-4 (#42 in the paper)

Let  $\tau_0 = [15, 15, 4] = -\frac{1}{2} + \frac{i}{2\sqrt{15}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = -48112 + 27776\sqrt{3} - 21504\sqrt{5} + 12416\sqrt{15}$$

has minimal polynomial  $T^4 + 192448T^3 + 10618368T^2 + 49266688T + 65536$ .

We have  $\mu(t) = \frac{8\sqrt{15}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(30m+15n)q^{64m^2+60mn+15n^2} \\ &= 30q^{15} - 60q^{19} + 60q^{31} - 60q^{51} + 60q^{79} - 60q^{115} - 90q^{135} + 180q^{139} - 180q^{151} + 60q^{159} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(240), \left(\frac{960}{\cdot}\right))$ .

**$\mu$ -T4-16-1 (#43 in the paper)**

Let  $\tau_0 = [16, -16, 11] = \frac{1}{2} + \frac{i\sqrt{7}}{4}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - \frac{51}{\sqrt{2}} - 15\sqrt{\frac{7}{2}}$$

has minimal polynomial  $T^4 - 32T^3 - 3792T^2 + 64768T + 1$ .

We have  $\mu(t) = \frac{\sqrt{7}}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-32m+16n)q^{11m^2-4mn+n^2} \\ &= 32q - 64q^8 - 96q^9 + 192q^{16} + 160q^{25} + 64q^{29} - 320q^{32} - 192q^{37} - 224q^{49} + 320q^{53} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(448), \left(\frac{112}{\cdot}\right))$ .

**$\mu$ -T4-16-2 (#43 in the paper)**

Let  $\tau_0 = [112, 112, 29] = -\frac{1}{2} + \frac{i}{4\sqrt{7}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - \frac{51}{\sqrt{2}} + 15\sqrt{\frac{7}{2}}$$

has minimal polynomial  $T^4 - 32T^3 - 3792T^2 + 64768T + 1$ .

We have  $\mu(t) = \frac{\sqrt{7}}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(224m+112n)q^{29m^2+28mn+7n^2} \\ &= 224q^7 - 448q^8 + 448q^{11} - 448q^{16} + 448q^{23} - 448q^{32} + 448q^{43} - 448q^{56} - 672q^{63} + 1344q^{64} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(448), \left(\frac{112}{\cdot}\right))$ .

**$\mu$ -T4-16-3 (#43 in the paper)**

Let  $\tau_0 = [112, 0, 1] = \frac{i}{4\sqrt{7}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + \frac{51}{\sqrt{2}} - 15\sqrt{\frac{7}{2}}$$

has minimal polynomial  $T^4 - 32T^3 - 3792T^2 + 64768T + 1$ .

We have  $\mu(t) = \frac{\sqrt{7}}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)112nq^{m^2+7n^2} \\ &= 224q^7 + 448q^8 + 448q^{11} + 448q^{16} + 448q^{23} + 448q^{32} + 448q^{43} + 448q^{56} - 672q^{63} - 1344q^{64} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(448), \left(\frac{112}{\cdot}\right))$ .

**$\mu$ -T4-16-4 (#43 in the paper)**

Let  $\tau_0 = [16, 0, 7] = \frac{i\sqrt{7}}{4}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + \frac{51}{\sqrt{2}} + 15\sqrt{\frac{7}{2}}$$

has minimal polynomial  $T^4 - 32T^3 - 3792T^2 + 64768T + 1$ .

We have  $\mu(t) = \frac{\sqrt{7}}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)16nq^{7m^2+n^2} \\ &= 32q + 64q^8 - 96q^9 - 192q^{16} + 160q^{25} + 64q^{29} + 320q^{32} - 192q^{37} - 224q^{49} + 320q^{53} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(448), \left(\frac{112}{\cdot}\right))$ .

**$\mu$ -T4-17-1 (#44 in the paper)**

Let  $\tau_0 = [4, -4, 29] = \frac{1}{2} + i\sqrt{7}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -4145152 - 2931072\sqrt{2} - 1566720\sqrt{7} - 1107840\sqrt{14}$$

has minimal polynomial  $T^4 + 16580608T^3 - 248512512T^2 - 536870912T + 4294967296$ .

We have  $\mu(t) = \frac{4\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-8m+4n)q^{116m^2-4mn+n^2} \\ &= 8q - 24q^9 + 40q^{25} - 56q^{49} + 72q^{81} - 16q^{113} - 40q^{121} - 80q^{137} + 112q^{161} + 104q^{169} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1792), \left(\frac{1792}{\cdot}\right))$ .

**$\mu$ -T4-17-2 (#44 in the paper)**

Let  $\tau_0 = [28, 28, 11] = -\frac{1}{2} + \frac{i}{\sqrt{7}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -4145152 - 2931072\sqrt{2} + 1566720\sqrt{7} + 1107840\sqrt{14}$$

has minimal polynomial  $T^4 + 16580608T^3 - 248512512T^2 - 536870912T + 4294967296$ .

We have  $\mu(t) = \frac{4\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(56m+28n)q^{44m^2+28mn+7n^2} \\ &= 56q^7 - 112q^{23} - 168q^{63} + 112q^{71} + 336q^{79} - 336q^{127} - 112q^{151} + 280q^{175} - 560q^{191} + 336q^{207} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1792), \left(\frac{1792}{\cdot}\right))$ .

**$\mu$ -T4-17-3 (#44 in the paper)**

Let  $\tau_0 = [44, 28, 7] = -\frac{7}{22} + \frac{i\sqrt{7}}{11}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -4145152 + 2931072\sqrt{2} + 1566720\sqrt{7} - 1107840\sqrt{14}$$

has minimal polynomial  $T^4 + 16580608T^3 - 248512512T^2 - 536870912T + 4294967296$ .

We have  $\mu(t) = \frac{4\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(56m+44n)q^{28m^2+28mn+11n^2} \\ &= 64q^{11} - 192q^{43} + 64q^{67} - 192q^{99} + 320q^{107} + 320q^{163} + 64q^{179} - 448q^{203} - 192q^{211} - 448q^{259} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1792), \left(\frac{1792}{\cdot}\right))$ .

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**$\mu$ -T4-17-4 (#44 in the paper)**

Let  $\tau_0 = [116, 4, 1] = -\frac{1}{58} + \frac{i\sqrt{7}}{29}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -4145152 + 2931072\sqrt{2} - 1566720\sqrt{7} + 1107840\sqrt{14}$$

has minimal polynomial  $T^4 + 16580608T^3 - 248512512T^2 - 536870912T + 4294967296$ .

We have  $\mu(t) = \frac{4\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 116n)q^{4m^2 + 4mn + 29n^2} \\ &= 448q^{29} + 448q^{37} + 448q^{53} + 448q^{77} + 448q^{109} + 448q^{149} + 448q^{197} - 896q^{253} - 1344q^{261} - 1344q^{277} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1792), (\frac{1792}{\cdot}))$ .

---

**$\mu$ -T4-18-1 (#45 in the paper)**

Let  $\tau_0 = [29, 28, 7] = -\frac{14}{29} + \frac{i\sqrt{7}}{29}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 4145168 - 2931072\sqrt{2} + 1566720\sqrt{7} - 1107840\sqrt{14}$$

has minimal polynomial  $T^4 - 16580672T^3 + 547358208T^2 - 4244652032T + 65536$ .

We have  $\mu(t) = \frac{16\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(56m + 29n)q^{112m^2 + 112mn + 29n^2} \\ &= 4q^{29} - 12q^{37} + 20q^{53} - 28q^{77} + 36q^{109} - 44q^{149} + 52q^{197} - 56q^{253} - 12q^{261} + 20q^{277} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(448), (\frac{1792}{\cdot}))$ .

---

**$\mu$ -T4-18-2 (#45 in the paper)**

Let  $\tau_0 = [11, 4, 1] = -\frac{2}{11} + \frac{i\sqrt{7}}{11}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 4145168 - 2931072\sqrt{2} - 1566720\sqrt{7} + 1107840\sqrt{14}$$

has minimal polynomial  $T^4 - 16580672T^3 + 547358208T^2 - 4244652032T + 65536$ .

We have  $\mu(t) = \frac{16\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 11n)q^{16m^2 + 16mn + 11n^2} \\ &= 28q^{11} + 28q^{43} - 84q^{67} - 84q^{99} + 28q^{107} - 84q^{163} + 140q^{179} + 28q^{203} + 140q^{211} - 84q^{259} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(448), (\frac{1792}{\cdot}))$ .

---

**$\mu$ -T4-18-3 (#45 in the paper)**

Let  $\tau_0 = [7, 0, 1] = \frac{i}{\sqrt{7}}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 4145168 + 2931072\sqrt{2} - 1566720\sqrt{7} - 1107840\sqrt{14}$$

has minimal polynomial  $T^4 - 16580672T^3 + 547358208T^2 - 4244652032T + 65536$ .

We have  $\mu(t) = \frac{16\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)7nq^{16m^2 + 7n^2} \\ &= 14q^7 + 28q^{23} - 42q^{63} + 28q^{71} - 84q^{79} - 84q^{127} + 28q^{151} + 70q^{175} + 140q^{191} - 84q^{207} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(448), (\frac{1792}{\cdot}))$ .

---

**$\mu$ -T4-18-4 (#45 in the paper)**

Let  $\tau_0 = [1, 0, 7] = i\sqrt{7}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 4145168 + 2931072\sqrt{2} + 1566720\sqrt{7} + 1107840\sqrt{14}$$

has minimal polynomial  $T^4 - 16580672T^3 + 547358208T^2 - 4244652032T + 65536$ .

We have  $\mu(t) = \frac{16\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)nq^{112m^2 + n^2} \\ &= 2q - 6q^9 + 10q^{25} - 14q^{49} + 18q^{81} + 4q^{113} - 34q^{121} + 20q^{137} - 28q^{161} + 26q^{169} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(448), (\frac{1792}{\cdot}))$ .

---

**$\mu$ -T4-19-1 (#46 in the paper)**

Let  $\tau_0 = [8, -8, 13] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{11}{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - 240\sqrt{11} - 168\sqrt{22}$$

has minimal polynomial  $T^4 - 32T^3 - 2508672T^2 + 40142848T + 4096$ .

We have  $\mu(t) = \frac{\sqrt{\frac{11}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-16m + 8n)q^{26m^2 - 4mn + n^2} \\ &= 16q - 48q^9 - 32q^{23} + 80q^{25} + 96q^{31} - 160q^{47} - 112q^{49} + 224q^{71} + 144q^{81} + 32q^{89} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(704), (\frac{352}{\cdot}))$ .

---

**$\mu$ -T4-19-2 (#46 in the paper)**

Let  $\tau_0 = [88, 88, 23] = -\frac{1}{2} + \frac{i}{2\sqrt{22}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - 240\sqrt{11} + 168\sqrt{22}$$

has minimal polynomial  $T^4 - 32T^3 - 2508672T^2 + 40142848T + 4096$ .

We have  $\mu(t) = \frac{\sqrt{\frac{11}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(176m + 88n)q^{46m^2 + 44mn + 11n^2} \\ &= 176q^{11} - 352q^{13} + 352q^{19} - 352q^{29} + 352q^{43} - 352q^{61} + 352q^{83} - 528q^{99} + 1056q^{101} - 1056q^{107} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(704), (\frac{352}{\cdot}))$ .

---

**$\mu$ -T4-19-3 (#46 in the paper)**

Let  $\tau_0 = [88, 0, 1] = \frac{i}{2\sqrt{22}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + 240\sqrt{11} - 168\sqrt{22}$$

has minimal polynomial  $T^4 - 32T^3 - 2508672T^2 + 40142848T + 4096$ .



We have  $\mu(t) = \frac{\sqrt{\frac{11}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 88nq^{2m^2+11n^2} \\ = 176q^{11} + 352q^{13} + 352q^{19} + 352q^{29} + 352q^{43} + 352q^{61} + 352q^{83} - 528q^{99} - 1056q^{101} - 1056q^{107} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(704), (\frac{352}{\cdot}))$ .

---

#### [μ-T4-19-4 \(#46 in the paper\)](#)

Let  $\tau_0 = [8, 0, 11] = \frac{1}{2}i\sqrt{\frac{11}{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + 240\sqrt{11} + 168\sqrt{22} \\ \text{has minimal polynomial } T^4 - 32T^3 - 2508672T^2 + 40142848T + 4096.$$

We have  $\mu(t) = \frac{\sqrt{\frac{11}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 8nq^{22m^2+n^2} \\ = 16q - 48q^9 + 32q^{23} + 80q^{25} - 96q^{31} + 160q^{47} - 112q^{49} - 224q^{71} + 144q^{81} + 32q^{89} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(704), (\frac{352}{\cdot}))$ .

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#### [μ-T4-20-1 \(#47 in the paper\)](#)

Let  $\tau_0 = [4, -4, 23] = \frac{1}{2} + i\sqrt{\frac{11}{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -627232 - 443520\sqrt{2} - 189120\sqrt{11} - 133728\sqrt{22} \\ \text{has minimal polynomial } T^4 + 2508928T^3 - 40138752T^2 - 131072T + 1048576.$$

We have  $\mu(t) = \frac{2\sqrt{22}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-8m + 4n)q^{92m^2-4mn+n^2} \\ = 8q - 24q^9 + 40q^{25} - 56q^{49} + 72q^{81} - 16q^{89} + 48q^{97} - 80q^{113} - 88q^{121} + 112q^{137} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1408), (\frac{1408}{\cdot}))$ .

---

#### [μ-T4-20-2 \(#47 in the paper\)](#)

Let  $\tau_0 = [44, 44, 13] = -\frac{1}{2} + \frac{i}{\sqrt{22}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -627232 + 443520\sqrt{2} - 189120\sqrt{11} + 133728\sqrt{22} \\ \text{has minimal polynomial } T^4 + 2508928T^3 - 40138752T^2 - 131072T + 1048576.$$

We have  $\mu(t) = \frac{2\sqrt{22}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(88m + 44n)q^{52m^2+44mn+11n^2} \\ = 88q^{11} - 176q^{19} + 176q^{43} - 176q^{83} - 264q^{99} + 528q^{107} - 528q^{131} + 176q^{139} + 528q^{171} - 176q^{211} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1408), (\frac{1408}{\cdot}))$ .

---

#### [μ-T4-20-3 \(#47 in the paper\)](#)

Let  $\tau_0 = [52, 44, 11] = -\frac{11}{26} + \frac{1}{13}i\sqrt{\frac{11}{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -627232 + 443520\sqrt{2} + 189120\sqrt{11} - 133728\sqrt{22} \\ \text{has minimal polynomial } T^4 + 2508928T^3 - 40138752T^2 - 131072T + 1048576.$$

We have  $\mu(t) = \frac{2\sqrt{22}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(88m + 52n)q^{44m^2+44mn+13n^2} \\ = 32q^{13} - 96q^{29} + 160q^{61} + 32q^{101} - 224q^{109} - 96q^{117} + 160q^{149} + 288q^{173} - 224q^{197} - 352q^{253} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1408), (\frac{1408}{\cdot}))$ .

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#### [μ-T4-20-4 \(#47 in the paper\)](#)

Let  $\tau_0 = [92, 4, 1] = -\frac{1}{46} + \frac{1}{23}i\sqrt{\frac{11}{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -627232 - 443520\sqrt{2} + 189120\sqrt{11} + 133728\sqrt{22} \\ \text{has minimal polynomial } T^4 + 2508928T^3 - 40138752T^2 - 131072T + 1048576.$$

We have  $\mu(t) = \frac{2\sqrt{22}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 92n)q^{4m^2+4mn+23n^2} \\ = 352q^{23} + 352q^{31} + 352q^{47} + 352q^{71} + 352q^{103} + 352q^{143} + 352q^{191} - 1056q^{199} - 1056q^{207} - 1056q^{223} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1408), (\frac{1408}{\cdot}))$ .

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#### [μ-T4-21-1 \(#48 in the paper\)](#)

Let  $\tau_0 = [46, 44, 11] = -\frac{11}{23} + \frac{1}{23}i\sqrt{\frac{11}{2}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 627248 + 443520\sqrt{2} - 189120\sqrt{11} - 133728\sqrt{22} \\ \text{has minimal polynomial } T^4 - 2508992T^3 + 80291328T^2 - 642301952T + 65536.$$

We have  $\mu(t) = \frac{4\sqrt{22}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(88m + 46n)q^{88m^2+88mn+23n^2} \\ = 8q^{23} - 24q^{31} + 40q^{47} - 56q^{71} + 72q^{103} - 88q^{143} + 104q^{191} + 8q^{199} - 24q^{207} + 40q^{223} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(704), (\frac{1408}{\cdot}))$ .

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#### [μ-T4-21-2 \(#48 in the paper\)](#)

Let  $\tau_0 = [26, 4, 1] = -\frac{1}{13} + \frac{1}{13}i\sqrt{\frac{11}{2}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 627248 - 443520\sqrt{2} - 189120\sqrt{11} + 133728\sqrt{22} \\ \text{has minimal polynomial } T^4 - 2508992T^3 + 80291328T^2 - 642301952T + 65536.$$

We have  $\mu(t) = \frac{4\sqrt{22}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 26n)q^{8m^2+8mn+13n^2} \\ = 88q^{13} + 88q^{29} + 88q^{61} - 264q^{101} + 88q^{109} - 264q^{117} - 264q^{149} + 88q^{173} - 264q^{197} + 88q^{253} + \dots$$



is in  $\mathcal{S}_2(\Gamma_0(704), (\frac{1408}{\cdot}))$ .

[μ-T4-21-3 \(#48 in the paper\)](#)

Let  $\tau_0 = [22, 0, 1] = \frac{i}{\sqrt{22}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 627248 - 443520\sqrt{2} + 189120\sqrt{11} - 133728\sqrt{22}$$

has minimal polynomial  $T^4 - 2508992T^3 + 80291328T^2 - 642301952T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{22}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 22nq^{8m^2+11n^2} \\ &= 44q^{11} + 88q^{19} + 88q^{43} + 88q^{83} - 132q^{99} - 264q^{107} - 264q^{131} + 88q^{139} - 264q^{171} + 88q^{211} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(704), (\frac{1408}{\cdot}))$ .

[μ-T4-21-4 \(#48 in the paper\)](#)

Let  $\tau_0 = [2, 0, 11] = i\sqrt{\frac{11}{2}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 627248 + 443520\sqrt{2} + 189120\sqrt{11} + 133728\sqrt{22}$$

has minimal polynomial  $T^4 - 2508992T^3 + 80291328T^2 - 642301952T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{22}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 2nq^{88m^2+n^2} \\ &= 4q - 12q^9 + 20q^{25} - 28q^{49} + 36q^{81} + 8q^{89} - 24q^{97} + 40q^{113} - 44q^{121} - 56q^{137} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(704), (\frac{1408}{\cdot}))$ .

[μ-T4-22-1 \(#49 in the paper\)](#)

Let  $\tau_0 = [8, -8, 31] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{29}{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - 55440\sqrt{2} - 10296\sqrt{58}$$

has minimal polynomial  $T^4 - 32T^3 - 24591257472T^2 + 393460123648T + 4096$ .

We have  $\mu(t) = \frac{\sqrt{\frac{29}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-16m + 8n) q^{62m^2 - 4mn + n^2} \\ &= 16q - 48q^9 + 80q^{25} - 112q^{49} - 32q^{59} + 96q^{67} + 144q^{81} - 160q^{83} + 224q^{107} - 176q^{121} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1856), (\frac{928}{\cdot}))$ .

[μ-T4-22-2 \(#49 in the paper\)](#)

Let  $\tau_0 = [232, 232, 59] = -\frac{1}{2} + \frac{i}{2\sqrt{58}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + 55440\sqrt{2} - 10296\sqrt{58}$$

has minimal polynomial  $T^4 - 32T^3 - 24591257472T^2 + 393460123648T + 4096$ .

We have  $\mu(t) = \frac{\sqrt{\frac{29}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (464m + 232n) q^{118m^2 + 116mn + 29n^2} \\ &= 464q^{29} - 928q^{31} + 928q^{37} - 928q^{47} + 928q^{61} - 928q^{79} + 928q^{101} - 928q^{127} + 928q^{157} - 928q^{191} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1856), (\frac{928}{\cdot}))$ .

[μ-T4-22-3 \(#49 in the paper\)](#)

Let  $\tau_0 = [232, 0, 1] = \frac{i}{2\sqrt{58}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - 55440\sqrt{2} + 10296\sqrt{58}$$

has minimal polynomial  $T^4 - 32T^3 - 24591257472T^2 + 393460123648T + 4096$ .

We have  $\mu(t) = \frac{\sqrt{\frac{29}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 232nq^{2m^2+29n^2} \\ &= 464q^{29} + 928q^{31} + 928q^{37} + 928q^{47} + 928q^{61} + 928q^{79} + 928q^{101} + 928q^{127} + 928q^{157} + 928q^{191} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1856), (\frac{928}{\cdot}))$ .

[μ-T4-22-4 \(#49 in the paper\)](#)

Let  $\tau_0 = [8, 0, 29] = \frac{1}{2}i\sqrt{\frac{29}{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + 55440\sqrt{2} + 10296\sqrt{58}$$

has minimal polynomial  $T^4 - 32T^3 - 24591257472T^2 + 393460123648T + 4096$ .

We have  $\mu(t) = \frac{\sqrt{\frac{29}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 8nq^{58m^2+n^2} \\ &= 16q - 48q^9 + 80q^{25} - 112q^{49} + 32q^{59} - 96q^{67} + 144q^{81} + 160q^{83} - 224q^{107} - 176q^{121} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1856), (\frac{928}{\cdot}))$ .

[μ-T4-23-1 \(#50 in the paper\)](#)

Let  $\tau_0 = [4, -4, 59] = \frac{1}{2} + i\sqrt{\frac{29}{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -6147814432 - 4347161280\sqrt{2} - 1141620480\sqrt{29} - 807247584\sqrt{58}$$

has minimal polynomial  $T^4 + 24591257728T^3 - 393460119552T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{58}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-8m + 4n) q^{236m^2 - 4mn + n^2} \\ &= 8q - 24q^9 + 40q^{25} - 56q^{49} + 72q^{81} - 88q^{121} + 104q^{169} - 120q^{225} - 16q^{233} + 48q^{241} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(3712), (\frac{3712}{\cdot}))$ .

[μ-T4-23-2 \(#50 in the paper\)](#)

Let  $\tau_0 = [116, 116, 31] = -\frac{1}{2} + \frac{i}{\sqrt{58}}$  with  $h(D) = 4$ . Then

$t = t_P(\tau_0) = -6147814432 + 4347161280\sqrt{2} + 1141620480\sqrt{29} - 807247584\sqrt{58}$   
has minimal polynomial  $T^4 + 24591257728T^3 - 393460119552T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{58}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(232m + 116n)q^{124m^2 + 116mn + 29n^2} \\ &= 232q^{29} - 464q^{37} + 464q^{61} - 464q^{101} + 464q^{157} - 464q^{229} - 696q^{261} + 1392q^{269} - 1392q^{293} + 464q^{317} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(3712), (\frac{3712}{\cdot}))$ .

#### $\mu$ -T4-23-3 (#50 in the paper)

Let  $\tau_0 = [124, 116, 29] = -\frac{29}{62} + \frac{1}{31}i\sqrt{\frac{29}{2}}$  with  $h(D) = 4$ . Then

$t = t_P(\tau_0) = -6147814432 + 4347161280\sqrt{2} + 1141620480\sqrt{29} + 807247584\sqrt{58}$   
has minimal polynomial  $T^4 + 24591257728T^3 - 393460119552T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{58}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(232m + 124n)q^{116m^2 + 116mn + 31n^2} \\ &= 32q^{31} - 96q^{47} + 160q^{79} - 224q^{127} + 288q^{191} + 32q^{263} - 352q^{271} - 96q^{279} + 160q^{311} - 224q^{359} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(3712), (\frac{3712}{\cdot}))$ .

#### $\mu$ -T4-23-4 (#50 in the paper)

Let  $\tau_0 = [236, 4, 1] = -\frac{1}{118} + \frac{1}{59}i\sqrt{\frac{29}{2}}$  with  $h(D) = 4$ . Then

$t = t_P(\tau_0) = -6147814432 + 4347161280\sqrt{2} - 1141620480\sqrt{29} + 807247584\sqrt{58}$   
has minimal polynomial  $T^4 + 24591257728T^3 - 393460119552T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{58}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 236n)q^{4m^2 + 4mn + 59n^2} \\ &= 928q^{59} + 928q^{67} + 928q^{83} + 928q^{107} + 928q^{139} + 928q^{179} + 928q^{227} + 928q^{283} + 928q^{347} + 928q^{419} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(3712), (\frac{3712}{\cdot}))$ .

#### $\mu$ -T4-24-1 (#51 in the paper)

Let  $\tau_0 = [118, 116, 29] = -\frac{29}{59} + \frac{1}{59}i\sqrt{\frac{29}{2}}$  with  $h(D) = 2$ . Then

$t = t_P(\tau_0) = 6147814448 - 4347161280\sqrt{2} + 1141620480\sqrt{29} - 807247584\sqrt{58}$   
has minimal polynomial  $T^4 - 24591257792T^3 + 786920252928T^2 - 6295361994752T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{58}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(232m + 118n)q^{232m^2 + 232mn + 59n^2} \\ &= 8q^{59} - 24q^{67} + 40q^{83} - 56q^{107} + 72q^{139} - 88q^{179} + 104q^{227} - 120q^{283} + 136q^{347} - 152q^{419} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1856), (\frac{3712}{\cdot}))$ .

#### $\mu$ -T4-24-2 (#51 in the paper)

Let  $\tau_0 = [62, 4, 1] = -\frac{1}{31} + \frac{1}{31}i\sqrt{\frac{29}{2}}$  with  $h(D) = 2$ . Then

$t = t_P(\tau_0) = 6147814448 + 4347161280\sqrt{2} - 1141620480\sqrt{29} - 807247584\sqrt{58}$   
has minimal polynomial  $T^4 - 24591257792T^3 + 786920252928T^2 - 6295361994752T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{58}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 62n)q^{8m^2 + 8mn + 31n^2} \\ &= 232q^{31} + 232q^{47} + 232q^{79} + 232q^{127} + 232q^{191} - 696q^{263} + 232q^{271} - 696q^{279} - 696q^{311} - 696q^{359} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1856), (\frac{3712}{\cdot}))$ .

#### $\mu$ -T4-24-3 (#51 in the paper)

Let  $\tau_0 = [58, 0, 1] = \frac{i}{\sqrt{58}}$  with  $h(D) = 2$ . Then

$t = t_P(\tau_0) = 6147814448 - 4347161280\sqrt{2} - 1141620480\sqrt{29} + 807247584\sqrt{58}$   
has minimal polynomial  $T^4 - 24591257792T^3 + 786920252928T^2 - 6295361994752T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{58}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)58nq^{8m^2 + 29n^2} \\ &= 116q^{29} + 232q^{37} + 232q^{61} + 232q^{101} + 232q^{157} + 232q^{229} - 348q^{261} - 696q^{269} - 696q^{293} + 232q^{317} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1856), (\frac{3712}{\cdot}))$ .

#### $\mu$ -T4-24-4 (#51 in the paper)

Let  $\tau_0 = [2, 0, 29] = i\sqrt{\frac{29}{2}}$  with  $h(D) = 2$ . Then

$t = t_P(\tau_0) = 6147814448 + 4347161280\sqrt{2} + 1141620480\sqrt{29} + 807247584\sqrt{58}$   
has minimal polynomial  $T^4 - 24591257792T^3 + 786920252928T^2 - 6295361994752T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{58}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)2nq^{232m^2 + n^2} \\ &= 4q - 12q^9 + 20q^{25} - 28q^{49} + 36q^{81} - 44q^{121} + 52q^{169} - 60q^{225} + 8q^{233} - 24q^{241} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1856), (\frac{3712}{\cdot}))$ .

#### $\mu$ -N4-1-1

Let  $\tau_0 = [8, 7, 2] = -\frac{7}{16} + \frac{i\sqrt{15}}{16}$  with  $h(D) = 2$ . Then

$t = t_P(\tau_0) = \frac{47}{4} + \frac{17i\sqrt{3}}{4} - \frac{21\sqrt{5}}{4} - \frac{7i\sqrt{15}}{4}$   
has minimal polynomial  $T^4 - 47T^3 + 753T^2 - 32T + 256$ .

We have  $\mu(t) = \frac{\sqrt{15}}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(14m+8n)q^{8m^2+7mn+2n^2} \\ &= 16q^2 - 12q^3 - 20q^5 + 8q^8 + 24q^{12} + 40q^{17} - 48q^{18} - 40q^{20} + 24q^{23} + 36q^{27} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(60), \left(\frac{60}{\cdot}\right))$ .

---

#### $\mu$ -N4-1-2

Let  $\tau_0 = [8, -7, 2] = \frac{7}{16} + \frac{i\sqrt{15}}{16}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = \frac{47}{4} - \frac{17i\sqrt{3}}{4} - \frac{21\sqrt{5}}{4} + \frac{7i\sqrt{15}}{4}$$

has minimal polynomial  $T^4 - 47T^3 + 753T^2 - 32T + 256$ .

We have  $\mu(t) = \frac{\sqrt{15}}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-14m+8n)q^{8m^2-7mn+2n^2} \\ &= 16q^2 - 12q^3 - 20q^5 + 8q^8 + 24q^{12} + 40q^{17} - 48q^{18} - 40q^{20} + 24q^{23} + 36q^{27} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(60), \left(\frac{60}{\cdot}\right))$ .

---

#### $\mu$ -N4-1-3

Let  $\tau_0 = [4, 1, 1] = -\frac{1}{8} + \frac{i\sqrt{15}}{8}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = \frac{47}{4} + \frac{17i\sqrt{3}}{4} + \frac{21\sqrt{5}}{4} + \frac{7i\sqrt{15}}{4}$$

has minimal polynomial  $T^4 - 47T^3 + 753T^2 - 32T + 256$ .

We have  $\mu(t) = \frac{\sqrt{15}}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(2m+4n)q^{4m^2+mn+n^2} \\ &= 8q + 4q^4 + 12q^6 - 24q^9 - 20q^{10} - 28q^{16} + 36q^{24} + 40q^{25} + 40q^{34} - 12q^{36} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(60), \left(\frac{60}{\cdot}\right))$ .

---

#### $\mu$ -N4-1-4

Let  $\tau_0 = [4, -1, 1] = \frac{1}{8} + \frac{i\sqrt{15}}{8}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = \frac{47}{4} - \frac{17i\sqrt{3}}{4} + \frac{21\sqrt{5}}{4} - \frac{7i\sqrt{15}}{4}$$

has minimal polynomial  $T^4 - 47T^3 + 753T^2 - 32T + 256$ .

We have  $\mu(t) = \frac{\sqrt{15}}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-2m+4n)q^{4m^2-mn+n^2} \\ &= 8q + 4q^4 + 12q^6 - 24q^9 - 20q^{10} - 28q^{16} + 36q^{24} + 40q^{25} + 40q^{34} - 12q^{36} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(60), \left(\frac{60}{\cdot}\right))$ .

---

#### $\mu$ -N4-2-1

Let  $\tau_0 = [2, 1, 2] = -\frac{1}{4} + \frac{i\sqrt{15}}{4}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 8 + 128i\sqrt{3} + 56i\sqrt{15}$$

has minimal polynomial  $T^4 - 32T^3 + 192768T^2 - 3080192T + 16777216$ .

We have  $\mu(t) = \frac{2\sqrt{15}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(2m+2n)q^{16m^2+2mn+n^2} \\ &= 4q - 12q^9 + 20q^{25} - 28q^{49} - 8q^{61} + 24q^{69} + 36q^{81} - 40q^{85} + 56q^{109} - 44q^{121} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(120), \left(\frac{240}{\cdot}\right))$ .

---

#### $\mu$ -N4-2-2

Let  $\tau_0 = [2, -1, 2] = \frac{1}{4} + \frac{i\sqrt{15}}{4}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 8 - 128i\sqrt{3} - 56i\sqrt{15}$$

has minimal polynomial  $T^4 - 32T^3 + 192768T^2 - 3080192T + 16777216$ .

We have  $\mu(t) = \frac{2\sqrt{15}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-2m+2n)q^{16m^2-2mn+n^2} \\ &= 4q - 12q^9 + 20q^{25} - 28q^{49} - 8q^{61} + 24q^{69} + 36q^{81} - 40q^{85} + 56q^{109} - 44q^{121} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(120), \left(\frac{240}{\cdot}\right))$ .

---

#### $\mu$ -N4-2-3

Let  $\tau_0 = [6, 3, 1] = -\frac{1}{4} + \frac{1}{4}i\sqrt{\frac{5}{3}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 8 + 128i\sqrt{3} - 56i\sqrt{15}$$

has minimal polynomial  $T^4 - 32T^3 + 192768T^2 - 3080192T + 16777216$ .

We have  $\mu(t) = \frac{2\sqrt{15}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(6m+6n)q^{8m^2+6mn+3n^2} \\ &= 12q^3 - 24q^{23} - 36q^{27} + 72q^{47} + 60q^{75} + 24q^{83} - 120q^{95} - 72q^{107} - 84q^{147} + 120q^{155} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(120), \left(\frac{240}{\cdot}\right))$ .

---

#### $\mu$ -N4-2-4

Let  $\tau_0 = [6, -3, 1] = \frac{1}{4} + \frac{1}{4}i\sqrt{\frac{5}{3}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = 8 - 128i\sqrt{3} + 56i\sqrt{15}$$

has minimal polynomial  $T^4 - 32T^3 + 192768T^2 - 3080192T + 16777216$ .

We have  $\mu(t) = \frac{2\sqrt{15}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-6m+6n)q^{8m^2-6mn+3n^2} \\ &= 12q^3 - 24q^{23} - 36q^{27} + 72q^{47} + 60q^{75} + 24q^{83} - 120q^{95} - 72q^{107} - 84q^{147} + 120q^{155} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(120), \left(\frac{240}{\cdot}\right))$ .

---

#### $\mu$ -N4-3-1

Let  $\tau_0 = [8, 6, 3] = -\frac{3}{8} + \frac{i\sqrt{15}}{8}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = \frac{17}{4} + \frac{17i\sqrt{3}}{4} - \frac{21\sqrt{5}}{4} + \frac{7i\sqrt{15}}{4}$$

has minimal polynomial  $T^4 - 17T^3 + 33T^2 - 4352T + 65536$ .

We have  $\mu(t) = \frac{\sqrt{15}}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(12m+8n)q^{6m^2+3mn+n^2} \\ &= 16q - 8q^4 - 24q^6 - 48q^9 + 40q^{10} + 56q^{16} - 72q^{24} + 80q^{25} - 80q^{34} + 24q^{36} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(120), \left(\frac{60}{\cdot}\right))$ .

---

#### $\mu$ -N4-3-2

Let  $\tau_0 = [8, -6, 3] = \frac{3}{8} + \frac{i\sqrt{15}}{8}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = \frac{17}{4} - \frac{17i\sqrt{3}}{4} - \frac{21\sqrt{5}}{4} - \frac{7i\sqrt{15}}{4}$$

has minimal polynomial  $T^4 - 17T^3 + 33T^2 - 4352T + 65536$ .

We have  $\mu(t) = \frac{\sqrt{15}}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-12m+8n)q^{6m^2-3mn+n^2} \\ &= 16q - 8q^4 - 24q^6 - 48q^9 + 40q^{10} + 56q^{16} - 72q^{24} + 80q^{25} - 80q^{34} + 24q^{36} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(120), \left(\frac{60}{\cdot}\right))$ .

---

#### $\mu$ -N4-3-3

Let  $\tau_0 = [16, 2, 1] = -\frac{1}{16} + \frac{i\sqrt{15}}{16}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = \frac{17}{4} + \frac{17i\sqrt{3}}{4} + \frac{21\sqrt{5}}{4} - \frac{7i\sqrt{15}}{4}$$

has minimal polynomial  $T^4 - 17T^3 + 33T^2 - 4352T + 65536$ .

We have  $\mu(t) = \frac{\sqrt{15}}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(4m+16n)q^{2m^2+mn+2n^2} \\ &= 32q^2 + 24q^3 + 40q^5 + 16q^8 + 48q^{12} - 80q^{17} - 96q^{18} - 80q^{20} - 48q^{23} - 72q^{27} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(120), \left(\frac{60}{\cdot}\right))$ .

---

#### $\mu$ -N4-3-4

Let  $\tau_0 = [16, -2, 1] = \frac{1}{16} + \frac{i\sqrt{15}}{16}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = \frac{17}{4} - \frac{17i\sqrt{3}}{4} + \frac{21\sqrt{5}}{4} + \frac{7i\sqrt{15}}{4}$$

has minimal polynomial  $T^4 - 17T^3 + 33T^2 - 4352T + 65536$ .

We have  $\mu(t) = \frac{\sqrt{15}}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-4m+16n)q^{2m^2-mn+2n^2} \\ &= 32q^2 + 24q^3 + 40q^5 + 16q^8 + 48q^{12} - 80q^{17} - 96q^{18} - 80q^{20} - 48q^{23} - 72q^{27} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(120), \left(\frac{60}{\cdot}\right))$ .

---

#### $\mu$ -N4-4-1

Let  $\tau_0 = [9, 8, 2] = -\frac{4}{9} + \frac{i\sqrt{2}}{9}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 1808 + 1280\sqrt{2} - 64\sqrt{2(799 + 565\sqrt{2})}$$

has minimal polynomial  $T^4 - 7232T^3 - 31232T^2 - 1851392T + 65536$ .

We have  $\mu(t) = \frac{16\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(16m+9n)q^{32m^2+32mn+9n^2} \\ &= 4q^9 - 12q^{17} + 20q^{33} - 28q^{57} + 4q^{73} - 12q^{81} + 36q^{89} + 20q^{97} - 28q^{121} - 44q^{129} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(128), \left(\frac{512}{\cdot}\right))$ .

---

#### $\mu$ -N4-4-2

Let  $\tau_0 = [1, 0, 2] = i\sqrt{2}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 1808 + 1280\sqrt{2} + 64\sqrt{2(799 + 565\sqrt{2})}$$

has minimal polynomial  $T^4 - 7232T^3 - 31232T^2 - 1851392T + 65536$ .

We have  $\mu(t) = \frac{16\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)nq^{32m^2+n^2} \\ &= 2q - 6q^9 + 10q^{25} + 4q^{33} - 12q^{41} - 14q^{49} + 20q^{57} - 10q^{81} + 36q^{113} - 22q^{121} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(128), \left(\frac{512}{\cdot}\right))$ .

---

#### $\mu$ -N4-4-3

Let  $\tau_0 = [3, 2, 1] = -\frac{1}{3} + \frac{i\sqrt{2}}{3}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 1808 - 1280\sqrt{2} + 64i\sqrt{2(-799 + 565\sqrt{2})}$$

has minimal polynomial  $T^4 - 7232T^3 - 31232T^2 - 1851392T + 65536$ .

We have  $\mu(t) = \frac{16\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(4m+3n)q^{16m^2+8mn+3n^2} \\ &= 6q^3 - 2q^{11} - 10q^{19} - 4q^{27} - 2q^{43} + 12q^{51} + 14q^{59} - 26q^{67} + 30q^{75} + 22q^{83} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(128), \left(\frac{512}{\cdot}\right))$ .

---

#### $\mu$ -N4-4-4

Let  $\tau_0 = [3, -2, 1] = \frac{1}{3} + \frac{i\sqrt{2}}{3}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 1808 - 1280\sqrt{2} - 64i\sqrt{2\left(-799 + 565\sqrt{2}\right)}$$

has minimal polynomial  $T^4 - 7232T^3 - 31232T^2 - 1851392T + 65536$ .

We have  $\mu(t) = \frac{16\sqrt{2}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-4m + 3n)q^{16m^2 - 8mn + 3n^2} \\ &= 6q^3 - 2q^{11} - 10q^{19} - 4q^{27} - 2q^{43} + 12q^{51} + 14q^{59} - 26q^{67} + 30q^{75} + 22q^{83} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(128), \left(\frac{512}{\cdot}\right))$ .

---

#### $\mu$ -N4-5-1

Let  $\tau_0 = [17, 16, 4] = -\frac{8}{17} + \frac{2i}{17}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 71696 - 60288\sqrt[4]{2} + 50688\sqrt{2} - 42624\sqrt[4]{8}$$

has minimal polynomial  $T^4 - 286784T^3 + 7079424T^2 - 73416704T + 65536$ .

We have  $\mu(t) = \frac{32}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(32m + 17n)q^{64m^2 + 64mn + 17n^2} \\ &= 4q^{17} - 12q^{25} + 20q^{41} - 28q^{65} + 36q^{97} - 44q^{137} + 4q^{145} - 12q^{153} + 20q^{169} + 52q^{185} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(256), \left(\frac{1024}{\cdot}\right))$ .

---

#### $\mu$ -N4-5-2

Let  $\tau_0 = [1, 0, 4] = 2i$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 71696 + 60288\sqrt[4]{2} + 50688\sqrt{2} + 42624\sqrt[4]{8}$$

has minimal polynomial  $T^4 - 286784T^3 + 7079424T^2 - 73416704T + 65536$ .

We have  $\mu(t) = \frac{32}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)nq^{64m^2 + n^2} \\ &= 2q - 6q^9 + 10q^{25} - 14q^{49} + 4q^{65} - 12q^{73} + 18q^{81} + 20q^{89} - 28q^{113} - 22q^{121} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(256), \left(\frac{1024}{\cdot}\right))$ .

---

#### $\mu$ -N4-5-3

Let  $\tau_0 = [5, 2, 1] = -\frac{1}{5} + \frac{2i}{5}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 71696 + 60288i\sqrt[4]{2} - 50688\sqrt{2} - 42624i\sqrt[4]{8}$$

has minimal polynomial  $T^4 - 286784T^3 + 7079424T^2 - 73416704T + 65536$ .

