Supplementary data for the paper

Determinants of Mahler measures and special values of L-functions

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In this data file, we compile a complete list of $t \in \mathsf{T}_{P,4}$ or $\mathsf{T}_{Q,4}$, together with the associated formulas that express $\mu(t)$ or $\nu(t)$ as L-values of cusp forms. We use the following labeling system:

- the 3 rational $t \in T_{P,4}$ are labeled from μ -R-1 to μ -R-3;
- the 12 pairs of real quadratic $t \in T_{P,4}$ are labeled from μ -R2-1-x to μ -R2-12-x;
- the 4 pairs of imaginary quadratic $t \in T_{P,4}$ are labeled from μ -I2-1-x to μ -I2-4-x;
- the 24 quartets of totally real quartic $t \in T_{P,4}$ are labeled from μ -T4-1-x to μ -T4-24-x;
- the 24 quartets of non-totally real quartic $t \in T_{P,4}$ are labeled from μ -N4-1-x to μ -N4-24-x;
- the 3 rational $t \in \mathsf{T}_{Q,4}$ are labeled from $\nu\text{-R-1}$ to $\nu\text{-R-3}$;

 μ -R2-1-2 (#1 in the paper)

 $t = t_P(\tau_0) = 8 - 6\sqrt{2}$

Let $\tau_0 = [16, 16, 5] = -\frac{1}{2} + \frac{i}{4}$ with h(D) = 2. Then

has minimal polynomial $T^2 - 16T - 8$.

- the 15 pairs of real quadratic $t \in T_{Q,4}$ are labeled from ν -R2-1-x to ν -R2-15-x;
- the 2 pairs of imaginary quadratic $t \in T_{Q,4}$ are labeled from ν -I2-1-x to ν -I2-2-x;
- the 4 trios of (non-totally real) cubic $t \in T_{Q,4}$ are labeled from ν -C-1-x to ν -C-4-x;
- the 24 quartets of totally real quartic $t \in T_{Q,4}$ are labeled from ν -T4-1-x to ν -T4-24-x;
- the 25 quartets of non-totally real quartic $t \in T_{Q,4}$ are labeled from ν -N4-1-x to ν -N4-25-x.