We have  $\mu(t) = \frac{32}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(4m + 5n)q^{16m^2 + 8mn + 5n^2} \\ &= 10q^5 + 2q^{13} + 18q^{29} - 22q^{37} - 30q^{45} - 6q^{53} - 14q^{61} - 12q^{85} + 42q^{101} + 34q^{109} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(256), \left(\frac{1024}{\cdot}\right))$ .

---

#### $\mu$ -N4-5-4

Let  $\tau_0 = [5, -2, 1] = \frac{1}{5} + \frac{2i}{5}$  with  $h(D) = 1$ . Then

$$t = t_P(\tau_0) = 71696 - 60288i\sqrt[4]{2} - 50688\sqrt{2} + 42624i\sqrt[4]{8}$$

has minimal polynomial  $T^4 - 286784T^3 + 7079424T^2 - 73416704T + 65536$ .

We have  $\mu(t) = \frac{32}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-4m + 5n)q^{16m^2 - 8mn + 5n^2} \\ &= 10q^5 + 2q^{13} + 18q^{29} - 22q^{37} - 30q^{45} - 6q^{53} - 14q^{61} - 12q^{85} + 42q^{101} + 34q^{109} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(256), \left(\frac{1024}{\cdot}\right))$ .

---

#### $\mu$ -N4-6-1

Let  $\tau_0 = [32, 32, 9] = -\frac{1}{2} + \frac{i}{4\sqrt{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - 20\sqrt{799 + 565\sqrt{2}} + 14\sqrt{2\left(799 + 565\sqrt{2}\right)}$$

has minimal polynomial  $T^4 - 32T^3 + 368T^2 - 1792T - 64$ .

We have  $\mu(t) = \frac{1}{2\sqrt{2}\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(64m + 32n)q^{9m^2 + 8mn + 2n^2} \\ &= 64q^2 - 128q^3 + 128q^6 - 128q^{11} - 64q^{18} + 384q^{19} - 384q^{22} + 256q^{27} - 384q^{34} + 128q^{38} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(128), \left(\frac{32}{\cdot}\right))$ .

---

#### $\mu$ -N4-6-2

Let  $\tau_0 = [32, 0, 1] = \frac{i}{4\sqrt{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + 20\sqrt{799 + 565\sqrt{2}} - 14\sqrt{2\left(799 + 565\sqrt{2}\right)}$$

has minimal polynomial  $T^4 - 32T^3 + 368T^2 - 1792T - 64$ .

We have  $\mu(t) = \frac{1}{2\sqrt{2}\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)32nq^{m^2 + 2n^2} \\ &= 64q^2 + 128q^3 + 128q^6 + 128q^{11} - 64q^{18} - 384q^{19} - 384q^{22} - 256q^{27} - 384q^{34} + 128q^{38} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(128), \left(\frac{32}{\cdot}\right))$ .

---

#### $\mu$ -N4-6-3

Let  $\tau_0 = [16, 8, 3] = -\frac{1}{4} + \frac{i}{2\sqrt{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + 20i\sqrt{-799 + 565\sqrt{2}} + 14i\sqrt{2\left(-799 + 565\sqrt{2}\right)}$$

has minimal polynomial  $T^4 - 32T^3 + 368T^2 - 1792T - 64$ .

We have  $\mu(t) = \frac{1}{2\sqrt{2}\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(16m + 16n)q^{3m^2 + 2mn + n^2}$$

$= 32q - 160q^9 + 192q^{17} + 160q^{25} - 256q^{33} - 192q^{41} - 224q^{49} + 768q^{57} - 64q^{73} + 32q^{81} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(128), \left(\frac{32}{\cdot}\right))$ .

---

**$\mu$ -N4-6-4**

Let  $\tau_0 = [16, -8, 3] = \frac{1}{4} + \frac{i}{2\sqrt{2}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - 20i\sqrt{-799 + 565\sqrt{2}} - 14i\sqrt{2(-799 + 565\sqrt{2})}$$

has minimal polynomial  $T^4 - 32T^3 + 368T^2 - 1792T - 64$ .

We have  $\mu(t) = \frac{1}{2\sqrt{2}\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-16m + 16n)q^{3m^2 - 2mn + n^2} \\ &= 32q - 160q^9 + 192q^{17} + 160q^{25} - 256q^{33} - 192q^{41} - 224q^{49} + 768q^{57} - 64q^{73} + 32q^{81} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(128), \left(\frac{32}{\cdot}\right))$ .

---

**$\mu$ -N4-7-1**

Let  $\tau_0 = [64, 64, 17] = -\frac{1}{2} + \frac{i}{8}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + 6\sqrt[4]{2} - 9\sqrt[4]{8}$$

has minimal polynomial  $T^4 - 32T^3 + 816T^2 - 8960T - 8$ .

We have  $\mu(t) = \frac{1}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(128m + 64n)q^{17m^2 + 16mn + 4n^2} \\ &= 128q^4 - 256q^5 + 256q^8 - 256q^{13} + 256q^{20} - 256q^{29} - 384q^{36} + 768q^{37} - 512q^{40} + 768q^{45} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(256), \left(\frac{64}{\cdot}\right))$ .

---

**$\mu$ -N4-7-2**

Let  $\tau_0 = [64, 0, 1] = \frac{i}{8}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - 6\sqrt[4]{2} + 9\sqrt[4]{8}$$

has minimal polynomial  $T^4 - 32T^3 + 816T^2 - 8960T - 8$ .

We have  $\mu(t) = \frac{1}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)64nq^{m^2 + 4n^2} \\ &= 128q^4 + 256q^5 + 256q^8 + 256q^{13} + 256q^{20} + 256q^{29} - 384q^{36} - 768q^{37} - 512q^{40} - 768q^{45} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(256), \left(\frac{64}{\cdot}\right))$ .

---

**$\mu$ -N4-7-3**

Let  $\tau_0 = [16, 8, 5] = -\frac{1}{4} + \frac{i}{2}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + 6i\sqrt[4]{2} + 9i\sqrt[4]{8}$$

has minimal polynomial  $T^4 - 32T^3 + 816T^2 - 8960T - 8$ .

We have  $\mu(t) = \frac{1}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(16m + 16n)q^{5m^2 + 2mn + n^2} \\ &= 32q - 96q^9 - 64q^{17} + 352q^{25} - 320q^{41} - 224q^{49} + 512q^{65} - 192q^{73} + 288q^{81} + 320q^{89} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(256), \left(\frac{64}{\cdot}\right))$ .

---

**$\mu$ -N4-7-4**

Let  $\tau_0 = [16, -8, 5] = \frac{1}{4} + \frac{i}{2}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - 6i\sqrt[4]{2} - 9i\sqrt[4]{8}$$

has minimal polynomial  $T^4 - 32T^3 + 816T^2 - 8960T - 8$ .

We have  $\mu(t) = \frac{1}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-16m + 16n)q^{5m^2 - 2mn + n^2} \\ &= 32q - 96q^9 - 64q^{17} + 352q^{25} - 320q^{41} - 224q^{49} + 512q^{65} - 192q^{73} + 288q^{81} + 320q^{89} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(256), \left(\frac{64}{\cdot}\right))$ .

---

**$\mu$ -N4-8-1**

Let  $\tau_0 = [2, -2, 3] = \frac{1}{2} + \frac{i\sqrt{5}}{2}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = -272 - 128\sqrt{5} - 64\sqrt{38 + 17\sqrt{5}}$$

has minimal polynomial  $T^4 + 1088T^3 - 31232T^2 + 278528T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{5}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-4m + 2n)q^{24m^2 - 4mn + n^2} \\ &= 4q - 12q^9 - 8q^{21} + 20q^{25} + 24q^{29} - 40q^{45} - 28q^{49} + 56q^{69} + 44q^{81} - 24q^{89} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(160), \left(\frac{320}{\cdot}\right))$ .

---

**$\mu$ -N4-8-2**

Let  $\tau_0 = [10, 10, 3] = -\frac{1}{2} + \frac{i}{2\sqrt{5}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = -272 - 128\sqrt{5} + 64\sqrt{38 + 17\sqrt{5}}$$

has minimal polynomial  $T^4 + 1088T^3 - 31232T^2 + 278528T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{5}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(20m + 10n)q^{24m^2 + 20mn + 5n^2} \\ &= 20q^5 - 40q^9 + 40q^{21} - 40q^{41} - 60q^{45} + 120q^{49} - 120q^{61} + 40q^{69} + 120q^{81} - 40q^{105} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(160), \left(\frac{320}{\cdot}\right))$ .

---

**$\mu$ -N4-8-3**

Let  $\tau_0 = [6, 2, 1] = -\frac{1}{6} + \frac{i\sqrt{5}}{6}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = -272 + 128\sqrt{5} + 64i\sqrt{-38 + 17\sqrt{5}}$$

has minimal polynomial  $T^4 + 1088T^3 - 31232T^2 + 278528T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{5}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(4m+6n)q^{8m^2+4mn+3n^2} \\ &= 12q^3 + 4q^7 + 20q^{15} - 28q^{23} - 40q^{27} - 20q^{35} + 28q^{43} - 44q^{47} + 28q^{63} + 44q^{67} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(160), \left(\frac{320}{\cdot}\right))$ .

---

#### $\mu$ -N4-8-4

Let  $\tau_0 = [6, -2, 1] = \frac{1}{6} + \frac{i\sqrt{5}}{6}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = -272 + 128\sqrt{5} - 64i\sqrt{-38 + 17\sqrt{5}}$$

has minimal polynomial  $T^4 + 1088T^3 - 31232T^2 + 278528T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{5}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-4m+6n)q^{8m^2-4mn+3n^2} \\ &= 12q^3 + 4q^7 + 20q^{15} - 28q^{23} - 40q^{27} - 20q^{35} + 28q^{43} - 44q^{47} + 28q^{63} + 44q^{67} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(160), \left(\frac{320}{\cdot}\right))$ .

---

#### $\mu$ -N4-9-1

Let  $\tau_0 = [2, -2, 5] = \frac{1}{2} + \frac{3i}{2}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = -3088 - 1664\sqrt[4]{12} - 1792\sqrt{3} - 960\sqrt[4]{108}$$

has minimal polynomial  $T^4 + 12352T^3 - 391680T^2 + 3162112T + 65536$ .

We have  $\mu(t) = \frac{12}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-4m+2n)q^{40m^2-4mn+n^2} \\ &= 4q - 12q^9 + 20q^{25} - 8q^{37} + 24q^{45} - 28q^{49} - 40q^{61} + 36q^{81} + 56q^{85} - 72q^{117} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(288), \left(\frac{576}{\cdot}\right))$ .

---

#### $\mu$ -N4-9-2

Let  $\tau_0 = [18, 18, 5] = -\frac{1}{2} + \frac{i}{6}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = -3088 + 1664\sqrt[4]{12} - 1792\sqrt{3} + 960\sqrt[4]{108}$$

has minimal polynomial  $T^4 + 12352T^3 - 391680T^2 + 3162112T + 65536$ .

We have  $\mu(t) = \frac{12}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(36m+18n)q^{40m^2+36mn+9n^2} \\ &= 36q^9 - 72q^{13} + 72q^{25} - 72q^{45} + 72q^{73} - 108q^{81} + 216q^{85} - 216q^{97} - 72q^{109} + 216q^{117} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(288), \left(\frac{576}{\cdot}\right))$ .

---

#### $\mu$ -N4-9-3

Let  $\tau_0 = [10, 2, 1] = -\frac{1}{10} + \frac{3i}{10}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = -3088 + 1664i\sqrt[4]{12} + 1792\sqrt{3} - 960i\sqrt[4]{108}$$

has minimal polynomial  $T^4 + 12352T^3 - 391680T^2 + 3162112T + 65536$ .

We have  $\mu(t) = \frac{12}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(4m+10n)q^{8m^2+4mn+5n^2} \\ &= 20q^5 + 12q^9 + 28q^{17} + 4q^{29} - 52q^{41} - 24q^{45} - 44q^{53} - 72q^{65} - 36q^{81} + 44q^{89} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(288), \left(\frac{576}{\cdot}\right))$ .

---

#### $\mu$ -N4-9-4

Let  $\tau_0 = [10, -2, 1] = \frac{1}{10} + \frac{3i}{10}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = -3088 - 1664i\sqrt[4]{12} + 1792\sqrt{3} + 960i\sqrt[4]{108}$$

has minimal polynomial  $T^4 + 12352T^3 - 391680T^2 + 3162112T + 65536$ .

We have  $\mu(t) = \frac{12}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-4m+10n)q^{8m^2-4mn+5n^2} \\ &= 20q^5 + 12q^9 + 28q^{17} + 4q^{29} - 52q^{41} - 24q^{45} - 44q^{53} - 72q^{65} - 36q^{81} + 44q^{89} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(288), \left(\frac{576}{\cdot}\right))$ .

---

#### $\mu$ -N4-10-1

Let  $\tau_0 = [2, -2, 7] = \frac{1}{2} + \frac{i\sqrt{13}}{2}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = -20752 - 5760\sqrt{13} - 192\sqrt{23382 + 6485\sqrt{13}}$$

has minimal polynomial  $T^4 + 83008T^3 - 2652672T^2 + 21250048T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{13}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-4m+2n)q^{56m^2-4mn+n^2} \\ &= 4q - 12q^9 + 20q^{25} - 28q^{49} - 8q^{53} + 24q^{61} - 40q^{77} + 36q^{81} + 56q^{101} - 44q^{121} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(416), \left(\frac{832}{\cdot}\right))$ .

---

#### $\mu$ -N4-10-2

Let  $\tau_0 = [26, 26, 7] = -\frac{1}{2} + \frac{i}{2\sqrt{13}}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = -20752 - 5760\sqrt{13} + 192\sqrt{23382 + 6485\sqrt{13}}$$

has minimal polynomial  $T^4 + 83008T^3 - 2652672T^2 + 21250048T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{13}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(52m+26n)q^{56m^2+52mn+13n^2} \\ &= 52q^{13} - 104q^{17} + 104q^{29} - 104q^{49} + 104q^{77} - 104q^{113} - 156q^{117} + 312q^{121} - 312q^{133} + 312q^{153} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(416), \left(\frac{832}{\cdot}\right))$ .

---

#### $\mu$ -N4-10-3

Let  $\tau_0 = [14, 2, 1] = -\frac{1}{14} + \frac{i\sqrt{13}}{14}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = -20752 + 5760\sqrt{13} + 192i\sqrt{-23382 + 6485\sqrt{13}}$$

has minimal polynomial  $T^4 + 83008T^3 - 2652672T^2 + 21250048T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{13}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(4m + 14n)q^{8m^2 + 4mn + 7n^2} \\ &= 28q^7 + 20q^{11} + 36q^{19} + 12q^{31} + 44q^{47} - 76q^{59} - 84q^{63} + 4q^{67} - 68q^{71} - 92q^{83} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(416), \left(\frac{832}{\cdot}\right))$ .

---

#### $\mu$ -N4-10-4

Let  $\tau_0 = [14, -2, 1] = \frac{1}{14} + \frac{i\sqrt{13}}{14}$  with  $h(D) = 2$ . Then

$$t = t_P(\tau_0) = -20752 + 5760\sqrt{13} - 192i\sqrt{-23382 + 6485\sqrt{13}}$$

has minimal polynomial  $T^4 + 83008T^3 - 2652672T^2 + 21250048T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{13}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-4m + 14n)q^{8m^2 - 4mn + 7n^2} \\ &= 28q^7 + 20q^{11} + 36q^{19} + 12q^{31} + 44q^{47} - 76q^{59} - 84q^{63} + 4q^{67} - 68q^{71} - 92q^{83} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(416), \left(\frac{832}{\cdot}\right))$ .

---

#### $\mu$ -N4-11-1

Let  $\tau_0 = [20, 0, 1] = \frac{i}{2\sqrt{5}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 288 + 128\sqrt{5} - 576\sqrt{-2 + \sqrt{5}} - 256\sqrt{5(-2 + \sqrt{5})}$$

has minimal polynomial  $T^4 - 1152T^3 + 22528T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{5}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)20nq^{4m^2 + 5n^2} \\ &= 40q^5 + 80q^9 + 80q^{21} + 80q^{41} - 120q^{45} - 240q^{49} - 240q^{61} + 80q^{69} - 240q^{81} + 80q^{105} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(320), \left(\frac{320}{\cdot}\right))$ .

---

#### $\mu$ -N4-11-2

Let  $\tau_0 = [4, 0, 5] = \frac{i\sqrt{5}}{2}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 288 + 128\sqrt{5} + 576\sqrt{-2 + \sqrt{5}} + 256\sqrt{5(-2 + \sqrt{5})}$$

has minimal polynomial  $T^4 - 1152T^3 + 22528T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{5}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)4nq^{20m^2 + n^2} \\ &= 8q - 24q^9 + 16q^{21} + 40q^{25} - 48q^{29} + 80q^{45} - 56q^{49} - 112q^{69} + 88q^{81} - 48q^{89} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(320), \left(\frac{320}{\cdot}\right))$ .

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#### $\mu$ -N4-11-3

Let  $\tau_0 = [12, 8, 3] = -\frac{1}{3} + \frac{i\sqrt{5}}{6}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 288 - 128\sqrt{5} + 576i\sqrt{2 + \sqrt{5}} - 256i\sqrt{5(2 + \sqrt{5})}$$

has minimal polynomial  $T^4 - 1152T^3 + 22528T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{5}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(16m + 12n)q^{12m^2 + 8mn + 3n^2} \\ &= 24q^3 - 8q^7 - 40q^{15} + 56q^{23} - 80q^{27} - 40q^{35} + 56q^{43} + 88q^{47} - 56q^{63} + 88q^{67} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(320), \left(\frac{320}{\cdot}\right))$ .

---

#### $\mu$ -N4-11-4

Let  $\tau_0 = [12, -8, 3] = \frac{1}{3} + \frac{i\sqrt{5}}{6}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 288 - 128\sqrt{5} - 576i\sqrt{2 + \sqrt{5}} + 256i\sqrt{5(2 + \sqrt{5})}$$

has minimal polynomial  $T^4 - 1152T^3 + 22528T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{5}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-16m + 12n)q^{12m^2 - 8mn + 3n^2} \\ &= 24q^3 - 8q^7 - 40q^{15} + 56q^{23} - 80q^{27} - 40q^{35} + 56q^{43} + 88q^{47} - 56q^{63} + 88q^{67} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(320), \left(\frac{320}{\cdot}\right))$ .

---

#### $\mu$ -N4-12-1

Let  $\tau_0 = [24, 20, 5] = -\frac{5}{12} + \frac{i\sqrt{5}}{12}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - 144\sqrt{38 + 17\sqrt{5}} + 64\sqrt{190 + 85\sqrt{5}}$$

has minimal polynomial  $T^4 - 32T^3 + 1408T^2 - 18432T + 4096$ .

We have  $\mu(t) = \frac{\sqrt{5}}{2\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(40m + 24n)q^{10m^2 + 10mn + 3n^2} \\ &= 16q^3 - 48q^7 + 80q^{15} + 16q^{23} - 160q^{27} + 80q^{35} + 144q^{43} - 112q^{47} - 16q^{63} - 48q^{67} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(160), \left(\frac{80}{\cdot}\right))$ .

---

#### $\mu$ -N4-12-2



Let  $\tau_0 = [24, 4, 1] = -\frac{1}{12} + \frac{i\sqrt{5}}{12}$  with  $h(D) = 4$ . Then  
 $t = t_P(\tau_0) = 8 + 144\sqrt{38 + 17\sqrt{5}} - 64\sqrt{190 + 85\sqrt{5}}$   
has minimal polynomial  $T^4 - 32T^3 + 1408T^2 - 18432T + 4096$ .  
We have  $\mu(t) = \frac{\sqrt{5}}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where  

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 24n)q^{2m^2 + 2mn + 3n^2}$$

$$= 80q^3 + 80q^7 + 80q^{15} - 240q^{23} - 160q^{27} - 240q^{35} + 80q^{43} - 240q^{47} + 240q^{63} + 400q^{67} + \dots$$
  
is in  $\mathcal{S}_2(\Gamma_0(160), \left(\frac{80}{\cdot}\right))$ .

---

#### $\mu$ -N4-12-3

Let  $\tau_0 = [8, 4, 3] = -\frac{1}{4} + \frac{i\sqrt{5}}{4}$  with  $h(D) = 4$ . Then  
 $t = t_P(\tau_0) = 8 + 144i\sqrt{-38 + 17\sqrt{5}} + 64i\sqrt{-190 + 85\sqrt{5}}$   
has minimal polynomial  $T^4 - 32T^3 + 1408T^2 - 18432T + 4096$ .  
We have  $\mu(t) = \frac{\sqrt{5}}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where  

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 8n)q^{6m^2 + 2mn + n^2}$$

$$= 16q - 48q^9 - 32q^{21} + 80q^{25} + 96q^{29} - 160q^{45} - 112q^{49} + 224q^{69} + 176q^{81} - 96q^{89} + \dots$$
  
is in  $\mathcal{S}_2(\Gamma_0(160), \left(\frac{80}{\cdot}\right))$ .

---

#### $\mu$ -N4-12-4

Let  $\tau_0 = [8, -4, 3] = \frac{1}{4} + \frac{i\sqrt{5}}{4}$  with  $h(D) = 4$ . Then  
 $t = t_P(\tau_0) = 8 - 144i\sqrt{-38 + 17\sqrt{5}} - 64i\sqrt{-190 + 85\sqrt{5}}$   
has minimal polynomial  $T^4 - 32T^3 + 1408T^2 - 18432T + 4096$ .  
We have  $\mu(t) = \frac{\sqrt{5}}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where  

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-8m + 8n)q^{6m^2 - 2mn + n^2}$$

$$= 16q - 48q^9 - 32q^{21} + 80q^{25} + 96q^{29} - 160q^{45} - 112q^{49} + 224q^{69} + 176q^{81} - 96q^{89} + \dots$$
  
is in  $\mathcal{S}_2(\Gamma_0(160), \left(\frac{80}{\cdot}\right))$ .

---

#### $\mu$ -N4-13-1

Let  $\tau_0 = [2, -2, 13] = \frac{1}{2} + \frac{5i}{2}$  with  $h(D) = 2$ . Then  
 $t = t_P(\tau_0) = -1658896 - 1109376\sqrt[4]{5} - 741888\sqrt{5} - 496128\sqrt[4]{125}$   
has minimal polynomial  $T^4 + 6635584T^3 - 212335104T^2 + 1698709504T + 65536$ .  
We have  $\mu(t) = \frac{20}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where  

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-4m + 2n)q^{104m^2 - 4mn + n^2}$$

$$= 4q - 12q^9 + 20q^{25} - 28q^{49} + 36q^{81} - 8q^{101} + 24q^{109} - 44q^{121} - 40q^{125} + 56q^{149} + \dots$$
  
is in  $\mathcal{S}_2(\Gamma_0(800), \left(\frac{1600}{\cdot}\right))$ .

---

#### $\mu$ -N4-13-2

Let  $\tau_0 = [50, 50, 13] = -\frac{1}{2} + \frac{i}{10}$  with  $h(D) = 2$ . Then  
 $t = t_P(\tau_0) = -1658896 + 1109376\sqrt[4]{5} - 741888\sqrt{5} + 496128\sqrt[4]{125}$   
has minimal polynomial  $T^4 + 6635584T^3 - 212335104T^2 + 1698709504T + 65536$ .  
We have  $\mu(t) = \frac{20}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where  

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(100m + 50n)q^{104m^2 + 100mn + 25n^2}$$

$$= 100q^{25} - 200q^{29} + 200q^{41} - 200q^{61} + 200q^{89} - 200q^{125} + 200q^{169} - 200q^{221} - 300q^{225} + 600q^{229} + \dots$$
  
is in  $\mathcal{S}_2(\Gamma_0(800), \left(\frac{1600}{\cdot}\right))$ .

---

#### $\mu$ -N4-13-3

Let  $\tau_0 = [26, 2, 1] = -\frac{1}{26} + \frac{5i}{26}$  with  $h(D) = 2$ . Then  
 $t = t_P(\tau_0) = -1658896 + 1109376i\sqrt[4]{5} + 741888\sqrt{5} - 496128i\sqrt[4]{125}$   
has minimal polynomial  $T^4 + 6635584T^3 - 212335104T^2 + 1698709504T + 65536$ .  
We have  $\mu(t) = \frac{20}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where  

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(4m + 26n)q^{8m^2 + 4mn + 13n^2}$$

$$= 52q^{13} + 44q^{17} + 60q^{25} + 36q^{37} + 68q^{53} + 28q^{73} + 76q^{97} - 148q^{113} - 156q^{117} - 120q^{125} + \dots$$
  
is in  $\mathcal{S}_2(\Gamma_0(800), \left(\frac{1600}{\cdot}\right))$ .

---

#### $\mu$ -N4-13-4

Let  $\tau_0 = [26, -2, 1] = \frac{1}{26} + \frac{5i}{26}$  with  $h(D) = 2$ . Then  
 $t = t_P(\tau_0) = -1658896 - 1109376i\sqrt[4]{5} + 741888\sqrt{5} + 496128i\sqrt[4]{125}$   
has minimal polynomial  $T^4 + 6635584T^3 - 212335104T^2 + 1698709504T + 65536$ .  
We have  $\mu(t) = \frac{20}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where  

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-4m + 26n)q^{8m^2 - 4mn + 13n^2}$$

$$= 52q^{13} + 44q^{17} + 60q^{25} + 36q^{37} + 68q^{53} + 28q^{73} + 76q^{97} - 148q^{113} - 156q^{117} - 120q^{125} + \dots$$
  
is in  $\mathcal{S}_2(\Gamma_0(800), \left(\frac{1600}{\cdot}\right))$ .

---

#### $\mu$ -N4-14-1

Let  $\tau_0 = [4, -4, 9] = \frac{1}{2} + i\sqrt{2}$  with  $h(D) = 4$ . Then  
 $t = t_P(\tau_0) = -1792 - 1280\sqrt{2} - 640\sqrt{1 + 5\sqrt{2}} - 448\sqrt{2 + 10\sqrt{2}}$   
has minimal polynomial  $T^4 + 7168T^3 - 376832T^2 + 8388608T - 67108864$ .  
We have  $\mu(t) = \frac{4\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where  

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-8m + 4n)q^{36m^2 - 4mn + n^2}$$

$$= 8q - 24q^9 + 40q^{25} - 16q^{33} + 48q^{41} - 56q^{49} - 80q^{57} + 184q^{81} - 144q^{113} - 88q^{121} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(512), \left(\frac{512}{\cdot}\right))$ .

---

**$\mu$ -N4-14-2**

Let  $\tau_0 = [36, 4, 1] = -\frac{1}{18} + \frac{i\sqrt{2}}{9}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -1792 - 1280\sqrt{2} + 640\sqrt{1+5\sqrt{2}} + 448\sqrt{2+10\sqrt{2}}$$

has minimal polynomial  $T^4 + 7168T^3 - 376832T^2 + 8388608T - 67108864$ .

We have  $\mu(t) = \frac{4\sqrt{2}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m+36n)q^{4m^2+4mn+9n^2} \\ &= 128q^9 + 128q^{17} + 128q^{33} + 128q^{57} - 384q^{73} - 384q^{81} + 128q^{89} - 384q^{97} - 384q^{121} + 128q^{129} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(512), \left(\frac{512}{\cdot}\right))$ .

---

**$\mu$ -N4-14-3**

Let  $\tau_0 = [12, 4, 3] = -\frac{1}{6} + \frac{i\sqrt{2}}{3}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -1792 + 1280\sqrt{2} + 640i\sqrt{-1+5\sqrt{2}} - 448i\sqrt{-2+10\sqrt{2}}$$

has minimal polynomial  $T^4 + 7168T^3 - 376832T^2 + 8388608T - 67108864$ .

We have  $\mu(t) = \frac{4\sqrt{2}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m+12n)q^{12m^2+4mn+3n^2} \\ &= 24q^3 + 8q^{11} + 40q^{19} - 128q^{27} - 8q^{43} - 128q^{51} + 56q^{59} + 104q^{67} + 120q^{75} + 88q^{83} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(512), \left(\frac{512}{\cdot}\right))$ .

---

**$\mu$ -N4-14-4**

Let  $\tau_0 = [12, -4, 3] = \frac{1}{6} + \frac{i\sqrt{2}}{3}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = -1792 + 1280\sqrt{2} - 640i\sqrt{-1+5\sqrt{2}} + 448i\sqrt{-2+10\sqrt{2}}$$

has minimal polynomial  $T^4 + 7168T^3 - 376832T^2 + 8388608T - 67108864$ .

We have  $\mu(t) = \frac{4\sqrt{2}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-8m+12n)q^{12m^2-4mn+3n^2} \\ &= 24q^3 + 8q^{11} + 40q^{19} - 128q^{27} - 8q^{43} - 128q^{51} + 56q^{59} + 104q^{67} + 120q^{75} + 88q^{83} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(512), \left(\frac{512}{\cdot}\right))$ .

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**$\mu$ -N4-15-1**

Let  $\tau_0 = [36, 0, 1] = \frac{i}{6}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 3104 - 1664\sqrt[4]{12} + 1792\sqrt{3} - 960\sqrt[4]{108}$$

has minimal polynomial  $T^4 + 7168T^3 - 376832T^2 + 8388608T - 67108864$ .

We have  $\mu(t) = \frac{6}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)36nq^{4m^2+9n^2} \\ &= 72q^9 + 144q^{13} + 144q^{25} + 144q^{45} + 144q^{73} - 216q^{81} - 432q^{85} - 432q^{97} + 144q^{109} - 432q^{117} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(576), \left(\frac{576}{\cdot}\right))$ .

---

**$\mu$ -N4-15-2**

Let  $\tau_0 = [4, 0, 9] = \frac{3i}{2}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 3104 + 1664\sqrt[4]{12} + 1792\sqrt{3} + 960\sqrt[4]{108}$$

has minimal polynomial  $T^4 - 12416T^3 + 202752T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{6}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)4nq^{36m^2+n^2} \\ &= 8q - 24q^9 + 40q^{25} + 16q^{37} - 48q^{45} - 56q^{49} + 80q^{61} + 72q^{81} - 112q^{85} + 144q^{117} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(576), \left(\frac{576}{\cdot}\right))$ .

---

**$\mu$ -N4-15-3**

Let  $\tau_0 = [20, 16, 5] = -\frac{2}{5} + \frac{3i}{10}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 3104 + 1664i\sqrt[4]{12} - 1792\sqrt{3} - 960i\sqrt[4]{108}$$

has minimal polynomial  $T^4 - 12416T^3 + 202752T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{6}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(32m+20n)q^{20m^2+16mn+5n^2} \\ &= 40q^5 - 24q^9 - 56q^{17} + 8q^{29} + 104q^{41} - 48q^{45} - 88q^{53} + 144q^{65} + 72q^{81} - 88q^{89} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(576), \left(\frac{576}{\cdot}\right))$ .

---

**$\mu$ -N4-15-4**

Let  $\tau_0 = [20, -16, 5] = \frac{2}{5} + \frac{3i}{10}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 3104 - 1664i\sqrt[4]{12} - 1792\sqrt{3} + 960i\sqrt[4]{108}$$

has minimal polynomial  $T^4 - 12416T^3 + 202752T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{6}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-32m+20n)q^{20m^2-16mn+5n^2} \\ &= 40q^5 - 24q^9 - 56q^{17} + 8q^{29} + 104q^{41} - 48q^{45} - 88q^{53} + 144q^{65} + 72q^{81} - 88q^{89} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(576), \left(\frac{576}{\cdot}\right))$ .

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**$\mu$ -N4-16-1**

Let  $\tau_0 = [40, 36, 9] = -\frac{9}{20} + \frac{3i}{20}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - 32\sqrt[4]{12} + 16\sqrt[4]{108}$$

has minimal polynomial  $T^4 - 32T^3 + 12672T^2 - 198656T + 4096$ .

We have  $\mu(t) = \frac{3}{2\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(72m+40n)q^{18m^2+18mn+5n^2}$   
 $= 16q^5 - 48q^9 + 80q^{17} - 112q^{29} + 16q^{41} + 96q^{45} + 80q^{53} - 288q^{65} + 144q^{81} + 208q^{89} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(288), (\frac{144}{\cdot}))$ .

#### $\mu$ -N4-16-2

Let  $\tau_0 = [40, 4, 1] = -\frac{1}{20} + \frac{3i}{20}$  with  $h(D) = 4$ . Then  
 $t = t_P(\tau_0) = 8 + 32\sqrt[4]{12} - 16\sqrt[4]{108}$   
has minimal polynomial  $T^4 - 32T^3 + 12672T^2 - 198656T + 4096$ .  
We have  $\mu(t) = \frac{3}{2\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where  
 $\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m+40n)q^{2m^2+2mn+5n^2}$   
 $= 144q^5 + 144q^9 + 144q^{17} + 144q^{29} - 432q^{41} - 288q^{45} - 432q^{53} - 288q^{65} - 432q^{81} + 144q^{89} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(288), (\frac{144}{\cdot}))$ .

#### $\mu$ -N4-16-3

Let  $\tau_0 = [8, 4, 5] = -\frac{1}{4} + \frac{3i}{4}$  with  $h(D) = 4$ . Then  
 $t = t_P(\tau_0) = 8 + 32i\sqrt[4]{12} + 16i\sqrt[4]{108}$   
has minimal polynomial  $T^4 - 32T^3 + 12672T^2 - 198656T + 4096$ .  
We have  $\mu(t) = \frac{3}{2\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where  
 $\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m+8n)q^{10m^2+2mn+n^2}$   
 $= 16q - 48q^9 + 80q^{25} - 32q^{37} + 96q^{45} - 112q^{49} - 160q^{61} + 144q^{81} + 224q^{85} - 288q^{117} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(288), (\frac{144}{\cdot}))$ .

#### $\mu$ -N4-16-4

Let  $\tau_0 = [8, -4, 5] = \frac{1}{4} + \frac{3i}{4}$  with  $h(D) = 4$ . Then  
 $t = t_P(\tau_0) = 8 - 32i\sqrt[4]{12} - 16i\sqrt[4]{108}$   
has minimal polynomial  $T^4 - 32T^3 + 12672T^2 - 198656T + 4096$ .  
We have  $\mu(t) = \frac{3}{2\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where  
 $\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-8m+8n)q^{10m^2-2mn+n^2}$   
 $= 16q - 48q^9 + 80q^{25} - 32q^{37} + 96q^{45} - 112q^{49} - 160q^{61} + 144q^{81} + 224q^{85} - 288q^{117} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(288), (\frac{144}{\cdot}))$ .

#### $\mu$ -N4-17-1

Let  $\tau_0 = [100, 0, 1] = \frac{i}{10}$  with  $h(D) = 4$ . Then  
 $t = t_P(\tau_0) = 1658912 - 1109376\sqrt[4]{5} + 741888\sqrt{5} - 496128\sqrt[4]{125}$   
has minimal polynomial  $T^4 - 6635648T^3 + 106174464T^2 - 131072T + 1048576$ .  
We have  $\mu(t) = \frac{10}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where  
 $\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)100nq^{4m^2+25n^2}$   
 $= 200q^{25} + 400q^{29} + 400q^{41} + 400q^{61} + 400q^{89} + 400q^{125} + 400q^{169} + 400q^{221} - 600q^{225} - 1200q^{229} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(1600), (\frac{1600}{\cdot}))$ .

#### $\mu$ -N4-17-2

Let  $\tau_0 = [4, 0, 25] = \frac{5i}{2}$  with  $h(D) = 4$ . Then  
 $t = t_P(\tau_0) = 1658912 + 1109376\sqrt[4]{5} + 741888\sqrt{5} + 496128\sqrt[4]{125}$   
has minimal polynomial  $T^4 - 6635648T^3 + 106174464T^2 - 131072T + 1048576$ .  
We have  $\mu(t) = \frac{10}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where  
 $\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)4nq^{100m^2+n^2}$   
 $= 8q - 24q^9 + 40q^{25} - 56q^{49} + 72q^{81} + 16q^{101} - 48q^{109} - 88q^{121} + 80q^{125} - 112q^{149} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(1600), (\frac{1600}{\cdot}))$ .

#### $\mu$ -N4-17-3

Let  $\tau_0 = [52, 48, 13] = -\frac{6}{13} + \frac{5i}{26}$  with  $h(D) = 4$ . Then  
 $t = t_P(\tau_0) = 1658912 + 1109376i\sqrt[4]{5} - 741888\sqrt{5} - 496128i\sqrt[4]{125}$   
has minimal polynomial  $T^4 - 6635648T^3 + 106174464T^2 - 131072T + 1048576$ .  
We have  $\mu(t) = \frac{10}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where  
 $\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(96m+52n)q^{52m^2+48mn+13n^2}$   
 $= 104q^{13} - 88q^{17} - 120q^{25} + 72q^{37} + 136q^{53} - 56q^{73} - 152q^{97} + 296q^{113} - 312q^{117} - 240q^{125} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(1600), (\frac{1600}{\cdot}))$ .

#### $\mu$ -N4-17-4

Let  $\tau_0 = [52, -48, 13] = \frac{6}{13} + \frac{5i}{26}$  with  $h(D) = 4$ . Then  
 $t = t_P(\tau_0) = 1658912 - 1109376i\sqrt[4]{5} - 741888\sqrt{5} + 496128i\sqrt[4]{125}$   
has minimal polynomial  $T^4 - 6635648T^3 + 106174464T^2 - 131072T + 1048576$ .  
We have  $\mu(t) = \frac{10}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where  
 $\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-96m+52n)q^{52m^2-48mn+13n^2}$   
 $= 104q^{13} - 88q^{17} - 120q^{25} + 72q^{37} + 136q^{53} - 56q^{73} - 152q^{97} + 296q^{113} - 312q^{117} - 240q^{125} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(1600), (\frac{1600}{\cdot}))$ .

#### $\mu$ -N4-18-1

Let  $\tau_0 = [2, -2, 19] = \frac{1}{2} + \frac{i\sqrt{37}}{2}$  with  $h(D) = 2$ . Then  
 $t = t_P(\tau_0) = -49787152 - 8184960\sqrt{37} - 1344\sqrt{2744518518 + 451196065\sqrt{37}}$   
has minimal polynomial  $T^4 + 199148608T^3 - 6372751872T^2 + 50982043648T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{37}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-4m+2n)q^{152m^2-4mn+n^2} \\ &= 4q - 12q^9 + 20q^{25} - 28q^{49} + 36q^{81} - 44q^{121} - 8q^{149} + 24q^{157} + 52q^{169} - 40q^{173} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1184), (\frac{2368}{\cdot}))$ .

---

#### $\mu$ -N4-18-2

Let  $\tau_0 = [74, 74, 19] = -\frac{1}{2} + \frac{i}{2\sqrt{37}}$  with  $h(D) = 2$ . Then

$t = t_P(\tau_0) = -49787152 - 8184960\sqrt{37} + 1344\sqrt{2744518518 + 451196065\sqrt{37}}$   
has minimal polynomial  $T^4 + 199148608T^3 - 6372751872T^2 + 50982043648T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{37}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(148m+74n)q^{152m^2+148mn+37n^2} \\ &= 148q^{37} - 296q^{41} + 296q^{53} - 296q^{73} + 296q^{101} - 296q^{137} + 296q^{181} - 296q^{233} + 296q^{293} - 444q^{333} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1184), (\frac{2368}{\cdot}))$ .

---

#### $\mu$ -N4-18-3

Let  $\tau_0 = [38, 2, 1] = -\frac{1}{38} + \frac{i\sqrt{37}}{38}$  with  $h(D) = 2$ . Then

$t = t_P(\tau_0) = -49787152 + 8184960\sqrt{37} + 1344i\sqrt{-2744518518 + 451196065\sqrt{37}}$   
has minimal polynomial  $T^4 + 199148608T^3 - 6372751872T^2 + 50982043648T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{37}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(4m+38n)q^{8m^2+4mn+19n^2} \\ &= 76q^{19} + 68q^{23} + 84q^{31} + 60q^{43} + 92q^{59} + 52q^{79} + 100q^{103} + 44q^{131} + 108q^{163} - 220q^{167} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1184), (\frac{2368}{\cdot}))$ .

---

#### $\mu$ -N4-18-4

Let  $\tau_0 = [38, -2, 1] = \frac{1}{38} + \frac{i\sqrt{37}}{38}$  with  $h(D) = 2$ . Then

$t = t_P(\tau_0) = -49787152 + 8184960\sqrt{37} - 1344i\sqrt{-2744518518 + 451196065\sqrt{37}}$   
has minimal polynomial  $T^4 + 199148608T^3 - 6372751872T^2 + 50982043648T + 65536$ .

We have  $\mu(t) = \frac{4\sqrt{37}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-4m+38n)q^{8m^2-4mn+19n^2} \\ &= 76q^{19} + 68q^{23} + 84q^{31} + 60q^{43} + 92q^{59} + 52q^{79} + 100q^{103} + 44q^{131} + 108q^{163} - 220q^{167} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1184), (\frac{2368}{\cdot}))$ .

---

#### $\mu$ -N4-19-1

Let  $\tau_0 = [52, 0, 1] = \frac{i}{2\sqrt{13}}$  with  $h(D) = 4$ . Then

$t = t_P(\tau_0) = 20768 + 5760\sqrt{13} - 124608\sqrt{-18+5\sqrt{13}} - 34560\sqrt{13(-18+5\sqrt{13})}$

has minimal polynomial  $T^4 - 83072T^3 + 1333248T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{13}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)52nq^{4m^2+13n^2} \\ &= 104q^{13} + 208q^{17} + 208q^{29} + 208q^{49} + 208q^{77} + 208q^{113} - 312q^{117} - 624q^{121} - 624q^{133} - 624q^{153} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(832), (\frac{832}{\cdot}))$ .

---

#### $\mu$ -N4-19-2

Let  $\tau_0 = [4, 0, 13] = \frac{i\sqrt{13}}{2}$  with  $h(D) = 4$ . Then

$t = t_P(\tau_0) = 20768 + 5760\sqrt{13} + 124608\sqrt{-18+5\sqrt{13}} + 34560\sqrt{13(-18+5\sqrt{13})}$

has minimal polynomial  $T^4 - 83072T^3 + 1333248T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{13}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)4nq^{52m^2+n^2} \\ &= 8q - 24q^9 + 40q^{25} - 56q^{49} + 16q^{53} - 48q^{61} + 80q^{77} + 72q^{81} - 112q^{101} - 88q^{121} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(832), (\frac{832}{\cdot}))$ .

---

#### $\mu$ -N4-19-3

Let  $\tau_0 = [28, 24, 7] = -\frac{3}{7} + \frac{i\sqrt{13}}{14}$  with  $h(D) = 4$ . Then

$t = t_P(\tau_0) = 20768 - 5760\sqrt{13} + 124608i\sqrt{18+5\sqrt{13}} - 34560i\sqrt{13(18+5\sqrt{13})}$

has minimal polynomial  $T^4 - 83072T^3 + 1333248T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{13}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(48m+28n)q^{28m^2+24mn+7n^2} \\ &= 56q^7 - 40q^{11} - 72q^{19} + 24q^{31} + 88q^{47} + 152q^{59} - 168q^{63} - 8q^{67} - 136q^{71} + 184q^{83} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(832), (\frac{832}{\cdot}))$ .

---

#### $\mu$ -N4-19-4

Let  $\tau_0 = [28, -24, 7] = \frac{3}{7} + \frac{i\sqrt{13}}{14}$  with  $h(D) = 4$ . Then

$t = t_P(\tau_0) = 20768 - 5760\sqrt{13} - 124608i\sqrt{18+5\sqrt{13}} + 34560i\sqrt{13(18+5\sqrt{13})}$

has minimal polynomial  $T^4 - 83072T^3 + 1333248T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{13}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where

$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-48m + 28n)q^{28m^2 - 24mn + 7n^2}$   
 $= 56q^7 - 40q^{11} - 72q^{19} + 24q^{31} + 88q^{47} + 152q^{59} - 168q^{63} - 8q^{67} - 136q^{71} + 184q^{83} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(832), \left(\frac{832}{\cdot}\right))$ .

---

#### $\mu$ -N4-20-1

Let  $\tau_0 = [56, 52, 13] = -\frac{13}{28} + \frac{i\sqrt{13}}{28}$  with  $h(D) = 4$ . Then  
 $t = t_P(\tau_0) = 8 - 31152\sqrt{23382} + 6485\sqrt{13} + 8640\sqrt{303966} + 84305\sqrt{13}$   
has minimal polynomial  $T^4 - 32T^3 + 83328T^2 - 1329152T + 4096$ .  
We have  $\mu(t) = \frac{\sqrt{13}}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where  
 $\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(104m + 56n)q^{26m^2 + 26mn + 7n^2}$   
 $= 16q^7 - 48q^{11} + 80q^{19} - 112q^{31} + 144q^{47} + 16q^{59} - 48q^{63} - 176q^{67} + 80q^{71} - 112q^{83} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(416), \left(\frac{208}{\cdot}\right))$ .

---

#### $\mu$ -N4-20-2

Let  $\tau_0 = [56, 4, 1] = -\frac{1}{28} + \frac{i\sqrt{13}}{28}$  with  $h(D) = 4$ . Then  
 $t = t_P(\tau_0) = 8 + 31152\sqrt{23382} + 6485\sqrt{13} - 8640\sqrt{303966} + 84305\sqrt{13}$   
has minimal polynomial  $T^4 - 32T^3 + 83328T^2 - 1329152T + 4096$ .  
We have  $\mu(t) = \frac{\sqrt{13}}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where  
 $\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 56n)q^{2m^2 + 2mn + 7n^2}$   
 $= 208q^7 + 208q^{11} + 208q^{19} + 208q^{31} + 208q^{47} - 624q^{59} - 624q^{63} + 208q^{67} - 624q^{71} - 624q^{83} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(416), \left(\frac{208}{\cdot}\right))$ .

---

#### $\mu$ -N4-20-3

Let  $\tau_0 = [8, 4, 7] = -\frac{1}{4} + \frac{i\sqrt{13}}{4}$  with  $h(D) = 4$ . Then  
 $t = t_P(\tau_0) = 8 + 31152i\sqrt{-23382} + 6485\sqrt{13} + 8640i\sqrt{-303966} + 84305\sqrt{13}$   
has minimal polynomial  $T^4 - 32T^3 + 83328T^2 - 1329152T + 4096$ .  
We have  $\mu(t) = \frac{\sqrt{13}}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where  
 $\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 8n)q^{14m^2 + 2mn + n^2}$   
 $= 16q - 48q^9 + 80q^{25} - 112q^{49} - 32q^{53} + 96q^{61} - 160q^{77} + 144q^{81} + 224q^{101} - 176q^{121} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(416), \left(\frac{208}{\cdot}\right))$ .

---

#### $\mu$ -N4-20-4

Let  $\tau_0 = [8, -4, 7] = \frac{1}{4} + \frac{i\sqrt{13}}{4}$  with  $h(D) = 4$ . Then  
 $t = t_P(\tau_0) = 8 - 31152i\sqrt{-23382} + 6485\sqrt{13} - 8640i\sqrt{-303966} + 84305\sqrt{13}$   
has minimal polynomial  $T^4 - 32T^3 + 83328T^2 - 1329152T + 4096$ .  
We have  $\mu(t) = \frac{\sqrt{13}}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where  
 $\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-8m + 8n)q^{14m^2 - 2mn + n^2}$   
 $= 16q - 48q^9 + 80q^{25} - 112q^{49} - 32q^{53} + 96q^{61} - 160q^{77} + 144q^{81} + 224q^{101} - 176q^{121} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(416), \left(\frac{208}{\cdot}\right))$ .

---

#### $\mu$ -N4-21-1

Let  $\tau_0 = [104, 100, 25] = -\frac{25}{52} + \frac{5i}{52}$  with  $h(D) = 4$ . Then  
 $t = t_P(\tau_0) = 8 - 864\sqrt[4]{5} + 384\sqrt[4]{125}$   
has minimal polynomial  $T^4 - 32T^3 + 6635904T^2 - 106170368T + 4096$ .  
We have  $\mu(t) = \frac{5}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where  
 $\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(200m + 104n)q^{50m^2 + 50mn + 13n^2}$   
 $= 16q^{13} - 48q^{17} + 80q^{25} - 112q^{37} + 144q^{53} - 176q^{73} + 208q^{97} + 16q^{113} - 48q^{117} - 160q^{125} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(800), \left(\frac{400}{\cdot}\right))$ .

---

#### $\mu$ -N4-21-2

Let  $\tau_0 = [104, 4, 1] = -\frac{1}{52} + \frac{5i}{52}$  with  $h(D) = 4$ . Then  
 $t = t_P(\tau_0) = 8 + 864\sqrt[4]{5} - 384\sqrt[4]{125}$   
has minimal polynomial  $T^4 - 32T^3 + 6635904T^2 - 106170368T + 4096$ .  
We have  $\mu(t) = \frac{5}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where  
 $\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 104n)q^{2m^2 + 2mn + 13n^2}$   
 $= 400q^{13} + 400q^{17} + 400q^{25} + 400q^{37} + 400q^{53} + 400q^{73} + 400q^{97} - 1200q^{113} - 1200q^{117} - 800q^{125} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(800), \left(\frac{400}{\cdot}\right))$ .

---

#### $\mu$ -N4-21-3

Let  $\tau_0 = [8, 4, 13] = -\frac{1}{4} + \frac{5i}{4}$  with  $h(D) = 4$ . Then  
 $t = t_P(\tau_0) = 8 + 864i\sqrt[4]{5} + 384i\sqrt[4]{125}$   
has minimal polynomial  $T^4 - 32T^3 + 6635904T^2 - 106170368T + 4096$ .  
We have  $\mu(t) = \frac{5}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$ , where  
 $\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 8n)q^{26m^2 + 2mn + n^2}$   
 $= 16q - 48q^9 + 80q^{25} - 112q^{49} + 144q^{81} - 32q^{101} + 96q^{109} - 176q^{121} - 160q^{125} + 224q^{149} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(800), \left(\frac{400}{\cdot}\right))$ .

---

#### $\mu$ -N4-21-4

Let  $\tau_0 = [8, -4, 13] = \frac{1}{4} + \frac{5i}{4}$  with  $h(D) = 4$ . Then

$t = t_P(\tau_0) = 8 - 864i\sqrt[4]{5} - 384i\sqrt[4]{125}$   
has minimal polynomial  $T^4 - 32T^3 + 6635904T^2 - 106170368T + 4096$ .

We have  $\mu(t) = \frac{5}{2\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-8m + 8n)q^{26m^2 - 2mn + n^2} \\ &= 16q - 48q^9 + 80q^{25} - 112q^{49} + 144q^{81} - 32q^{101} + 96q^{109} - 176q^{121} - 160q^{125} + 224q^{149} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(800), (\frac{400}{\cdot}))$ .

---

#### $\mu$ -N4-22-1

Let  $\tau_0 = [152, 148, 37] = -\frac{37}{76} + \frac{i\sqrt{37}}{76}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - 522765264\sqrt{2744518518 + 451196065\sqrt{37}} + 85942080\sqrt{37(2744518518 + 451196065\sqrt{37})}$$

has minimal polynomial  $T^4 - 32T^3 + 199148928T^2 - 3186378752T + 4096$ .

We have  $\mu(t) = \frac{\sqrt{37}}{2\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(296m + 152n)q^{74m^2 + 74mn + 19n^2} \\ &= 16q^{19} - 48q^{23} + 80q^{31} - 112q^{43} + 144q^{59} - 176q^{79} + 208q^{103} - 240q^{131} + 272q^{163} + 16q^{167} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1184), (\frac{592}{\cdot}))$ .

---

#### $\mu$ -N4-22-2

Let  $\tau_0 = [152, 4, 1] = -\frac{1}{76} + \frac{i\sqrt{37}}{76}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + 522765264\sqrt{2744518518 + 451196065\sqrt{37}} - 85942080\sqrt{37(2744518518 + 451196065\sqrt{37})}$$

has minimal polynomial  $T^4 - 32T^3 + 199148928T^2 - 3186378752T + 4096$ .

We have  $\mu(t) = \frac{\sqrt{37}}{2\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 152n)q^{2m^2 + 2mn + 19n^2} \\ &= 592q^{19} + 592q^{23} + 592q^{31} + 592q^{43} + 592q^{59} + 592q^{79} + 592q^{103} + 592q^{131} + 592q^{163} - 1776q^{167} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1184), (\frac{592}{\cdot}))$ .

---

#### $\mu$ -N4-22-3

Let  $\tau_0 = [8, 4, 19] = -\frac{1}{4} + \frac{i\sqrt{37}}{4}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 + 522765264i\sqrt{-2744518518 + 451196065\sqrt{37}} + 85942080i\sqrt{37(-2744518518 + 451196065\sqrt{37})}$$

has minimal polynomial  $T^4 - 32T^3 + 199148928T^2 - 3186378752T + 4096$ .

We have  $\mu(t) = \frac{\sqrt{37}}{2\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 8n)q^{38m^2 + 2mn + n^2} \\ &= 16q - 48q^9 + 80q^{25} - 112q^{49} + 144q^{81} - 176q^{121} - 32q^{149} + 96q^{157} + 208q^{169} - 160q^{173} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1184), (\frac{592}{\cdot}))$ .

---

#### $\mu$ -N4-22-4

Let  $\tau_0 = [8, -4, 19] = \frac{1}{4} + \frac{i\sqrt{37}}{4}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 8 - 522765264i\sqrt{-2744518518 + 451196065\sqrt{37}} - 85942080i\sqrt{37(-2744518518 + 451196065\sqrt{37})}$$

has minimal polynomial  $T^4 - 32T^3 + 199148928T^2 - 3186378752T + 4096$ .

We have  $\mu(t) = \frac{\sqrt{37}}{2\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-8m + 8n)q^{38m^2 - 2mn + n^2} \\ &= 16q - 48q^9 + 80q^{25} - 112q^{49} + 144q^{81} - 176q^{121} - 32q^{149} + 96q^{157} + 208q^{169} - 160q^{173} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1184), (\frac{592}{\cdot}))$ .

---

#### $\mu$ -N4-23-1

Let  $\tau_0 = [148, 0, 1] = \frac{i}{2\sqrt{37}}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 49787168 + 8184960\sqrt{37} - 2091061056\sqrt{-882 + 145\sqrt{37}} - 343768320\sqrt{37(-882 + 145\sqrt{37})}$$

has minimal polynomial  $T^4 - 199148672T^3 + 3186382848T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{37}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)148nq^{4m^2 + 37n^2} \\ &= 296q^{37} + 592q^{41} + 592q^{53} + 592q^{73} + 592q^{101} + 592q^{137} + 592q^{181} + 592q^{233} + 592q^{293} - 888q^{333} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(2368), (\frac{2368}{\cdot}))$ .

---

#### $\mu$ -N4-23-2

Let  $\tau_0 = [4, 0, 37] = \frac{i\sqrt{37}}{2}$  with  $h(D) = 4$ . Then

$$t = t_P(\tau_0) = 49787168 + 8184960\sqrt{37} + 2091061056\sqrt{-882 + 145\sqrt{37}} + 343768320\sqrt{37(-882 + 145\sqrt{37})}$$

has minimal polynomial  $T^4 - 199148672T^3 + 3186382848T^2 - 131072T + 1048576$ .

We have  $\mu(t) = \frac{2\sqrt{37}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)4nq^{148m^2 + n^2} \\ &= 8q - 24q^9 + 40q^{25} - 56q^{49} + 72q^{81} - 88q^{121} + 16q^{149} - 48q^{157} + 104q^{169} + 80q^{173} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(2368), (\frac{2368}{\cdot}))$ .

---

#### $\mu$ -N4-23-3

Let  $\tau_0 = [76, 72, 19] = -\frac{9}{19} + \frac{i\sqrt{37}}{38}$  with  $h(D) = 4$ . Then

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 6nq^{3m^2+n^2}$$

$= 12q - 48q^7 + 24q^{13} + 96q^{19} - 60q^{25} - 48q^{31} - 120q^{37} + 96q^{43} + 108q^{49} + 168q^{61} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(36), \left(\frac{36}{\cdot}\right))$ .

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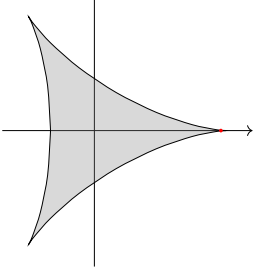
**$\nu$ -R-3**

Let  $\tau_0 = [9, 3, 1] = -\frac{1}{6} + \frac{i}{2\sqrt{3}}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = 24$$

has minimal polynomial  $T - 24$  and

$k = \sqrt[3]{t} \approx 2.884499140614816764643$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{3}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 18n)q^{m^2+mn+n^2} \\ &= 54q - 108q^4 - 54q^7 + 270q^{13} + 216q^{16} - 378q^{19} - 270q^{25} + 108q^{28} - 216q^{31} + 594q^{37} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(27), \left(\frac{9}{\cdot}\right))$ .

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**$\nu$ -R2-1-1 (#13 in the paper)**

Let  $\tau_0 = [3, 0, 2] = i\sqrt{\frac{2}{3}}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = 108 + 54\sqrt{2}$$

has minimal polynomial  $T^2 - 216T + 5832$  and

$k = \sqrt[3]{t} \approx 5.691518445260100656540$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9}{4\sqrt{2}\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)6nq^{6m^2+n^2} \\ &= 12q - 24q^4 + 24q^7 - 48q^{10} + 48q^{16} + 96q^{22} - 36q^{25} - 48q^{28} - 120q^{31} + 96q^{40} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(72), \left(\frac{72}{\cdot}\right))$ .

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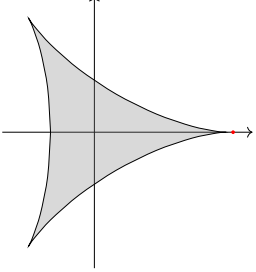
**$\nu$ -R2-1-2 (#13 in the paper)**

Let  $\tau_0 = [6, 0, 1] = \frac{i}{\sqrt{6}}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = 108 - 54\sqrt{2}$$

has minimal polynomial  $T^2 - 216T + 5832$  and

$k = \sqrt[3]{t} \approx 3.162600661514223642574$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9}{4\sqrt{2}\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)12nq^{3m^2+2n^2} \\ &= 24q^2 + 48q^5 - 48q^8 - 96q^{11} + 48q^{14} - 96q^{20} + 48q^{29} + 96q^{32} + 96q^{35} + 192q^{44} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(72), \left(\frac{72}{\cdot}\right))$ .

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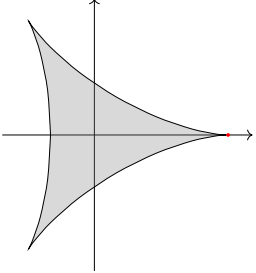
**$\nu$ -R2-2-1 (#14 in the paper)**

Let  $\tau_0 = [9, 0, 1] = \frac{i}{3}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = 18 + 6\sqrt{3}$$

has minimal polynomial  $T^2 - 36T + 216$  and

$k = \sqrt[3]{t} \approx 3.050705018258479587421$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{\sqrt{3}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

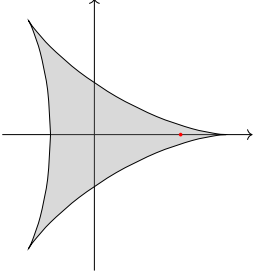
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)18nq^{m^2+n^2}$$



$= 36q + 72q^2 - 72q^4 - 72q^5 - 144q^8 + 72q^{10} - 144q^{13} + 144q^{16} + 360q^{17} + 144q^{20} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(36), (\frac{12}{\cdot}))$ .

**$\nu$ -R2-2-2 (#14 in the paper)**

Let  $\tau_0 = [9, 6, 2] = -\frac{1}{3} + \frac{i}{3}$  with  $h(D) = 2$ . Then  
 $t = t_Q(\tau_0) = 18 - 6\sqrt{3}$   
has minimal polynomial  $T^2 - 36T + 216$  and  
 $k = \sqrt[3]{t} \approx 1.966758491591281434257$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{\sqrt{3}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where  
 $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(18m + 18n)q^{2m^2 + 2mn + n^2}$   
 $= 36q - 36q^2 - 72q^4 + 36q^5 + 72q^8 + 72q^{10} - 144q^{13} + 144q^{16} - 180q^{17} - 72q^{20} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(36), (\frac{12}{\cdot}))$ .

**$\nu$ -R2-3-1 (#15 in the paper)**

Let  $\tau_0 = [1, 0, 1] = i$  with  $h(D) = 1$ . Then  
 $t = t_Q(\tau_0) = 270 + 162\sqrt{3}$   
has minimal polynomial  $T^2 - 540T - 5832$  and  
 $k = \sqrt[3]{t} \approx 8.196152422706631880582$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{27\sqrt{3}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where  
 $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)2nq^{9m^2 + n^2}$   
 $= 4q - 8q^4 + 8q^{10} - 16q^{13} + 16q^{16} + 12q^{25} - 40q^{34} + 8q^{37} - 16q^{40} + 28q^{49} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(36), (\frac{108}{\cdot}))$ .

**$\nu$ -R2-3-2 (#15 in the paper)**

Let  $\tau_0 = [2, 2, 1] = -\frac{1}{2} + \frac{i}{2}$  with  $h(D) = 1$ . Then  
 $t = t_Q(\tau_0) = 270 - 162\sqrt{3}$   
has minimal polynomial  $T^2 - 540T - 5832$  and  
 $k = \sqrt[3]{t} \approx -2.196152422706631880582$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{27\sqrt{3}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where  
 $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(6m + 4n)q^{9m^2 + 6mn + 2n^2}$   
 $= 8q^2 - 8q^5 - 16q^8 + 40q^{17} + 16q^{20} - 32q^{26} - 56q^{29} + 32q^{32} - 8q^{41} + 24q^{50} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(36), (\frac{108}{\cdot}))$ .

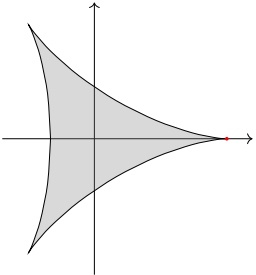
**$\nu$ -R2-4-1 (#16 in the paper)**

Let  $\tau_0 = [3, 0, 4] = \frac{2i}{\sqrt{3}}$  with  $h(D) = 2$ . Then  
 $t = t_Q(\tau_0) = 729 + 405\sqrt{3}$   
has minimal polynomial  $T^2 - 1458T + 39366$  and  
 $k = \sqrt[3]{t} \approx 11.26749364485270551553$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where  
 $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)6nq^{12m^2 + n^2}$   
 $= 12q - 24q^4 + 24q^{13} - 60q^{25} + 96q^{28} - 120q^{37} + 108q^{49} - 48q^{52} + 168q^{61} - 120q^{73} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(144), (\frac{144}{\cdot}))$ .

**$\nu$ -R2-4-2 (#16 in the paper)**

Let  $\tau_0 = [12, 0, 1] = \frac{i}{2\sqrt{3}}$  with  $h(D) = 2$ . Then  
 $t = t_Q(\tau_0) = 729 - 405\sqrt{3}$   
has minimal polynomial  $T^2 - 1458T + 39366$  and  
 $k = \sqrt[3]{t} \approx 3.019115822861089757232$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where  
 $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)24nq^{3m^2 + 4n^2}$   
 $= 48q^4 + 96q^7 - 192q^{19} - 192q^{28} + 96q^{31} - 192q^{43} + 96q^{52} + 384q^{67} + 384q^{76} + 96q^{79} + \dots$

is in  $\mathcal{S}_2(\Gamma_0(144), (\frac{144}{\cdot}))$ .

[ν-R2-5-1 \(#17 in the paper\)](#)

Let  $\tau_0 = [3, -3, 2] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{5}{3}}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = -\frac{27}{2} - \frac{27\sqrt{5}}{2}$$

has minimal polynomial  $T^2 + 27T - 729$  and

$k = \sqrt[3]{t} \approx -3.521954990115985529900$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{5}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m + 6n)q^{6m^2 - 3mn + n^2} \\ &= 12q - 36q^4 + 60q^{10} - 12q^{16} - 48q^{19} - 60q^{25} + 96q^{31} + 120q^{34} - 60q^{40} - 240q^{46} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(45), (\frac{45}{\cdot}))$ .

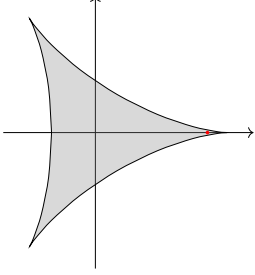
[ν-R2-5-2 \(#17 in the paper\)](#)

Let  $\tau_0 = [6, 3, 1] = -\frac{1}{4} + \frac{1}{4}i\sqrt{\frac{5}{3}}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = -\frac{27}{2} + \frac{27\sqrt{5}}{2}$$

has minimal polynomial  $T^2 + 27T - 729$  and

$k = \sqrt[3]{t} \approx 2.555398926237728751166$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{5}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 12n)q^{3m^2 + 3mn + 2n^2} \\ &= 30q^2 - 30q^5 - 30q^8 - 60q^{17} + 90q^{20} + 120q^{23} - 90q^{32} - 120q^{38} + 120q^{47} - 150q^{50} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(45), (\frac{45}{\cdot}))$ .

[ν-R2-6-1 \(#18 in the paper\)](#)

Let  $\tau_0 = [3, 0, 5] = i\sqrt{\frac{5}{3}}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = \frac{3375}{2} + \frac{1485\sqrt{5}}{2}$$

has minimal polynomial  $T^2 - 3375T + 91125$  and

$k = \sqrt[3]{t} \approx 14.95956587864062048278$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{5}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)6nq^{15m^2 + n^2} \\ &= 12q - 24q^4 + 72q^{16} - 48q^{19} - 60q^{25} + 96q^{31} - 120q^{40} + 84q^{49} + 24q^{61} + 24q^{64} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(180), (\frac{180}{\cdot}))$ .

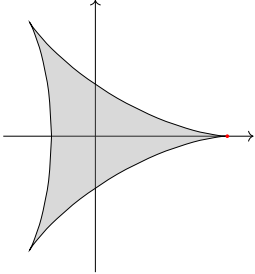
[ν-R2-6-2 \(#18 in the paper\)](#)

Let  $\tau_0 = [15, 0, 1] = \frac{i}{\sqrt{15}}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = \frac{3375}{2} - \frac{1485\sqrt{5}}{2}$$

has minimal polynomial  $T^2 - 3375T + 91125$  and

$k = \sqrt[3]{t} \approx 3.008108682100951536972$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{5}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)30nq^{3m^2 + 5n^2} \\ &= 60q^5 + 120q^8 + 120q^{17} - 120q^{20} - 240q^{23} - 120q^{32} - 240q^{47} + 120q^{53} - 240q^{68} + 360q^{80} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(180), (\frac{180}{\cdot}))$ .

[ν-R2-7-1 \(#19 in the paper\)](#)

Let  $\tau_0 = [3, -3, 7] = \frac{1}{2} + \frac{5i}{2\sqrt{3}}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = -4320 - 1944\sqrt{5}$$

has minimal polynomial  $T^2 + 8640T - 233280$  and

$k = \sqrt[3]{t} \approx -20.54099758874507351758$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{45}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m+6n)q^{21m^2-3mn+n^2}$$

$$= 12q - 24q^4 + 48q^{16} - 12q^{19} - 84q^{31} + 216q^{49} - 156q^{61} - 96q^{64} + 24q^{76} - 48q^{79} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(225), \left(\frac{225}{\cdot}\right))$ .

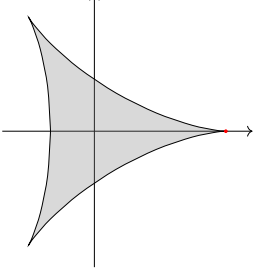
[ν-R2-7-2 \(#19 in the paper\)](#)

Let  $\tau_0 = [21, 3, 1] = -\frac{1}{14} + \frac{5i}{14\sqrt{3}}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = -4320 + 1944\sqrt{5}$$

has minimal polynomial  $T^2 + 8640T - 233280$  and

$k = \sqrt[3]{t} \approx 2.996891159467877124543$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{45}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m+42n)q^{3m^2+3mn+7n^2}$$

$$= 150q^7 + 150q^{13} - 300q^{28} - 300q^{37} + 150q^{43} - 300q^{52} + 150q^{67} - 300q^{73} + 150q^{97} + 600q^{103} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(225), \left(\frac{225}{\cdot}\right))$ .

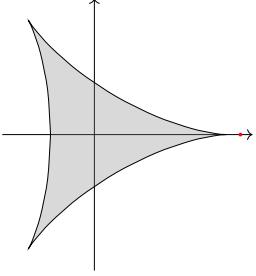
[ν-R2-8-1 \(#20 in the paper\)](#)

Let  $\tau_0 = [9, 0, 2] = \frac{i\sqrt{2}}{3}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = 32 + 2\sqrt{6}$$

has minimal polynomial  $T^2 - 64T + 1000$  and

$k = \sqrt[3]{t} \approx 3.329186449936013683413$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{\sqrt{\frac{3}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)18nq^{2m^2+n^2}$$

$$= 36q + 72q^3 - 72q^4 - 144q^6 + 72q^9 - 144q^{12} + 144q^{16} + 288q^{18} + 72q^{19} - 144q^{22} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(72), \left(\frac{24}{\cdot}\right))$ .

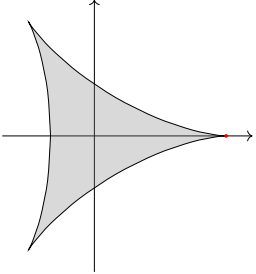
[ν-R2-8-2 \(#20 in the paper\)](#)

Let  $\tau_0 = [18, 0, 1] = \frac{i}{3\sqrt{2}}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = 32 - 2\sqrt{6}$$

has minimal polynomial  $T^2 - 64T + 1000$  and

$k = \sqrt[3]{t} \approx 3.003736843934408677123$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{\sqrt{\frac{3}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)36nq^{m^2+2n^2}$$

$$= 72q^2 + 144q^3 + 144q^6 - 144q^8 - 288q^9 + 144q^{11} - 288q^{12} - 288q^{17} + 144q^{18} - 288q^{24} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(72), \left(\frac{24}{\cdot}\right))$ .

[ν-R2-9-1 \(#21 in the paper\)](#)

Let  $\tau_0 = [1, 0, 2] = i\sqrt{2}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = 3672 + 1458\sqrt{6}$$

has minimal polynomial  $T^2 - 7344T + 729000$  and

$k = \sqrt[3]{t} \approx 19.34846922834953429459$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{27\sqrt{\frac{3}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 2nq^{18m^2+n^2} \\ = 4q - 8q^4 + 16q^{16} + 8q^{19} - 16q^{22} - 20q^{25} + 32q^{34} - 40q^{43} + 28q^{49} - 32q^{64} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(72), (\frac{216}{\cdot}))$ .

#### [ν-R2-9-2 \(#21 in the paper\)](#)

Let  $\tau_0 = [2, 0, 1] = \frac{i}{\sqrt{2}}$  with  $h(D) = 1$ . Then

$t = t_Q(\tau_0) = 3672 - 1458\sqrt{6}$   
has minimal polynomial  $T^2 - 7344T + 729000$  and  
 $k = \sqrt[3]{t} \approx 4.651530771650465705408$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{27\sqrt{\frac{3}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 4nq^{9m^2+2n^2} \\ = 8q^2 - 16q^8 + 16q^{11} - 32q^{17} + 32q^{32} + 16q^{38} + 64q^{41} - 32q^{44} - 40q^{50} - 80q^{59} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(72), (\frac{216}{\cdot}))$ .

#### [ν-R2-10-1 \(#22 in the paper\)](#)

Let  $\tau_0 = [3, -3, 5] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{17}{3}}$  with  $h(D) = 2$ . Then

$t = t_Q(\tau_0) = -864 - 216\sqrt{17}$   
has minimal polynomial  $T^2 + 1728T - 46656$  and  
 $k = \sqrt[3]{t} \approx -12.06123975553198228290$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{17}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

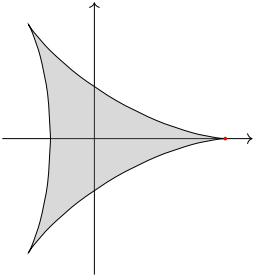
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-9m + 6n) q^{15m^2-3mn+n^2} \\ = 12q - 24q^4 - 12q^{13} + 48q^{16} + 60q^{19} - 144q^{25} + 132q^{43} + 84q^{49} + 24q^{52} - 204q^{55} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(153), (\frac{153}{\cdot}))$ .

#### [ν-R2-10-2 \(#22 in the paper\)](#)

Let  $\tau_0 = [15, 3, 1] = -\frac{1}{10} + \frac{1}{10}i\sqrt{\frac{17}{3}}$  with  $h(D) = 2$ . Then

$t = t_Q(\tau_0) = -864 + 216\sqrt{17}$   
has minimal polynomial  $T^2 + 1728T - 46656$  and  
 $k = \sqrt[3]{t} \approx 2.984767795822010538755$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{17}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (9m + 30n) q^{3m^2+3mn+5n^2} \\ = 102q^5 + 102q^{11} - 102q^{17} - 204q^{20} + 102q^{23} - 204q^{29} + 102q^{41} - 204q^{44} - 102q^{65} + 204q^{68} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(153), (\frac{153}{\cdot}))$ .

#### [ν-R2-11-1 \(#23 in the paper\)](#)

Let  $\tau_0 = [3, -3, 13] = \frac{1}{2} + \frac{7i}{2\sqrt{3}}$  with  $h(D) = 2$ . Then

$t = t_Q(\tau_0) = -163296 - 35640\sqrt{21}$   
has minimal polynomial  $T^2 + 326592T - 8817984$  and  
 $k = \sqrt[3]{t} \approx -68.86742011984472704913$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{63}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

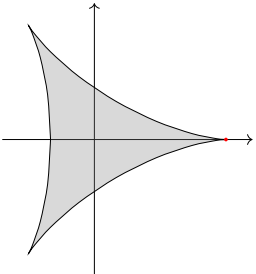
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-9m + 6n) q^{39m^2-3mn+n^2} \\ = 12q - 24q^4 + 48q^{16} - 60q^{25} - 12q^{37} + 60q^{43} - 96q^{64} + 132q^{67} - 156q^{79} + 120q^{100} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(441), (\frac{441}{\cdot}))$ .

#### [ν-R2-11-2 \(#23 in the paper\)](#)

Let  $\tau_0 = [39, 3, 1] = -\frac{1}{26} + \frac{7i}{26\sqrt{3}}$  with  $h(D) = 2$ . Then

$t = t_Q(\tau_0) = -163296 + 35640\sqrt{21}$   
has minimal polynomial  $T^2 + 326592T - 8817984$  and  
 $k = \sqrt[3]{t} \approx 2.999917339431239738841$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{63}{8\pi^2}L(\Theta_Q, \tau_0, 2)$ , where

$$\begin{aligned}\Theta_{Q, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-3}(n)(9m + 78n)q^{3m^2 + 3mn + 13n^2} \\ &= 294q^{13} + 294q^{19} + 294q^{31} - 588q^{52} - 588q^{61} + 294q^{73} - 588q^{76} - 588q^{97} + 294q^{103} - 588q^{124} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(441), (\frac{441}{\cdot}))$ .

[ν-R2-12-1 \(#24 in the paper\)](#)

Let  $\tau_0 = [9, -9, 5] = \frac{1}{2} + \frac{i\sqrt{11}}{6}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = 4 - 4\sqrt{33}$$

has minimal polynomial  $T^2 - 8T - 512$  and

$k = \sqrt[3]{t} \approx -2.667383081517582729725$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{\sqrt{33}}{16\pi^2}L(\Theta_Q, \tau_0, 2)$ , where

$$\begin{aligned}\Theta_{Q, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-3}(n)(-27m + 18n)q^{5m^2 - 3mn + n^2} \\ &= 36q - 36q^3 - 72q^4 + 180q^9 + 72q^{12} - 396q^{15} + 144q^{16} - 216q^{25} + 288q^{27} + 180q^{31} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(99), (\frac{33}{\cdot}))$ .

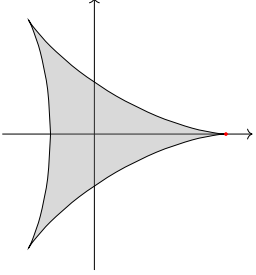
[ν-R2-12-2 \(#24 in the paper\)](#)

Let  $\tau_0 = [27, 3, 1] = -\frac{1}{18} + \frac{i\sqrt{11}}{18}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = 4 + 4\sqrt{33}$$

has minimal polynomial  $T^2 - 8T - 512$  and

$k = \sqrt[3]{t} \approx 2.999194249762008163998$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{\sqrt{33}}{8\pi^2}L(\Theta_Q, \tau_0, 2)$ , where

$$\begin{aligned}\Theta_{Q, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-3}(n)(9m + 54n)q^{m^2 + mn + 3n^2} \\ &= 198q^3 + 198q^5 + 198q^9 - 198q^{11} - 396q^{12} - 198q^{15} - 396q^{20} + 198q^{23} - 396q^{27} + 198q^{33} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(99), (\frac{33}{\cdot}))$ .

[ν-R2-13-1 \(#25 in the paper\)](#)

Let  $\tau_0 = [1, -1, 3] = \frac{1}{2} + \frac{i\sqrt{11}}{2}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = -16740 - 2916\sqrt{33}$$

has minimal polynomial  $T^2 + 33480T - 373248$  and

$k = \sqrt[3]{t} \approx -32.23368793961408597955$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{27\sqrt{33}}{16\pi^2}L(\Theta_Q, \tau_0, 2)$ , where

$$\begin{aligned}\Theta_{Q, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-3}(n)(-3m + 2n)q^{27m^2 - 3mn + n^2} \\ &= 4q - 8q^4 + 16q^{16} - 24q^{25} + 20q^{31} - 28q^{37} + 28q^{49} + 44q^{55} - 32q^{64} - 52q^{67} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(99), (\frac{297}{\cdot}))$ .

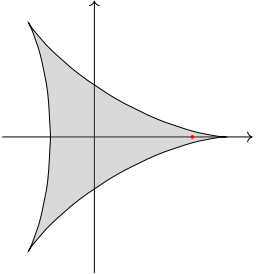
[ν-R2-13-2 \(#25 in the paper\)](#)

Let  $\tau_0 = [5, 3, 1] = -\frac{3}{10} + \frac{i\sqrt{11}}{10}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = -16740 + 2916\sqrt{33}$$

has minimal polynomial  $T^2 + 33480T - 373248$  and

$k = \sqrt[3]{t} \approx 2.233687939614085979552$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{27\sqrt{33}}{8\pi^2}L(\Theta_Q, \tau_0, 2)$ , where

$$\begin{aligned}\Theta_{Q, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-3}(n)(9m + 10n)q^{9m^2 + 9mn + 5n^2} \\ &= 22q^5 - 22q^{11} - 44q^{20} + 22q^{23} + 44q^{44} - 44q^{47} + 88q^{53} + 22q^{59} - 110q^{71} + 88q^{80} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(99), (\frac{297}{\cdot}))$ .

[ν-R2-14-1 \(#26 in the paper\)](#)

Let  $\tau_0 = [3, -3, 11] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{41}{3}}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = -55296 - 8640\sqrt{41}$$

has minimal polynomial  $T^2 + 110592T - 2985984$  and

$k = \sqrt[3]{t} \approx -48.00390497909782603051$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{41}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m+6n)q^{33m^2-3mn+n^2} \\ &= 12q - 24q^4 + 48q^{16} - 60q^{25} - 12q^{31} + 60q^{37} - 84q^{43} + 84q^{49} + 132q^{61} - 96q^{64} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(369), (\frac{369}{\cdot}))$ .

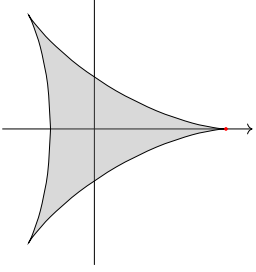
**$\nu$ -R2-14-2 (#26 in the paper)**

Let  $\tau_0 = [33, 3, 1] = -\frac{1}{22} + \frac{1}{22}i\sqrt{\frac{41}{3}}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = -55296 + 8640\sqrt{41}$$

has minimal polynomial  $T^2 + 110592T - 2985984$  and

$k = \sqrt[3]{t} \approx 2.999755958660059445454$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{41}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m+66n)q^{3m^2+3mn+11n^2} \\ &= 246q^{11} + 246q^{17} + 246q^{29} - 246q^{41} - 492q^{44} + 246q^{47} - 492q^{53} - 492q^{68} + 246q^{71} - 492q^{89} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(369), (\frac{369}{\cdot}))$ .

**$\nu$ -R2-15-1 (#27 in the paper)**

Let  $\tau_0 = [3, -3, 23] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{89}{3}}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = -13500000 - 1431000\sqrt{89}$$

has minimal polynomial  $T^2 + 27000000T - 729000000$  and

$k = \sqrt[3]{t} \approx -300.0000999998666669519$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{89}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m+6n)q^{69m^2-3mn+n^2} \\ &= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} - 12q^{67} + 60q^{73} - 84q^{79} + 132q^{97} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(801), (\frac{801}{\cdot}))$ .

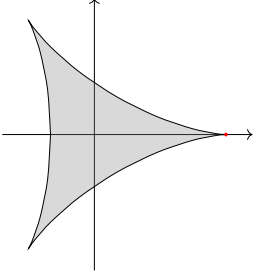
**$\nu$ -R2-15-2 (#27 in the paper)**

Let  $\tau_0 = [69, 3, 1] = -\frac{1}{46} + \frac{1}{46}i\sqrt{\frac{89}{3}}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = -13500000 + 1431000\sqrt{89}$$

has minimal polynomial  $T^2 + 27000000T - 729000000$  and

$k = \sqrt[3]{t} \approx 2.999999000001666662815$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{89}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m+138n)q^{3m^2+3mn+23n^2} \\ &= 534q^{23} + 534q^{29} + 534q^{41} + 534q^{59} + 534q^{83} - 534q^{89} - 1068q^{92} - 1068q^{101} + 534q^{113} - 1068q^{116} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(801), (\frac{801}{\cdot}))$ .

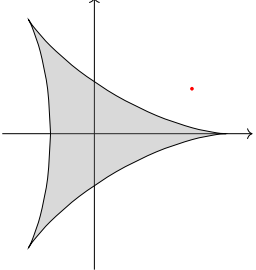
**$\nu$ -I2-1-1**

Let  $\tau_0 = [3, 2, 1] = -\frac{1}{3} + \frac{i\sqrt{2}}{3}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = 4 + 10i\sqrt{2}$$

has minimal polynomial  $T^2 - 8T + 216$  and

$k = \sqrt[3]{t} \approx 2.224744871391589049099 + 1.024944026382329769127i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{3\sqrt{\frac{3}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(6m+6n)q^{3m^2+2mn+n^2} \\ &= 12q - 12q^3 - 24q^4 + 24q^6 - 12q^9 + 24q^{12} + 48q^{16} - 48q^{18} + 24q^{19} - 48q^{22} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(24), \left(\frac{24}{\cdot}\right))$ .

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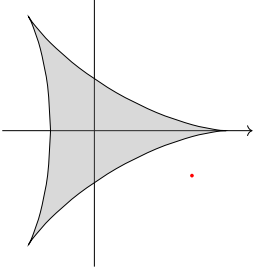
#### $\nu$ -I2-1-2

Let  $\tau_0 = [3, -2, 1] = \frac{1}{3} + \frac{i\sqrt{2}}{3}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = 4 - 10i\sqrt{2}$$

has minimal polynomial  $T^2 - 8T + 216$  and

$k = \sqrt[3]{t} \approx 2.224744871391589049099 - 1.024944026382329769127i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{3\sqrt{\frac{3}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-6m+6n)q^{3m^2-2mn+n^2} \\ &= 12q - 12q^3 - 24q^4 + 24q^6 - 12q^9 + 24q^{12} + 48q^{16} - 48q^{18} + 24q^{19} - 48q^{22} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(24), \left(\frac{24}{\cdot}\right))$ .