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\mu-R-1
Let \tau_0 = [2, 2, 1] = -\frac{1}{2} + \frac{i}{2} with h(D) = 1. Then
t = t_P(\tau_0) = -16
has minimal polynomial T + 16.
We have \mu(t) = \frac{4}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (4m + 2n) q^{8m^2 + 4mn + n^2}
              = 4q - 8q^5 - 12q^9 + 24q^{13} + 8q^{17} - 4q^{25} - 40q^{29} - 8q^{37} + 40q^{41} + 24q^{45} + \cdots
is in S_2\left(\Gamma_0(32), \left(\frac{64}{\cdot}\right)\right).
\mu-R-2
Let \tau_0 = [8, 4, 1] = -\frac{1}{4} + \frac{i}{4} with h(D) = 1. Then
t = t_P(\tau_0) = 8
has minimal polynomial T-8.
We have \mu(t) = \frac{1}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 8n)q^{2m^2 + 2mn + n^2}
              = 16q - 32q^5 - 48q^9 + 96q^{13} + 32q^{17} - 16q^{25} - 160q^{29} - 32q^{37} + 160q^{41} + 96q^{45} + \cdots
is in S_2\left(\Gamma_0(32), \left(\frac{16}{\cdot}\right)\right).
\mu-R-3
Let \tau_0 = [4, 0, 1] = \frac{i}{2} with h(D) = 1. Then
t = t_P(\tau_0) = 32
has minimal polynomial T-32.
We have \mu(t) = \frac{2}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 4nq^{4m^2 + n^2}
              = 8q + 16q^5 - 24q^9 - 48q^{13} + 16q^{17} - 8q^{25} + 80q^{29} + 16q^{37} + 80q^{41} - 48q^{45} + \cdots
is in S_2(\Gamma_0(64), (\frac{64}{\cdot})).
\mu-R2-1-1 (#1 in the paper)
Let \tau_0 = [16, 0, 1] = \frac{i}{4} with h(D) = 2. Then
t = t_P(\tau_0) = 8 + 6\sqrt{2}
has minimal polynomial T^2 - 16T - 8.
We have \mu(t) = \frac{1}{4\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 16nq^{m^2 + n^2}
              = 32q + 64q^2 + 64q^5 - 96q^9 - 128q^{10} - 192q^{13} + 64q^{17} - 192q^{18} - 32q^{25} + 384q^{26} + \cdots
is in S_2(\Gamma_0(64), (\frac{16}{})).
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We have \mu(t) = \frac{1}{4\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(32m + 16n)q^{5m^2 + 4mn + n^2}
              =32q-64q^2+64q^5-96q^9+128q^{10}-192q^{13}+64q^{17}+192q^{18}-32q^{25}-384q^{26}+\cdots
is in S_2\left(\Gamma_0(64), \left(\frac{16}{\cdot}\right)\right).
\mu\text{-R2-2-1} (#2 in the paper)
Let \tau_0 = [8, 0, 1] = \frac{i}{2\sqrt{2}} with h(D) = 2. Then
t = t_P(\tau_0) = 8 + 8\sqrt{2}
has minimal polynomial T^2 - 16T - 64.
We have \mu(t) = \frac{1}{\sqrt{2\pi^2}} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 8nq^{2m^2+n^2}
              = 16q + 32q^3 - 16q^9 - 96q^{11} - 96q^{17} + 32q^{19} + 80q^{25} + 64q^{27} + 192q^{33} - 96q^{41} + \cdots
is in S_2(\Gamma_0(64), (\frac{32}{4})).
t = t_P(\tau_0) = 8 - 8\sqrt{2}
has minimal polynomial T^2 - 16T - 64.
We have \mu(t) = \frac{1}{\sqrt{2}\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (16m + 8n) q^{6m^2 + 4mn + n^2}
              = 16q - 32q^3 - 16q^9 + 96q^{11} - 96q^{17} - 32q^{19} + 80q^{25} - 64q^{27} + 192q^{33} - 96q^{41} + \cdots
is in S_2\left(\Gamma_0(64), \left(\frac{32}{\cdot}\right)\right).
\mu\text{-R2-3-1} (#3 in the paper)
Let \tau_0 = [12, 4, 1] = -\frac{1}{6} + \frac{i}{3\sqrt{2}} with h(D) = 2. Then
t = t_P(\tau_0) = -32 + 32\sqrt{2}
has minimal polynomial T^2 + 64T - 1024.
We have \mu(t) = \frac{2\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(8m+12n)q^{4m^2+4mn+3n^2}
              =32q^3+32q^{11}-96q^{19}-64q^{27}-96q^{43}+192q^{51}+160q^{59}-96q^{67}+160q^{75}+32q^{83}+\cdots
is in S_2(\Gamma_0(128), (\frac{128}{.}))
\mu-R2-3-2 (#3 in the paper)
Let \tau_0 = [4, -4, 3] = \frac{1}{2} + \frac{i}{\sqrt{2}} with h(D) = 2. Then
t = t_P(\tau_0) = -32 - 32\sqrt{2}
has minimal polynomial T^2 + 64T - 1024.
We have \mu(t)=\frac{2\sqrt{2}}{\pi^2}L(\Theta_{P,\tau_0},2), where \Theta_{P,\tau_0}(\tau)=\sum_{m,n\in\mathbb{Z}}\chi_{-4}(n)(-8m+4n)q^{12m^2-4mn+n^2}
              =8q-40q^9+48q^{17}+40q^{25}-64q^{33}-48q^{41}-56q^{49}+192q^{57}-16q^{73}+8q^{81}+\cdots