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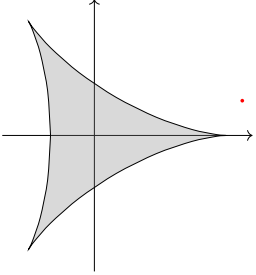
#### $\nu$ -I2-2-1

Let  $\tau_0 = [3, 1, 1] = -\frac{1}{6} + \frac{i\sqrt{11}}{6}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = 32 + 8i\sqrt{11}$$

has minimal polynomial  $T^2 - 64T + 1728$  and

$k = \sqrt[3]{t} \approx 3.372281323269014329925 + 0.792286991393261277794i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{3\sqrt{33}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(3m+6n)q^{3m^2+mn+n^2} \\ &= 12q + 6q^3 - 24q^4 - 30q^9 - 12q^{12} + 66q^{15} + 48q^{16} - 72q^{25} - 48q^{27} + 60q^{31} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(33), \left(\frac{33}{\cdot}\right))$ .

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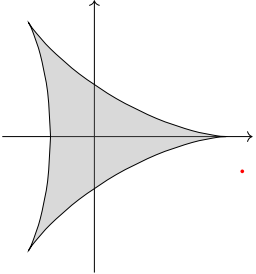
#### $\nu$ -I2-2-2

Let  $\tau_0 = [3, -1, 1] = \frac{1}{6} + \frac{i\sqrt{11}}{6}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = 32 - 8i\sqrt{11}$$

has minimal polynomial  $T^2 - 64T + 1728$  and

$k = \sqrt[3]{t} \approx 3.372281323269014329925 - 0.792286991393261277794i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{3\sqrt{33}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-3m+6n)q^{3m^2-mn+n^2} \\ &= 12q + 6q^3 - 24q^4 - 30q^9 - 12q^{12} + 66q^{15} + 48q^{16} - 72q^{25} - 48q^{27} + 60q^{31} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(33), \left(\frac{33}{\cdot}\right))$ .

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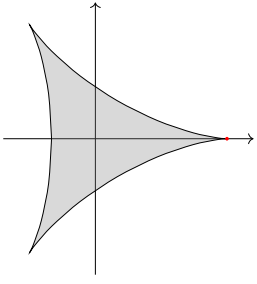
#### $\nu$ -C-1-1

Let  $\tau_0 = [27, 0, 1] = \frac{i}{3\sqrt{3}}$  with  $h(D) = 3$ . Then

$$t = t_Q(\tau_0) = 6 - 6\sqrt[3]{2} + 18\sqrt[3]{4}$$

has minimal polynomial  $T^3 - 18T^2 + 756T - 27000$  and

$k = \sqrt[3]{t} \approx 3.000507048964448844951$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{3}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 54nq^{m^2+3n^2} \\ &= 108q^3 + 216q^4 + 216q^7 - 432q^{13} - 432q^{16} + 216q^{19} - 432q^{21} - 216q^{28} - 432q^{37} + 216q^{39} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(108), \left(\frac{36}{\cdot}\right))$ .

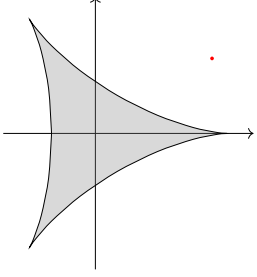
#### $\nu$ -C-1-2

Let  $\tau_0 = [9, 6, 4] = -\frac{1}{3} + \frac{i}{\sqrt{3}}$  with  $h(D) = 3$ . Then

$$t = t_Q(\tau_0) = 6 + 3\sqrt[3]{2} - 9\sqrt[3]{4} + 3i\sqrt[3]{2}\sqrt{3} + 9i\sqrt[3]{4}\sqrt{3}$$

has minimal polynomial  $T^3 - 18T^2 + 756T - 27000$  and

$k = \sqrt[3]{t} \approx 2.659914121621583806585 + 1.709727167708387368149i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{3}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (18m + 18n) q^{4m^2+2mn+n^2} \\ &= 36q - 108q^4 + 72q^7 - 36q^{13} + 216q^{16} - 144q^{19} - 180q^{25} + 108q^{28} - 144q^{31} + 180q^{37} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(108), \left(\frac{36}{\cdot}\right))$ .

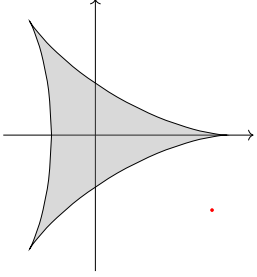
#### $\nu$ -C-1-3

Let  $\tau_0 = [9, -6, 4] = \frac{1}{3} + \frac{i}{\sqrt{3}}$  with  $h(D) = 3$ . Then

$$t = t_Q(\tau_0) = 6 + 3\sqrt[3]{2} - 9\sqrt[3]{4} - 3i\sqrt[3]{2}\sqrt{3} - 9i\sqrt[3]{4}\sqrt{3}$$

has minimal polynomial  $T^3 - 18T^2 + 756T - 27000$  and

$k = \sqrt[3]{t} \approx 2.659914121621583806585 - 1.709727167708387368149i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{3}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-18m + 18n) q^{4m^2-2mn+n^2} \\ &= 36q - 108q^4 + 72q^7 - 36q^{13} + 216q^{16} - 144q^{19} - 180q^{25} + 108q^{28} - 144q^{31} + 180q^{37} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(108), \left(\frac{36}{\cdot}\right))$ .

#### $\nu$ -C-2-1

Let  $\tau_0 = [1, 0, 3] = i\sqrt{3}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = 17766 + 14094\sqrt[3]{2} + 11178\sqrt[3]{4}$$

has minimal polynomial  $T^3 - 53298T^2 + 1635876T - 19683000$  and

$k = \sqrt[3]{t} \approx 37.62589891676765375567$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{81}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 2nq^{27m^2+n^2} \\ &= 4q - 8q^4 + 16q^{16} - 20q^{25} + 8q^{28} - 16q^{31} + 32q^{43} + 28q^{49} - 40q^{52} - 32q^{64} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(108), \left(\frac{324}{\cdot}\right))$ .

#### $\nu$ -C-2-2

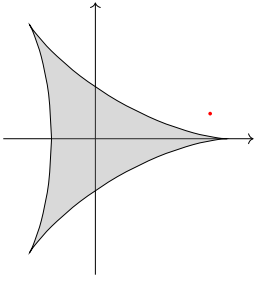
Let  $\tau_0 = [4, 2, 1] = -\frac{1}{4} + \frac{i\sqrt{3}}{4}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = 17766 - 7047\sqrt[3]{2} - 5589\sqrt[3]{4} + 7047i\sqrt[3]{2}\sqrt{3} - 5589i\sqrt[3]{4}\sqrt{3}$$

has minimal polynomial  $T^3 - 53298T^2 + 1635876T - 19683000$  and

$k = \sqrt[3]{t} \approx 2.616964743186866031313 + 0.572192121667771129124i$  is not in  $\mathcal{K}_Q^\circ$ .





We have  $\nu(t) = \frac{81}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(6m+8n)q^{9m^2+6mn+4n^2} \\ &= 16q^4 + 4q^7 - 20q^{13} - 32q^{16} + 28q^{19} - 16q^{28} - 44q^{37} + 52q^{49} + 80q^{52} + 4q^{61} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(108), (\frac{324}{\cdot}))$ .

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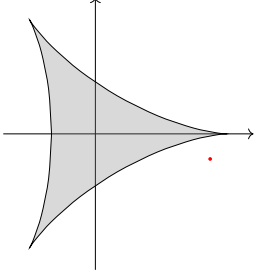
#### $\nu$ -C-2-3

Let  $\tau_0 = [4, -2, 1] = \frac{1}{4} + \frac{i\sqrt{3}}{4}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = 17766 - 7047\sqrt[3]{2} - 5589\sqrt[3]{4} - 7047i\sqrt[3]{2}\sqrt{3} + 5589i\sqrt[3]{4}\sqrt{3}$$

has minimal polynomial  $T^3 - 53298T^2 + 1635876T - 19683000$  and

$k = \sqrt[3]{t} \approx 2.616964743186866031313 - 0.572192121667771129124i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{81}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-6m+8n)q^{9m^2-6mn+4n^2} \\ &= 16q^4 + 4q^7 - 20q^{13} - 32q^{16} + 28q^{19} - 16q^{28} - 44q^{37} + 52q^{49} + 80q^{52} + 4q^{61} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(108), (\frac{324}{\cdot}))$ .

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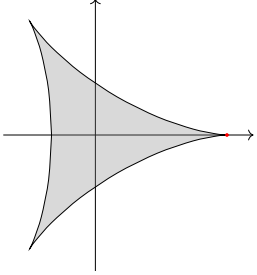
#### $\nu$ -C-3-1

Let  $\tau_0 = [63, 3, 1] = -\frac{1}{42} + \frac{i\sqrt{3}}{14}$  with  $h(D) = 3$ . Then

$$t = t_Q(\tau_0) = 96 + 56\sqrt[3]{3} - 72\sqrt[3]{9}$$

has minimal polynomial  $T^3 - 288T^2 + 63936T - 1536000$  and

$k = \sqrt[3]{t} \approx 2.999997802867938024843$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m+126n)q^{m^2+mn+7n^2} \\ &= 486q^7 + 486q^9 + 486q^{13} + 486q^{19} - 972q^{28} - 972q^{31} - 972q^{36} + 486q^{37} - 972q^{43} + 486q^{49} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(243), (\frac{81}{\cdot}))$ .

---

#### $\nu$ -C-3-2

Let  $\tau_0 = [9, 3, 7] = -\frac{1}{6} + \frac{i\sqrt{3}}{2}$  with  $h(D) = 3$ . Then

$$t = t_Q(\tau_0) = 96 + 108i\sqrt[6]{3} - 28\sqrt[3]{3} + 36\sqrt[3]{9} + 28i\sqrt[6]{243}$$

has minimal polynomial  $T^3 - 288T^2 + 63936T - 1536000$  and

$k = \sqrt[3]{t} \approx 5.865745215438474359242 + 2.013218760179034262415i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m+18n)q^{7m^2+mn+n^2} \\ &= 36q - 72q^4 + 18q^7 - 90q^{13} + 144q^{16} + 126q^{19} - 180q^{25} - 36q^{28} + 72q^{31} - 198q^{37} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(243), (\frac{81}{\cdot}))$ .

---

#### $\nu$ -C-3-3

Let  $\tau_0 = [9, -3, 7] = \frac{1}{6} + \frac{i\sqrt{3}}{2}$  with  $h(D) = 3$ . Then

$$t = t_Q(\tau_0) = 96 - 108i\sqrt[6]{3} - 28\sqrt[3]{3} + 36\sqrt[3]{9} - 28i\sqrt[6]{243}$$

has minimal polynomial  $T^3 - 288T^2 + 63936T - 1536000$  and

$k = \sqrt[3]{t} \approx 5.865745215438474359242 - 2.013218760179034262415i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m+18n)q^{7m^2-mn+n^2}$$

$$= 36q - 72q^4 + 18q^7 - 90q^{13} + 144q^{16} + 126q^{19} - 180q^{25} - 36q^{28} + 72q^{31} - 198q^{37} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(243), (\frac{81}{\cdot}))$ .

#### $\nu$ -C-4-1

Let  $\tau_0 = [1, -1, 7] = \frac{1}{2} + \frac{3i\sqrt{3}}{2}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = -4096224 - 2840184 \sqrt[3]{3} - 1969272 \sqrt[3]{9}$$

has minimal polynomial  $T^3 + 12288672T^2 - 700259904T + 10077696000$  and

$$k = \sqrt[3]{t} \approx -230.7644944264686007685 \text{ is not in } \mathcal{K}_Q^\circ.$$

We have  $\nu(t) = \frac{243}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-3m+2n)q^{63m^2-3mn+n^2}$$

$$= 4q - 8q^4 + 16q^{16} - 20q^{25} + 28q^{49} - 4q^{61} - 32q^{64} + 20q^{67} - 28q^{73} + 44q^{91} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(243), (\frac{729}{\cdot}))$ .

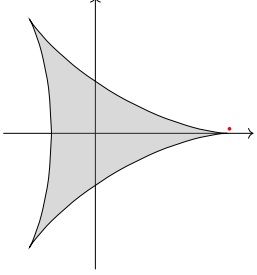
#### $\nu$ -C-4-2

Let  $\tau_0 = [7, 1, 1] = -\frac{1}{14} + \frac{3i\sqrt{3}}{14}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = -4096224 - 2953908i \sqrt[3]{3} + 1420092 \sqrt[3]{3} + 984636 \sqrt[3]{9} + 1420092i \sqrt[6]{243}$$

has minimal polynomial  $T^3 + 12288672T^2 - 700259904T + 10077696000$  and

$$k = \sqrt[3]{t} \approx 3.057720768431478199602 + 0.1026441050822645592587i \text{ is not in } \mathcal{K}_Q^\circ.$$



We have  $\nu(t) = \frac{243}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(3m+14n)q^{9m^2+3mn+7n^2}$$

$$= 28q^7 + 22q^{13} + 34q^{19} - 56q^{28} - 50q^{31} + 16q^{37} - 62q^{43} + 40q^{49} - 44q^{52} - 68q^{76} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(243), (\frac{729}{\cdot}))$ .

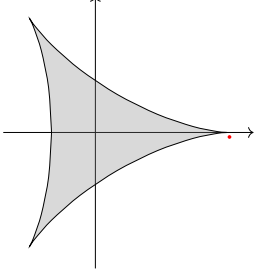
#### $\nu$ -C-4-3

Let  $\tau_0 = [7, -1, 1] = \frac{1}{14} + \frac{3i\sqrt{3}}{14}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = -4096224 + 2953908i \sqrt[3]{3} + 1420092 \sqrt[3]{3} + 984636 \sqrt[3]{9} - 1420092i \sqrt[6]{243}$$

has minimal polynomial  $T^3 + 12288672T^2 - 700259904T + 10077696000$  and

$$k = \sqrt[3]{t} \approx 3.057720768431478199602 - 0.1026441050822645592587i \text{ is not in } \mathcal{K}_Q^\circ.$$



We have  $\nu(t) = \frac{243}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-3m+14n)q^{9m^2-3mn+7n^2}$$

$$= 28q^7 + 22q^{13} + 34q^{19} - 56q^{28} - 50q^{31} + 16q^{37} - 62q^{43} + 40q^{49} - 44q^{52} - 68q^{76} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(243), (\frac{729}{\cdot}))$ .

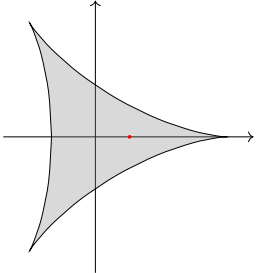
#### $\nu$ -T4-1-1 (#52 in the paper)

Let  $\tau_0 = [27, 24, 8] = -\frac{4}{9} + \frac{2i\sqrt{2}}{9}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 109 - 70\sqrt{2} - 55\sqrt{3} + 35\sqrt{6}$$

has minimal polynomial  $T^4 - 436T^3 + 18836T^2 - 214016T + 97336$  and

$$k = \sqrt[3]{t} \approx 0.7799151863765605325094 \text{ is in } \mathcal{K}_Q^\circ.$$



We have  $\nu(t) = -\frac{\sqrt{\frac{3}{2}}}{\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(72m+54n)q^{8m^2+8mn+3n^2}$$

$$= 72q^3 - 72q^4 - 144q^{12} + 144q^{16} + 72q^{19} + 288q^{24} - 360q^{27} - 144q^{36} - 360q^{43} + 288q^{48} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(288), (\frac{96}{\cdot}))$ .

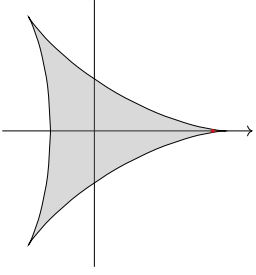
[ν-T4-1-2 \(#52 in the paper\)](#)

Let  $\tau_0 = [27, 12, 4] = -\frac{2}{9} + \frac{2i\sqrt{2}}{9}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 109 - 70\sqrt{2} + 55\sqrt{3} - 35\sqrt{6}$$

has minimal polynomial  $T^4 - 436T^3 + 18836T^2 - 214016T + 97336$  and

$k = \sqrt[3]{t} \approx 2.693248060916926080203$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{\sqrt{\frac{3}{2}}}{\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(36m+54n)q^{4m^2+4mn+3n^2}$$

$$= 144q^3 - 144q^8 + 144q^{11} - 288q^{12} - 288q^{24} + 144q^{27} + 288q^{32} + 576q^{36} - 288q^{44} + 576q^{48} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(288), (\frac{96}{\cdot}))$ .

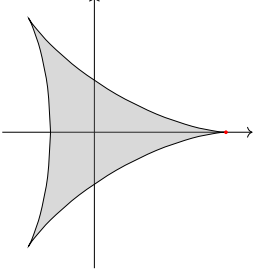
[ν-T4-1-3 \(#52 in the paper\)](#)

Let  $\tau_0 = [72, 0, 1] = \frac{i}{6\sqrt{2}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 109 + 70\sqrt{2} - 55\sqrt{3} - 35\sqrt{6}$$

has minimal polynomial  $T^4 - 436T^3 + 18836T^2 - 214016T + 97336$  and

$k = \sqrt[3]{t} \approx 3.000000516756102426046$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{\sqrt{\frac{3}{2}}}{2\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)144nq^{m^2+8n^2}$$

$$= 288q^8 + 576q^9 + 576q^{12} + 576q^{17} + 576q^{24} - 576q^{32} - 576q^{33} - 1152q^{36} - 1152q^{41} + 576q^{44} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(288), (\frac{96}{\cdot}))$ .

[ν-T4-1-4 \(#52 in the paper\)](#)

Let  $\tau_0 = [9, 0, 8] = \frac{2i\sqrt{2}}{3}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 109 + 70\sqrt{2} + 55\sqrt{3} + 35\sqrt{6}$$

has minimal polynomial  $T^4 - 436T^3 + 18836T^2 - 214016T + 97336$  and

$k = \sqrt[3]{t} \approx 7.299830388126588534035$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{\sqrt{\frac{3}{2}}}{2\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)18nq^{8m^2+n^2}$$

$$= 36q - 72q^4 + 72q^9 - 144q^{12} + 144q^{16} + 288q^{24} - 180q^{25} - 288q^{33} - 144q^{36} + 288q^{48} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(288), (\frac{96}{\cdot}))$ .

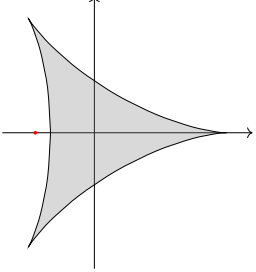
[ν-T4-2-1 \(#53 in the paper\)](#)

Let  $\tau_0 = [12, 12, 5] = -\frac{1}{2} + \frac{i}{\sqrt{6}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 7155 + 5049\sqrt{2} - 4131\sqrt{3} - 2916\sqrt{6}$$

has minimal polynomial  $T^4 - 28620T^3 + 766908T^2 + 314928T - 4251528$  and

$k = \sqrt[3]{t} \approx -1.348044649977884605914$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9}{2\sqrt{2}\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

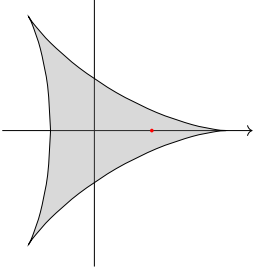
$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(36m+24n)q^{15m^2+12mn+4n^2} \\ &= 48q^4 - 48q^7 - 96q^{16} + 96q^{28} + 240q^{31} - 192q^{40} - 384q^{55} + 192q^{64} + 240q^{79} + 384q^{88} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(288), \left(\frac{288}{\cdot}\right))$ .

[ν-T4-2-2 \(#53 in the paper\)](#)

Let  $\tau_0 = [15, 12, 4] = -\frac{2}{5} + \frac{2}{5}i\sqrt{\frac{2}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 7155 - 5049\sqrt{2} - 4131\sqrt{3} + 2916\sqrt{6}$   
has minimal polynomial  $T^4 - 28620T^3 + 766908T^2 + 314928T - 4251528$  and  
 $k = \sqrt[3]{t} \approx 1.309579622892052421771$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9}{\sqrt{2}\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

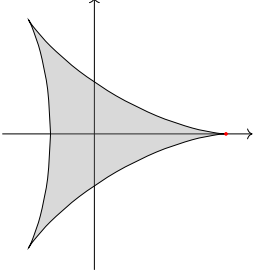
$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(36m+30n)q^{12m^2+12mn+5n^2} \\ &= 48q^5 - 48q^8 - 96q^{20} + 48q^{29} + 96q^{32} + 192q^{44} - 240q^{53} - 96q^{56} - 192q^{77} + 192q^{80} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(288), \left(\frac{288}{\cdot}\right))$ .

[ν-T4-2-3 \(#53 in the paper\)](#)

Let  $\tau_0 = [24, 0, 1] = \frac{i}{2\sqrt{6}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 7155 - 5049\sqrt{2} + 4131\sqrt{3} - 2916\sqrt{6}$   
has minimal polynomial  $T^4 - 28620T^3 + 766908T^2 + 314928T - 4251528$  and  
 $k = \sqrt[3]{t} \approx 3.000944876058051906767$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9}{2\sqrt{2}\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)48nq^{3m^2+8n^2} \\ &= 96q^8 + 192q^{11} + 192q^{20} - 192q^{32} - 192q^{35} - 384q^{44} + 192q^{56} - 384q^{59} - 384q^{80} + 192q^{83} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(288), \left(\frac{288}{\cdot}\right))$ .

[ν-T4-2-4 \(#53 in the paper\)](#)

Let  $\tau_0 = [3, 0, 8] = 2i\sqrt{\frac{2}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 7155 + 5049\sqrt{2} + 4131\sqrt{3} + 2916\sqrt{6}$   
has minimal polynomial  $T^4 - 28620T^3 + 766908T^2 + 314928T - 4251528$  and  
 $k = \sqrt[3]{t} \approx 30.57882620119961435692$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9}{2\sqrt{2}\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)6nq^{24m^2+n^2} \\ &= 12q - 24q^4 + 48q^{16} - 36q^{25} - 48q^{28} + 96q^{40} - 36q^{49} - 96q^{64} + 168q^{73} - 192q^{88} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(288), \left(\frac{288}{\cdot}\right))$ .

[ν-T4-3-1 \(#54 in the paper\)](#)

Let  $\tau_0 = [12, -12, 7] = \frac{1}{2} + \frac{i}{\sqrt{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 500877 - 408969\sqrt{\frac{3}{2}} - \frac{708345}{\sqrt{2}} + 289170\sqrt{3}$   
has minimal polynomial  $T^4 - 2003508T^3 + 31313466T^2 + 1230187500T - 16607531250$  and  
 $k = \sqrt[3]{t} \approx -2.888747638054569328812$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9}{2\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-36m+24n)q^{21m^2-12mn+4n^2} \\ &= 48q^4 - 48q^{13} - 96q^{16} + 240q^{37} + 96q^{52} - 336q^{61} - 240q^{100} - 48q^{109} + 384q^{112} + 768q^{133} + \dots\end{aligned}$$

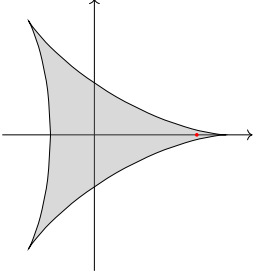
is in  $\mathcal{S}_2(\Gamma_0(576), \left(\frac{576}{\cdot}\right))$ .

[ν-T4-3-2 \(#54 in the paper\)](#)

Let  $\tau_0 = [21, 12, 4] = -\frac{2}{7} + \frac{4i}{7\sqrt{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 500877 - 408969\sqrt{\frac{3}{2}} + \frac{708345}{\sqrt{2}} - 289170\sqrt{3}$

has minimal polynomial  $T^4 - 2003508T^3 + 31313466T^2 + 1230187500T - 16607531250$  and  $k = \sqrt[3]{t} \approx 2.335283725576904820241$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9}{\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(36m+42n)q^{12m^2+12mn+7n^2} \\ &= 96q^7 - 96q^{16} - 192q^{28} + 96q^{31} + 384q^{76} + 96q^{79} - 480q^{103} + 384q^{112} - 192q^{124} - 480q^{127} + \dots \end{aligned}$$

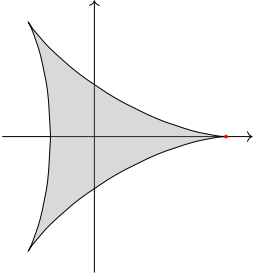
is in  $\mathcal{S}_2(\Gamma_0(576), (\frac{576}{\cdot}))$ .

**$\nu$ -T4-3-3 (#54 in the paper)**

Let  $\tau_0 = [48, 0, 1] = \frac{i}{4\sqrt{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 500877 + 408969\sqrt{\frac{3}{2}} - \frac{708345}{\sqrt{2}} - 289170\sqrt{3}$$

has minimal polynomial  $T^4 - 2003508T^3 + 31313466T^2 + 1230187500T - 16607531250$  and  $k = \sqrt[3]{t} \approx 3.000013476588670790643$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9}{2\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)96nq^{3m^2+16n^2} \\ &= 192q^{16} + 384q^{19} + 384q^{28} + 384q^{43} - 768q^{67} - 768q^{76} - 384q^{91} - 768q^{112} + 384q^{124} - 768q^{139} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(576), (\frac{576}{\cdot}))$ .

**$\nu$ -T4-3-4 (#54 in the paper)**

Let  $\tau_0 = [3, 0, 16] = \frac{4i}{\sqrt{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 500877 + 408969\sqrt{\frac{3}{2}} + \frac{708345}{\sqrt{2}} + 289170\sqrt{3}$$

has minimal polynomial  $T^4 - 2003508T^3 + 31313466T^2 + 1230187500T - 16607531250$  and  $k = \sqrt[3]{t} \approx 126.0653975251660565242$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9}{2\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)6nq^{48m^2+n^2} \\ &= 12q - 24q^4 + 48q^{16} - 60q^{25} + 108q^{49} - 48q^{52} - 120q^{73} + 168q^{97} + 120q^{100} - 192q^{112} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(576), (\frac{576}{\cdot}))$ .

**$\nu$ -T4-4-1 (#55 in the paper)**

Let  $\tau_0 = [4, -4, 3] = \frac{1}{2} + \frac{i}{\sqrt{2}}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = 13062249 - 9236430\sqrt{2} - 7541505\sqrt{3} + 5332635\sqrt{6}$$

has minimal polynomial  $T^4 - 52248996T^3 - 2201460444T^2 + 106073419104T + 51728341176$  and  $k = \sqrt[3]{t} \approx -4.134293714534110962198$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{27\sqrt{\frac{3}{2}}}{2\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-12m+8n)q^{27m^2-12mn+4n^2} \\ &= 16q^4 - 32q^{16} - 16q^{19} + 80q^{43} + 64q^{64} - 112q^{67} + 32q^{76} - 64q^{88} - 80q^{100} + 128q^{136} + \dots \end{aligned}$$

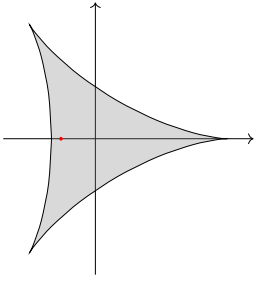
is in  $\mathcal{S}_2(\Gamma_0(288), (\frac{864}{\cdot}))$ .

**$\nu$ -T4-4-2 (#55 in the paper)**

Let  $\tau_0 = [8, 8, 3] = -\frac{1}{2} + \frac{i}{2\sqrt{2}}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = 13062249 - 9236430\sqrt{2} + 7541505\sqrt{3} - 5332635\sqrt{6}$$

has minimal polynomial  $T^4 - 52248996T^3 - 2201460444T^2 + 106073419104T + 51728341176$  and  $k = \sqrt[3]{t} \approx -0.7845372058337206260300$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{27\sqrt{\frac{3}{2}}}{2\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(24m+16n)q^{27m^2+24mn+8n^2} \\ &= 32q^8 - 32q^{11} - 64q^{32} + 64q^{44} + 160q^{59} - 128q^{68} - 32q^{83} - 224q^{107} + 128q^{128} + 160q^{131} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(288), \left(\frac{864}{\cdot}\right))$ .

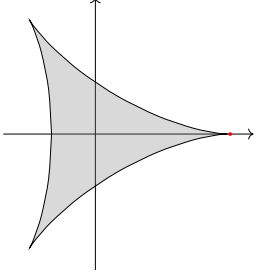
[ν-T4-4-3 \(#55 in the paper\)](#)

Let  $\tau_0 = [8, 0, 1] = \frac{i}{2\sqrt{2}}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = 13062249 + 9236430\sqrt{2} - 7541505\sqrt{3} - 5332635\sqrt{6}$$

has minimal polynomial  $T^4 - 52248996T^3 - 2201460444T^2 + 106073419104T + 51728341176$  and

$k = \sqrt[3]{t} \approx 3.072806497095363261234$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{27\sqrt{\frac{3}{2}}}{2\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)16nq^{9m^2+8n^2} \\ &= 32q^8 + 64q^{17} - 64q^{32} - 128q^{41} + 64q^{44} - 128q^{68} + 64q^{89} - 128q^{113} + 128q^{128} + 256q^{137} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(288), \left(\frac{864}{\cdot}\right))$ .

[ν-T4-4-4 \(#55 in the paper\)](#)

Let  $\tau_0 = [1, 0, 8] = 2i\sqrt{2}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = 13062249 + 9236430\sqrt{2} + 7541505\sqrt{3} + 5332635\sqrt{6}$$

has minimal polynomial  $T^4 - 52248996T^3 - 2201460444T^2 + 106073419104T + 51728341176$  and

$k = \sqrt[3]{t} \approx 373.8460244232724683270$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{27\sqrt{\frac{3}{2}}}{2\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)2nq^{72m^2+n^2} \\ &= 4q - 8q^4 + 16q^{16} - 20q^{25} + 28q^{49} - 32q^{64} + 8q^{73} - 16q^{76} + 32q^{88} - 40q^{97} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(288), \left(\frac{864}{\cdot}\right))$ .

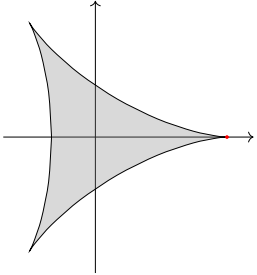
[ν-T4-5-1 \(#56 in the paper\)](#)

Let  $\tau_0 = [30, 0, 1] = \frac{i}{\sqrt{30}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 24084 - 17010\sqrt{2} - 10692\sqrt{5} + 7560\sqrt{10}$$

has minimal polynomial  $T^4 - 96336T^3 + 36613296T^2 - 1836660096T + 24794911296$  and

$k = \sqrt[3]{t} \approx 3.000281435776754682265$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{\frac{5}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)60nq^{3m^2+10n^2} \\ &= 120q^{10} + 240q^{13} + 240q^{22} + 240q^{37} - 240q^{40} - 480q^{43} - 480q^{52} + 240q^{58} - 480q^{67} + 240q^{85} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(360), \left(\frac{360}{\cdot}\right))$ .

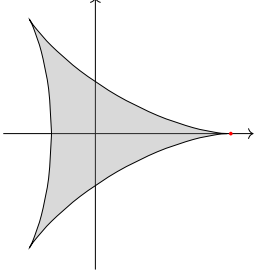
[ν-T4-5-2 \(#56 in the paper\)](#)

Let  $\tau_0 = [15, 0, 2] = i\sqrt{\frac{2}{15}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 24084 - 17010\sqrt{2} + 10692\sqrt{5} - 7560\sqrt{10}$$

has minimal polynomial  $T^4 - 96336T^3 + 36613296T^2 - 1836660096T + 24794911296$  and

$k = \sqrt[3]{t} \approx 3.088022061272761611574$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{\frac{5}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 30nq^{6m^2+5n^2} \\ = 60q^5 + 120q^{11} - 120q^{20} - 240q^{26} + 120q^{29} - 240q^{44} + 120q^{59} - 240q^{74} + 240q^{80} + 480q^{86} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(360), (\frac{360}{\cdot}))$ .

[ν-T4-5-3 \(#56 in the paper\)](#)

Let  $\tau_0 = [6, 0, 5] = i\sqrt{\frac{5}{6}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 24084 + 17010\sqrt{2} - 10692\sqrt{5} - 7560\sqrt{10}$$

has minimal polynomial  $T^4 - 96336T^3 + 36613296T^2 - 1836660096T + 24794911296$  and

$k = \sqrt[3]{t} \approx 6.874743269597869965463$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{\frac{5}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 12nq^{15m^2+2n^2} \\ = 24q^2 - 48q^8 + 48q^{17} - 96q^{23} + 96q^{32} + 192q^{47} - 120q^{50} + 48q^{62} - 240q^{65} - 96q^{68} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(360), (\frac{360}{\cdot}))$ .

[ν-T4-5-4 \(#56 in the paper\)](#)

Let  $\tau_0 = [3, 0, 10] = i\sqrt{\frac{10}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 24084 + 17010\sqrt{2} + 10692\sqrt{5} + 7560\sqrt{10}$$

has minimal polynomial  $T^4 - 96336T^3 + 36613296T^2 - 1836660096T + 24794911296$  and

$k = \sqrt[3]{t} \approx 45.78135537484650292819$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{\frac{5}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 6nq^{30m^2+n^2} \\ = 12q - 24q^4 + 48q^{16} - 60q^{25} + 24q^{31} - 48q^{34} + 96q^{46} + 84q^{49} - 120q^{55} - 96q^{64} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(360), (\frac{360}{\cdot}))$ .

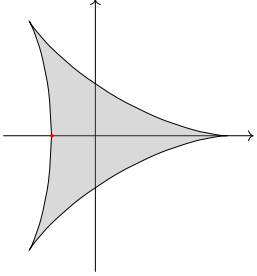
[ν-T4-6-1 \(#57 in the paper\)](#)

Let  $\tau_0 = [18, 18, 7] = -\frac{1}{2} + \frac{i\sqrt{5}}{6}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 44 + 10\sqrt{3} - 14\sqrt{5} - 8\sqrt{15}$$

has minimal polynomial  $T^4 - 176T^3 + 7136T^2 - 80896T - 85184$  and

$k = \sqrt[3]{t} \approx -0.9893232047286543709252$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{\sqrt{15}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (54m + 36n) q^{7m^2+6mn+2n^2} \\ = 72q^2 - 72q^3 - 144q^8 + 144q^{12} + 360q^{15} - 288q^{18} - 72q^{23} - 504q^{27} + 288q^{32} + 360q^{35} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(180), (\frac{60}{\cdot}))$ .

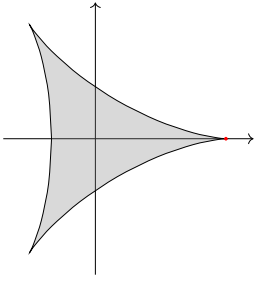
[ν-T4-6-2 \(#57 in the paper\)](#)

Let  $\tau_0 = [27, 6, 2] = -\frac{1}{9} + \frac{i\sqrt{5}}{9}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 44 - 10\sqrt{3} - 14\sqrt{5} + 8\sqrt{15}$$

has minimal polynomial  $T^4 - 176T^3 + 7136T^2 - 80896T - 85184$  and

$k = \sqrt[3]{t} \approx 2.976046549728605636849$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{\sqrt{15}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

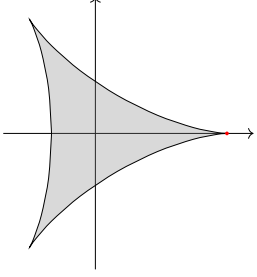
$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(18m + 54n)q^{2m^2+2mn+3n^2} \\ &= 180q^3 + 180q^7 - 180q^{10} - 360q^{12} + 180q^{15} - 360q^{18} + 180q^{27} - 360q^{28} + 360q^{40} + 360q^{42} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(180), (\frac{60}{\cdot}))$ .

[ν-T4-6-3 \(#57 in the paper\)](#)

Let  $\tau_0 = [45, 0, 1] = \frac{i}{3\sqrt{5}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 44 - 10\sqrt{3} + 14\sqrt{5} - 8\sqrt{15}$   
has minimal polynomial  $T^4 - 176T^3 + 7136T^2 - 80896T - 85184$  and  
 $k = \sqrt[3]{t} \approx 3.000021364279298004162$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{\sqrt{15}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)90nq^{m^2+5n^2} \\ &= 180q^5 + 360q^6 + 360q^9 + 360q^{14} - 360q^{20} - 360q^{21} - 720q^{24} - 720q^{29} + 360q^{30} - 720q^{36} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(180), (\frac{60}{\cdot}))$ .

[ν-T4-6-4 \(#57 in the paper\)](#)

Let  $\tau_0 = [9, 0, 5] = \frac{i\sqrt{5}}{3}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 44 + 10\sqrt{3} + 14\sqrt{5} + 8\sqrt{15}$   
has minimal polynomial  $T^4 - 176T^3 + 7136T^2 - 80896T - 85184$  and  
 $k = \sqrt[3]{t} \approx 4.981388495408017008014$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{\sqrt{15}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)18nq^{5m^2+n^2} \\ &= 36q - 72q^4 + 72q^6 - 144q^9 + 144q^{16} + 360q^{21} - 144q^{24} - 180q^{25} - 360q^{30} + 288q^{36} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(180), (\frac{60}{\cdot}))$ .

[ν-T4-7-1 \(#58 in the paper\)](#)

Let  $\tau_0 = [2, -2, 3] = \frac{1}{2} + \frac{i\sqrt{5}}{2}$  with  $h(D) = 2$ . Then

$t = t_Q(\tau_0) = 315684 + 182250\sqrt{3} - 141426\sqrt{5} - 81648\sqrt{15}$   
has minimal polynomial  $T^4 - 1262736T^3 - 1357059744T^2 + 49698157824T - 45270270144$  and  
 $k = \sqrt[3]{t} \approx -10.35160689062294407775$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{27\sqrt{15}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

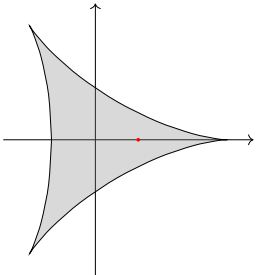
$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-6m + 4n)q^{27m^2-6mn+2n^2} \\ &= 8q^2 - 16q^8 - 8q^{23} + 32q^{32} + 40q^{35} - 56q^{47} - 40q^{50} + 88q^{83} + 16q^{92} + 24q^{98} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(180), (\frac{540}{\cdot}))$ .

[ν-T4-7-2 \(#58 in the paper\)](#)

Let  $\tau_0 = [7, 6, 2] = -\frac{3}{7} + \frac{i\sqrt{5}}{7}$  with  $h(D) = 2$ . Then

$t = t_Q(\tau_0) = 315684 - 182250\sqrt{3} - 141426\sqrt{5} + 81648\sqrt{15}$   
has minimal polynomial  $T^4 - 1262736T^3 - 1357059744T^2 + 49698157824T - 45270270144$  and  
 $k = \sqrt[3]{t} \approx 0.9777714981286222747001$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{27\sqrt{15}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where



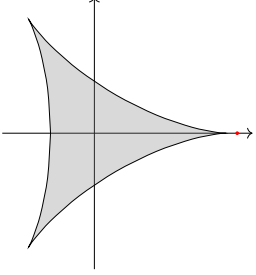
$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(18m+14n)q^{18m^2+18mn+7n^2} \\ &= 20q^7 - 20q^{10} - 40q^{28} + 40q^{40} + 20q^{43} + 80q^{58} - 100q^{67} - 40q^{82} - 100q^{103} + 80q^{112} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(180), (\frac{540}{\cdot}))$ .

#### [ν-T4-7-3 \(#58 in the paper\)](#)

Let  $\tau_0 = [5, 0, 1] = \frac{i}{\sqrt{5}}$  with  $h(D) = 2$ . Then

$t = t_Q(\tau_0) = 315684 - 182250\sqrt{3} + 141426\sqrt{5} - 81648\sqrt{15}$   
has minimal polynomial  $T^4 - 1262736T^3 - 1357059744T^2 + 49698157824T - 45270270144$  and  
 $k = \sqrt[3]{t} \approx 3.256856697511690800059$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{27\sqrt{15}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)10nq^{9m^2+5n^2} \\ &= 20q^5 + 40q^{14} - 40q^{20} - 80q^{29} + 40q^{41} - 80q^{56} + 80q^{80} + 40q^{86} + 160q^{89} - 80q^{101} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(180), (\frac{540}{\cdot}))$ .

#### [ν-T4-7-4 \(#58 in the paper\)](#)

Let  $\tau_0 = [1, 0, 5] = i\sqrt{5}$  with  $h(D) = 2$ . Then

$t = t_Q(\tau_0) = 315684 + 182250\sqrt{3} + 141426\sqrt{5} + 81648\sqrt{15}$   
has minimal polynomial  $T^4 - 1262736T^3 - 1357059744T^2 + 49698157824T - 45270270144$  and  
 $k = \sqrt[3]{t} \approx 108.1169786949826310030$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{27\sqrt{15}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

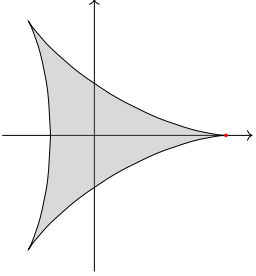
$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)2nq^{45m^2+n^2} \\ &= 4q - 8q^4 + 16q^{16} - 20q^{25} + 8q^{46} + 12q^{49} + 32q^{61} - 32q^{64} - 40q^{70} + 56q^{94} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(180), (\frac{540}{\cdot}))$ .

#### [ν-T4-8-1 \(#59 in the paper\)](#)

Let  $\tau_0 = [60, 0, 1] = \frac{i}{2\sqrt{15}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 2776869 + \frac{3206385\sqrt{3}}{2} - 1241838\sqrt{5} - \frac{1433943\sqrt{15}}{2}$   
has minimal polynomial  $T^4 - 11107476T^3 + 1591369821T^2 - 69739270326T + 941480149401$  and  
 $k = \sqrt[3]{t} \approx 3.000002430830552643351$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{5}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

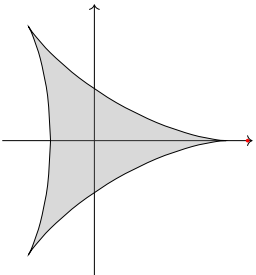
$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)120nq^{3m^2+20n^2} \\ &= 240q^{20} + 480q^{23} + 480q^{32} + 480q^{47} + 480q^{68} - 480q^{80} - 960q^{83} - 960q^{92} + 480q^{95} - 960q^{107} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(720), (\frac{720}{\cdot}))$ .

#### [ν-T4-8-2 \(#59 in the paper\)](#)

Let  $\tau_0 = [15, 0, 4] = \frac{2i}{\sqrt{15}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 2776869 - \frac{3206385\sqrt{3}}{2} - 1241838\sqrt{5} + \frac{1433943\sqrt{15}}{2}$   
has minimal polynomial  $T^4 - 11107476T^3 + 1591369821T^2 - 69739270326T + 941480149401$  and  
 $k = \sqrt[3]{t} \approx 3.493316486262842119150$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{5}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 30nq^{12m^2+5n^2} \\ = 60q^5 + 120q^{17} - 120q^{20} - 240q^{32} + 120q^{53} - 240q^{68} + 240q^{80} + 480q^{92} + 120q^{113} - 300q^{125} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(720), \left(\frac{720}{\cdot}\right))$ .

[ν-T4-8-3 \(#59 in the paper\)](#)

Let  $\tau_0 = [12, 0, 5] = \frac{1}{2}i\sqrt{\frac{5}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 2776869 - \frac{3206385\sqrt{3}}{2} + 1241838\sqrt{5} - \frac{1433943\sqrt{15}}{2}$$

has minimal polynomial  $T^4 - 11107476T^3 + 1591369821T^2 - 69739270326T + 941480149401$  and  $k = \sqrt[3]{t} \approx 4.191546178842404673881$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{5}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 24nq^{15m^2+4n^2} \\ = 48q^4 - 96q^{16} + 96q^{19} - 192q^{31} + 288q^{64} - 192q^{76} + 384q^{79} - 240q^{100} - 480q^{115} + 384q^{124} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(720), \left(\frac{720}{\cdot}\right))$ .

[ν-T4-8-4 \(#59 in the paper\)](#)

Let  $\tau_0 = [3, 0, 20] = 2i\sqrt{\frac{5}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 2776869 + \frac{3206385\sqrt{3}}{2} + 1241838\sqrt{5} + \frac{1433943\sqrt{15}}{2}$$

has minimal polynomial  $T^4 - 11107476T^3 + 1591369821T^2 - 69739270326T + 941480149401$  and  $k = \sqrt[3]{t} \approx 223.1190200886623003702$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{5}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 6nq^{60m^2+n^2} \\ = 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} + 24q^{61} - 144q^{64} + 96q^{76} - 120q^{85} + 120q^{100} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(720), \left(\frac{720}{\cdot}\right))$ .

[ν-T4-9-1 \(#60 in the paper\)](#)

Let  $\tau_0 = [6, -6, 5] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{7}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 3672 + 2106\sqrt{3} - 1404\sqrt{7} - 810\sqrt{21}$$

has minimal polynomial  $T^4 - 14688T^3 - 863136T^2 + 68024448T - 918330048$  and  $k = \sqrt[3]{t} \approx -4.744827653771052283184$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{7}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-18m + 12n)q^{15m^2-6mn+2n^2} \\ = 24q^2 - 48q^8 - 24q^{11} + 120q^{23} + 96q^{32} - 168q^{35} + 48q^{44} - 216q^{50} + 264q^{71} + 192q^{74} + \dots$$

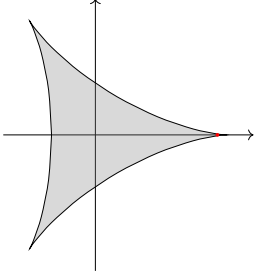
is in  $\mathcal{S}_2(\Gamma_0(252), \left(\frac{252}{\cdot}\right))$ .

[ν-T4-9-2 \(#60 in the paper\)](#)

Let  $\tau_0 = [15, 6, 2] = -\frac{1}{5} + \frac{1}{5}i\sqrt{\frac{7}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 3672 - 2106\sqrt{3} - 1404\sqrt{7} + 810\sqrt{21}$$

has minimal polynomial  $T^4 - 14688T^3 - 863136T^2 + 68024448T - 918330048$  and  $k = \sqrt[3]{t} \approx 2.782909133110217016772$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{7}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(18m + 30n)q^{6m^2+6mn+5n^2} \\ = 84q^5 - 84q^{14} + 84q^{17} - 168q^{20} - 168q^{38} + 84q^{41} + 168q^{56} + 336q^{62} - 168q^{68} + 84q^{77} + \dots$$

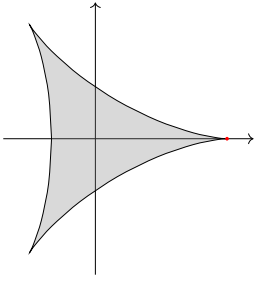
is in  $\mathcal{S}_2(\Gamma_0(252), \left(\frac{252}{\cdot}\right))$ .

[ν-T4-9-3 \(#60 in the paper\)](#)

Let  $\tau_0 = [21, 0, 1] = \frac{i}{\sqrt{21}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 3672 - 2106\sqrt{3} + 1404\sqrt{7} - 810\sqrt{21}$$

has minimal polynomial  $T^4 - 14688T^3 - 863136T^2 + 68024448T - 918330048$  and  $k = \sqrt[3]{t} \approx 3.001833215856110191252$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{7}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 42nq^{3m^2+7n^2} \\ = 84q^7 + 168q^{10} + 168q^{19} - 168q^{28} - 336q^{31} + 168q^{34} - 336q^{40} - 168q^{55} - 336q^{76} + 168q^{82} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(252), (\frac{252}{\cdot}))$ .

[ν-T4-9-4 \(#60 in the paper\)](#)

Let  $\tau_0 = [3, 0, 7] = i\sqrt{\frac{7}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 3672 + 2106\sqrt{3} + 1404\sqrt{7} + 810\sqrt{21}$   
has minimal polynomial  $T^4 - 14688T^3 - 863136T^2 + 68024448T - 918330048$  and  
 $k = \sqrt[3]{t} \approx 24.52224560359002972316$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{7}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 6nq^{21m^2+n^2} \\ = 12q - 24q^4 + 48q^{16} + 24q^{22} - 108q^{25} + 96q^{37} - 120q^{46} + 84q^{49} - 96q^{64} + 168q^{70} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(252), (\frac{252}{\cdot}))$ .

[ν-T4-10-1 \(#61 in the paper\)](#)

Let  $\tau_0 = [6, -6, 7] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{11}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 41904 - 24300\sqrt{3} - 12690\sqrt{11} + 7290\sqrt{33}$   
has minimal polynomial  $T^4 - 167616T^3 - 57573504T^2 + 3353353344T - 45270270144$  and  
 $k = \sqrt[3]{t} \approx -7.336871811011252320169$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{11}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

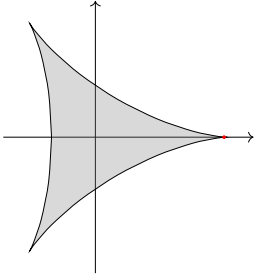
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-18m + 12n) q^{21m^2-6mn+2n^2} \\ = 24q^2 - 48q^8 - 24q^{17} + 120q^{29} + 96q^{32} - 168q^{41} - 120q^{50} + 48q^{68} - 96q^{74} + 264q^{77} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(396), (\frac{396}{\cdot}))$ .

[ν-T4-10-2 \(#61 in the paper\)](#)

Let  $\tau_0 = [21, 6, 2] = -\frac{1}{7} + \frac{1}{7}i\sqrt{\frac{11}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 41904 - 24300\sqrt{3} + 12690\sqrt{11} - 7290\sqrt{33}$   
has minimal polynomial  $T^4 - 167616T^3 - 57573504T^2 + 3353353344T - 45270270144$  and  
 $k = \sqrt[3]{t} \approx 2.934594492492810551258$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{11}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

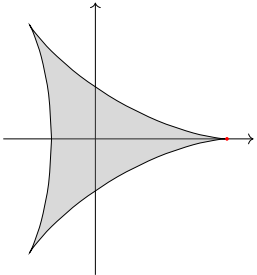
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (18m + 42n) q^{6m^2+6mn+7n^2} \\ = 132q^7 + 132q^{19} - 132q^{22} - 264q^{28} + 132q^{43} - 264q^{46} - 264q^{76} + 132q^{79} + 264q^{88} + 528q^{94} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(396), (\frac{396}{\cdot}))$ .

[ν-T4-10-3 \(#61 in the paper\)](#)

Let  $\tau_0 = [33, 0, 1] = \frac{i}{\sqrt{33}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 41904 + 24300\sqrt{3} - 12690\sqrt{11} - 7290\sqrt{33}$   
has minimal polynomial  $T^4 - 167616T^3 - 57573504T^2 + 3353353344T - 45270270144$  and  
 $k = \sqrt[3]{t} \approx 3.000160771067964258678$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{11}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 66nq^{3m^2+11n^2} \\ &= 132q^{11} + 264q^{14} + 264q^{23} + 264q^{38} - 264q^{44} - 528q^{47} - 528q^{56} + 264q^{59} - 528q^{71} + 264q^{86} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(396), \left(\frac{396}{\cdot}\right))$ .

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[ν-T4-10-4 \(#61 in the paper\)](#)

Let  $\tau_0 = [3, 0, 11] = i\sqrt{\frac{11}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 41904 + 24300\sqrt{3} + 12690\sqrt{11} + 7290\sqrt{33}$   
has minimal polynomial  $T^4 - 167616T^3 - 57573504T^2 + 3353353344T - 45270270144$  and  
 $k = \sqrt[3]{t} \approx 55.17395774420300781742$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{11}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 6nq^{33m^2+n^2} \\ &= 12q - 24q^4 + 48q^{16} - 60q^{25} + 24q^{34} - 48q^{37} + 180q^{49} - 120q^{58} - 96q^{64} + 168q^{82} + \dots\end{aligned}$$

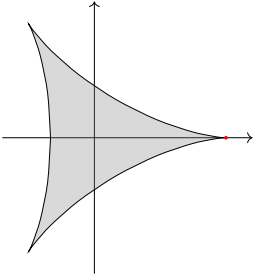
is in  $\mathcal{S}_2(\Gamma_0(396), \left(\frac{396}{\cdot}\right))$ .

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[ν-T4-11-1 \(#62 in the paper\)](#)

Let  $\tau_0 = [78, 0, 1] = \frac{i}{\sqrt{78}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 26990604 - 19085220\sqrt{2} + 7484400\sqrt{13} - 5292270\sqrt{26}$   
has minimal polynomial  $T^4 - 107962416T^3 + 1129077404496T^2 - 60812770640256T + 820972403643456$  and  
 $k = \sqrt[3]{t} \approx 3.000000250111302002348$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{\frac{13}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 156nq^{3m^2+26n^2} \\ &= 312q^{26} + 624q^{29} + 624q^{38} + 624q^{53} + 624q^{74} + 624q^{101} - 624q^{104} - 1248q^{107} - 1248q^{116} - 1248q^{131} + \dots\end{aligned}$$

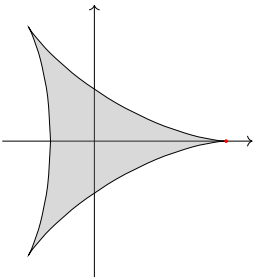
is in  $\mathcal{S}_2(\Gamma_0(936), \left(\frac{936}{\cdot}\right))$ .

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[ν-T4-11-2 \(#62 in the paper\)](#)

Let  $\tau_0 = [39, 0, 2] = i\sqrt{\frac{2}{39}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 26990604 - 19085220\sqrt{2} - 7484400\sqrt{13} + 5292270\sqrt{26}$   
has minimal polynomial  $T^4 - 107962416T^3 + 1129077404496T^2 - 60812770640256T + 820972403643456$  and  
 $k = \sqrt[3]{t} \approx 3.002599405003644741088$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{\frac{13}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 78nq^{6m^2+13n^2} \\ &= 156q^{13} + 312q^{19} + 312q^{37} - 312q^{52} - 624q^{58} + 312q^{67} - 624q^{76} - 624q^{106} + 312q^{109} - 624q^{148} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(936), \left(\frac{936}{\cdot}\right))$ .

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[ν-T4-11-3 \(#62 in the paper\)](#)

Let  $\tau_0 = [6, 0, 13] = i\sqrt{\frac{13}{6}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 26990604 + 19085220\sqrt{2} - 7484400\sqrt{13} - 5292270\sqrt{26}$   
has minimal polynomial  $T^4 - 107962416T^3 + 1129077404496T^2 - 60812770640256T + 820972403643456$  and  
 $k = \sqrt[3]{t} \approx 21.83135346928720715226$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{\frac{13}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 12nq^{39m^2+2n^2} \\ &= 24q^2 - 48q^8 + 96q^{32} + 48q^{41} - 96q^{47} - 120q^{50} + 192q^{71} - 240q^{89} + 168q^{98} - 192q^{128} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(936), \left(\frac{936}{\cdot}\right))$ .

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[ν-T4-11-4 \(#62 in the paper\)](#)

Let  $\tau_0 = [3, 0, 26] = i\sqrt{\frac{26}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 26990604 + 19085220\sqrt{2} + 7484400\sqrt{13} + 5292270\sqrt{26}$   
has minimal polynomial  $T^4 - 107962416T^3 + 1129077404496T^2 - 60812770640256T + 820972403643456$  and  
 $k = \sqrt[3]{t} \approx 476.1496906574249660186$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{\frac{13}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 6nq^{78m^2+n^2} \\ = 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 24q^{79} - 48q^{82} + 96q^{94} + 120q^{100} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(936), (\frac{936}{\cdot}))$ .

#### [ν-T4-12-1 \(#63 in the paper\)](#)

Let  $\tau_0 = [3, -3, 17] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{65}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -560736 - 250776\sqrt{5} - 155520\sqrt{13} - 69552\sqrt{65}$   
has minimal polynomial  $T^4 + 2242944T^3 - 57573504T^2 - 161243136T + 2176782336$  and  
 $k = \sqrt[3]{t} \approx -130.9004481970993878129$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{65}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

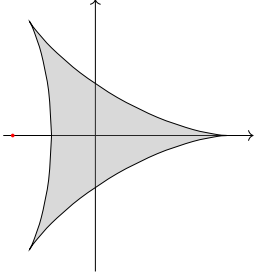
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m + 6n)q^{51m^2-3mn+n^2} \\ = 12q - 24q^4 + 48q^{16} - 60q^{25} + 72q^{49} + 60q^{55} - 84q^{61} - 96q^{64} + 132q^{79} - 156q^{91} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(585), (\frac{585}{\cdot}))$ .

#### [ν-T4-12-2 \(#63 in the paper\)](#)

Let  $\tau_0 = [15, 15, 7] = -\frac{1}{2} + \frac{1}{2}i\sqrt{\frac{13}{15}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -560736 - 250776\sqrt{5} + 155520\sqrt{13} + 69552\sqrt{65}$   
has minimal polynomial  $T^4 + 2242944T^3 - 57573504T^2 - 161243136T + 2176782336$  and  
 $k = \sqrt[3]{t} \approx -1.885002489933845045468$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{65}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

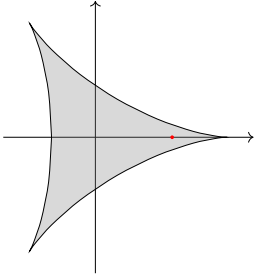
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(45m + 30n)q^{21m^2+15mn+5n^2} \\ = 60q^5 - 60q^{11} - 120q^{20} + 300q^{41} + 120q^{44} - 240q^{59} - 420q^{71} + 240q^{80} - 60q^{89} + 780q^{119} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(585), (\frac{585}{\cdot}))$ .

#### [ν-T4-12-3 \(#63 in the paper\)](#)

Let  $\tau_0 = [21, 15, 5] = -\frac{5}{14} + \frac{1}{14}i\sqrt{\frac{65}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -560736 + 250776\sqrt{5} + 155520\sqrt{13} - 69552\sqrt{65}$   
has minimal polynomial  $T^4 + 2242944T^3 - 57573504T^2 - 161243136T + 2176782336$  and  
 $k = \sqrt[3]{t} \approx 1.750783430031650074781$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{65}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

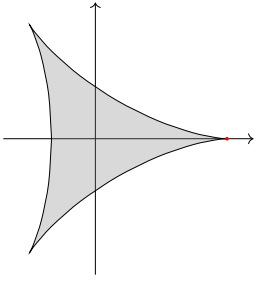
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(45m + 42n)q^{15m^2+15mn+7n^2} \\ = 78q^7 - 78q^{13} - 156q^{28} + 78q^{37} + 156q^{52} + 312q^{67} - 156q^{73} - 390q^{85} + 78q^{97} + 312q^{112} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(585), (\frac{585}{\cdot}))$ .

#### [ν-T4-12-4 \(#63 in the paper\)](#)

Let  $\tau_0 = [51, 3, 1] = -\frac{1}{34} + \frac{1}{34}i\sqrt{\frac{65}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -560736 + 250776\sqrt{5} - 155520\sqrt{13} + 69552\sqrt{65}$   
has minimal polynomial  $T^4 + 2242944T^3 - 57573504T^2 - 161243136T + 2176782336$  and  
 $k = \sqrt[3]{t} \approx 2.999987962483975845344$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{65}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

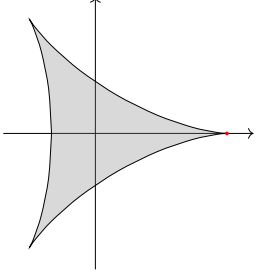
$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 102n)q^{3m^2+3mn+17n^2} \\ &= 390q^{17} + 390q^{23} + 390q^{35} + 390q^{53} - 390q^{65} - 780q^{68} - 390q^{77} - 780q^{92} + 390q^{107} - 780q^{113} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(585), (\frac{585}{\cdot}))$ .

#### $\nu$ -T4-13-1 (#64 in the paper)

Let  $\tau_0 = [102, 0, 1] = \frac{i}{\sqrt{102}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 383973804 - 271496610\sqrt{2} - 93127320\sqrt{17} + 65847600\sqrt{34}$   
has minimal polynomial  $T^4 - 1535895216T^3 + 60296198732496T^2 - 3253755396329856T + 43925697850453056$  and  
 $k = \sqrt[3]{t} \approx 3.000000017579773802155$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{\frac{17}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

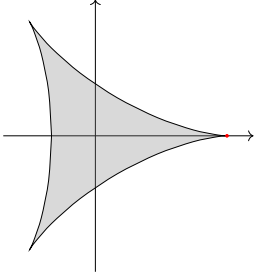
$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)204nq^{3m^2+34n^2} \\ &= 408q^{34} + 816q^{37} + 816q^{46} + 816q^{61} + 816q^{82} + 816q^{109} - 816q^{136} - 1632q^{139} + 816q^{142} - 1632q^{148} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1224), (\frac{1224}{\cdot}))$ .

#### $\nu$ -T4-13-2 (#64 in the paper)

Let  $\tau_0 = [51, 0, 2] = i\sqrt{\frac{2}{51}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 383973804 + 271496610\sqrt{2} - 93127320\sqrt{17} - 65847600\sqrt{34}$   
has minimal polynomial  $T^4 - 1535895216T^3 + 60296198732496T^2 - 3253755396329856T + 43925697850453056$  and  
 $k = \sqrt[3]{t} \approx 3.000689004041877768997$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{\frac{17}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)102nq^{6m^2+17n^2} \\ &= 204q^{17} + 408q^{23} + 408q^{41} - 408q^{68} + 408q^{71} - 816q^{74} - 816q^{92} + 408q^{113} - 816q^{122} - 816q^{164} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1224), (\frac{1224}{\cdot}))$ .

#### $\nu$ -T4-13-3 (#64 in the paper)

Let  $\tau_0 = [6, 0, 17] = i\sqrt{\frac{17}{6}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 383973804 - 271496610\sqrt{2} + 93127320\sqrt{17} - 65847600\sqrt{34}$   
has minimal polynomial  $T^4 - 1535895216T^3 + 60296198732496T^2 - 3253755396329856T + 43925697850453056$  and  
 $k = \sqrt[3]{t} \approx 33.97142910118434756295$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{\frac{17}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)12nq^{51m^2+2n^2} \\ &= 24q^2 - 48q^8 + 96q^{32} - 120q^{50} + 48q^{53} - 96q^{59} + 192q^{83} + 168q^{98} - 240q^{101} - 192q^{128} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1224), (\frac{1224}{\cdot}))$ .

#### $\nu$ -T4-13-4 (#64 in the paper)

Let  $\tau_0 = [3, 0, 34] = i\sqrt{\frac{34}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 383973804 + 271496610\sqrt{2} + 93127320\sqrt{17} + 65847600\sqrt{34}$   
has minimal polynomial  $T^4 - 1535895216T^3 + 60296198732496T^2 - 3253755396329856T + 43925697850453056$  and

$k = \sqrt[3]{t} \approx 1153.763588107886395599$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{\frac{17}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 6nq^{102m^2+n^2} \\ &= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 120q^{100} + 24q^{103} - 48q^{106} + 96q^{118} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1224), (\frac{1224}{\cdot}))$ .

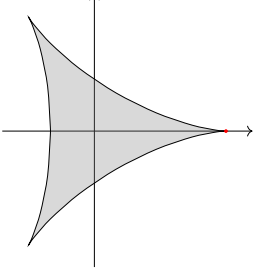
[ν-T4-14-1 \(#65 in the paper\)](#)

Let  $\tau_0 = [42, 0, 1] = \frac{i}{\sqrt{42}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 196452 - 80190\sqrt{6} - 52380\sqrt{14} + 42768\sqrt{21}$$

has minimal polynomial  $T^4 - 785808T^3 + 750216816T^2 - 39366000000T + 531441000000$  and

$k = \sqrt[3]{t} \approx 3.000034402170826129903$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{\frac{7}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 84nq^{3m^2+14n^2} \\ &= 168q^{14} + 336q^{17} + 336q^{26} + 336q^{41} - 336q^{56} - 672q^{59} + 336q^{62} - 672q^{68} - 672q^{83} + 336q^{89} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(504), (\frac{504}{\cdot}))$ .

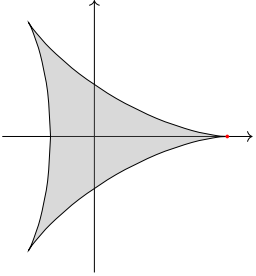
[ν-T4-14-2 \(#65 in the paper\)](#)

Let  $\tau_0 = [21, 0, 2] = i\sqrt{\frac{2}{21}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 196452 - 80190\sqrt{6} + 52380\sqrt{14} - 42768\sqrt{21}$$

has minimal polynomial  $T^4 - 785808T^3 + 750216816T^2 - 39366000000T + 531441000000$  and

$k = \sqrt[3]{t} \approx 3.030580680157893721226$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{\frac{7}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 42nq^{6m^2+7n^2} \\ &= 84q^7 + 168q^{13} - 168q^{28} + 168q^{31} - 336q^{34} - 336q^{52} + 168q^{61} - 336q^{82} + 168q^{103} + 336q^{112} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(504), (\frac{504}{\cdot}))$ .

[ν-T4-14-3 \(#65 in the paper\)](#)

Let  $\tau_0 = [6, 0, 7] = i\sqrt{\frac{7}{6}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 196452 + 80190\sqrt{6} - 52380\sqrt{14} - 42768\sqrt{21}$$

has minimal polynomial  $T^4 - 785808T^3 + 750216816T^2 - 39366000000T + 531441000000$  and

$k = \sqrt[3]{t} \approx 9.658365615485717586681$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{\frac{7}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 12nq^{21m^2+2n^2} \\ &= 24q^2 - 48q^8 + 48q^{23} - 96q^{29} + 96q^{32} - 120q^{50} + 192q^{53} - 240q^{71} + 48q^{86} - 96q^{92} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(504), (\frac{504}{\cdot}))$ .

[ν-T4-14-4 \(#65 in the paper\)](#)

Let  $\tau_0 = [3, 0, 14] = i\sqrt{\frac{14}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 196452 + 80190\sqrt{6} + 52380\sqrt{14} + 42768\sqrt{21}$$

has minimal polynomial  $T^4 - 785808T^3 + 750216816T^2 - 39366000000T + 531441000000$  and

$k = \sqrt[3]{t} \approx 92.24212347817280636972$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{\frac{7}{2}}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 6nq^{42m^2+n^2} \\ &= 12q - 24q^4 + 48q^{16} - 60q^{25} + 24q^{43} - 48q^{46} + 84q^{49} + 96q^{58} - 96q^{64} - 120q^{67} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(504), (\frac{504}{\cdot}))$ .

[ν-T4-15-1 \(#66 in the paper\)](#)

Let  $\tau_0 = [6, -6, 11] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{19}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 1840752 - 1063530\sqrt{3} - 422604\sqrt{19} + 243810\sqrt{57}$$

has minimal polynomial  $T^4 - 7363008T^3 - 19484198784T^2 + 1062882000000T - 14348907000000$  and  $k = \sqrt[3]{t} \approx -13.92303144171231222418$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{19}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-18m + 12n)q^{33m^2 - 6mn + 2n^2} \\ = 24q^2 - 48q^8 - 24q^{29} + 96q^{32} + 120q^{41} - 120q^{50} - 168q^{53} + 264q^{89} + 168q^{98} - 312q^{113} + \dots$$

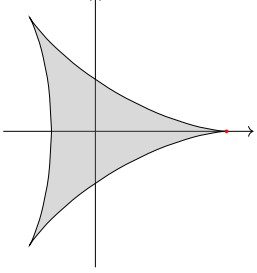
is in  $\mathcal{S}_2(\Gamma_0(684), \left(\frac{684}{\cdot}\right))$ .

[ν-T4-15-2 \(#66 in the paper\)](#)

Let  $\tau_0 = [33, 6, 2] = -\frac{1}{11} + \frac{1}{11}i\sqrt{\frac{19}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 1840752 + 1063530\sqrt{3} - 422604\sqrt{19} - 243810\sqrt{57}$$

has minimal polynomial  $T^4 - 7363008T^3 - 19484198784T^2 + 1062882000000T - 14348907000000$  and  $k = \sqrt[3]{t} \approx 2.990062462715962105781$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{19}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(18m + 66n)q^{6m^2 + 6mn + 11n^2} \\ = 228q^{11} + 228q^{23} - 228q^{38} - 456q^{44} + 228q^{47} - 456q^{62} + 228q^{83} - 456q^{92} + 228q^{131} - 456q^{134} + \dots$$

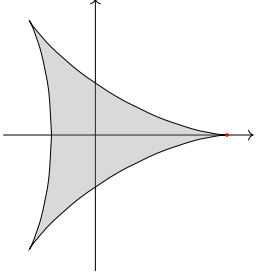
is in  $\mathcal{S}_2(\Gamma_0(684), \left(\frac{684}{\cdot}\right))$ .

[ν-T4-15-3 \(#66 in the paper\)](#)

Let  $\tau_0 = [57, 0, 1] = \frac{i}{\sqrt{57}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 1840752 - 1063530\sqrt{3} + 422604\sqrt{19} - 243810\sqrt{57}$$

has minimal polynomial  $T^4 - 7363008T^3 - 19484198784T^2 + 1062882000000T - 14348907000000$  and  $k = \sqrt[3]{t} \approx 3.000003665671602468610$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{19}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)114nq^{3m^2 + 19n^2} \\ = 228q^{19} + 456q^{22} + 456q^{31} + 456q^{46} + 456q^{67} - 456q^{76} - 912q^{79} - 912q^{88} + 456q^{94} - 912q^{103} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(684), \left(\frac{684}{\cdot}\right))$ .

[ν-T4-15-4 \(#66 in the paper\)](#)

Let  $\tau_0 = [3, 0, 19] = i\sqrt{\frac{19}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 1840752 + 1063530\sqrt{3} + 422604\sqrt{19} + 243810\sqrt{57}$$

has minimal polynomial  $T^4 - 7363008T^3 - 19484198784T^2 + 1062882000000T - 14348907000000$  and  $k = \sqrt[3]{t} \approx 194.5675559011432169189$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{19}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)6nq^{57m^2 + n^2} \\ = 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} + 24q^{58} - 48q^{61} - 96q^{64} + 96q^{73} - 120q^{82} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(684), \left(\frac{684}{\cdot}\right))$ .

[ν-T4-16-1 \(#67 in the paper\)](#)

Let  $\tau_0 = [9, -9, 11] = \frac{1}{2} + \frac{i\sqrt{35}}{6}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -108 - 52\sqrt{5} - 28\sqrt{21} - 12\sqrt{105}$$

has minimal polynomial  $T^4 + 432T^3 - 20224T^2 + 230400T - 512000$  and  $k = \sqrt[3]{t} \approx -7.805469883833490659350$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{\sqrt{105}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-27m + 18n)q^{11m^2 - 3mn + n^2} \\ = 36q - 72q^4 - 36q^9 + 180q^{15} + 144q^{16} - 252q^{21} - 180q^{25} + 72q^{36} + 252q^{39} + 252q^{49} + \dots$$



is in  $\mathcal{S}_2(\Gamma_0(315), (\frac{105}{\cdot}))$ .

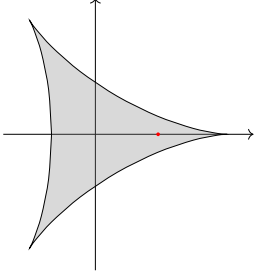
**$\nu$ -T4-16-2 (#67 in the paper)**

Let  $\tau_0 = [27, 21, 7] = -\frac{7}{18} + \frac{i\sqrt{35}}{18}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -108 + 52\sqrt{5} - 28\sqrt{21} + 12\sqrt{105}$$

has minimal polynomial  $T^4 + 432T^3 - 20224T^2 + 230400T - 512000$  and

$k = \sqrt[3]{t} \approx 1.430426554208694631300$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{\sqrt{105}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(63m + 54n)q^{7m^2 + 7mn + 3n^2} \\ &= 90q^3 - 90q^5 - 180q^{12} + 90q^{17} + 180q^{20} + 360q^{27} - 630q^{33} + 90q^{45} - 450q^{47} + 360q^{48} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(315), (\frac{105}{\cdot}))$ .

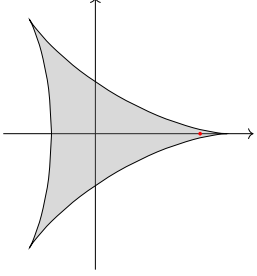
**$\nu$ -T4-16-3 (#67 in the paper)**

Let  $\tau_0 = [27, 15, 5] = -\frac{5}{18} + \frac{i\sqrt{35}}{18}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -108 + 52\sqrt{5} + 28\sqrt{21} - 12\sqrt{105}$$

has minimal polynomial  $T^4 + 432T^3 - 20224T^2 + 230400T - 512000$  and

$k = \sqrt[3]{t} \approx 2.388383978853552004590$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{\sqrt{105}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(45m + 54n)q^{5m^2 + 5mn + 3n^2} \\ &= 126q^3 - 126q^7 - 252q^{12} + 126q^{13} - 252q^{27} + 252q^{28} + 630q^{33} - 630q^{45} + 504q^{48} - 252q^{52} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(315), (\frac{105}{\cdot}))$ .

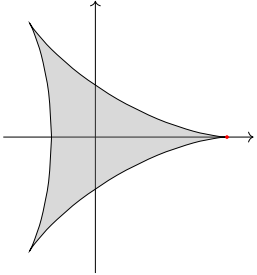
**$\nu$ -T4-16-4 (#67 in the paper)**

Let  $\tau_0 = [81, 3, 1] = -\frac{1}{54} + \frac{i\sqrt{35}}{54}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -108 - 52\sqrt{5} + 28\sqrt{21} + 12\sqrt{105}$$

has minimal polynomial  $T^4 + 432T^3 - 20224T^2 + 230400T - 512000$  and

$k = \sqrt[3]{t} \approx 2.999999771121820929736$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{\sqrt{105}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 162n)q^{m^2 + mn + 9n^2} \\ &= 630q^9 + 630q^{11} + 630q^{15} + 630q^{21} + 630q^{29} - 630q^{35} - 1260q^{36} - 630q^{39} - 1260q^{44} - 630q^{51} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(315), (\frac{105}{\cdot}))$ .

**$\nu$ -T4-17-1 (#68 in the paper)**

Let  $\tau_0 = [1, -1, 9] = \frac{1}{2} + \frac{i\sqrt{35}}{2}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = -29491668 - 13189068\sqrt{5} - 6435612\sqrt{21} - 2878092\sqrt{105}$$

has minimal polynomial  $T^4 + 117966672T^3 + 617538816T^2 - 82133222400T - 272097792000$  and

$k = \sqrt[3]{t} \approx -490.4406237403115043826$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{27\sqrt{105}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-3m + 2n)q^{81m^2 - 3mn + n^2} \\ &= 4q - 8q^4 + 16q^{16} - 20q^{25} + 28q^{49} - 32q^{64} - 4q^{79} + 20q^{85} - 28q^{91} + 40q^{100} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(315), (\frac{945}{\cdot}))$ .

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**$\nu$ -T4-17-2 (#68 in the paper)**

Let  $\tau_0 = [5, -5, 3] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{7}{5}}$  with  $h(D) = 2$ . Then

$t = t_Q(\tau_0) = -29491668 + 13189068\sqrt{5} - 6435612\sqrt{21} + 2878092\sqrt{105}$   
has minimal polynomial  $T^4 + 117966672T^3 + 617538816T^2 - 82133222400T - 272097792000$  and  
 $k = \sqrt[3]{t} \approx -3.018463787303855973129$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{27\sqrt{105}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-15m + 10n)q^{27m^2 - 15mn + 5n^2} \\ &= 20q^5 - 20q^{17} - 40q^{20} + 100q^{47} + 40q^{68} - 140q^{77} + 80q^{80} - 80q^{83} - 100q^{125} + 140q^{143} + \dots \end{aligned}$$

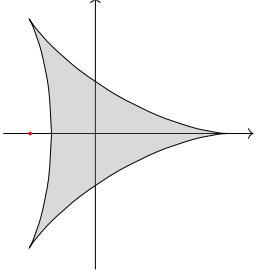
is in  $\mathcal{S}_2(\Gamma_0(315), (\frac{945}{\cdot}))$ .

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**$\nu$ -T4-17-3 (#68 in the paper)**

Let  $\tau_0 = [7, 7, 3] = -\frac{1}{2} + \frac{1}{2}i\sqrt{\frac{5}{7}}$  with  $h(D) = 2$ . Then

$t = t_Q(\tau_0) = -29491668 + 13189068\sqrt{5} + 6435612\sqrt{21} - 2878092\sqrt{105}$   
has minimal polynomial  $T^4 + 117966672T^3 + 617538816T^2 - 82133222400T - 272097792000$  and  
 $k = \sqrt[3]{t} \approx -1.486194642718856814680$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{27\sqrt{105}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(21m + 14n)q^{27m^2 + 21mn + 7n^2} \\ &= 28q^7 - 28q^{13} - 56q^{28} + 56q^{52} + 140q^{55} - 112q^{73} - 196q^{97} - 28q^{103} + 112q^{112} + 140q^{145} + \dots \end{aligned}$$

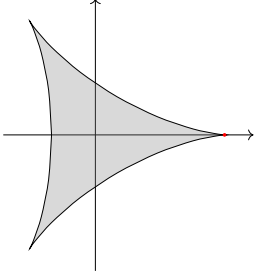
is in  $\mathcal{S}_2(\Gamma_0(315), (\frac{945}{\cdot}))$ .

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**$\nu$ -T4-17-4 (#68 in the paper)**

Let  $\tau_0 = [11, 3, 1] = -\frac{3}{22} + \frac{i\sqrt{35}}{22}$  with  $h(D) = 2$ . Then

$t = t_Q(\tau_0) = -29491668 - 13189068\sqrt{5} + 6435612\sqrt{21} + 2878092\sqrt{105}$   
has minimal polynomial  $T^4 + 117966672T^3 + 617538816T^2 - 82133222400T - 272097792000$  and  
 $k = \sqrt[3]{t} \approx 2.945282170334217170434$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{27\sqrt{105}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 22n)q^{9m^2 + 9mn + 11n^2} \\ &= 70q^{11} + 70q^{29} - 70q^{35} - 140q^{44} + 70q^{65} - 140q^{71} - 140q^{116} + 70q^{119} + 140q^{140} + 280q^{149} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(315), (\frac{945}{\cdot}))$ .

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**$\nu$ -T4-18-1 (#69 in the paper)**

Let  $\tau_0 = [3, -3, 37] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{145}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -764507376 - 341898084\sqrt{5} - 141965460\sqrt{29} - 63488880\sqrt{145}$   
has minimal polynomial  $T^4 + 3058029504T^3 + 56747272896T^2 - 7522959753216T + 101559956668416$  and  
 $k = \sqrt[3]{t} \approx -1451.489469928476265374$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{145}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m + 6n)q^{11m^2 - 3mn + n^2} \\ &= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 120q^{100} - 12q^{109} + 60q^{115} - 216q^{121} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1305), (\frac{1305}{\cdot}))$ .

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**$\nu$ -T4-18-2 (#69 in the paper)**

Let  $\tau_0 = [15, -15, 11] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{29}{15}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -764507376 + 341898084\sqrt{5} - 141965460\sqrt{29} + 63488880\sqrt{145}$   
has minimal polynomial  $T^4 + 3058029504T^3 + 56747272896T^2 - 7522959753216T + 101559956668416$  and  
 $k = \sqrt[3]{t} \approx -4.012422491869414401188$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{145}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-45m + 30n)q^{33m^2 - 15mn + 5n^2}$$

$= 60q^5 - 120q^{20} - 60q^{23} + 300q^{53} + 240q^{80} - 420q^{83} + 120q^{92} - 240q^{107} - 300q^{125} + 480q^{167} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(1305), (\frac{1305}{\cdot}))$ .

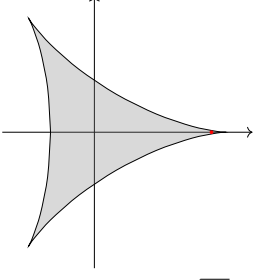
**$\nu$ -T4-18-3 (#69 in the paper)**

Let  $\tau_0 = [33, 15, 5] = -\frac{5}{22} + \frac{1}{22}i\sqrt{\frac{145}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -764507376 - 341898084\sqrt{5} + 141965460\sqrt{29} + 63488880\sqrt{145}$

has minimal polynomial  $T^4 + 3058029504T^3 + 56747272896T^2 - 7522959753216T + 101559956668416$  and

$k = \sqrt[3]{t} \approx 2.670334723648260851547$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{145}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(45m + 66n)q^{15m^2 + 15mn + 11n^2} \\ &= 174q^{11} - 174q^{29} + 174q^{41} - 348q^{44} - 348q^{89} + 174q^{101} + 348q^{116} + 696q^{131} - 348q^{164} + 696q^{176} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1305), (\frac{1305}{\cdot}))$ .

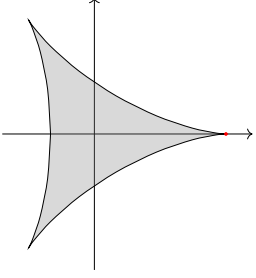
**$\nu$ -T4-18-4 (#69 in the paper)**

Let  $\tau_0 = [111, 3, 1] = -\frac{1}{74} + \frac{1}{74}i\sqrt{\frac{145}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -764507376 + 341898084\sqrt{5} + 141965460\sqrt{29} - 63488880\sqrt{145}$

has minimal polynomial  $T^4 + 3058029504T^3 + 56747272896T^2 - 7522959753216T + 101559956668416$  and

$k = \sqrt[3]{t} \approx 2.999999991170784987653$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{145}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 222n)q^{3m^2 + 3mn + 37n^2} \\ &= 870q^{37} + 870q^{43} + 870q^{55} + 870q^{73} + 870q^{97} + 870q^{127} - 870q^{145} - 1740q^{148} - 1740q^{157} + 870q^{163} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1305), (\frac{1305}{\cdot}))$ .

**$\nu$ -T4-19-1 (#70 in the paper)**

Let  $\tau_0 = [6, -6, 17] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{31}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 147786552 + 85324590\sqrt{3} - 26545428\sqrt{31} - 15326010\sqrt{93}$

has minimal polynomial  $T^4 - 591146208T^3 - 14332946052384T^2 + 774840978000000T - 10460353203000000$  and

$k = \sqrt[3]{t} \approx -28.96428380372967473288$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{31}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-18m + 12n)q^{51m^2 - 6mn + 2n^2} \\ &= 24q^2 - 48q^8 + 96q^{32} - 24q^{47} - 120q^{50} + 120q^{59} - 168q^{71} + 168q^{98} + 264q^{107} - 192q^{128} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1116), (\frac{1116}{\cdot}))$ .

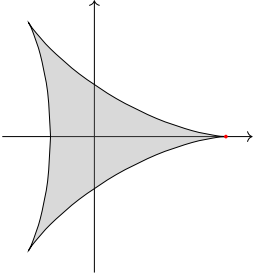
**$\nu$ -T4-19-2 (#70 in the paper)**

Let  $\tau_0 = [51, 6, 2] = -\frac{1}{17} + \frac{1}{17}i\sqrt{\frac{31}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 147786552 - 85324590\sqrt{3} - 26545428\sqrt{31} + 15326010\sqrt{93}$

has minimal polynomial  $T^4 - 591146208T^3 - 14332946052384T^2 + 774840978000000T - 10460353203000000$  and

$k = \sqrt[3]{t} \approx 2.998889665517662550346$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{31}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(18m + 102n)q^{6m^2 + 6mn + 17n^2}$$

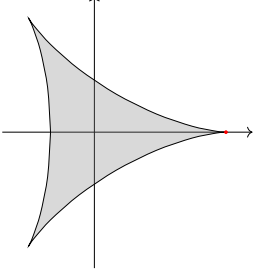
$= 372q^{17} + 372q^{29} + 372q^{53} - 372q^{62} - 744q^{68} - 744q^{86} + 372q^{89} - 744q^{116} + 372q^{137} - 744q^{158} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(1116), (\frac{1116}{\cdot}))$ .