is in S_2\left(\Gamma_0(128), \left(\frac{128}{\cdot}\right)\right).
\mu\text{-R2-4-1} (#4 in the paper)
Let \tau_0 = [2, 0, 1] = \frac{i}{\sqrt{2}} with h(D) = 1. Then
t = t_P(\tau_0) = 48 + 32\sqrt{2}
has minimal polynomial T^2 - 96T + 256.
We have \mu(t) = \frac{4\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 2nq^{8m^2 + n^2}
              = 4q - 4q^9 - 24q^{17} + 20q^{25} + 48q^{33} - 24q^{41} - 28q^{49} - 16q^{57} + 8q^{73} - 44q^{81} + \cdots
is in S_2(\Gamma_0(64), (\frac{128}{.})).
\mu-R2-4-2 (#4 in the paper)
Let \tau_0 = [6, -4, 1] = \frac{1}{3} + \frac{i}{3\sqrt{2}} with h(D) = 1. Then
t = t_P(\tau_0) = 48 - 32\sqrt{2}
has minimal polynomial T^2 - 96T + 256.
We have \mu(t) = \frac{4\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-8m+6n)q^{8m^2-8mn+3n^2}
              =8q^3-24q^{11}+8q^{19}+16q^{27}+40q^{43}-48q^{51}-24q^{59}-56q^{67}+40q^{75}+72q^{83}+\cdots
is in S_2(\Gamma_0(64), (\frac{128}{.})).
\mu\text{-R2-5-1} (#5 in the paper)
Let \tau_0 = [20, 4, 1] = -\frac{1}{10} + \frac{i}{5} with h(D) = 2. Then
t = t_P(\tau_0) = -256 + 192\sqrt{2}
has minimal polynomial T^2 + 512T - 8192.
We have \mu(t) = \frac{4}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(8m+20n)q^{4m^2+4mn+5n^2}
              =64q^5+64q^{13}+64q^{29}-192q^{37}-192q^{45}+64q^{53}-192q^{61}-128q^{85}+320q^{101}+320q^{109}+\cdots
is in S_2(\Gamma_0(256), (\frac{256}{.})).
\mu-R2-5-2 (#5 in the paper)
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Let \tau_0 = [4, -4, 5] = \frac{1}{2} + i with h(D) = 2. Then
t = t_P(\tau_0) = -256 - 192\sqrt{2}
has minimal polynomial T^2 + 512T - 8192.
We have \mu(t) = \frac{4}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-8m+4n)q^{20m^2-4mn+n^2}
               = 8q - 24q^9 - 16q^{17} + 88q^{25} - 80q^{41} - 56q^{49} + 128q^{65} - 48q^{73} + 72q^{81} + 80q^{89} + \cdots
is in S_2\left(\Gamma_0(256), \left(\frac{256}{\cdot}\right)\right).
\mu-R2-6-1 (#6 in the paper)
Let \tau_0 = [1, 0, 1] = i \text{ with } h(D) = 1. Then
t = t_P(\tau_0) = 272 + 192\sqrt{2}
has minimal polynomial T^2 - 544T + 256.
We have \mu(t) = \frac{16}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) n q^{16m^2 + n^2}
               = 2q - 6q^9 + 4q^{17} - 2q^{25} + 20q^{41} - 14q^{49} - 24q^{65} - 12q^{73} + 18q^{81} + 20q^{89} + \cdots
is in S_2(\Gamma_0(64), (\frac{256}{.})).
\mu-R2-6-2 (#6 in the paper)
Let \tau_0 = [5, 4, 1] = -\frac{2}{5} + \frac{i}{5} with h(D) = 1. Then
t = t_P(\tau_0) = 272 - 192\sqrt{2}
has minimal polynomial T^2 - 544T + 256.
We have \mu(t) = \frac{16}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(8m+5n)q^{16m^2+16mn+5n^2}
               =4q^{5}-12q^{13}+20q^{29}+4q^{37}-12q^{45}-28q^{53}+20q^{61}+8q^{85}+4q^{101}-12q^{109}+\cdots
is in S_2\left(\Gamma_0(64), \left(\frac{256}{\cdot}\right)\right).
\mu\text{-R2-7-1} (#7 in the paper)
Let \tau_0 = [16, 4, 1] = -\frac{1}{8} + \frac{i\sqrt{3}}{8} with h(D) = 2. Then
t = t_P(\tau_0) = 8 + 4\sqrt{3}
has minimal polynomial T^2 - 16T + 16.
We have \mu(t) = \frac{\sqrt{3}}{8\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(8m+16n)q^{m^2+mn+n^2}
               =48q+48q^3-96q^7-144q^9-96q^{13}+96q^{19}+288q^{21}+240q^{25}-144q^{27}+288q^{31}+\cdots
is in S_2\left(\Gamma_0(48), \left(\frac{12}{\cdot}\right)\right).
\mu-R2-7-2 (#7 in the paper)
Let \tau_0 = [16, 12, 3] = -\frac{3}{8} + \frac{i\sqrt{3}}{8} with h(D) = 2. Then
t = t_P(\tau_0) = 8 - 4\sqrt{3}
has minimal polynomial T^2 - 16T + 16.
We have \mu(t) = \frac{\sqrt{3}}{8\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (24m + 16n) q^{3m^2 + 3mn + n^2}
               = 16q - 48q^3 + 96q^7 - 48q^9 - 32q^{13} - 96q^{19} + 96q^{21} + 80q^{25} + 144q^{27} - 288q^{31} + \cdots
is in S_2\left(\Gamma_0(48), \left(\frac{12}{\cdot}\right)\right).