[ν-T4-19-3 \(#70 in the paper\)](#)

Let  $\tau_0 = [93, 0, 1] = \frac{i}{\sqrt{93}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 147786552 - 85324590\sqrt{3} + 26545428\sqrt{31} - 15326010\sqrt{93}$$

has minimal polynomial  $T^4 - 591146208T^3 - 14332946052384T^2 + 774840978000000T - 10460353203000000$  and  $k = \sqrt[3]{t} \approx 3.000000045672108076038$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{31}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 186nq^{3m^2+31n^2} \\ &= 372q^{31} + 744q^{34} + 744q^{43} + 744q^{58} + 744q^{79} + 744q^{106} - 744q^{124} - 1488q^{127} - 1488q^{136} + 744q^{139} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1116), (\frac{1116}{\cdot}))$ .

[ν-T4-19-4 \(#70 in the paper\)](#)

Let  $\tau_0 = [3, 0, 31] = i\sqrt{\frac{31}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 147786552 + 85324590\sqrt{3} + 26545428\sqrt{31} + 15326010\sqrt{93}$$

has minimal polynomial  $T^4 - 591146208T^3 - 14332946052384T^2 + 774840978000000T - 10460353203000000$  and  $k = \sqrt[3]{t} \approx 839.2749095354222491264$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{31}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 6nq^{93m^2+n^2} \\ &= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 24q^{94} - 48q^{97} + 120q^{100} + 96q^{109} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1116), (\frac{1116}{\cdot}))$ .

[ν-T4-20-1 \(#71 in the paper\)](#)

Let  $\tau_0 = [3, -3, 47] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{185}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -12945890016 - 5789578032\sqrt{5} - 2128291200\sqrt{37} - 951800760\sqrt{185}$$

has minimal polynomial  $T^4 + 51783560064T^3 + 3891696679296T^2 - 285652051255296T + 3856302691946496$  and  $k = \sqrt[3]{t} \approx -3727.325334919065611177$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{185}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-9m + 6n)q^{141m^2-3mn+n^2} \\ &= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 120q^{100} - 132q^{121} - 12q^{139} + 60q^{145} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1665), (\frac{1665}{\cdot}))$ .

[ν-T4-20-2 \(#71 in the paper\)](#)

Let  $\tau_0 = [15, -15, 13] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{37}{15}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -12945890016 - 5789578032\sqrt{5} + 2128291200\sqrt{37} + 951800760\sqrt{185}$$

has minimal polynomial  $T^4 + 51783560064T^3 + 3891696679296T^2 - 285652051255296T + 3856302691946496$  and  $k = \sqrt[3]{t} \approx -4.991131435766682204562$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{185}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-45m + 30n)q^{39m^2-15mn+5n^2} \\ &= 60q^5 - 120q^{20} - 60q^{29} + 300q^{59} + 240q^{80} - 420q^{89} + 120q^{116} - 300q^{125} - 240q^{131} + 660q^{179} + \dots \end{aligned}$$

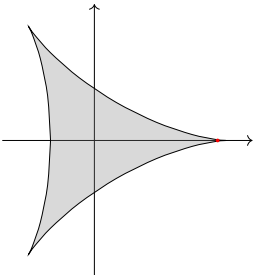
is in  $\mathcal{S}_2(\Gamma_0(1665), (\frac{1665}{\cdot}))$ .

[ν-T4-20-3 \(#71 in the paper\)](#)

Let  $\tau_0 = [39, 15, 5] = -\frac{5}{26} + \frac{1}{26}i\sqrt{\frac{185}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -12945890016 + 5789578032\sqrt{5} + 2128291200\sqrt{37} - 951800760\sqrt{185}$$

has minimal polynomial  $T^4 + 51783560064T^3 + 3891696679296T^2 - 285652051255296T + 3856302691946496$  and  $k = \sqrt[3]{t} \approx 2.809783185779822031690$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{185}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(45m + 78n)q^{15m^2+15mn+13n^2}$$

$$= 222q^{13} - 222q^{37} + 222q^{43} - 444q^{52} - 444q^{97} + 222q^{103} + 444q^{148} + 888q^{163} - 444q^{172} + 222q^{193} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1665), (\frac{1665}{\cdot}))$ .

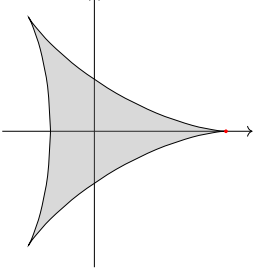
[ν-T4-20-4 \(#71 in the paper\)](#)

Let  $\tau_0 = [141, 3, 1] = -\frac{1}{94} + \frac{1}{94}i\sqrt{\frac{185}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -12945890016 + 5789578032\sqrt{5} - 2128291200\sqrt{37} + 951800760\sqrt{185}$$

has minimal polynomial  $T^4 + 51783560064T^3 + 3891696679296T^2 - 285652051255296T + 3856302691946496$  and

$k = \sqrt[3]{t} \approx 2.999999999478598999442$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{185}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 282n)q^{3m^2+3mn+47n^2}$$

$$= 1110q^{47} + 1110q^{53} + 1110q^{65} + 1110q^{83} + 1110q^{107} + 1110q^{137} + 1110q^{173} - 1110q^{185} - 2220q^{188} - 2220q^{197} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1665), (\frac{1665}{\cdot}))$ .

[ν-T4-21-1 \(#72 in the paper\)](#)

Let  $\tau_0 = [3, -3, 67] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{265}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -1663957782000 - 744144542460\sqrt{5} - 228562179300\sqrt{53} - 102216114000\sqrt{265}$$

has minimal polynomial  $T^4 + 6655831128000T^3 + 1997074895544000T^2 - 117546246144000000T + 1586874322944000000$  and

$k = \sqrt[3]{t} \approx -18810.51841576643192570$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{265}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m + 6n)q^{201m^2-3mn+n^2}$$

$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 120q^{100} - 132q^{121} + 156q^{169} - 168q^{196} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(2385), (\frac{2385}{\cdot}))$ .

[ν-T4-21-2 \(#72 in the paper\)](#)

Let  $\tau_0 = [15, -15, 17] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{53}{15}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -1663957782000 - 744144542460\sqrt{5} + 228562179300\sqrt{53} + 102216114000\sqrt{265}$$

has minimal polynomial  $T^4 + 6655831128000T^3 + 1997074895544000T^2 - 117546246144000000T + 1586874322944000000$  and

$k = \sqrt[3]{t} \approx -7.061539506556527015363$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{265}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-45m + 30n)q^{51m^2-15mn+5n^2}$$

$$= 60q^5 - 120q^{20} - 60q^{41} + 300q^{71} + 240q^{80} - 420q^{101} - 300q^{125} + 120q^{164} - 240q^{179} + 660q^{191} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(2385), (\frac{2385}{\cdot}))$ .

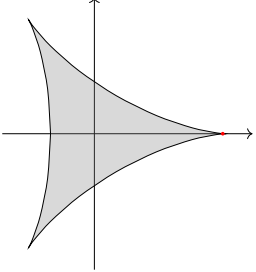
[ν-T4-21-3 \(#72 in the paper\)](#)

Let  $\tau_0 = [51, 15, 5] = -\frac{5}{34} + \frac{1}{34}i\sqrt{\frac{265}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -1663957782000 + 744144542460\sqrt{5} - 228562179300\sqrt{53} + 102216114000\sqrt{265}$$

has minimal polynomial  $T^4 + 6655831128000T^3 + 1997074895544000T^2 - 117546246144000000T + 1586874322944000000$  and

$k = \sqrt[3]{t} \approx 2.927022752085963462815$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{265}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(45m + 102n)q^{15m^2+15mn+17n^2}$$

$$= 318q^{17} + 318q^{47} - 318q^{53} - 636q^{68} + 318q^{107} - 636q^{113} - 636q^{188} + 318q^{197} + 636q^{212} + 1272q^{227} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(2385), (\frac{2385}{\cdot}))$ .

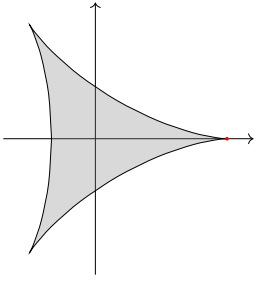
[ν-T4-21-4 \(#72 in the paper\)](#)

Let  $\tau_0 = [201, 3, 1] = -\frac{1}{134} + \frac{1}{134}i\sqrt{\frac{265}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -1663957782000 + 744144542460\sqrt{5} + 228562179300\sqrt{53} - 102216114000\sqrt{265}$$

has minimal polynomial  $T^4 + 6655831128000T^3 + 1997074895544000T^2 - 117546246144000000T + 1586874322944000000$  and

$k = \sqrt[3]{t} \approx 2.999999999995943406694$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{265}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 402n)q^{3m^2+3mn+67n^2} \\ &= 1590q^{67} + 1590q^{73} + 1590q^{85} + 1590q^{103} + 1590q^{127} + 1590q^{157} + 1590q^{193} + 1590q^{235} - 1590q^{265} - 3180q^{268} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(2385), (\frac{2385}{\cdot}))$ .

[ν-T4-22-1 \(#73 in the paper\)](#)

Let  $\tau_0 = [6, -6, 31] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{59}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 315624577104 - 182226258900\sqrt{3} - 41090893110\sqrt{59} + 23723795970\sqrt{177}$$

has minimal polynomial  $T^4 - 1262498308416T^3 - 1418469875132085504T^2 + 76599213979666287744T - 1034089388725494884544$  and  $k = \sqrt[3]{t} \approx -103.9608765556193856071$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{59}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-18m + 12n)q^{93m^2-6mn+2n^2} \\ &= 24q^2 - 48q^8 + 96q^{32} - 120q^{50} - 24q^{89} + 168q^{98} + 120q^{101} - 168q^{113} - 192q^{128} + 264q^{149} + \dots\end{aligned}$$

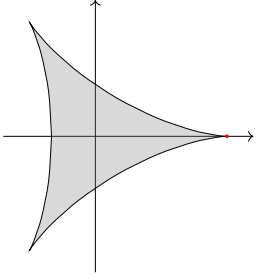
is in  $\mathcal{S}_2(\Gamma_0(2124), (\frac{2124}{\cdot}))$ .

[ν-T4-22-2 \(#73 in the paper\)](#)

Let  $\tau_0 = [93, 6, 2] = -\frac{1}{31} + \frac{1}{31}i\sqrt{\frac{59}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 315624577104 - 182226258900\sqrt{3} + 41090893110\sqrt{59} - 23723795970\sqrt{177}$$

has minimal polynomial  $T^4 - 1262498308416T^3 - 1418469875132085504T^2 + 76599213979666287744T - 1034089388725494884544$  and  $k = \sqrt[3]{t} \approx 2.999975970374139770994$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{59}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(18m + 186n)q^{6m^2+6mn+31n^2} \\ &= 708q^{31} + 708q^{43} + 708q^{67} + 708q^{103} - 708q^{118} - 1416q^{124} - 1416q^{142} + 708q^{151} - 1416q^{172} + 708q^{211} + \dots\end{aligned}$$

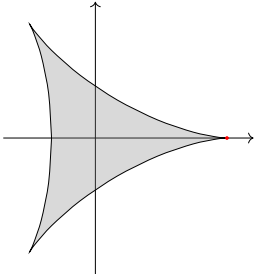
is in  $\mathcal{S}_2(\Gamma_0(2124), (\frac{2124}{\cdot}))$ .

[ν-T4-22-3 \(#73 in the paper\)](#)

Let  $\tau_0 = [177, 0, 1] = \frac{i}{\sqrt{177}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 315624577104 + 182226258900\sqrt{3} - 41090893110\sqrt{59} - 23723795970\sqrt{177}$$

has minimal polynomial  $T^4 - 1262498308416T^3 - 1418469875132085504T^2 + 76599213979666287744T - 1034089388725494884544$  and  $k = \sqrt[3]{t} \approx 3.000000000021386148237$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{59}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)354nq^{3m^2+59n^2} \\ &= 708q^{59} + 1416q^{62} + 1416q^{71} + 1416q^{86} + 1416q^{107} + 1416q^{134} + 1416q^{167} + 1416q^{206} - 1416q^{236} - 2832q^{239} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(2124), (\frac{2124}{\cdot}))$ .

[ν-T4-22-4 \(#73 in the paper\)](#)

Let  $\tau_0 = [3, 0, 59] = i\sqrt{\frac{59}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 315624577104 + 182226258900\sqrt{3} + 41090893110\sqrt{59} + 23723795970\sqrt{177}$$

has minimal polynomial  $T^4 - 1262498308416T^3 - 1418469875132085504T^2 + 76599213979666287744T - 1034089388725494884544$  and  $k = \sqrt[3]{t} \approx 10807.96004376491659607$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{59}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 6nq^{177m^2+n^2} \\ = 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 120q^{100} - 132q^{121} + 156q^{169} + 24q^{178} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(2124), (\frac{2124}{\cdot}))$ .

[ν-T4-23-1 \(#74 in the paper\)](#)

Let  $\tau_0 = [3, -3, 41] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{161}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -2471866848 - 539405568\sqrt{21} - 297577800\sqrt{69} - 194810400\sqrt{161}$   
has minimal polynomial  $T^4 + 9887467392T^3 - 220305619584T^2 - 2519424000000T + 34012224000000$  and  
 $k = \sqrt[3]{t} \approx -2146.322714573342218474$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{161}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m + 6n)q^{123m^2-3mn+n^2} \\ = 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 120q^{100} - 144q^{121} + 60q^{127} - 84q^{133} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1449), (\frac{1449}{\cdot}))$ .

[ν-T4-23-2 \(#74 in the paper\)](#)

Let  $\tau_0 = [21, -21, 11] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{23}{21}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -2471866848 + 539405568\sqrt{21} + 297577800\sqrt{69} - 194810400\sqrt{161}$   
has minimal polynomial  $T^4 + 9887467392T^3 - 220305619584T^2 - 2519424000000T + 34012224000000$  and  
 $k = \sqrt[3]{t} \approx -2.403852339561353514411$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{161}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

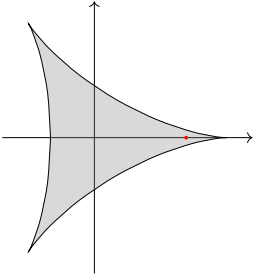
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-63m + 42n)q^{33m^2-21mn+7n^2} \\ = 84q^7 - 84q^{19} - 168q^{28} + 420q^{61} + 168q^{76} - 336q^{97} - 588q^{103} + 336q^{112} - 84q^{157} - 420q^{175} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1449), (\frac{1449}{\cdot}))$ .

[ν-T4-23-3 \(#74 in the paper\)](#)

Let  $\tau_0 = [33, 21, 7] = -\frac{7}{22} + \frac{1}{22}i\sqrt{\frac{161}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -2471866848 + 539405568\sqrt{21} - 297577800\sqrt{69} + 194810400\sqrt{161}$   
has minimal polynomial  $T^4 + 9887467392T^3 - 220305619584T^2 - 2519424000000T + 34012224000000$  and  
 $k = \sqrt[3]{t} \approx 2.093249264186714483631$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{161}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

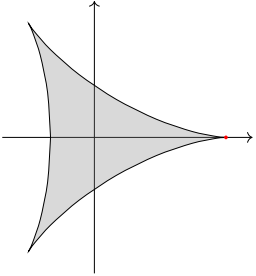
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(63m + 66n)q^{21m^2+21mn+11n^2} \\ = 138q^{11} - 138q^{23} - 276q^{44} + 138q^{53} + 276q^{92} - 276q^{107} + 552q^{113} + 138q^{137} - 690q^{149} + 552q^{176} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1449), (\frac{1449}{\cdot}))$ .

[ν-T4-23-4 \(#74 in the paper\)](#)

Let  $\tau_0 = [123, 3, 1] = -\frac{1}{82} + \frac{1}{82}i\sqrt{\frac{161}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -2471866848 - 539405568\sqrt{21} + 297577800\sqrt{69} + 194810400\sqrt{161}$   
has minimal polynomial  $T^4 + 9887467392T^3 - 220305619584T^2 - 2519424000000T + 34012224000000$  and  
 $k = \sqrt[3]{t} \approx 2.999999997269270398621$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{161}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 246n)q^{3m^2+3mn+41n^2} \\ = 966q^{41} + 966q^{47} + 966q^{59} + 966q^{77} + 966q^{101} + 966q^{131} - 966q^{161} - 1932q^{164} + 966q^{167} - 1932q^{173} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1449), (\frac{1449}{\cdot}))$ .

[ν-T4-24-1 \(#75 in the paper\)](#)

Let  $\tau_0 = [3, -3, 53] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{209}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -61082965296 - 10633179420\sqrt{33} - 8090636400\sqrt{57} - 4225197060\sqrt{209}$

has minimal polynomial  $T^4 + 244331861184T^3 - 6584729661504T^2 - 660451885056T + 8916100448256$  and  $k = \sqrt[3]{t} \approx -6251.631456432370137700$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{209}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

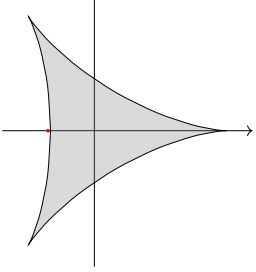
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m + 6n)q^{159m^2 - 3mn + n^2} \\ = 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 120q^{100} - 132q^{121} - 12q^{157} + 60q^{163} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1881), (\frac{1881}{\cdot}))$ .

#### [ν-T4-24-2 \(#75 in the paper\)](#)

Let  $\tau_0 = [33, 33, 13] = -\frac{1}{2} + \frac{1}{2}i\sqrt{\frac{19}{33}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -61082965296 - 10633179420\sqrt{33} + 8090636400\sqrt{57} + 4225197060\sqrt{209}$   
has minimal polynomial  $T^4 + 244331861184T^3 - 6584729661504T^2 - 660451885056T + 8916100448256$  and  $k = \sqrt[3]{t} \approx -1.059062443227299151536$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{209}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

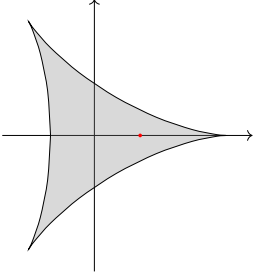
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(99m + 66n)q^{39m^2 + 33mn + 11n^2} \\ = 132q^{11} - 132q^{17} - 264q^{44} + 264q^{68} + 660q^{83} - 528q^{101} - 132q^{131} - 924q^{149} + 528q^{176} + 660q^{197} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1881), (\frac{1881}{\cdot}))$ .

#### [ν-T4-24-3 \(#75 in the paper\)](#)

Let  $\tau_0 = [39, 33, 11] = -\frac{11}{26} + \frac{1}{26}i\sqrt{\frac{209}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -61082965296 + 10633179420\sqrt{33} + 8090636400\sqrt{57} - 4225197060\sqrt{209}$   
has minimal polynomial  $T^4 + 244331861184T^3 - 6584729661504T^2 - 660451885056T + 8916100448256$  and  $k = \sqrt[3]{t} \approx 1.043971864736952165677$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{209}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

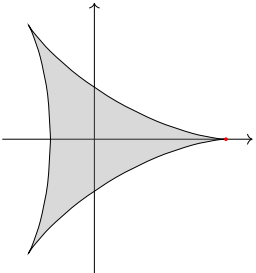
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(99m + 78n)q^{33m^2 + 33mn + 13n^2} \\ = 114q^{13} - 114q^{19} - 228q^{52} + 228q^{76} + 114q^{79} + 456q^{109} - 570q^{127} - 228q^{151} - 570q^{193} + 456q^{208} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1881), (\frac{1881}{\cdot}))$ .

#### [ν-T4-24-4 \(#75 in the paper\)](#)

Let  $\tau_0 = [159, 3, 1] = -\frac{1}{106} + \frac{1}{106}i\sqrt{\frac{209}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -61082965296 + 10633179420\sqrt{33} - 8090636400\sqrt{57} + 4225197060\sqrt{209}$   
has minimal polynomial  $T^4 + 244331861184T^3 - 6584729661504T^2 - 660451885056T + 8916100448256$  and  $k = \sqrt[3]{t} \approx 2.999999999889494559309$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{209}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 318n)q^{3m^2 + 3mn + 53n^2} \\ = 1254q^{53} + 1254q^{59} + 1254q^{71} + 1254q^{89} + 1254q^{113} + 1254q^{143} + 1254q^{179} - 1254q^{209} - 2508q^{212} - 1254q^{221} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1881), (\frac{1881}{\cdot}))$ .

#### [ν-N4-1-1](#)

Let  $\tau_0 = [1, -1, 2] = \frac{1}{2} + \frac{i\sqrt{7}}{2}$  with  $h(D) = 1$ . Then



$$t = t_Q(\tau_0) = -\frac{4023}{4} - \frac{891\sqrt{21}}{4} - \frac{243\sqrt[4]{3}\sqrt{93\sqrt{3}+61\sqrt{7}}}{2\sqrt{2}}$$

has minimal polynomial  $T^4 + 4023T^3 - 133407T^2 + 17458821T - 66430125$  and  $k = \sqrt[3]{t} \approx -15.94898512139843890669$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{27\sqrt{21}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-3m+2n)q^{18m^2-3mn+n^2} \\ &= 4q - 8q^4 + 12q^{16} + 20q^{22} - 20q^{25} - 28q^{28} + 44q^{46} + 28q^{49} - 52q^{58} - 24q^{64} + \dots\end{aligned}$$

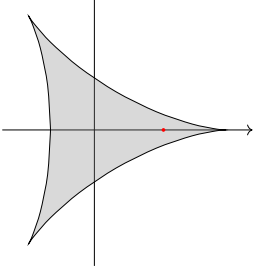
is in  $\mathcal{S}_2(\Gamma_0(63), (\frac{189}{\cdot}))$ .

#### $\nu$ -N4-1-2

Let  $\tau_0 = [4, 3, 1] = -\frac{3}{8} + \frac{i\sqrt{7}}{8}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = -\frac{4023}{4} - \frac{891\sqrt{21}}{4} + \frac{243\sqrt[4]{3}\sqrt{93\sqrt{3}+61\sqrt{7}}}{2\sqrt{2}}$$

has minimal polynomial  $T^4 + 4023T^3 - 133407T^2 + 17458821T - 66430125$  and  $k = \sqrt[3]{t} \approx 1.575121578964678896813$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{27\sqrt{21}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m+8n)q^{9m^2+9mn+4n^2} \\ &= 14q^4 - 14q^7 - 28q^{16} + 14q^{22} + 28q^{28} + 56q^{37} - 28q^{43} - 70q^{46} + 14q^{58} - 14q^{64} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(63), (\frac{189}{\cdot}))$ .

#### $\nu$ -N4-1-3

Let  $\tau_0 = [2, 1, 1] = -\frac{1}{4} + \frac{i\sqrt{7}}{4}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = -\frac{4023}{4} + \frac{891\sqrt{21}}{4} + \frac{243i\sqrt[4]{3}\sqrt{-93\sqrt{3}+61\sqrt{7}}}{2\sqrt{2}}$$

has minimal polynomial  $T^4 + 4023T^3 - 133407T^2 + 17458821T - 66430125$  and  $k = \sqrt[3]{t} \approx 3.623187281867491681756 + 1.730348100185317455035i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{27\sqrt{21}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(3m+4n)q^{9m^2+3mn+2n^2} \\ &= 8q^2 - 14q^8 - 10q^{11} + 14q^{14} - 22q^{23} + 26q^{29} + 24q^{32} + 6q^{44} - 40q^{50} + 38q^{53} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(63), (\frac{189}{\cdot}))$ .

#### $\nu$ -N4-1-4

Let  $\tau_0 = [2, -1, 1] = \frac{1}{4} + \frac{i\sqrt{7}}{4}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = -\frac{4023}{4} + \frac{891\sqrt{21}}{4} - \frac{243i\sqrt[4]{3}\sqrt{-93\sqrt{3}+61\sqrt{7}}}{2\sqrt{2}}$$

has minimal polynomial  $T^4 + 4023T^3 - 133407T^2 + 17458821T - 66430125$  and  $k = \sqrt[3]{t} \approx 3.623187281867491681756 - 1.730348100185317455035i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{27\sqrt{21}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-3m+4n)q^{9m^2-3mn+2n^2} \\ &= 8q^2 - 14q^8 - 10q^{11} + 14q^{14} - 22q^{23} + 26q^{29} + 24q^{32} + 6q^{44} - 40q^{50} + 38q^{53} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(63), (\frac{189}{\cdot}))$ .

#### $\nu$ -N4-2-1

Let  $\tau_0 = [1, -1, 5] = \frac{1}{2} + \frac{i\sqrt{19}}{2}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = -221346 - 29322\sqrt{57} - 486\sqrt[4]{3}\sqrt{138318\sqrt{3}+54962\sqrt{19}}$$

has minimal polynomial  $T^4 + 885384T^3 - 71523648T^2 + 1944995328T - 17414258688$  and  $k = \sqrt[3]{t} \approx -96.02635191438447995375$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{27\sqrt{57}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-3m+2n)q^{45m^2-3mn+n^2} \\ &= 4q - 8q^4 + 16q^{16} - 20q^{25} - 4q^{43} + 48q^{49} - 28q^{55} - 32q^{64} + 44q^{73} - 52q^{85} + \dots\end{aligned}$$

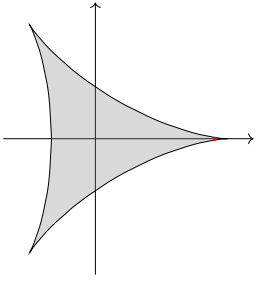
is in  $\mathcal{S}_2(\Gamma_0(171), (\frac{513}{\cdot}))$ .

#### $\nu$ -N4-2-2

Let  $\tau_0 = [7, 3, 1] = -\frac{3}{14} + \frac{i\sqrt{19}}{14}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = -221346 - 29322\sqrt{57} + 486\sqrt[4]{3}\sqrt{138318\sqrt{3}+54962\sqrt{19}}$$

has minimal polynomial  $T^4 + 885384T^3 - 71523648T^2 + 1944995328T - 17414258688$  and  $k = \sqrt[3]{t} \approx 2.727345302759981770335$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{27\sqrt{57}}{8\pi^2}L(\Theta_Q, \tau_0, 2)$ , where

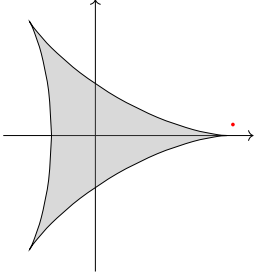
$$\begin{aligned}\Theta_{Q, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-3}(n)(9m + 14n)q^{9m^2 + 9mn + 7n^2} \\ &= 38q^7 - 38q^{19} + 38q^{25} - 76q^{28} - 76q^{55} + 38q^{61} + 76q^{76} + 152q^{85} - 76q^{100} + 152q^{112} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(171), (\frac{513}{\cdot}))$ .

#### $\nu$ -N4-2-3

Let  $\tau_0 = [5, 1, 1] = -\frac{1}{10} + \frac{i\sqrt{19}}{10}$  with  $h(D) = 1$ . Then

$t = t_Q(\tau_0) = -221346 + 29322\sqrt{57} + 486i\sqrt[4]{3}\sqrt{-138318\sqrt{3} + 54962\sqrt{19}}$   
has minimal polynomial  $T^4 + 885384T^3 - 71523648T^2 + 1944995328T - 17414258688$  and  
 $k = \sqrt[3]{t} \approx 3.135915712951375547117 + 0.251102521170458487535i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{27\sqrt{57}}{16\pi^2}L(\Theta_Q, \tau_0, 2)$ , where

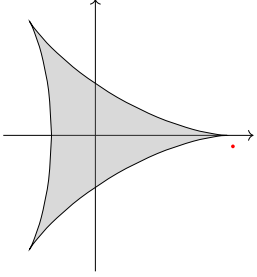
$$\begin{aligned}\Theta_{Q, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-3}(n)(3m + 10n)q^{9m^2 + 3mn + 5n^2} \\ &= 20q^5 + 14q^{11} + 26q^{17} - 40q^{20} - 34q^{23} - 38q^{35} - 28q^{44} + 32q^{47} - 52q^{68} + 76q^{77} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(171), (\frac{513}{\cdot}))$ .

#### $\nu$ -N4-2-4

Let  $\tau_0 = [5, -1, 1] = \frac{1}{10} + \frac{i\sqrt{19}}{10}$  with  $h(D) = 1$ . Then

$t = t_Q(\tau_0) = -221346 + 29322\sqrt{57} - 486i\sqrt[4]{3}\sqrt{-138318\sqrt{3} + 54962\sqrt{19}}$   
has minimal polynomial  $T^4 + 885384T^3 - 71523648T^2 + 1944995328T - 17414258688$  and  
 $k = \sqrt[3]{t} \approx 3.135915712951375547117 - 0.251102521170458487535i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{27\sqrt{57}}{16\pi^2}L(\Theta_Q, \tau_0, 2)$ , where

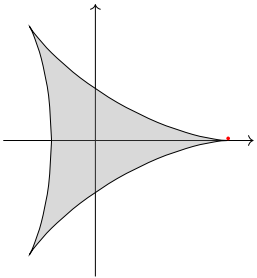
$$\begin{aligned}\Theta_{Q, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-3}(n)(-3m + 10n)q^{9m^2 - 3mn + 5n^2} \\ &= 20q^5 + 14q^{11} + 26q^{17} - 40q^{20} - 34q^{23} - 38q^{35} - 28q^{44} + 32q^{47} - 52q^{68} + 76q^{77} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(171), (\frac{513}{\cdot}))$ .

#### $\nu$ -N4-3-1

Let  $\tau_0 = [9, 1, 1] = -\frac{1}{18} + \frac{i\sqrt{35}}{18}$  with  $h(D) = 2$ . Then

$t = t_Q(\tau_0) = 144 - 52\sqrt{5} - 80i\sqrt{7} + 36i\sqrt{35}$   
has minimal polynomial  $T^4 - 576T^3 + 277696T^2 - 14155776T + 191102976$  and  
 $k = \sqrt[3]{t} \approx 3.027355824977709699703 + 0.047968520678194350363i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{3\sqrt{105}}{16\pi^2}L(\Theta_Q, \tau_0, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(3m+18n)q^{3m^2+mn+3n^2}$$

$$= 36q^3 + 30q^5 + 42q^7 - 72q^{12} - 42q^{13} - 30q^{17} - 60q^{20} + 18q^{27} - 84q^{28} - 90q^{45} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(105), \left(\frac{105}{\cdot}\right))$ .

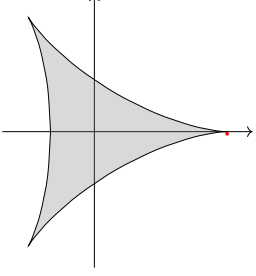
#### $\nu$ -N4-3-2

Let  $\tau_0 = [9, -1, 1] = \frac{1}{18} + \frac{i\sqrt{35}}{18}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = 144 - 52\sqrt{5} + 80i\sqrt{7} - 36i\sqrt{35}$$

has minimal polynomial  $T^4 - 576T^3 + 277696T^2 - 14155776T + 191102976$  and

$k = \sqrt[3]{t} \approx 3.027355824977709699703 - 0.047968520678194350363i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{3\sqrt{105}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-3m+18n)q^{3m^2-mn+3n^2}$$

$$= 36q^3 + 30q^5 + 42q^7 - 72q^{12} - 42q^{13} - 30q^{17} - 60q^{20} + 18q^{27} - 84q^{28} - 90q^{45} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(105), \left(\frac{105}{\cdot}\right))$ .

#### $\nu$ -N4-3-3

Let  $\tau_0 = [3, 1, 3] = -\frac{1}{6} + \frac{i\sqrt{35}}{6}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = 144 + 52\sqrt{5} + 80i\sqrt{7} + 36i\sqrt{35}$$

has minimal polynomial  $T^4 - 576T^3 + 277696T^2 - 14155776T + 191102976$  and

$k = \sqrt[3]{t} \approx 7.472135954999579392818 + 2.645751311064590590502i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{3\sqrt{105}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(3m+6n)q^{9m^2+mn+n^2}$$

$$= 12q - 24q^4 + 6q^9 - 30q^{15} + 48q^{16} + 42q^{21} - 60q^{25} - 12q^{36} - 42q^{39} + 84q^{49} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(105), \left(\frac{105}{\cdot}\right))$ .

#### $\nu$ -N4-3-4

Let  $\tau_0 = [3, -1, 3] = \frac{1}{6} + \frac{i\sqrt{35}}{6}$  with  $h(D) = 2$ . Then

$$t = t_Q(\tau_0) = 144 + 52\sqrt{5} - 80i\sqrt{7} - 36i\sqrt{35}$$

has minimal polynomial  $T^4 - 576T^3 + 277696T^2 - 14155776T + 191102976$  and

$k = \sqrt[3]{t} \approx 7.472135954999579392818 - 2.645751311064590590502i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{3\sqrt{105}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-3m+6n)q^{9m^2-mn+n^2}$$

$$= 12q - 24q^4 + 6q^9 - 30q^{15} + 48q^{16} + 42q^{21} - 60q^{25} - 12q^{36} - 42q^{39} + 84q^{49} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(105), \left(\frac{105}{\cdot}\right))$ .

#### $\nu$ -N4-4-1

Let  $\tau_0 = [1, -1, 11] = \frac{1}{2} + \frac{i\sqrt{43}}{2}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = -221184162 - 19474182\sqrt{129} - 1458\sqrt{46028106018 + 4052548766\sqrt{129}}$$

has minimal polynomial  $T^4 + 884736648T^3 - 71663476032T^2 + 1934927709696T - 17414258688000$  and

$k = \sqrt[3]{t} \approx -960.0002636717221142411$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{27\sqrt{129}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-3m+2n)q^{9m^2-3mn+n^2}$$

$$= 4q - 8q^4 + 16q^{16} - 20q^{25} + 28q^{49} - 32q^{64} - 4q^{97} + 40q^{100} + 20q^{103} - 28q^{109} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(387), \left(\frac{1161}{\cdot}\right))$ .

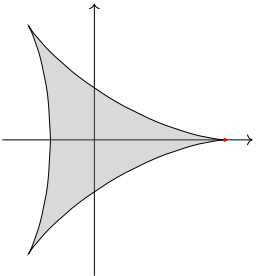
#### $\nu$ -N4-4-2

Let  $\tau_0 = [13, 3, 1] = -\frac{3}{26} + \frac{i\sqrt{43}}{26}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = -221184162 - 19474182\sqrt{129} + 1458\sqrt{46028106018 + 4052548766\sqrt{129}}$$

has minimal polynomial  $T^4 + 884736648T^3 - 71663476032T^2 + 1934927709696T - 17414258688000$  and

$k = \sqrt[3]{t} \approx 2.971962624499130926145$  is **IN**  $\mathcal{K}_Q^\circ$ .



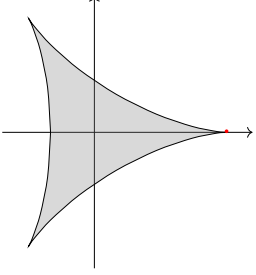
We have  $\nu(t) = -\frac{27\sqrt{129}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m+26n)q^{9m^2+9mn+13n^2}$   
 $= 86q^{13} + 86q^{31} - 86q^{43} - 172q^{52} + 86q^{67} - 172q^{79} + 86q^{121} - 172q^{124} + 172q^{172} + 344q^{181} + \dots$   
is in  $\mathcal{S}_2(\Gamma_0(387), \left(\frac{1161}{\cdot}\right))$ .

#### $\nu$ -N4-4-3

Let  $\tau_0 = [11, 1, 1] = -\frac{1}{22} + \frac{i\sqrt{43}}{22}$  with  $h(D) = 1$ . Then

$t = t_Q(\tau_0) = -221184162 + 19474182\sqrt{129} + 1458i\sqrt{-46028106018 + 4052548766\sqrt{129}}$   
has minimal polynomial  $T^4 + 884736648T^3 - 71663476032T^2 + 1934927709696T - 17414258688000$  and  
 $k = \sqrt[3]{t} \approx 3.014018275763481888956 + 0.024433072518158402813i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{27\sqrt{129}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(3m+22n)q^{9m^2+3mn+11n^2}$$

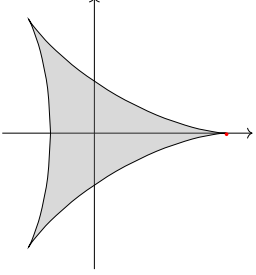
$$= 44q^{11} + 38q^{17} + 50q^{23} + 32q^{41} - 88q^{44} - 82q^{47} + 56q^{53} - 94q^{59} - 76q^{68} + 26q^{83} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(387), \left(\frac{1161}{\cdot}\right))$ .

#### $\nu$ -N4-4-4

Let  $\tau_0 = [11, -1, 1] = \frac{1}{22} + \frac{i\sqrt{43}}{22}$  with  $h(D) = 1$ . Then

$t = t_Q(\tau_0) = -221184162 + 19474182\sqrt{129} - 1458i\sqrt{-46028106018 + 4052548766\sqrt{129}}$   
has minimal polynomial  $T^4 + 884736648T^3 - 71663476032T^2 + 1934927709696T - 17414258688000$  and  
 $k = \sqrt[3]{t} \approx 3.014018275763481888956 - 0.024433072518158402813i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{27\sqrt{129}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-3m+22n)q^{9m^2-3mn+11n^2}$$

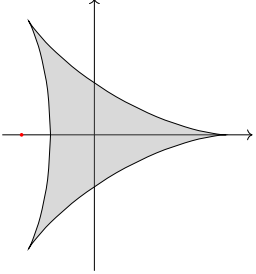
$$= 44q^{11} + 38q^{17} + 50q^{23} + 32q^{41} - 88q^{44} - 82q^{47} + 56q^{53} - 94q^{59} - 76q^{68} + 26q^{83} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(387), \left(\frac{1161}{\cdot}\right))$ .

#### $\nu$ -N4-5-1

Let  $\tau_0 = [9, 9, 4] = -\frac{1}{2} + \frac{i\sqrt{7}}{6}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 18 - \frac{3\sqrt{21}}{2} - \frac{9}{2}\sqrt[4]{3}\sqrt{\sqrt{3}+2\sqrt{7}}$   
has minimal polynomial  $T^4 - 72T^3 + 1728T^2 - 10449T - 91125$  and  
 $k = \sqrt[3]{t} \approx -1.659386141097061711062$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{\sqrt{21}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(27m+18n)q^{4m^2+3mn+n^2}$$

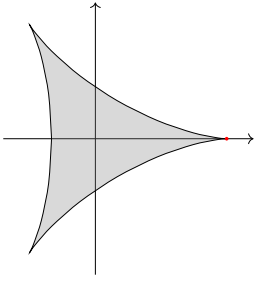
$$= 36q - 36q^2 - 72q^4 + 252q^8 - 144q^{11} - 252q^{14} + 108q^{16} + 180q^{22} + 288q^{23} - 180q^{25} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(63), \left(\frac{21}{\cdot}\right))$ .

#### $\nu$ -N4-5-2

Let  $\tau_0 = [18, 3, 1] = -\frac{1}{12} + \frac{i\sqrt{7}}{12}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 18 - \frac{3\sqrt{21}}{2} + \frac{9}{2}\sqrt[4]{3}\sqrt{\sqrt{3}+2\sqrt{7}}$   
has minimal polynomial  $T^4 - 72T^3 + 1728T^2 - 10449T - 91125$  and  
 $k = \sqrt[3]{t} \approx 2.993374122716392032259$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{\sqrt{21}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 36n)q^{m^2+mn+2n^2} \\ &= 126q^2 + 126q^4 - 126q^7 - 126q^8 - 252q^{11} + 126q^{14} - 252q^{16} + 126q^{22} - 252q^{23} + 252q^{28} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(63), (\frac{21}{\cdot}))$ .

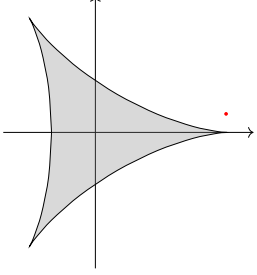
#### $\nu$ -N4-5-3

Let  $\tau_0 = [9, 3, 2] = -\frac{1}{6} + \frac{i\sqrt{7}}{6}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 18 + \frac{3\sqrt{21}}{2} + \frac{9}{2}i\sqrt{-3 + 2\sqrt{21}}$$

has minimal polynomial  $T^4 - 72T^3 + 1728T^2 - 10449T - 91125$  and

$k = \sqrt[3]{t} \approx 2.980143505070886883246 + 0.422186045999587410327i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{\sqrt{21}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 18n)q^{2m^2+mn+n^2} \\ &= 36q + 18q^2 - 72q^4 - 126q^8 + 72q^{11} + 126q^{14} + 108q^{16} + 180q^{22} - 144q^{23} - 180q^{25} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(63), (\frac{21}{\cdot}))$ .

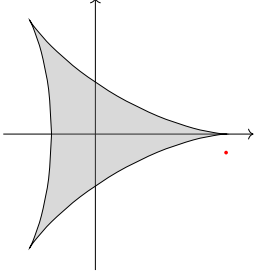
#### $\nu$ -N4-5-4

Let  $\tau_0 = [9, -3, 2] = \frac{1}{6} + \frac{i\sqrt{7}}{6}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 18 + \frac{3\sqrt{21}}{2} - \frac{9}{2}i\sqrt{-3 + 2\sqrt{21}}$$

has minimal polynomial  $T^4 - 72T^3 + 1728T^2 - 10449T - 91125$  and

$k = \sqrt[3]{t} \approx 2.980143505070886883246 - 0.422186045999587410327i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{\sqrt{21}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m + 18n)q^{2m^2-mn+n^2} \\ &= 36q + 18q^2 - 72q^4 - 126q^8 + 72q^{11} + 126q^{14} + 108q^{16} + 180q^{22} - 144q^{23} - 180q^{25} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(63), (\frac{21}{\cdot}))$ .

#### $\nu$ -N4-6-1

Let  $\tau_0 = [1, -1, 17] = \frac{1}{2} + \frac{i\sqrt{67}}{2}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = -36799488162 - 2595635766\sqrt{201} - 3402\sqrt{234015667197498 + 16506192489998\sqrt{201}}$$

has minimal polynomial  $T^4 + 147197952648T^3 - 11923033972032T^2 + 321921931101696T - 2897297289216000$  and

$k = \sqrt[3]{t} \approx -5280.000008716425589457$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{27\sqrt{201}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-3m + 2n)q^{153m^2-3mn+n^2} \\ &= 4q - 8q^4 + 16q^{16} - 20q^{25} + 28q^{49} - 32q^{64} + 40q^{100} - 44q^{121} - 4q^{151} + 20q^{157} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(603), (\frac{1809}{\cdot}))$ .

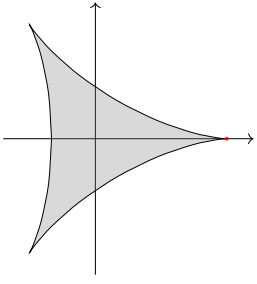
#### $\nu$ -N4-6-2

Let  $\tau_0 = [19, 3, 1] = -\frac{3}{38} + \frac{i\sqrt{67}}{38}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = -36799488162 - 2595635766\sqrt{201} + 3402\sqrt{234015667197498 + 16506192489998\sqrt{201}}$$

has minimal polynomial  $T^4 + 147197952648T^3 - 11923033972032T^2 + 321921931101696T - 2897297289216000$  and

$k = \sqrt[3]{t} \approx 2.994889267470101707326$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{27\sqrt{201}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

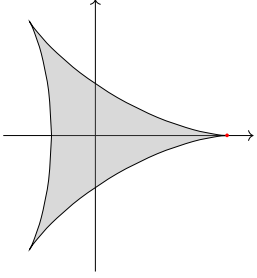
$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m+38n)q^{9m^2+9mn+19n^2} \\ &= 134q^{19} + 134q^{37} - 134q^{67} + 134q^{73} - 268q^{76} - 268q^{103} + 134q^{127} - 268q^{148} + 134q^{199} - 268q^{211} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(603), (\frac{1809}{\cdot}))$ .

#### $\nu$ -N4-6-3

Let  $\tau_0 = [17, 1, 1] = -\frac{1}{34} + \frac{i\sqrt{67}}{34}$  with  $h(D) = 1$ . Then

$t = t_Q(\tau_0) = -36799488162 + 2595635766\sqrt{201} + 3402i\sqrt{-234015667197498 + 16506192489998\sqrt{201}}$   
has minimal polynomial  $T^4 + 147197952648T^3 - 11923033972032T^2 + 321921931101696T - 2897297289216000$  and  
 $k = \sqrt[3]{t} \approx 3.002555363788691880693 + 0.004431055203832080052i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{27\sqrt{201}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

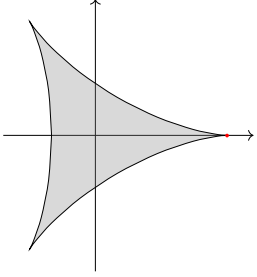
$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(3m+34n)q^{9m^2+3mn+17n^2} \\ &= 68q^{17} + 62q^{23} + 74q^{29} + 56q^{47} + 80q^{59} - 136q^{68} - 130q^{71} - 142q^{83} + 50q^{89} - 124q^{92} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(603), (\frac{1809}{\cdot}))$ .

#### $\nu$ -N4-6-4

Let  $\tau_0 = [17, -1, 1] = \frac{1}{34} + \frac{i\sqrt{67}}{34}$  with  $h(D) = 1$ . Then

$t = t_Q(\tau_0) = -36799488162 + 2595635766\sqrt{201} - 3402i\sqrt{-234015667197498 + 16506192489998\sqrt{201}}$   
has minimal polynomial  $T^4 + 147197952648T^3 - 11923033972032T^2 + 321921931101696T - 2897297289216000$  and  
 $k = \sqrt[3]{t} \approx 3.002555363788691880693 - 0.004431055203832080052i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{27\sqrt{201}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-3m+34n)q^{9m^2-3mn+17n^2} \\ &= 68q^{17} + 62q^{23} + 74q^{29} + 56q^{47} + 80q^{59} - 136q^{68} - 130q^{71} - 142q^{83} + 50q^{89} - 124q^{92} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(603), (\frac{1809}{\cdot}))$ .

#### $\nu$ -N4-7-1

Let  $\tau_0 = [1, -1, 41] = \frac{1}{2} + \frac{i\sqrt{163}}{2}$  with  $h(D) = 1$ . Then

$t = t_Q(\tau_0) = -65634353160192162 - 2968088047202178\sqrt{489} - 37422\sqrt{6152313052585724680358658 + 278217213315981187416074\sqrt{489}}$   
has minimal polynomial  $T^4 + 262537412640768648T^3 - 21265530423902068032T^2 + 574169321445369693696T - 5167523893008236544000$  and  
 $k = \sqrt[3]{t} \approx -640320.000000005926689$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{27\sqrt{489}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

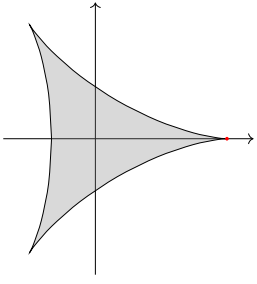
$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-3m+2n)q^{369m^2-3mn+n^2} \\ &= 4q - 8q^4 + 16q^{16} - 20q^{25} + 28q^{49} - 32q^{64} + 40q^{100} - 44q^{121} + 52q^{169} - 56q^{196} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1467), (\frac{4401}{\cdot}))$ .

#### $\nu$ -N4-7-2

Let  $\tau_0 = [43, 3, 1] = -\frac{3}{86} + \frac{i\sqrt{163}}{86}$  with  $h(D) = 1$ . Then

$t = t_Q(\tau_0) = -65634353160192162 - 2968088047202178\sqrt{489} + 37422\sqrt{6152313052585724680358658 + 278217213315981187416074\sqrt{489}}$   
has minimal polynomial  $T^4 + 262537412640768648T^3 - 21265530423902068032T^2 + 574169321445369693696T - 5167523893008236544000$  and  
 $k = \sqrt[3]{t} \approx 2.999957833780763770823$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{27\sqrt{489}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$ , where

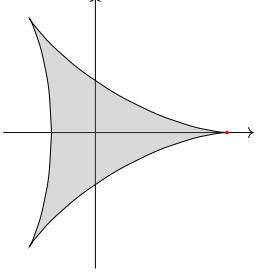
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m+86n)q^{9m^2+9mn+43n^2} \\ = 326q^{43} + 326q^{61} + 326q^{97} + 326q^{151} - 326q^{163} - 652q^{172} - 652q^{199} + 326q^{223} - 652q^{244} - 652q^{307} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1467), (\frac{4401}{\cdot}))$ .

#### $\nu$ -N4-7-3

Let  $\tau_0 = [41, 1, 1] = -\frac{1}{82} + \frac{i\sqrt{163}}{82}$  with  $h(D) = 1$ . Then

$t = t_Q(\tau_0) = -65634353160192162 + 2968088047202178\sqrt{489} + 37422\sqrt{6152313052585724680358658 - 278217213315981187416074\sqrt{489}}$  has minimal polynomial  $T^4 + 262537412640768648T^3 - 21265530423902068032T^2 + 574169321445369693696T - 5167523893008236544000$  and  $k = \sqrt[3]{t} \approx 3.000021083109616726215 + 0.000036517359216866885i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{27\sqrt{489}}{16\pi^2}L(\Theta_{Q,\tau_0},2)$ , where

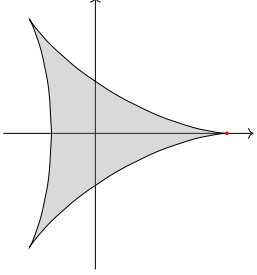
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(3m+82n)q^{9m^2+3mn+41n^2} \\ = 164q^{41} + 158q^{47} + 170q^{53} + 152q^{71} + 176q^{83} + 146q^{113} + 182q^{131} - 328q^{164} - 322q^{167} + 140q^{173} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1467), (\frac{4401}{\cdot}))$ .

#### $\nu$ -N4-7-4

Let  $\tau_0 = [41, -1, 1] = \frac{1}{82} + \frac{i\sqrt{163}}{82}$  with  $h(D) = 1$ . Then

$t = t_Q(\tau_0) = -65634353160192162 + 2968088047202178\sqrt{489} - 37422\sqrt{6152313052585724680358658 - 278217213315981187416074\sqrt{489}}$  has minimal polynomial  $T^4 + 262537412640768648T^3 - 21265530423902068032T^2 + 574169321445369693696T - 5167523893008236544000$  and  $k = \sqrt[3]{t} \approx 3.000021083109616726215 - 0.000036517359216866885i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{27\sqrt{489}}{16\pi^2}L(\Theta_{Q,\tau_0},2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-3m+82n)q^{9m^2-3mn+41n^2} \\ = 164q^{41} + 158q^{47} + 170q^{53} + 152q^{71} + 176q^{83} + 146q^{113} + 182q^{131} - 328q^{164} - 322q^{167} + 140q^{173} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1467), (\frac{4401}{\cdot}))$ .

#### $\nu$ -N4-8-1

Let  $\tau_0 = [9, -9, 7] = \frac{1}{2} + \frac{i\sqrt{19}}{6}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 18 - 6\sqrt{57} - 18\sqrt{2(-3 + \sqrt{57})}$$

has minimal polynomial  $T^4 - 72T^3 + 1728T^2 + 870912T - 23887872$  and  $k = \sqrt[3]{t} \approx -4.337355981044898204485$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{\sqrt{57}}{16\pi^2}L(\Theta_{Q,\tau_0},2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-27m+18n)q^{7m^2-3mn+n^2} \\ = 36q - 72q^4 - 36q^5 + 180q^{11} + 144q^{16} - 252q^{17} + 72q^{20} - 144q^{23} - 180q^{25} + 684q^{35} + \dots$$

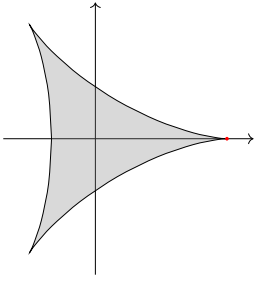
is in  $\mathcal{S}_2(\Gamma_0(171), (\frac{57}{\cdot}))$ .

#### $\nu$ -N4-8-2

Let  $\tau_0 = [45, 3, 1] = -\frac{1}{30} + \frac{i\sqrt{19}}{30}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 18 - 6\sqrt{57} + 18\sqrt{2(-3 + \sqrt{57})}$$

has minimal polynomial  $T^4 - 72T^3 + 1728T^2 + 870912T - 23887872$  and  $k = \sqrt[3]{t} \approx 2.999969508159073701732$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{\sqrt{57}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m+90n)q^{m^2+mn+5n^2} \\ = 342q^5 + 342q^7 + 342q^{11} + 342q^{17} - 342q^{19} - 684q^{20} - 684q^{23} + 342q^{25} - 684q^{28} - 342q^{35} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(171), (\frac{57}{\cdot}))$ .

#### $\nu$ -N4-8-3

Let  $\tau_0 = [9, 3, 5] = -\frac{1}{6} + \frac{i\sqrt{19}}{6}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 18 + 6\sqrt{57} + 18i\sqrt[4]{3}\sqrt{2(\sqrt{3} + \sqrt{19})}$$

has minimal polynomial  $T^4 - 72T^3 + 1728T^2 + 870912T - 23887872$  and  $k = \sqrt[3]{t} \approx 4.486362111210192137840 + 1.416361433154947478404i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{\sqrt{57}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m+18n)q^{5m^2+mn+n^2} \\ = 36q - 72q^4 + 18q^5 - 90q^{11} + 144q^{16} + 126q^{17} - 36q^{20} + 72q^{23} - 180q^{25} - 342q^{35} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(171), (\frac{57}{\cdot}))$ .

#### $\nu$ -N4-8-4

Let  $\tau_0 = [9, -3, 5] = \frac{1}{6} + \frac{i\sqrt{19}}{6}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 18 + 6\sqrt{57} - 18i\sqrt[4]{3}\sqrt{2(\sqrt{3} + \sqrt{19})}$$

has minimal polynomial  $T^4 - 72T^3 + 1728T^2 + 870912T - 23887872$  and  $k = \sqrt[3]{t} \approx 4.486362111210192137840 - 1.416361433154947478404i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{\sqrt{57}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m+18n)q^{5m^2-mn+n^2} \\ = 36q - 72q^4 + 18q^5 - 90q^{11} + 144q^{16} + 126q^{17} - 36q^{20} + 72q^{23} - 180q^{25} - 342q^{35} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(171), (\frac{57}{\cdot}))$ .

#### $\nu$ -N4-9-1

Let  $\tau_0 = [9, -9, 13] = \frac{1}{2} + \frac{i\sqrt{43}}{6}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 18 - 42\sqrt{129} - 54\sqrt[4]{3}\sqrt{-26\sqrt{3} + 14\sqrt{43}}$$

has minimal polynomial  $T^4 - 72T^3 + 1728T^2 + 884722176T - 23887872000$  and  $k = \sqrt[3]{t} \approx -9.813394848885976189306$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{\sqrt{129}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-27m+18n)q^{13m^2-3mn+n^2} \\ = 36q - 72q^4 - 36q^{11} + 144q^{16} + 180q^{17} - 252q^{23} - 180q^{25} + 396q^{41} + 72q^{44} - 144q^{47} + \dots$$

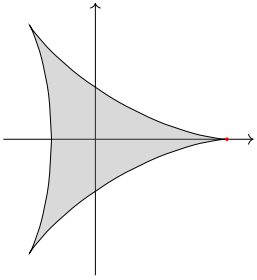
is in  $\mathcal{S}_2(\Gamma_0(387), (\frac{129}{\cdot}))$ .

#### $\nu$ -N4-9-2

Let  $\tau_0 = [99, 3, 1] = -\frac{1}{66} + \frac{i\sqrt{43}}{66}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 18 - 42\sqrt{129} + 54\sqrt[4]{3}\sqrt{-26\sqrt{3} + 14\sqrt{43}}$$

has minimal polynomial  $T^4 - 72T^3 + 1728T^2 + 884722176T - 23887872000$  and  $k = \sqrt[3]{t} \approx 2.999999969482447641562$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{\sqrt{129}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m+198n)q^{m^2+mn+11n^2} \\ = 774q^{11} + 774q^{13} + 774q^{17} + 774q^{23} + 774q^{31} + 774q^{41} - 774q^{43} - 1548q^{44} - 1548q^{47} - 1548q^{52} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(387), (\frac{129}{\cdot}))$ .

#### $\nu$ -N4-9-3

Let  $\tau_0 = [9, 3, 11] = -\frac{1}{6} + \frac{i\sqrt{43}}{6}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 18 + 42\sqrt{129} + 54\sqrt[4]{3}\sqrt{26\sqrt{3} + 14\sqrt{43}}$$

has minimal polynomial  $T^4 - 72T^3 + 1728T^2 + 884722176T - 23887872000$  and



$k = \sqrt[3]{t} \approx 9.309297906367732047008 + 3.341024875163763351406i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{\sqrt{129}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m+18n)q^{11m^2+mn+n^2} \\ &= 36q - 72q^4 + 18q^{11} + 144q^{16} - 90q^{17} + 126q^{23} - 180q^{25} - 198q^{41} - 36q^{44} + 72q^{47} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(387), (\frac{129}{\cdot}))$ .

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#### $\nu$ -N4-9-4

Let  $\tau_0 = [9, -3, 11] = \frac{1}{6} + \frac{i\sqrt{43}}{6}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 18 + 42\sqrt{129} - 54i\sqrt[4]{3}\sqrt{26\sqrt{3} + 14\sqrt{43}}$$

has minimal polynomial  $T^4 - 72T^3 + 1728T^2 + 884722176T - 23887872000$  and

$k = \sqrt[3]{t} \approx 9.309297906367732047008 - 3.341024875163763351406i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{\sqrt{129}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m+18n)q^{11m^2-mn+n^2} \\ &= 36q - 72q^4 + 18q^{11} + 144q^{16} - 90q^{17} + 126q^{23} - 180q^{25} - 198q^{41} - 36q^{44} + 72q^{47} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(387), (\frac{129}{\cdot}))$ .

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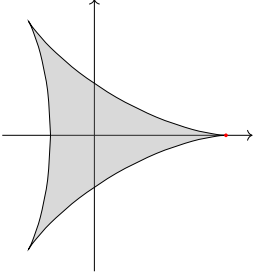
#### $\nu$ -N4-10-1

Let  $\tau_0 = [36, 0, 1] = \frac{i}{6}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 18 - \frac{45\sqrt[4]{3}}{\sqrt{2}} + 21\sqrt{3} + \frac{9\sqrt[4]{27}}{\sqrt{2}}$$

has minimal polynomial  $T^4 - 72T^3 + 1728T^2 - 301320T + 7762392$  and

$k = \sqrt[3]{t} \approx 3.000094159228714042426$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{\sqrt{3}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)72nq^{m^2+4n^2} \\ &= 144q^4 + 288q^5 + 288q^8 + 288q^{13} - 288q^{16} - 576q^{17} - 288q^{20} - 576q^{25} + 288q^{29} - 576q^{32} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(144), (\frac{48}{\cdot}))$ .

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#### $\nu$ -N4-10-2

Let  $\tau_0 = [9, 0, 4] = \frac{2i}{3}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 18 + \frac{45\sqrt[4]{3}}{\sqrt{2}} + 21\sqrt{3} - \frac{9\sqrt[4]{27}}{\sqrt{2}}$$

has minimal polynomial  $T^4 - 72T^3 + 1728T^2 - 301320T + 7762392$  and

$k = \sqrt[3]{t} \approx 4.339948457607474796175$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{\sqrt{3}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)18nq^{4m^2+n^2} \\ &= 36q - 72q^4 + 72q^5 - 144q^8 + 144q^{16} + 72q^{17} + 144q^{20} - 180q^{25} - 360q^{29} + 288q^{32} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(144), (\frac{48}{\cdot}))$ .

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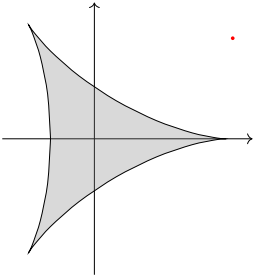
#### $\nu$ -N4-10-3

Let  $\tau_0 = [9, 6, 5] = -\frac{1}{3} + \frac{2i}{3}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 18 + \frac{45i\sqrt[4]{3}}{\sqrt{2}} - 21\sqrt{3} + \frac{9i\sqrt[4]{27}}{\sqrt{2}}$$

has minimal polynomial  $T^4 - 72T^3 + 1728T^2 - 301320T + 7762392$  and

$k = \sqrt[3]{t} \approx 3.154215655068299027894 + 2.293032966296685370141i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{\sqrt{3}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(18m+18n)q^{5m^2+2mn+n^2} \\ &= 36q - 72q^4 - 36q^5 + 72q^8 + 144q^{16} - 36q^{17} - 72q^{20} - 180q^{25} + 180q^{29} - 144q^{32} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(144), (\frac{48}{\cdot}))$ .

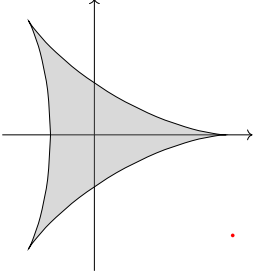
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#### $\nu$ -N4-10-4

Let  $\tau_0 = [9, -6, 5] = \frac{1}{3} + \frac{2i}{3}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 18 - \frac{45i\sqrt[4]{3}}{\sqrt{2}} - 21\sqrt{3} - \frac{9i\sqrt[4]{27}}{\sqrt{2}}$$

has minimal polynomial  $T^4 - 72T^3 + 1728T^2 - 301320T + 7762392$  and  $k = \sqrt[3]{t} \approx 3.154215655068299027894 - 2.293032966296685370141i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{\sqrt{3}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-18m + 18n) q^{5m^2 - 2mn + n^2} \\ &= 36q - 72q^4 - 36q^5 + 72q^8 + 144q^{16} - 36q^{17} - 72q^{20} - 180q^{25} + 180q^{29} - 144q^{32} + \dots \end{aligned}$$

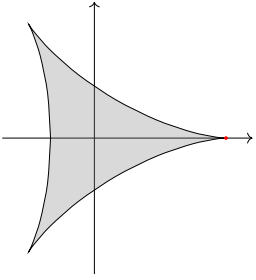
is in  $\mathcal{S}_2(\Gamma_0(144), \left(\frac{48}{\cdot}\right))$ .

#### $\nu$ -N4-11-1

Let  $\tau_0 = [63, 0, 1] = \frac{i}{3\sqrt{7}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 18 + \frac{57\sqrt{21}}{2} - \frac{27}{2}\sqrt{-93 + 38\sqrt{21}}$$

has minimal polynomial  $T^4 - 72T^3 + 1728T^2 - 16595199T + 447697125$  and  $k = \sqrt[3]{t} \approx 3.000001628406355859161$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{\sqrt{21}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 126nq^{m^2 + 7n^2} \\ &= 252q^7 + 504q^8 + 504q^{11} + 504q^{16} + 504q^{23} - 504q^{28} - 1008q^{29} - 504q^{32} - 1008q^{37} + 504q^{43} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(252), \left(\frac{84}{\cdot}\right))$ .

#### $\nu$ -N4-11-2

Let  $\tau_0 = [9, 0, 7] = \frac{i\sqrt{7}}{3}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 18 + \frac{57\sqrt{21}}{2} + \frac{27}{2}\sqrt{-93 + 38\sqrt{21}}$$

has minimal polynomial  $T^4 - 72T^3 + 1728T^2 - 16595199T + 447697125$  and  $k = \sqrt[3]{t} \approx 6.464953552253781031509$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{\sqrt{21}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 18nq^{7m^2 + n^2} \\ &= 36q - 72q^4 + 72q^8 - 144q^{11} + 144q^{16} + 288q^{23} - 180q^{25} + 72q^{29} - 504q^{32} + 288q^{44} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(252), \left(\frac{84}{\cdot}\right))$ .

#### $\nu$ -N4-11-3

Let  $\tau_0 = [9, 6, 8] = -\frac{1}{3} + \frac{i\sqrt{7}}{3}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 18 - \frac{57\sqrt{21}}{2} + \frac{27}{2}i\sqrt{93 + 38\sqrt{21}}$$

has minimal polynomial  $T^4 - 72T^3 + 1728T^2 - 16595199T + 447697125$  and  $k = \sqrt[3]{t} \approx 4.879947060114502569811 + 3.953420558712342562452i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{\sqrt{21}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (18m + 18n) q^{8m^2 + 2mn + n^2} \\ &= 36q - 72q^4 - 36q^8 + 72q^{11} + 144q^{16} - 144q^{23} - 180q^{25} - 36q^{29} + 252q^{32} - 144q^{44} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(252), \left(\frac{84}{\cdot}\right))$ .

#### $\nu$ -N4-11-4

Let  $\tau_0 = [9, -6, 8] = \frac{1}{3} + \frac{i\sqrt{7}}{3}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = 18 - \frac{57\sqrt{21}}{2} - \frac{27}{2}i\sqrt{93 + 38\sqrt{21}}$$

has minimal polynomial  $T^4 - 72T^3 + 1728T^2 - 16595199T + 447697125$  and  $k = \sqrt[3]{t} \approx 4.879947060114502569811 - 3.953420558712342562452i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{\sqrt{21}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-18m + 18n) q^{8m^2 - 2mn + n^2} \\ &= 36q - 72q^4 - 36q^8 + 72q^{11} + 144q^{16} - 144q^{23} - 180q^{25} - 36q^{29} + 252q^{32} - 144q^{44} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(252), \left(\frac{84}{\cdot}\right))$ .

#### $\nu$ -N4-12-1

Let  $\tau_0 = [3, 2, 2] = -\frac{1}{3} + \frac{i\sqrt{5}}{3}$  with  $h(D) = 2$ . Then

$t = t_Q(\tau_0) = (-8 + 44i) - (14 - 22i)\sqrt{5}$   
has minimal polynomial  $T^4 + 32T^3 + 7136T^2 - 432000T + 5832000$  and  
 $k = \sqrt[3]{t} \approx 3.690317755422442060866 + 2.844250129034582322015i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{3\sqrt{15}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (6m + 6n) q^{6m^2 + 2mn + n^2} \\ &= 12q - 24q^4 - 12q^6 + 24q^9 + 48q^{16} - 60q^{21} + 24q^{24} - 60q^{25} + 60q^{30} - 48q^{36} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(60), (\frac{60}{\cdot}))$ .

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#### $\nu$ -N4-12-2

Let  $\tau_0 = [3, -2, 2] = \frac{1}{3} + \frac{i\sqrt{5}}{3}$  with  $h(D) = 2$ . Then

$t = t_Q(\tau_0) = (-8 - 44i) - (14 + 22i)\sqrt{5}$   
has minimal polynomial  $T^4 + 32T^3 + 7136T^2 - 432000T + 5832000$  and  
 $k = \sqrt[3]{t} \approx 3.690317755422442060866 - 2.844250129034582322015i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{3\sqrt{15}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-6m + 6n) q^{6m^2 - 2mn + n^2} \\ &= 12q - 24q^4 - 12q^6 + 24q^9 + 48q^{16} - 60q^{21} + 24q^{24} - 60q^{25} + 60q^{30} - 48q^{36} + \dots\end{aligned}$$

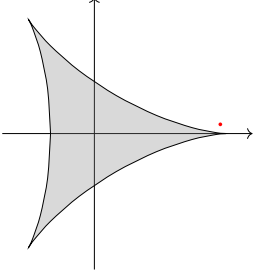
is in  $\mathcal{S}_2(\Gamma_0(60), (\frac{60}{\cdot}))$ .

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#### $\nu$ -N4-12-3

Let  $\tau_0 = [6, 2, 1] = -\frac{1}{6} + \frac{i\sqrt{5}}{6}$  with  $h(D) = 2$ . Then

$t = t_Q(\tau_0) = (-8 - 44i) + (14 + 22i)\sqrt{5}$   
has minimal polynomial  $T^4 + 32T^3 + 7136T^2 - 432000T + 5832000$  and  
 $k = \sqrt[3]{t} \approx 2.871860071068628466481 + 0.210275532819020968779i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{3\sqrt{15}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (6m + 12n) q^{3m^2 + 2mn + 2n^2} \\ &= 24q^2 + 12q^3 - 48q^8 - 24q^{12} - 60q^{15} + 48q^{18} - 24q^{23} + 84q^{27} + 96q^{32} + 120q^{35} + \dots\end{aligned}$$

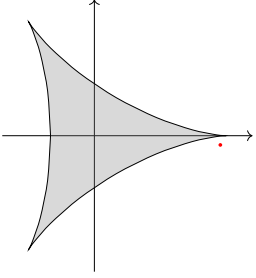
is in  $\mathcal{S}_2(\Gamma_0(60), (\frac{60}{\cdot}))$ .

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#### $\nu$ -N4-12-4

Let  $\tau_0 = [6, -2, 1] = \frac{1}{6} + \frac{i\sqrt{5}}{6}$  with  $h(D) = 2$ . Then

$t = t_Q(\tau_0) = (-8 + 44i) + (14 - 22i)\sqrt{5}$   
has minimal polynomial  $T^4 + 32T^3 + 7136T^2 - 432000T + 5832000$  and  
 $k = \sqrt[3]{t} \approx 2.871860071068628466481 - 0.210275532819020968779i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{3\sqrt{15}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-6m + 12n) q^{3m^2 - 2mn + 2n^2} \\ &= 24q^2 + 12q^3 - 48q^8 - 24q^{12} - 60q^{15} + 48q^{18} - 24q^{23} + 84q^{27} + 96q^{32} + 120q^{35} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(60), (\frac{60}{\cdot}))$ .

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#### $\nu$ -N4-13-1

Let  $\tau_0 = [9, -9, 19] = \frac{1}{2} + \frac{i\sqrt{67}}{6}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 18 - 186\sqrt{201} - 126\sqrt{-438 + 62\sqrt{201}}$   
has minimal polynomial  $T^4 - 72T^3 + 1728T^2 - 432000T - 3974344704000$  and  
 $k = \sqrt[3]{t} \approx -17.39668127009333151704$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{\sqrt{201}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-27m + 18n) q^{19m^2 - 3mn + n^2} \\ &= 36q - 72q^4 + 144q^{16} - 36q^{17} + 180q^{23} - 180q^{25} - 252q^{29} + 396q^{47} + 252q^{49} - 468q^{59} + \dots\end{aligned}$$

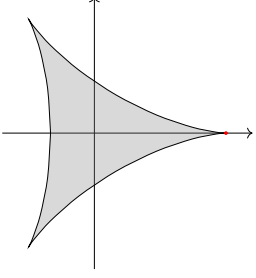
is in  $\mathcal{S}_2(\Gamma_0(603), (\frac{201}{\cdot}))$ .

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#### $\nu$ -N4-13-2

Let  $\tau_0 = [153, 3, 1] = -\frac{1}{102} + \frac{i\sqrt{67}}{102}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 18 - 186\sqrt{201} + 126\sqrt{-438 + 62\sqrt{201}}$   
has minimal polynomial  $T^4 - 72T^3 + 1728T^2 + 147197938176T - 3974344704000$  and  
 $k = \sqrt[3]{t} \approx 2.999999999816573535867$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{\sqrt{201}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 306n)q^{m^2 + mn + 17n^2}$$

$$= 1206q^{17} + 1206q^{19} + 1206q^{23} + 1206q^{29} + 1206q^{37} + 1206q^{47} + 1206q^{59} - 1206q^{67} - 2412q^{68} - 2412q^{71} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(603), (\frac{201}{\cdot}))$ .

#### $\nu$ -N4-13-3

Let  $\tau_0 = [9, 3, 17] = -\frac{1}{6} + \frac{i\sqrt{67}}{6}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 18 + 186\sqrt{201} + 126i\sqrt{438 + 62\sqrt{201}}$   
has minimal polynomial  $T^4 - 72T^3 + 1728T^2 + 147197938176T - 3974344704000$  and  
 $k = \sqrt[3]{t} \approx 16.37566506023077055824 + 5.94506164250723133745i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{\sqrt{201}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 18n)q^{17m^2 + mn + n^2}$$

$$= 36q - 72q^4 + 144q^{16} + 18q^{17} - 90q^{23} - 180q^{25} + 126q^{29} - 198q^{47} + 252q^{49} + 234q^{59} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(603), (\frac{201}{\cdot}))$ .

#### $\nu$ -N4-13-4

Let  $\tau_0 = [9, -3, 17] = \frac{1}{6} + \frac{i\sqrt{67}}{6}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 18 + 186\sqrt{201} - 126i\sqrt{438 + 62\sqrt{201}}$   
has minimal polynomial  $T^4 - 72T^3 + 1728T^2 + 147197938176T - 3974344704000$  and  
 $k = \sqrt[3]{t} \approx 16.37566506023077055824 - 5.94506164250723133745i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{\sqrt{201}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m + 18n)q^{17m^2 - mn + n^2}$$

$$= 36q - 72q^4 + 144q^{16} + 18q^{17} - 90q^{23} - 180q^{25} + 126q^{29} - 198q^{47} + 252q^{49} + 234q^{59} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(603), (\frac{201}{\cdot}))$ .

#### $\nu$ -N4-14-1

Let  $\tau_0 = [9, -9, 43] = \frac{1}{2} + \frac{i\sqrt{163}}{6}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 18 - 14478\sqrt{489} - 1386\sqrt{-53358 + 4826\sqrt{489}}$   
has minimal polynomial  $T^4 - 72T^3 + 1728T^2 + 262537412640754176T - 7088510141300736000$  and  
 $k = \sqrt[3]{t} \approx -86.19107506807197279940$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{\sqrt{489}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-27m + 18n)q^{43m^2 - 3mn + n^2}$$

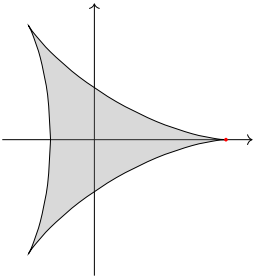
$$= 36q - 72q^4 + 144q^{16} - 180q^{25} - 36q^{41} + 180q^{47} + 252q^{49} - 252q^{53} - 288q^{64} + 396q^{71} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1467), (\frac{489}{\cdot}))$ .

#### $\nu$ -N4-14-2

Let  $\tau_0 = [369, 3, 1] = -\frac{1}{246} + \frac{i\sqrt{163}}{246}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 18 - 14478\sqrt{489} + 1386\sqrt{-53358 + 4826\sqrt{489}}$   
has minimal polynomial  $T^4 - 72T^3 + 1728T^2 + 262537412640754176T - 7088510141300736000$  and  
 $k = \sqrt[3]{t} \approx 2.999999999999999999897158$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{\sqrt{489}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 738n)q^{m^2 + mn + 41n^2}$$

$$= 2934q^{41} + 2934q^{43} + 2934q^{47} + 2934q^{53} + 2934q^{61} + 2934q^{71} + 2934q^{83} + 2934q^{97} + 2934q^{113} + 2934q^{131} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1467), (\frac{489}{\cdot}))$ .

#### $\nu$ -N4-14-3

Let  $\tau_0 = [9, 3, 41] = -\frac{1}{6} + \frac{i\sqrt{163}}{6}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 18 + 14478\sqrt{489} + 1386i\sqrt{53358 + 4826\sqrt{489}}$   
has minimal polynomial  $T^4 - 72T^3 + 1728T^2 + 262537412640754176T - 7088510141300736000$  and  
 $k = \sqrt[3]{t} \approx 80.99426524403682211188 + 29.47888142264088778219i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{\sqrt{489}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 18n)q^{41m^2 + mn + n^2} \\ = 36q - 72q^4 + 144q^{16} - 180q^{25} + 18q^{41} - 90q^{47} + 252q^{49} + 126q^{53} - 288q^{64} - 198q^{71} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1467), (\frac{489}{\cdot}))$ .

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#### $\nu$ -N4-14-4

Let  $\tau_0 = [9, -3, 41] = \frac{1}{6} + \frac{i\sqrt{163}}{6}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = 18 + 14478\sqrt{489} - 1386i\sqrt{53358 + 4826\sqrt{489}}$   
has minimal polynomial  $T^4 - 72T^3 + 1728T^2 + 262537412640754176T - 7088510141300736000$  and  
 $k = \sqrt[3]{t} \approx 80.99426524403682211188 - 29.47888142264088778219i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{\sqrt{489}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m + 18n)q^{41m^2 - mn + n^2} \\ = 36q - 72q^4 + 144q^{16} - 180q^{25} + 18q^{41} - 90q^{47} + 252q^{49} + 126q^{53} - 288q^{64} - 198q^{71} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(1467), (\frac{489}{\cdot}))$ .

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#### $\nu$ -N4-15-1

Let  $\tau_0 = [3, 2, 3] = -\frac{1}{3} + \frac{2i\sqrt{2}}{3}$  with  $h(D) = 2$ . Then

$t = t_Q(\tau_0) = (-73 + 161i) - (70 - 115i)\sqrt{2}$   
has minimal polynomial  $T^4 + 292T^3 + 117116T^2 - 6750000T + 91125000$  and  
 $k = \sqrt[3]{t} \approx 5.535533905932737622004 + 4.535533905932737622004i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{3\sqrt{\frac{3}{2}}}{2\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(6m + 6n)q^{9m^2 + 2mn + n^2} \\ = 12q - 24q^4 - 12q^9 + 24q^{12} + 48q^{16} - 48q^{24} - 60q^{25} + 48q^{33} + 24q^{36} - 48q^{48} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(96), (\frac{96}{\cdot}))$ .

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#### $\nu$ -N4-15-2

Let  $\tau_0 = [3, -2, 3] = \frac{1}{3} + \frac{2i\sqrt{2}}{3}$  with  $h(D) = 2$ . Then

$t = t_Q(\tau_0) = (-73 - 161i) - (70 + 115i)\sqrt{2}$   
has minimal polynomial  $T^4 + 292T^3 + 117116T^2 - 6750000T + 91125000$  and  
 $k = \sqrt[3]{t} \approx 5.535533905932737622004 - 4.535533905932737622004i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{3\sqrt{\frac{3}{2}}}{2\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-6m + 6n)q^{9m^2 - 2mn + n^2} \\ = 12q - 24q^4 - 12q^9 + 24q^{12} + 48q^{16} - 48q^{24} - 60q^{25} + 48q^{33} + 24q^{36} - 48q^{48} + \dots$$

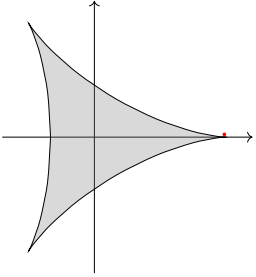
is in  $\mathcal{S}_2(\Gamma_0(96), (\frac{96}{\cdot}))$ .

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#### $\nu$ -N4-15-3

Let  $\tau_0 = [9, 2, 1] = -\frac{1}{9} + \frac{2i\sqrt{2}}{9}$  with  $h(D) = 2$ . Then

$t = t_Q(\tau_0) = (-73 - 161i) + (70 + 115i)\sqrt{2}$   
has minimal polynomial  $T^4 + 292T^3 + 117116T^2 - 6750000T + 91125000$  and  
 $k = \sqrt[3]{t} \approx 2.963603727660902786985 + 0.062044417943726518217i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{3\sqrt{\frac{3}{2}}}{2\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(6m + 18n)q^{3m^2 + 2mn + 3n^2} \\ = 36q^3 + 24q^4 + 48q^8 - 48q^{11} - 72q^{12} - 48q^{16} - 24q^{19} - 36q^{27} - 96q^{32} + 72q^{36} + \dots$$

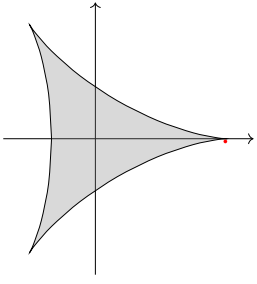
is in  $\mathcal{S}_2(\Gamma_0(96), (\frac{96}{\cdot}))$ .

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#### $\nu$ -N4-15-4

Let  $\tau_0 = [9, -2, 1] = \frac{1}{9} + \frac{2i\sqrt{2}}{9}$  with  $h(D) = 2$ . Then

$t = t_Q(\tau_0) = (-73 + 161i) + (70 - 115i)\sqrt{2}$   
has minimal polynomial  $T^4 + 292T^3 + 117116T^2 - 6750000T + 91125000$  and  
 $k = \sqrt[3]{t} \approx 2.963603727660902786985 - 0.062044417943726518217i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{3\sqrt{\frac{3}{2}}}{2\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-6m + 18n)q^{3m^2 - 2mn + 3n^2} \\ &= 36q^3 + 24q^4 + 48q^8 - 48q^{11} - 72q^{12} - 48q^{16} - 24q^{19} - 36q^{27} - 96q^{32} + 72q^{36} + \dots\end{aligned}$$

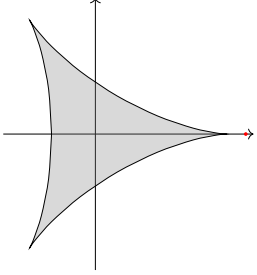
is in  $\mathcal{S}_2(\Gamma_0(96), (\frac{96}{\cdot}))$ .

#### $\nu$ -N4-16-1

Let  $\tau_0 = [4, 0, 1] = \frac{i}{2}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = 71712 - \frac{77031\sqrt[4]{3}}{\sqrt{2}} + 41391\sqrt{3} - \frac{44469\sqrt[4]{27}}{\sqrt{2}}$$

has minimal polynomial  $T^4 - 286848T^3 + 23427144T^2 - 618676056T + 5658783768$  and  $k = \sqrt[3]{t} \approx 3.428951936017602887126$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{27\sqrt{3}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)8nq^{9m^2 + 4n^2} \\ &= 16q^4 + 32q^{13} - 32q^{16} - 64q^{25} + 32q^{40} - 64q^{52} + 64q^{64} + 128q^{73} + 32q^{85} - 64q^{97} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(144), (\frac{432}{\cdot}))$ .

#### $\nu$ -N4-16-2

Let  $\tau_0 = [1, 0, 4] = 2i$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = 71712 + \frac{77031\sqrt[4]{3}}{\sqrt{2}} + 41391\sqrt{3} + \frac{44469\sqrt[4]{27}}{\sqrt{2}}$$

has minimal polynomial  $T^4 - 286848T^3 + 23427144T^2 - 618676056T + 5658783768$  and  $k = \sqrt[3]{t} \approx 65.94411502292882027695$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{27\sqrt{3}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)2nq^{36m^2 + n^2} \\ &= 4q - 8q^4 + 16q^{16} - 20q^{25} + 8q^{37} - 16q^{40} + 28q^{49} + 32q^{52} - 40q^{61} - 32q^{64} + \dots\end{aligned}$$

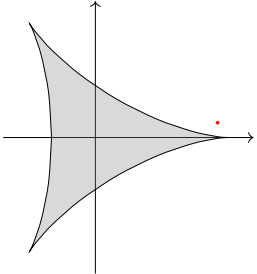
is in  $\mathcal{S}_2(\Gamma_0(144), (\frac{432}{\cdot}))$ .