\overline{\mu\text{-R2-8-1}}
 (#8 in the paper)

Let \tau_0 = [3, 3, 1] = -\frac{1}{2} + \frac{i}{2\sqrt{3}} with h(D) = 1. Then
t = t_P(\tau_0) = -112 + 64\sqrt{3}
has minimal polynomial T^2 + 224T + 256.
We have \mu(t) = \frac{8\sqrt{3}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(6m+3n)q^{16m^2+12mn+3n^2}
               =6q^{3}-12q^{7}+12q^{19}-18q^{27}+36q^{31}-12q^{39}-36q^{43}+36q^{63}+12q^{67}+30q^{75}+\cdots
is in S_2(\Gamma_0(48), (\frac{192}{10})).
\mu-R2-8-2 (#8 in the paper)
Let \tau_0 = [1, -1, 1] = \frac{1}{2} + \frac{i\sqrt{3}}{2} with h(D) = 1. Then
t = t_P(\tau_0) = -112 - 64\sqrt{3}
has minimal polynomial T^2 + 224T + 256.
We have \mu(t) = \frac{8\sqrt{3}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P, \tau_0}(	au) = \sum_{m, n \in \mathbb{Z}} \chi_{-4}(n) (-2m+n) q^{16m^2 - 4mn + n^2}
               = 2q - 6q^9 - 4q^{13} + 12q^{21} + 10q^{25} - 20q^{37} - 10q^{49} - 12q^{57} + 28q^{61} + 20q^{73} + \cdots
is in S_2(\Gamma_0(48), (\frac{192}{2})).
\mu-R2-9-1 (#9 in the paper)
Let \tau_0 = [4, 0, 3] = \frac{i\sqrt{3}}{2} with h(D) = 2. Then
t = t_P(\tau_0) = 128 + 64\sqrt{3}
has minimal polynomial T^2 - 256T + 4096.
We have \mu(t) = \frac{2\sqrt{3}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 4nq^{12m^2 + n^2}
               = 8q - 24q^9 + 16q^{13} - 48q^{21} + 40q^{25} + 80q^{37} - 40q^{49} - 48q^{57} - 112q^{61} + 80q^{73} + \cdots
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is in S_2(\Gamma_0(192), (\frac{192}{\cdot})).
\mu-R2-9-2 (#9 in the paper)
Let \tau_0 = [12, 0, 1] = \frac{i}{2\sqrt{3}} with h(D) = 2. Then
t=t_P(\tau_0)=128-64\sqrt{3} has minimal polynomial T^2-256T+4096.
We have \mu(t) = \frac{2\sqrt{3}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 12 n q^{4m^2 + 3n^2}
               = 24q^3 + 48q^7 + 48q^{19} - 72q^{27} - 144q^{31} + 48q^{39} - 144q^{43} - 144q^{63} + 48q^{67} + 120q^{75} + \cdots
is in S_2(\Gamma_0(192), (\frac{192}{2})).
\mu\text{-R2-}10\text{-}1 (#10 in the paper)
Let \tau_0 = [32, 4, 1] = -\frac{1}{16} + \frac{i\sqrt{7}}{16} with h(D) = 2. Then t = t_P(\tau_0) = 8 + 3\sqrt{7}
has minimal polynomial T^2 - 16T + 1.
We have \mu(t) = \frac{\sqrt{7}}{8\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 32n)q^{m^2 + mn + 2n^2}
               =112q^2+112q^4+112q^8+112q^{14}-336q^{16}-336q^{18}-224q^{22}-336q^{28}+112q^{32}-336q^{36}+\cdots
is in S_2\left(\Gamma_0(112), \left(\frac{28}{\cdot}\right)\right).
\mu-R2-10-2 (#10 in the paper)
Let \tau_0 = [32, 28, 7] = -\frac{7}{16} + \frac{i\sqrt{7}}{16} with h(D) = 2. Then
t = t_P(\tau_0) = 8 - 3\sqrt{7}
has minimal polynomial T^2 - 16T + 1.
We have \mu(t) = \frac{\sqrt{7}}{8\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(56m + 32n)q^{7m^2 + 7mn + 2n^2}
               =16q^2-48q^4+80q^8-112q^{14}+16q^{16}-48q^{18}+224q^{22}-112q^{28}-176q^{32}+144q^{36}+\cdots
is in S_2\left(\Gamma_0(112), \left(\frac{28}{\cdot}\right)\right).
\mu\text{-R2-11-1} (#11 in the paper)
Let \tau_0 = [7, 7, 2] = -\frac{1}{2} + \frac{i}{2\sqrt{7}} with h(D) = 1. Then
t = t_P(\tau_0) = -2032 + 768\sqrt{7}
has minimal polynomial T^2 + 4064T + 256.
We have \mu(t) = \frac{8\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(14m + 7n)q^{32m^2 + 28mn + 7n^2}
               = 14q^7 - 28q^{11} + 28q^{23} - 28q^{43} - 42q^{63} + 84q^{67} + 28q^{71} - 84q^{79} + 84q^{99} - 28q^{107} + \cdots
is in S_2(\Gamma_0(112), (\frac{448}{i})).
\mu-R2-11-2 (#11 in the paper)
Let \tau_0 = [1, -1, 2] = \frac{1}{2} + \frac{i\sqrt{7}}{2} with h(D) = 1. Then
t = t_P(\tau_0) = -2032 - 768\sqrt{7}
has minimal polynomial T^2 + 4064T + 256.
We have \mu(t) = \frac{8\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-2m+n) q^{32m^2-4mn+n^2}
               =2q-6q^9+10q^{25}-4q^{29}+12q^{37}-14q^{49}-20q^{53}+28q^{77}+18q^{81}-36q^{109}+\cdots
is in S_2\left(\Gamma_0(112), \left(\frac{448}{\cdot}\right)\right).
\mu-R2-12-1 (#12 in the paper)
Let \tau_0 = [4, 0, 7] = \frac{i\sqrt{7}}{2} with h(D) = 2. Then
t = t_P(\tau_0) = 2048 + 768\sqrt{7}
has minimal polynomial T^2 - 4096T + 65536.
We have \mu(t) = \frac{2\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 4nq^{28m^2 + n^2}
               = 8q - 24q^9 + 40q^{25} + 16q^{29} - 48q^{37} - 56q^{49} + 80q^{53} - 112q^{77} + 72q^{81} + 144q^{109} + \cdots
is in S_2(\Gamma_0(448), (\frac{448}{2})).
\mu\text{-R2-12-2} (#12 in the paper)
Let \tau_0 = [28, 0, 1] = \frac{i}{2\sqrt{7}} with h(D) = 2. Then
t = t_P(\tau_0) = 2048 - 768\sqrt{7}
has minimal polynomial T^2 - 4096T + 65536.
We have \mu(t) = \frac{2\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \tilde{\chi}_{-4}(n) 28nq^{4m^2 + 7n^2}
               =56q^{7}+112q^{11}+112q^{23}+112q^{43}-168q^{63}-336q^{67}+112q^{71}-336q^{79}-336q^{99}+112q^{107}+\cdots
is in S_2\left(\Gamma_0(448), \left(\frac{448}{\cdot}\right)\right).
Let \tau_0 = [4, 2, 1] = -\frac{1}{4} + \frac{i\sqrt{3}}{4} with h(D) = 1. Then
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 $t = t_P(\tau_0) = 8 + 8i\sqrt{3}$