#### $\nu$ -N4-16-3

Let  $\tau_0 = [5, 2, 1] = -\frac{1}{5} + \frac{2i}{5}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = 71712 + \frac{77031i\sqrt[4]{3}}{\sqrt{2}} - 41391\sqrt{3} - \frac{44469i\sqrt[4]{27}}{\sqrt{2}}$$

has minimal polynomial  $T^4 - 286848T^3 + 23427144T^2 - 618676056T + 5658783768$  and  $k = \sqrt[3]{t} \approx 2.786795221476077119283 + 0.338484145446773894913i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{27\sqrt{3}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(6m + 10n)q^{9m^2 + 6mn + 5n^2} \\ &= 20q^5 + 8q^8 - 28q^{17} - 8q^{20} - 4q^{29} - 16q^{32} - 52q^{41} + 44q^{53} + 64q^{65} + 40q^{68} + \dots\end{aligned}$$

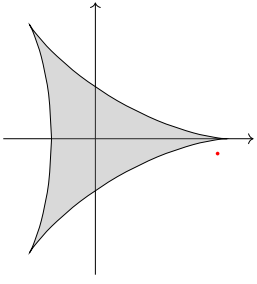
is in  $\mathcal{S}_2(\Gamma_0(144), (\frac{432}{\cdot}))$ .

#### $\nu$ -N4-16-4

Let  $\tau_0 = [5, -2, 1] = \frac{1}{5} + \frac{2i}{5}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = 71712 - \frac{77031i\sqrt[4]{3}}{\sqrt{2}} - 41391\sqrt{3} + \frac{44469i\sqrt[4]{27}}{\sqrt{2}}$$

has minimal polynomial  $T^4 - 286848T^3 + 23427144T^2 - 618676056T + 5658783768$  and  $k = \sqrt[3]{t} \approx 2.786795221476077119283 - 0.338484145446773894913i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{27\sqrt{3}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-6m + 10n)q^{9m^2 - 6mn + 5n^2} \\ &= 20q^5 + 8q^8 - 28q^{17} - 8q^{20} - 4q^{29} - 16q^{32} - 52q^{41} + 44q^{53} + 64q^{65} + 40q^{68} + \dots\end{aligned}$$

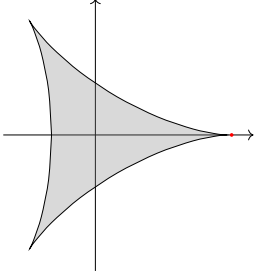
is in  $\mathcal{S}_2(\Gamma_0(144), (\frac{432}{\cdot}))$ .

#### [ν-N4-17-1](#)

Let  $\tau_0 = [7, 0, 1] = \frac{i}{\sqrt{7}}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = \frac{16580727}{4} + \frac{3618189\sqrt{21}}{4} - \frac{729}{4} \sqrt{1034609022 + 225770198\sqrt{21}}$$

has minimal polynomial  $T^4 - 16580727T^3 + 1343231343T^2 - 36253389429T + 326371204125$  and  $k = \sqrt[3]{t} \approx 3.107144446086423167471$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{27\sqrt{21}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)14nq^{9m^2 + 7n^2} \\ &= 28q^7 + 56q^{16} - 56q^{28} - 112q^{37} + 56q^{43} - 112q^{64} + 56q^{88} - 112q^{109} + 112q^{112} + 224q^{121} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(252), (\frac{756}{\cdot}))$ .

#### [ν-N4-17-2](#)

Let  $\tau_0 = [1, 0, 7] = i\sqrt{7}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = \frac{16580727}{4} + \frac{3618189\sqrt{21}}{4} + \frac{729}{4} \sqrt{1034609022 + 225770198\sqrt{21}}$$

has minimal polynomial  $T^4 - 16580727T^3 + 1343231343T^2 - 36253389429T + 326371204125$  and  $k = \sqrt[3]{t} \approx 254.9962628601550170203$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{27\sqrt{21}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)2nq^{63m^2 + n^2} \\ &= 4q - 8q^4 + 16q^{16} - 20q^{25} + 28q^{49} - 24q^{64} - 16q^{67} + 32q^{79} - 40q^{88} + 40q^{100} + \dots\end{aligned}$$

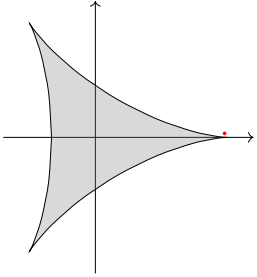
is in  $\mathcal{S}_2(\Gamma_0(252), (\frac{756}{\cdot}))$ .

#### [ν-N4-17-3](#)

Let  $\tau_0 = [8, 2, 1] = -\frac{1}{8} + \frac{i\sqrt{7}}{8}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = \frac{16580727}{4} - \frac{3618189\sqrt{21}}{4} + \frac{729}{4} i \sqrt{-1034609022 + 225770198\sqrt{21}}$$

has minimal polynomial  $T^4 - 16580727T^3 + 1343231343T^2 - 36253389429T + 326371204125$  and  $k = \sqrt[3]{t} \approx 2.946449760454525199770 + 0.090619481311418379611i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{27\sqrt{21}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(6m + 16n)q^{9m^2 + 6mn + 8n^2} \\ &= 32q^8 + 20q^{11} + 44q^{23} - 52q^{29} - 56q^{32} - 40q^{44} - 76q^{53} + 56q^{56} - 4q^{71} - 28q^{77} + \dots\end{aligned}$$

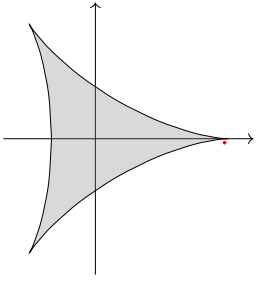
is in  $\mathcal{S}_2(\Gamma_0(252), (\frac{756}{\cdot}))$ .

#### [ν-N4-17-4](#)

Let  $\tau_0 = [8, -2, 1] = \frac{1}{8} + \frac{i\sqrt{7}}{8}$  with  $h(D) = 1$ . Then

$$t = t_Q(\tau_0) = \frac{16580727}{4} - \frac{3618189\sqrt{21}}{4} - \frac{729}{4} i \sqrt{-1034609022 + 225770198\sqrt{21}}$$

has minimal polynomial  $T^4 - 16580727T^3 + 1343231343T^2 - 36253389429T + 326371204125$  and  $k = \sqrt[3]{t} \approx 2.946449760454525199770 - 0.090619481311418379611i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{27\sqrt{21}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-6m + 16n)q^{9m^2 - 6mn + 8n^2} \\ &= 32q^8 + 20q^{11} + 44q^{23} - 52q^{29} - 56q^{32} - 40q^{44} - 76q^{53} + 56q^{56} - 4q^{71} - 28q^{77} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(252), (\frac{756}{\cdot}))$ .

#### $\nu$ -N4-18-1

Let  $\tau_0 = [3, -3, 4] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{13}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -\frac{621}{4} - \frac{189\sqrt{13}}{4} - \frac{81}{2}\sqrt{\frac{3}{2}(25 + 7\sqrt{13})}$$

has minimal polynomial  $T^4 + 621T^3 - 36450T^2 + 1062882T - 14348907$  and  $k = \sqrt[3]{t} \approx -8.781527031312423193580$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{13}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m + 6n)q^{12m^2 - 3mn + n^2} \\ &= 12q - 24q^4 - 12q^{10} + 108q^{16} - 84q^{22} - 60q^{25} + 156q^{40} - 48q^{43} + 84q^{49} - 156q^{52} + \dots\end{aligned}$$

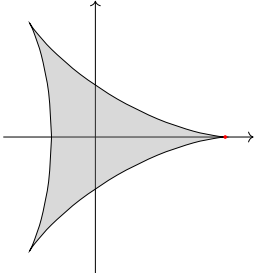
is in  $\mathcal{S}_2(\Gamma_0(117), (\frac{117}{\cdot}))$ .

#### $\nu$ -N4-18-2

Let  $\tau_0 = [12, 3, 1] = -\frac{1}{8} + \frac{1}{8}i\sqrt{\frac{13}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -\frac{621}{4} - \frac{189\sqrt{13}}{4} + \frac{81}{2}\sqrt{\frac{3}{2}(25 + 7\sqrt{13})}$$

has minimal polynomial  $T^4 + 621T^3 - 36450T^2 + 1062882T - 14348907$  and  $k = \sqrt[3]{t} \approx 2.961157292411807669795$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{13}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 24n)q^{3m^2 + 3mn + 4n^2} \\ &= 78q^4 + 78q^{10} - 78q^{13} - 156q^{16} + 78q^{22} - 156q^{25} - 78q^{40} + 156q^{52} + 312q^{55} - 156q^{61} + \dots\end{aligned}$$

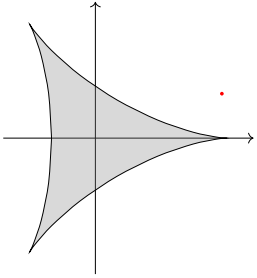
is in  $\mathcal{S}_2(\Gamma_0(117), (\frac{117}{\cdot}))$ .

#### $\nu$ -N4-18-3

Let  $\tau_0 = [6, 3, 2] = -\frac{1}{4} + \frac{1}{4}i\sqrt{\frac{13}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -\frac{621}{4} + \frac{189\sqrt{13}}{4} + \frac{81}{2}i\sqrt{\frac{3}{2}(-25 + 7\sqrt{13})}$$

has minimal polynomial  $T^4 + 621T^3 - 36450T^2 + 1062882T - 14348907$  and  $k = \sqrt[3]{t} \approx 2.884198072361761680992 + 1.013066403227640218407i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{13}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 12n)q^{6m^2 + 3mn + 2n^2} \\ &= 24q^2 + 6q^5 - 78q^8 + 42q^{11} - 90q^{20} + 78q^{26} + 216q^{32} - 102q^{41} - 162q^{44} - 30q^{47} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(117), (\frac{117}{\cdot}))$ .

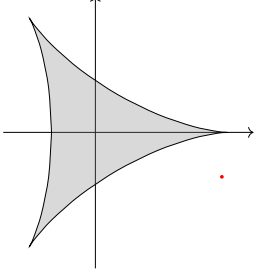
#### $\nu$ -N4-18-4

Let  $\tau_0 = [6, -3, 2] = \frac{1}{4} + \frac{1}{4}i\sqrt{\frac{13}{3}}$  with  $h(D) = 4$ . Then



$$t = t_Q(\tau_0) = -\frac{621}{4} + \frac{189\sqrt{13}}{4} - \frac{81}{2}i\sqrt{\frac{3}{2}(-25 + 7\sqrt{13})}$$

has minimal polynomial  $T^4 + 621T^3 - 36450T^2 + 1062882T - 14348907$  and  $k = \sqrt[3]{t} \approx 2.884198072361761680992 - 1.013066403227640218407i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{13}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m + 12n)q^{6m^2 - 3mn + 2n^2} \\ &= 24q^2 + 6q^5 - 78q^8 + 42q^{11} - 90q^{20} + 78q^{26} + 216q^{32} - 102q^{41} - 162q^{44} - 30q^{47} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(117), (\frac{117}{\cdot}))$ .

#### $\nu$ -N4-19-1

Let  $\tau_0 = [3, -3, 19] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{73}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -1343520 - 157248\sqrt{73} - 648\sqrt{8597643 + 1006278\sqrt{73}}$   
has minimal polynomial  $T^4 + 5374080T^3 - 225721728T^2 + 4353564672T - 58773123072$  and  $k = \sqrt[3]{t} \approx -175.1603645856597943265$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{73}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

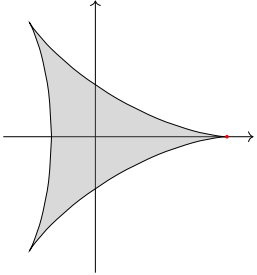
$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m + 6n)q^{57m^2 - 3mn + n^2} \\ &= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 12q^{55} + 60q^{61} - 96q^{64} - 84q^{67} + 132q^{85} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(657), (\frac{657}{\cdot}))$ .

#### $\nu$ -N4-19-2

Let  $\tau_0 = [57, 3, 1] = -\frac{1}{38} + \frac{1}{38}i\sqrt{\frac{73}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -1343520 - 157248\sqrt{73} + 648\sqrt{8597643 + 1006278\sqrt{73}}$   
has minimal polynomial  $T^4 + 5374080T^3 - 225721728T^2 + 4353564672T - 58773123072$  and  $k = \sqrt[3]{t} \approx 2.999994975940338128580$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{73}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

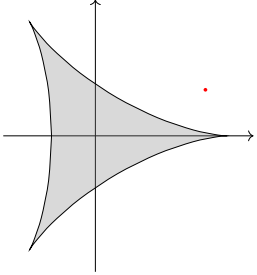
$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 114n)q^{3m^2 + 3mn + 19n^2} \\ &= 438q^{19} + 438q^{25} + 438q^{37} + 438q^{55} - 438q^{73} - 876q^{76} + 438q^{79} - 876q^{85} - 876q^{100} + 438q^{109} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(657), (\frac{657}{\cdot}))$ .

#### $\nu$ -N4-19-3

Let  $\tau_0 = [15, 9, 5] = -\frac{3}{10} + \frac{1}{10}i\sqrt{\frac{73}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -1343520 + 157248\sqrt{73} + 648i\sqrt{-8597643 + 1006278\sqrt{73}}$   
has minimal polynomial  $T^4 + 5374080T^3 - 225721728T^2 + 4353564672T - 58773123072$  and  $k = \sqrt[3]{t} \approx 2.509288293114881198229 + 1.049961325166505352713i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{73}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

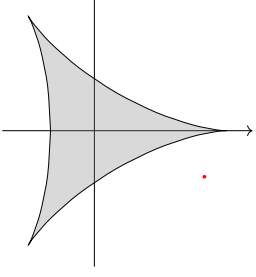
$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(27m + 30n)q^{15m^2 + 9mn + 5n^2} \\ &= 60q^5 + 6q^{11} - 66q^{17} - 120q^{20} + 114q^{29} - 12q^{44} - 48q^{47} - 174q^{53} + 186q^{59} + 132q^{68} + \dots\end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(657), (\frac{657}{\cdot}))$ .

#### $\nu$ -N4-19-4

Let  $\tau_0 = [15, -9, 5] = \frac{3}{10} + \frac{1}{10}i\sqrt{\frac{73}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -1343520 + 157248\sqrt{73} - 648i\sqrt{-8597643 + 1006278\sqrt{73}}$   
has minimal polynomial  $T^4 + 5374080T^3 - 225721728T^2 + 4353564672T - 58773123072$  and  
 $k = \sqrt[3]{t} \approx 2.509288293114881198229 - 1.049961325166505352713i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{73}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-27m + 30n)q^{15m^2 - 9mn + 5n^2} \\ = 60q^5 + 6q^{11} - 66q^{17} - 120q^{20} + 114q^{29} - 12q^{44} - 48q^{47} - 174q^{53} + 186q^{59} + 132q^{68} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(657), (\frac{657}{\cdot}))$ .

#### $\nu$ -N4-20-1

Let  $\tau_0 = [3, -3, 25] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{97}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -14325552 - 1454544\sqrt{97} - 324\sqrt{6(651648859 + 66164917\sqrt{97})}$   
has minimal polynomial  $T^4 + 57302208T^3 - 6706939968T^2 + 278628139008T - 3761479876608$  and  
 $k = \sqrt[3]{t} \approx -385.5293220589523968077$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{97}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

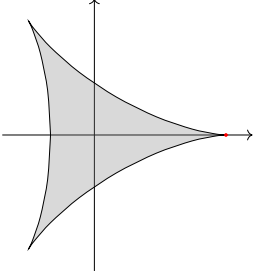
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m + 6n)q^{75m^2 - 3mn + n^2} \\ = 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} - 12q^{73} + 60q^{79} - 84q^{85} + 120q^{100} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(873), (\frac{873}{\cdot}))$ .

#### $\nu$ -N4-20-2

Let  $\tau_0 = [75, 3, 1] = -\frac{1}{50} + \frac{1}{50}i\sqrt{\frac{97}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -14325552 - 1454544\sqrt{97} + 324\sqrt{6(651648859 + 66164917\sqrt{97})}$   
has minimal polynomial  $T^4 + 57302208T^3 - 6706939968T^2 + 278628139008T - 3761479876608$  and  
 $k = \sqrt[3]{t} \approx 2.999999528815078663627$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{97}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 150n)q^{3m^2 + 3mn + 25n^2} \\ = 582q^{25} + 582q^{31} + 582q^{43} + 582q^{61} + 582q^{85} - 582q^{97} - 1164q^{100} - 1164q^{109} + 582q^{115} - 1164q^{124} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(873), (\frac{873}{\cdot}))$ .

#### $\nu$ -N4-20-3

Let  $\tau_0 = [15, 3, 5] = -\frac{1}{10} + \frac{1}{10}i\sqrt{\frac{97}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -14325552 + 1454544\sqrt{97} + 324i\sqrt{6(-651648859 + 66164917\sqrt{97})}$   
has minimal polynomial  $T^4 + 57302208T^3 - 6706939968T^2 + 278628139008T - 3761479876608$  and  
 $k = \sqrt[3]{t} \approx 3.631066876272990679995 + 0.511667372258937333181i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{97}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 30n)q^{15m^2 + 3mn + 5n^2} \\ = 60q^5 + 42q^{17} - 120q^{20} + 78q^{23} - 102q^{29} - 138q^{41} + 24q^{59} - 84q^{68} + 96q^{71} + 240q^{80} + \dots$$

is in  $\mathcal{S}_2(\Gamma_0(873), (\frac{873}{\cdot}))$ .

#### $\nu$ -N4-20-4

Let  $\tau_0 = [15, -3, 5] = \frac{1}{10} + \frac{1}{10}i\sqrt{\frac{97}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -14325552 + 1454544\sqrt{97} - 324i\sqrt{6(-651648859 + 66164917\sqrt{97})}$   
has minimal polynomial  $T^4 + 57302208T^3 - 6706939968T^2 + 278628139008T - 3761479876608$  and  
 $k = \sqrt[3]{t} \approx 3.631066876272990679995 - 0.511667372258937333181i$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{97}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m+30n)q^{15m^2-3mn+5n^2} \\ &= 60q^5 + 42q^{17} - 120q^{20} + 78q^{23} - 102q^{29} - 138q^{41} + 24q^{59} - 84q^{68} + 96q^{71} + 240q^{80} + \dots \\ &\text{is in } \mathcal{S}_2(\Gamma_0(873), \left(\frac{873}{\cdot}\right)).\end{aligned}$$

#### $\nu\text{-N4-21-1}$

Let  $\tau_0 = [3, -3, 31] = \frac{1}{2} + \frac{11i}{2\sqrt{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -115582896 - 20120400\sqrt{33} - 4860\sqrt{2\left(565606921 + 98459527\sqrt{33}\right)}$$

has minimal polynomial  $T^4 + 462331584T^3 - 14585085504T^2 + 113515167744T - 1532454764544$  and  $k = \sqrt[3]{t} \approx -773.2463243485238180888$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{99}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m+6n)q^{93m^2-3mn+n^2} \\ &= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} - 12q^{91} + 60q^{97} + 120q^{100} - 84q^{103} + \dots\end{aligned}$$

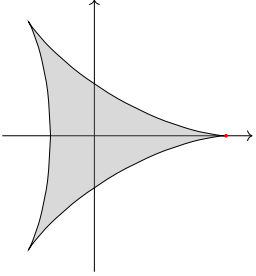
is in  $\mathcal{S}_2(\Gamma_0(1089), \left(\frac{1089}{\cdot}\right))$ .

#### $\nu\text{-N4-21-2}$

Let  $\tau_0 = [93, 3, 1] = -\frac{1}{62} + \frac{11i}{62\sqrt{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -115582896 - 20120400\sqrt{33} + 4860\sqrt{2\left(565606921 + 98459527\sqrt{33}\right)}$$

has minimal polynomial  $T^4 + 462331584T^3 - 14585085504T^2 + 113515167744T - 1532454764544$  and  $k = \sqrt[3]{t} \approx 2.999999941600362075881$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{99}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m+186n)q^{3m^2+3mn+31n^2} \\ &= 726q^{31} + 726q^{37} + 726q^{49} + 726q^{67} + 726q^{91} - 1452q^{124} - 1452q^{133} - 1452q^{148} + 726q^{157} - 1452q^{169} + \dots\end{aligned}$$

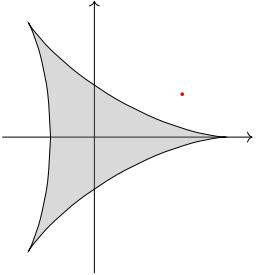
is in  $\mathcal{S}_2(\Gamma_0(1089), \left(\frac{1089}{\cdot}\right))$ .

#### $\nu\text{-N4-21-3}$

Let  $\tau_0 = [21, 15, 7] = -\frac{5}{14} + \frac{11i}{14\sqrt{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -115582896 + 20120400\sqrt{33} + 4860i\sqrt{2\left(-565606921 + 98459527\sqrt{33}\right)}$$

has minimal polynomial  $T^4 + 462331584T^3 - 14585085504T^2 + 113515167744T - 1532454764544$  and  $k = \sqrt[3]{t} \approx 2.002817764643678099769 + 0.979145264759923260540i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{99}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned}\Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(45m+42n)q^{21m^2+15mn+7n^2} \\ &= 84q^7 - 6q^{13} - 78q^{19} - 168q^{28} + 174q^{43} + 12q^{52} - 96q^{61} + 246q^{73} + 156q^{76} - 258q^{79} + \dots\end{aligned}$$

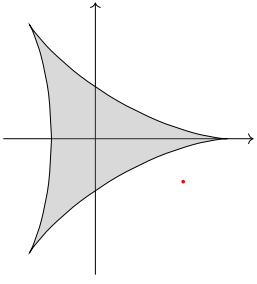
is in  $\mathcal{S}_2(\Gamma_0(1089), \left(\frac{1089}{\cdot}\right))$ .

#### $\nu\text{-N4-21-4}$

Let  $\tau_0 = [21, -15, 7] = \frac{5}{14} + \frac{11i}{14\sqrt{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -115582896 + 20120400\sqrt{33} - 4860i\sqrt{2\left(-565606921 + 98459527\sqrt{33}\right)}$$

has minimal polynomial  $T^4 + 462331584T^3 - 14585085504T^2 + 113515167744T - 1532454764544$  and  $k = \sqrt[3]{t} \approx 2.002817764643678099769 - 0.979145264759923260540i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{99}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-45m + 42n)q^{21m^2 - 15mn + 7n^2} \\ &= 84q^7 - 6q^{13} - 78q^{19} - 168q^{28} + 174q^{43} + 12q^{52} - 96q^{61} + 246q^{73} + 156q^{76} - 258q^{79} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1089), (\frac{1089}{\cdot}))$ .

#### $\nu$ -N4-22-1

Let  $\tau_0 = [3, -3, 43] = \frac{1}{2} + \frac{13i}{2\sqrt{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -4348502496 - 1206057600\sqrt{13} - 3240\sqrt{3602628061773 + 999189246396\sqrt{13}}$   
has minimal polynomial  $T^4 + 17394009984T^3 - 1233688869504T^2 + 41258732396544T - 556992887353344$  and  
 $k = \sqrt[3]{t} \approx -2590.994984257351217105$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{117}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

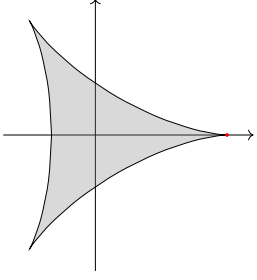
$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m + 6n)q^{129m^2 - 3mn + n^2} \\ &= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 120q^{100} - 132q^{121} - 12q^{127} + 60q^{133} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1521), (\frac{1521}{\cdot}))$ .

#### $\nu$ -N4-22-2

Let  $\tau_0 = [129, 3, 1] = -\frac{1}{86} + \frac{13i}{86\sqrt{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -4348502496 - 1206057600\sqrt{13} + 3240\sqrt{3602628061773 + 999189246396\sqrt{13}}$   
has minimal polynomial  $T^4 + 17394009984T^3 - 1233688869504T^2 + 41258732396544T - 556992887353344$  and  
 $k = \sqrt[3]{t} \approx 2.999999998447741499125$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{117}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

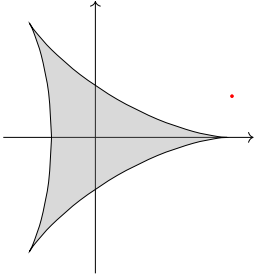
$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 258n)q^{3m^2 + 3mn + 43n^2} \\ &= 1014q^{43} + 1014q^{49} + 1014q^{61} + 1014q^{79} + 1014q^{103} + 1014q^{133} - 2028q^{172} - 2028q^{181} - 2028q^{196} + 1014q^{211} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1521), (\frac{1521}{\cdot}))$ .

#### $\nu$ -N4-22-3

Let  $\tau_0 = [21, 9, 7] = -\frac{3}{14} + \frac{13i}{14\sqrt{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -4348502496 + 1206057600\sqrt{13} + 3240i\sqrt{-3602628061773 + 999189246396\sqrt{13}}$   
has minimal polynomial  $T^4 + 17394009984T^3 - 1233688869504T^2 + 41258732396544T - 556992887353344$  and  
 $k = \sqrt[3]{t} \approx 3.114748570755876067671 + 0.939921106262329843702i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{117}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$ , where

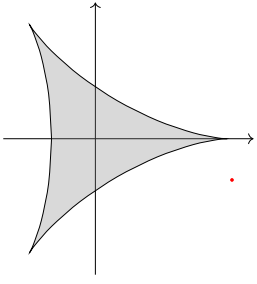
$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(27m + 42n)q^{21m^2 + 9mn + 7n^2} \\ &= 84q^7 + 30q^{19} - 168q^{28} - 114q^{31} + 138q^{37} - 222q^{67} - 24q^{73} - 60q^{76} + 282q^{97} + 192q^{109} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1521), (\frac{1521}{\cdot}))$ .

#### $\nu$ -N4-22-4

Let  $\tau_0 = [21, -9, 7] = \frac{3}{14} + \frac{13i}{14\sqrt{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -4348502496 + 1206057600\sqrt{13} - 3240i\sqrt{-3602628061773 + 999189246396\sqrt{13}}$   
has minimal polynomial  $T^4 + 17394009984T^3 - 1233688869504T^2 + 41258732396544T - 556992887353344$  and  
 $k = \sqrt[3]{t} \approx 3.114748570755876067671 - 0.939921106262329843702i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{117}{16\pi^2} L(\Theta_Q, \tau_0, 2)$ , where

$$\begin{aligned} \Theta_{Q, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-3}(n)(-27m + 42n)q^{21m^2 - 9mn + 7n^2} \\ &= 84q^7 + 30q^{19} - 168q^{28} - 114q^{31} + 138q^{37} - 222q^{67} - 24q^{73} - 60q^{76} + 282q^{97} + 192q^{109} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(1521), (\frac{1521}{\cdot}))$ .

#### $\nu$ -N4-23-1

Let  $\tau_0 = [3, -3, 61] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{241}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -423340998048 - 27269790912\sqrt{241} - 3240\sqrt{34144490291088327 + 2199439967608338\sqrt{241}}$   
has minimal polynomial  $T^4 + 1693363992192T^3 - 46980539789184T^2 + 68024448000000T - 918330048000000$  and  
 $k = \sqrt[3]{t} \approx -11919.28232822148138230$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{241}}{16\pi^2} L(\Theta_Q, \tau_0, 2)$ , where

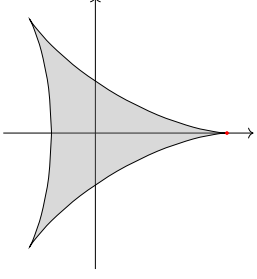
$$\begin{aligned} \Theta_{Q, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-3}(n)(-9m + 6n)q^{183m^2 - 3mn + n^2} \\ &= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 120q^{100} - 132q^{121} + 156q^{169} - 12q^{181} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(2169), (\frac{2169}{\cdot}))$ .

#### $\nu$ -N4-23-2

Let  $\tau_0 = [183, 3, 1] = -\frac{1}{122} + \frac{1}{122}i\sqrt{\frac{241}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -423340998048 - 27269790912\sqrt{241} + 3240\sqrt{34144490291088327 + 2199439967608338\sqrt{241}}$   
has minimal polynomial  $T^4 + 1693363992192T^3 - 46980539789184T^2 + 68024448000000T - 918330048000000$  and  
 $k = \sqrt[3]{t} \approx 2.99999999984055406797$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{241}}{8\pi^2} L(\Theta_Q, \tau_0, 2)$ , where

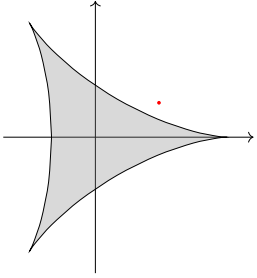
$$\begin{aligned} \Theta_{Q, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-3}(n)(9m + 366n)q^{3m^2 + 3mn + 61n^2} \\ &= 1446q^{61} + 1446q^{67} + 1446q^{79} + 1446q^{97} + 1446q^{121} + 1446q^{151} + 1446q^{187} + 1446q^{229} - 1446q^{241} - 2892q^{244} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(2169), (\frac{2169}{\cdot}))$ .

#### $\nu$ -N4-23-3

Let  $\tau_0 = [33, 27, 11] = -\frac{9}{22} + \frac{1}{22}i\sqrt{\frac{241}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -423340998048 + 27269790912\sqrt{241} + 3240i\sqrt{-34144490291088327 + 2199439967608338\sqrt{241}}$   
has minimal polynomial  $T^4 + 1693363992192T^3 - 46980539789184T^2 + 68024448000000T - 918330048000000$  and  
 $k = \sqrt[3]{t} \approx 1.450116699490707386282 + 0.784503680032144004279i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{241}}{16\pi^2} L(\Theta_Q, \tau_0, 2)$ , where

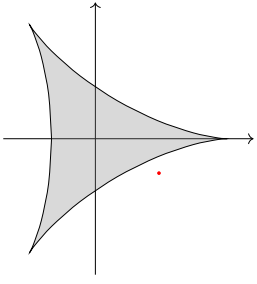
$$\begin{aligned} \Theta_{Q, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-3}(n)(81m + 66n)q^{33m^2 + 27mn + 11n^2} \\ &= 132q^{11} - 30q^{17} - 102q^{23} - 264q^{44} + 60q^{68} + 294q^{71} - 192q^{89} + 204q^{92} + 366q^{101} - 426q^{131} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(2169), (\frac{2169}{\cdot}))$ .

#### $\nu$ -N4-23-4

Let  $\tau_0 = [33, -27, 11] = \frac{9}{22} + \frac{1}{22}i\sqrt{\frac{241}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -423340998048 + 27269790912\sqrt{241} - 3240i\sqrt{-34144490291088327 + 2199439967608338\sqrt{241}}$   
has minimal polynomial  $T^4 + 1693363992192T^3 - 46980539789184T^2 + 68024448000000T - 918330048000000$  and  
 $k = \sqrt[3]{t} \approx 1.450116699490707386282 - 0.784503680032144004279i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{241}}{16\pi^2} L(\Theta_Q, \tau_0, 2)$ , where

$$\begin{aligned} \Theta_{Q, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-3}(n)(-81m + 66n)q^{33m^2 - 27mn + 11n^2} \\ &= 132q^{11} - 30q^{17} - 102q^{23} - 264q^{44} + 60q^{68} + 294q^{71} - 192q^{89} + 204q^{92} + 366q^{101} - 426q^{131} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(2169), (\frac{2169}{\cdot}))$ .

#### $\nu$ -N4-24-1

Let  $\tau_0 = [3, -3, 103] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{409}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -2131340796546048 - 105388019702784\sqrt{409} - 12960\sqrt{6 \left( 9015182817623475045667 + 445772100805075082311\sqrt{409} \right)}$$

has minimal polynomial  $T^4 + 8525363186184192T^3 - 560410748554973184T^2 + 17832200896512000000T - 240734712102912000000$  and  $k = \sqrt[3]{t} \approx -204285.5413317431374449$  is not in  $\mathcal{K}_Q^\circ$ .

We have  $\nu(t) = \frac{9\sqrt{409}}{16\pi^2} L(\Theta_Q, \tau_0, 2)$ , where

$$\begin{aligned} \Theta_{Q, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-3}(n)(-9m + 6n)q^{309m^2 - 3mn + n^2} \\ &= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 120q^{100} - 132q^{121} + 156q^{169} - 168q^{196} + \dots \end{aligned}$$

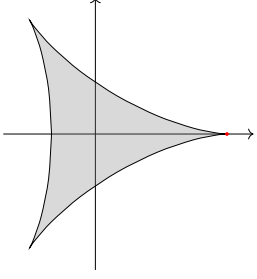
is in  $\mathcal{S}_2(\Gamma_0(3681), (\frac{3681}{\cdot}))$ .

#### $\nu$ -N4-24-2

Let  $\tau_0 = [309, 3, 1] = -\frac{1}{206} + \frac{1}{206}i\sqrt{\frac{409}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -2131340796546048 - 105388019702784\sqrt{409} + 12960\sqrt{6 \left( 9015182817623475045667 + 445772100805075082311\sqrt{409} \right)}$$

has minimal polynomial  $T^4 + 8525363186184192T^3 - 560410748554973184T^2 + 17832200896512000000T - 240734712102912000000$  and  $k = \sqrt[3]{t} \approx 2.9999999999999996832979$  is **IN**  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = -\frac{9\sqrt{409}}{8\pi^2} L(\Theta_Q, \tau_0, 2)$ , where

$$\begin{aligned} \Theta_{Q, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-3}(n)(9m + 618n)q^{3m^2 + 3mn + 103n^2} \\ &= 2454q^{103} + 2454q^{109} + 2454q^{121} + 2454q^{139} + 2454q^{163} + 2454q^{193} + 2454q^{229} + 2454q^{271} + 2454q^{319} + 2454q^{373} + \dots \end{aligned}$$

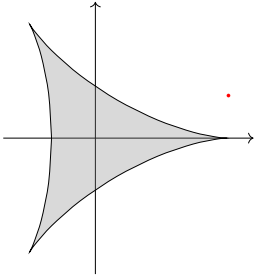
is in  $\mathcal{S}_2(\Gamma_0(3681), (\frac{3681}{\cdot}))$ .

#### $\nu$ -N4-24-3

Let  $\tau_0 = [33, 15, 11] = -\frac{5}{22} + \frac{1}{22}i\sqrt{\frac{409}{3}}$  with  $h(D) = 4$ . Then

$$t = t_Q(\tau_0) = -2131340796546048 + 105388019702784\sqrt{409} + 12960i\sqrt{6 \left( -9015182817623475045667 + 445772100805075082311\sqrt{409} \right)}$$

has minimal polynomial  $T^4 + 8525363186184192T^3 - 560410748554973184T^2 + 17832200896512000000T - 240734712102912000000$  and  $k = \sqrt[3]{t} \approx 3.034541370202187095363 + 0.970596093498807327403i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{409}}{16\pi^2} L(\Theta_Q, \tau_0, 2)$ , where

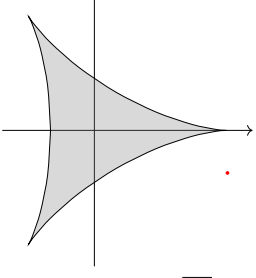
$$\begin{aligned} \Theta_{Q, \tau_0}(\tau) &= \sum_{m, n \in \mathbb{Z}} \chi_{-3}(n)(45m + 66n)q^{33m^2 + 15mn + 11n^2} \\ &= 132q^{11} + 42q^{29} - 264q^{44} - 174q^{47} + 222q^{59} - 354q^{107} - 48q^{113} - 84q^{116} + 438q^{149} + 312q^{173} + \dots \end{aligned}$$

is in  $\mathcal{S}_2(\Gamma_0(3681), (\frac{3681}{\cdot}))$ .

#### $\nu$ -N4-24-4

Let  $\tau_0 = [33, -15, 11] = \frac{5}{22} + \frac{1}{22}i\sqrt{\frac{409}{3}}$  with  $h(D) = 4$ . Then

$t = t_Q(\tau_0) = -2131340796546048 + 105388019702784\sqrt{409} - 12960i\sqrt{6\left(-9015182817623475045667 + 445772100805075082311\sqrt{409}\right)}$   
has minimal polynomial  $T^4 + 8525363186184192T^3 - 560410748554973184T^2 + 17832200896512000000T - 240734712102912000000$  and  
 $k = \sqrt[3]{t} \approx 3.034541370202187095363 - 0.970596093498807327403i$  is not in  $\mathcal{K}_Q^\circ$ .



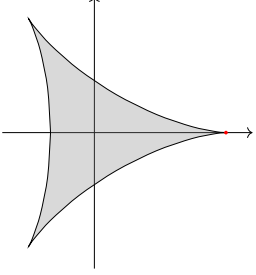
We have  $\nu(t) = \frac{9\sqrt{409}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where  

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-45m + 66n)q^{33m^2 - 15mn + 11n^2}$$

$$= 132q^{11} + 42q^{29} - 264q^{44} - 174q^{47} + 222q^{59} - 354q^{107} - 48q^{113} - 84q^{116} + 438q^{149} + 312q^{173} + \dots$$
  
is in  $\mathcal{S}_2(\Gamma_0(3681), \left(\frac{3681}{\cdot}\right))$ .

#### [ν-N4-25-1](#)

Let  $\tau_0 = [39, 0, 1] = \frac{i}{\sqrt{39}}$  with  $h(D) = 4$ . Then  
 $t = t_Q(\tau_0) = \frac{479061}{4} + \frac{132867\sqrt{13}}{4} - \frac{243}{4}\sqrt{7771398 + 2155398\sqrt{13}}$   
has minimal polynomial  $T^4 - 479061T^3 + 13466088T^2 - 28697814T + 387420489$  and  
 $k = \sqrt[3]{t} \approx 3.000056365680104470817$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{13}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where  

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)78nq^{3m^2 + 13n^2}$$

$$= 156q^{13} + 312q^{16} + 312q^{25} + 312q^{40} - 312q^{52} - 624q^{55} + 312q^{61} - 624q^{64} - 624q^{79} + 312q^{88} + \dots$$
  
is in  $\mathcal{S}_2(\Gamma_0(468), \left(\frac{468}{\cdot}\right))$ .

#### [ν-N4-25-2](#)

Let  $\tau_0 = [3, 0, 13] = i\sqrt{\frac{13}{3}}$  with  $h(D) = 4$ . Then  
 $t = t_Q(\tau_0) = \frac{479061}{4} + \frac{132867\sqrt{13}}{4} + \frac{243}{4}\sqrt{7771398 + 2155398\sqrt{13}}$   
has minimal polynomial  $T^4 - 479061T^3 + 13466088T^2 - 28697814T + 387420489$  and  
 $k = \sqrt[3]{t} \approx 78.24473259220120996252$  is not in  $\mathcal{K}_Q^\circ$ .

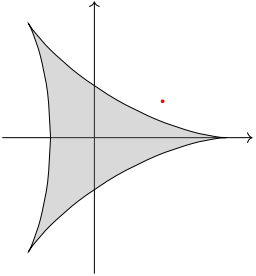
We have  $\nu(t) = \frac{9\sqrt{13}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where  

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)6nq^{39m^2 + n^2}$$

$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 24q^{40} - 48q^{43} + 84q^{49} + 96q^{55} - 216q^{64} + 168q^{88} + \dots$$
  
is in  $\mathcal{S}_2(\Gamma_0(468), \left(\frac{468}{\cdot}\right))$ .

#### [ν-N4-25-3](#)

Let  $\tau_0 = [15, 12, 5] = -\frac{2}{5} + \frac{1}{5}i\sqrt{\frac{13}{3}}$  with  $h(D) = 4$ . Then  
 $t = t_Q(\tau_0) = \frac{479061}{4} - \frac{132867\sqrt{13}}{4} + \frac{243}{4}i\sqrt{-7771398 + 2155398\sqrt{13}}$   
has minimal polynomial  $T^4 - 479061T^3 + 13466088T^2 - 28697814T + 387420489$  and  
 $k = \sqrt[3]{t} \approx 1.555106354180089945361 + 0.8289911002049984849582i$  is not in  $\mathcal{K}_Q^\circ$ .



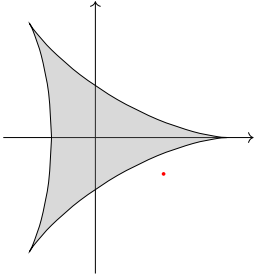
We have  $\nu(t) = \frac{9\sqrt{13}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$ , where  

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(36m + 30n)q^{15m^2 + 12mn + 5n^2}$$

$$= 60q^5 - 12q^8 - 48q^{11} - 120q^{20} + 156q^{32} - 84q^{41} + 96q^{44} + 168q^{47} - 192q^{59} - 156q^{65} + \dots$$
  
is in  $\mathcal{S}_2(\Gamma_0(468), \left(\frac{468}{\cdot}\right))$ .

#### [ν-N4-25-4](#)

Let  $\tau_0 = [15, -12, 5] = \frac{2}{5} + \frac{1}{5}i\sqrt{\frac{13}{3}}$  with  $h(D) = 4$ . Then  
 $t = t_Q(\tau_0) = \frac{479061}{4} - \frac{132867\sqrt{13}}{4} - \frac{243}{4}i\sqrt{-7771398 + 2155398\sqrt{13}}$   
has minimal polynomial  $T^4 - 479061T^3 + 13466088T^2 - 28697814T + 387420489$  and  
 $k = \sqrt[3]{t} \approx 1.555106354180089945361 - 0.828991100204998484958i$  is not in  $\mathcal{K}_Q^\circ$ .



We have  $\nu(t) = \frac{9\sqrt{13}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$ , where  
 $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(-36m+30n)q^{15m^2-12mn+5n^2}$   
 $\qquad\qquad\qquad = 60q^5 - 12q^8 - 48q^{11} - 120q^{20} + 156q^{32} - 84q^{41} + 96q^{44} + 168q^{47} - 192q^{59} - 156q^{65} + \dots$   
is in  $\mathcal{S}_2\left(\Gamma_0(468),\left(\frac{468}{\cdot}\right)\right)$ .

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