has minimal polynomial $T^2 - 16T + 256$.

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We have \mu(t) = \frac{\sqrt{3}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(4m+4n)q^{4m^2+2mn+n^2}
              = 8q - 24q^9 - 16q^{13} + 48q^{21} + 40q^{25} - 80q^{37} - 40q^{49} - 48q^{57} + 112q^{61} + 80q^{73} + \cdots
is in S_2\left(\Gamma_0(48), \left(\frac{48}{\cdot}\right)\right).
\mu-I2-1-2
Let \tau_0 = [4, -2, 1] = \frac{1}{4} + \frac{i\sqrt{3}}{4} with h(D) = 1. Then
t = t_P(\tau_0) = 8 - 8i\sqrt{3}
has minimal polynomial T^2 - 16T + 256.
We have \mu(t) = \frac{\sqrt{3}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-4m+4n)q^{4m^2-2mn+n^2}
               = 8q - 24q^9 - 16q^{13} + 48q^{21} + 40q^{25} - 80q^{37} - 40q^{49} - 48q^{57} + 112q^{61} + 80q^{73} + \cdots
is in S_2(\Gamma_0(48), (\frac{48}{48})).
\mu-I2-2-1
Let \tau_0 = [2, 1, 1] = -\frac{1}{4} + \frac{i\sqrt{7}}{4} with h(D) = 1. Then
t = t_P(\tau_0) = 8 + 24i\sqrt{7}
has minimal polynomial T^2 - 16T + 4096.
We have \mu(t) = \frac{2\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(2m+2n)q^{8m^2+2mn+n^2}
              = 4q - 12q^9 + 20q^{25} - 8q^{29} + 24q^{37} - 28q^{49} - 40q^{53} + 56q^{77} + 36q^{81} - 72q^{109} + \cdots
is in S_2(\Gamma_0(56), (\frac{112}{...})).
\mu-I2-2-2
Let \tau_0 = [2, -1, 1] = \frac{1}{4} + \frac{i\sqrt{7}}{4} with h(D) = 1. Then
t = t_P(\tau_0) = 8 - 24i\sqrt{7}
has minimal polynomial T^2 - 16T + 4096.
We have \mu(t) = \frac{2\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-2m+2n)q^{8m^2-2mn+n^2}
              =4q-12q^9+20q^{25}-8q^{29}+24q^{37}-28q^{49}-40q^{53}+56q^{77}+36q^{81}-72q^{109}+\cdots
is in S_2(\Gamma_0(56), (\frac{112}{.})).
Let \tau_0 = [4,3,1] = -\frac{3}{8} + \frac{i\sqrt{7}}{8} with h(D) = 1. Then t = t_P(\tau_0) = \frac{1}{2} + \frac{3i\sqrt{7}}{2}
has minimal polynomial T^2 - T + 16.
We have \mu(t) = \frac{\sqrt{7}}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(6m+4n)q^{4m^2+3mn+n^2}
              = 8q - 4q^2 - 12q^4 + 20q^8 - 24q^9 + 28q^{14} + 4q^{16} + 12q^{18} - 56q^{22} + 40q^{25} + \cdots
is in S_2\left(\Gamma_0(28), \left(\frac{28}{\cdot}\right)\right).
Let \tau_0 = [4, -3, 1] = \frac{3}{8} + \frac{i\sqrt{7}}{8} with h(D) = 1. Then
t = t_P(\tau_0) = \frac{1}{2} - \frac{3i\sqrt{7}}{2}
has minimal polynomial T^2 - T + 16.
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Let
$$\tau_0 = [4, -3, 1] = \frac{8}{8} + \frac{1}{8}$$
 with $h(D) = 1$. Then $t = t_P(\tau_0) = \frac{1}{2} - \frac{3i\sqrt{7}}{2}$ has minimal polynomial $T^2 - T + 16$. We have $\mu(t) = \frac{\sqrt{7}}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$, where $\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-6m + 4n)q^{4m^2 - 3mn + n^2}$ $= 8q - 4q^2 - 12q^4 + 20q^8 - 24q^9 + 28q^{14} + 4q^{16} + 12q^{18} - 56q^{22} + 40q^{25} + \cdots$ is in $S_2\left(\Gamma_0(28), \left(\frac{28}{\cdot}\right)\right)$.

μ -I2-4-1

Let
$$\tau_0 = [8, 2, 1] = -\frac{1}{8} + \frac{i\sqrt{7}}{8}$$
 with $h(D) = 1$. Then $t = t_P(\tau_0) = \frac{31}{2} + \frac{3i\sqrt{7}}{2}$ has minimal polynomial $T^2 - 31T + 256$. We have $\mu(t) = \frac{\sqrt{7}}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$, where $\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(4m + 8n)q^{2m^2 + mn + n^2}$
$$= 16q + 8q^2 + 24q^4 - 40q^8 - 48q^9 - 56q^{14} - 8q^{16} - 24q^{18} + 112q^{22} + 80q^{25} + \cdots$$
 is in $S_2\left(\Gamma_0(56), \left(\frac{28}{\cdot}\right)\right)$.

и-I2-4-2

$$t = t_P(\tau_0) = \frac{31}{2} - \frac{3i\sqrt{7}}{2}$$
 has minimal polynomial $T^2 - 31T + 256$. We have $\mu(t) = \frac{\sqrt{7}}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$, where
$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-4m + 8n) q^{2m^2 - mn + n^2}$$

$$= 16q + 8q^2 + 24q^4 - 40q^8 - 48q^9 - 56q^{14} - 8q^{16} - 24q^{18} + 112q^{22} + 80q^{25} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(56), \left(\frac{28}{\cdot}\right)\right)$.

Let $\tau_0 = [8, -2, 1] = \frac{1}{8} + \frac{i\sqrt{7}}{8}$ with h(D) = 1. Then

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\mu-T4-1-1 (#28 in the paper)
Let \tau_0 = [16, 16, 7] = -\frac{1}{2} + \frac{i\sqrt{3}}{4} with h(D) = 4. Then
t = t_P(\tau_0) = 8 - 3\sqrt{2} - 5\sqrt{6}
has minimal polynomial T^4 - 32T^3 + 48T^2 + 3328T + 16.
We have \mu(t) = \frac{\sqrt{3}}{4\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(32m + 16n)q^{7m^2 + 4mn + n^2}
                 =32q-64q^4-96q^9+192q^{12}+64q^{13}-192q^{21}+160q^{25}-384q^{28}+192q^{36}+320q^{37}+\cdots
is in S_2\left(\Gamma_0(192), \left(\frac{48}{\cdot}\right)\right).

\overline{\mu\text{-T4-1-2}}
 (#28 in the paper)

Let \tau_0 = [48, 48, 13] = -\frac{1}{2} + \frac{i}{4\sqrt{3}} with h(D) = 4. Then
t = t_P(\tau_0) = 8 + 3\sqrt{2} - 5\sqrt{6}
has minimal polynomial T^4 - 32T^3 + 48T^2 + 3328T + 16.
We have \mu(t) = \frac{\sqrt{3}}{4\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(96m + 48n)q^{13m^2 + 12mn + 3n^2}
                =96q^3-192q^4+192q^7-192q^{12}+192q^{19}-288q^{27}+384q^{28}-576q^{31}+576q^{36}+192q^{39}+\cdots
is in S_2(\Gamma_0(192), (\frac{48}{\cdot})).
\mu-T4-1-3 (#28 in the paper)

Let \tau_0 = [48, 0, 1] = \frac{i}{4\sqrt{3}} with h(D) = 4. Then t = t_P(\tau_0) = 8 - 3\sqrt{2} + 5\sqrt{6}
has minimal polynomial T^4 - 32T^3 + 48T^2 + 3328T + 16.
We have \mu(t) = \frac{\sqrt{3}}{4\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 48nq^{m^2+3n^2}
                =96q^3+192q^4+192q^7+192q^{12}+192q^{19}-288q^{27}-384q^{28}-576q^{31}-576q^{36}+192q^{39}+\cdots
is in S_2\left(\Gamma_0(192), \left(\frac{48}{\cdot}\right)\right).
\mu\text{-}\mathrm{T}4\text{-}1\text{-}4 (#28 in the paper)
Let \tau_0 = [16, 0, 3] = \frac{i\sqrt{3}}{4} with h(D) = 4. Then
t = t_P(\tau_0) = 8 + 3\sqrt{2} + 5\sqrt{6}
has minimal polynomial T^4 - 32T^3 + 48T^2 + 3328T + 16.
We have \mu(t) = \frac{\sqrt{3}}{4\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 16nq^{3m^2+n^2}
                =32q+64q^4-96q^9-192q^{12}+64q^{13}-192q^{21}+160q^{25}+384q^{28}-192q^{36}+320q^{37}+\cdots
is in S_2\left(\Gamma_0(192), \left(\frac{48}{\cdot}\right)\right).
\mu-T4-2-1 (#29 in the paper)
Let \tau_0 = [8, -8, 5] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{3}{2}} with h(D) = 4. Then
t = t_P(\tau_0) = 8 - 16\sqrt{3} - 8\sqrt{6}
has minimal polynomial T^4 - 32T^3 - 1920T^2 + 34816T + 4096.
We have \mu(t) = \frac{\sqrt{\frac{3}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-16m+8n)q^{10m^2-4mn+n^2}
                = 16q - 32q^{7} - 48q^{9} + 96q^{15} + 112q^{25} - 160q^{31} - 96q^{33} + 48q^{49} + 192q^{55} + 96q^{63} + \cdots
is in S_2\left(\Gamma_0(192), \left(\frac{96}{\cdot}\right)\right).
\mu-T4-2-2 (#29 in the paper)
Let \tau_0 = [24, 24, 7] = -\frac{1}{2} + \frac{i}{2\sqrt{6}} with h(D) = 4. Then t = t_P(\tau_0) = 8 - 16\sqrt{3} + 8\sqrt{6} has minimal polynomial T^4 - 32T^3 - 1920T^2 + 34816T + 4096.
We have \mu(t)=\frac{\sqrt{\frac{3}{2}}}{\pi^2}L(\Theta_{P,\tau_0},2), where \Theta_{P,\tau_0}(\tau)=\sum_{\substack{n,n\in\mathbb{Z}\\ r=2}}\chi_{-4}(n)(48m+24n)q^{14m^2+12mn+3n^2}
                =48q^3-96q^5+96q^{11}-96q^{21}-144q^{27}+288q^{29}-192q^{35}+288q^{45}-96q^{53}-288q^{59}+\cdots
is in S_2\left(\Gamma_0(192), \left(\frac{96}{\cdot}\right)\right).
\mu\text{-}\mathrm{T}4\text{-}2\text{-}3 (#29 in the paper)
Let \tau_0 = [24, 0, 1] = \frac{i}{2\sqrt{6}} with h(D) = 4. Then
t = t_P(\tau_0) = 8 + 16\sqrt{3} - 8\sqrt{6}
has minimal polynomial T^4 - 32T^3 - 1920T^2 + 34816T + 4096.
We have \mu(t) = \frac{\sqrt{\frac{3}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 24nq^{2m^2 + 3n^2}
                =48q^3+96q^5+96q^{11}+96q^{21}-144q^{27}-288q^{29}-192q^{35}-288q^{45}+96q^{53}-288q^{59}+\cdots
is in S_2\left(\Gamma_0(192), \left(\frac{96}{\cdot}\right)\right).
\mu-T4-2-4 (#29 in the paper)
Let \tau_0 = [8, 0, 3] = \frac{1}{2}i\sqrt{\frac{3}{2}} with h(D) = 4. Then
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 $t = t_P(\tau_0) = 8 + 16\sqrt{3} + 8\sqrt{6}$

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has minimal polynomial T^4 - 32T^3 - 1920T^2 + 34816T + 4096.
We have \mu(t) = \frac{\sqrt{\frac{3}{2}}}{\pi^2}L(\Theta_{P,\tau_0},2), where \Theta_{P,\tau_0}(\tau) = \sum\limits_{m,n\in\mathbb{Z}}\chi_{-4}(n)8nq^{6m^2+n^2}
               =16q+32q^{7}-48q^{9}-96q^{15}+112q^{25}+160q^{31}-96q^{33}+48q^{49}-192q^{55}-96q^{63}+\cdots
is in S_2\left(\Gamma_0(192), \left(\frac{96}{\cdot}\right)\right).
\mu-T4-3-1 (#30 in the paper)
Let \tau_0 = [8, -8, 11] = \frac{1}{2} + \frac{3i}{2\sqrt{2}} with h(D) = 4. Then
t = t_P(\tau_0) = 8 - 280\sqrt{2} - 224\sqrt{3}
has minimal polynomial T^4 - 32T^3 - 614272T^2 + 9832448T + 4096.
We have \mu(t) = \frac{3}{\sqrt{2\pi^2}} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-16m + 8n) q^{22m^2 - 4mn + n^2}
               = 16q - 48q^9 - 32q^{19} + 80q^{25} + 96q^{27} - 160q^{43} - 112q^{49} + 224q^{67} + 32q^{73} + 48q^{81} + \cdots
is in S_2(\Gamma_0(576), (\frac{288}{.})).
\mu\text{-T}4\text{-}3\text{-}2 (#30 in the paper)
Let \tau_0 = [72, 72, 19] = -\frac{1}{2} + \frac{i}{6\sqrt{2}} with h(D) = 4. Then
t = t_P(\tau_0) = 8 - 280\sqrt{2} + 224\sqrt{3}
has minimal polynomial T^4 - 32T^3 - 614272T^2 + 9832448T + 4096.
We have \mu(t) = \frac{3}{\sqrt{2\pi^2}} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(144m + 72n)q^{38m^2 + 36mn + 9n^2}
               =144q^9-288q^{11}+288q^{17}-288q^{27}+288q^{41}-288q^{59}-144q^{81}+864q^{83}-864q^{89}+864q^{99}+\cdots
is in S_2(\Gamma_0(576), (\frac{288}{\cdot})).
\mu-T4-3-3 (#30 in the paper)
Let \tau_0 = [72, 0, 1] = \frac{i}{6\sqrt{2}} with h(D) = 4. Then
t = t_P(\tau_0) = 8 + 280\sqrt{2} - 224\sqrt{3}
has minimal polynomial T^4 - 32T^3 - 614272T^2 + 9832448T + 4096
We have \mu(t) = \frac{3}{\sqrt{2\pi^2}} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 72nq^{2m^2+9n^2}
                = 144q^9 + 288q^{11} + 288q^{17} + 288q^{27} + 288q^{41} + 288q^{59} - 144q^{81} - 864q^{83} - 864q^{89} - 864q^{99} + \cdots
is in S_2(\Gamma_0(576), (\frac{288}{.})).
\mu\text{-T4-3-4} (#30 in the paper) Let \tau_0=[8,0,9]=\frac{3i}{2\sqrt{2}} with h(D)=4. Then
t = t_P(\tau_0) = 8 + 280\sqrt{2} + 224\sqrt{3}
has minimal polynomial T^4 - 32T^3 - 614272T^2 + 9832448T + 4096.
We have \mu(t) = \frac{3}{\sqrt{2}\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(	au) = \sum_{m,n \in \mathbb{Z}} \dot{\chi}_{-4}(n) 8nq^{18m^2 + n^2}
               = 16q - 48q^9 + 32q^{19} + 80q^{25} - 96q^{27} + 160q^{43} - 112q^{49} - 224q^{67} + 32q^{73} + 48q^{81} + \cdots
is in S_2(\Gamma_0(576), (\frac{288}{.})).
\mu-T4-4-1 (#31 in the paper)
Let \tau_0 = [4, -4, 7] = \frac{1}{2} + i\sqrt{\frac{3}{2}} with h(D) = 4. Then
t=t_P(\tau_0)=-544-384\sqrt{2}-320\sqrt{3}-224\sqrt{6} has minimal polynomial T^4+2176T^3-30720T^2-131072T+1048576.
We have \mu(t) = \frac{2\sqrt{6}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-8m+4n)q^{28m^2-4mn+n^2}
               = 8q - 24q^9 + 24q^{25} + 48q^{33} - 136q^{49} + 112q^{73} + 72q^{81} + 16q^{97} - 192q^{105} - 8q^{121} + \cdots
is in S_2(\Gamma_0(384), (\frac{384}{.}))
\mu\text{-}\mathrm{T}4\text{-}4\text{-}2 (#31 in the paper)
Let \tau_0 = [12, 12, 5] = -\frac{1}{2} + \frac{i}{\sqrt{6}} with h(D) = 4. Then
t=t_P(\tau_0)=-544+384\sqrt{2}-320\sqrt{3}+224\sqrt{6} has minimal polynomial T^4+2176T^3-30720T^2-131072T+1048576.
We have \mu(t) = \frac{2\sqrt{6}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(24m + 12n)q^{20m^2 + 12mn + 3n^2}
               =24q^{3}-48q^{11}-72q^{27}+192q^{35}-144q^{59}+72q^{75}-240q^{83}+144q^{99}+240q^{107}+48q^{131}+\cdots
is in S_2(\Gamma_0(384), (\frac{384}{.}))
\mu-T4-4-3 (#31 in the paper)
Let \tau_0 = [20, 12, 3] = -\frac{3}{10} + \frac{1}{5}i\sqrt{\frac{3}{2}} with h(D) = 4. Then
t = t_P(\tau_0) = -544 + 384\sqrt{2} + 320\sqrt{3} - 224\sqrt{6}
has minimal polynomial T^4 + 2176T^3 - 30720T^2 - 131072T + 1048576.
We have \mu(t) = \frac{2\sqrt{6}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(24m+20n)q^{12m^2+12mn+5n^2}
               =32q^{5}-96q^{21}+32q^{29}-96q^{45}+160q^{53}+192q^{77}-96q^{93}-224q^{101}-64q^{125}+32q^{149}+\cdots
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is in S_2\left(\Gamma_0(384), \left(\frac{384}{\cdot}\right)\right).
\mu-T4-4-4 (#31 in the paper)
Let \tau_0 = [28, 4, 1] = -\frac{1}{14} + \frac{1}{7}i\sqrt{\frac{3}{2}} with h(D) = 4. Then
t = t_P(\tau_0) = -544 - 384\sqrt{2} + 320\sqrt{3} + 224\sqrt{6}
has minimal polynomial T^4 + 2176T^3 - 30720T^2 - 131072T + 1048576.
We have \mu(t) = \frac{2\sqrt{6}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 28n)q^{4m^2 + 4mn + 7n^2}
                =96q^{7}+96q^{15}+96q^{31}-192q^{55}-288q^{63}-288q^{79}+96q^{87}-288q^{103}+96q^{127}-288q^{135}+\cdots
is in S_2(\Gamma_0(384), (\frac{384}{.})).
\mu-T4-5-1 (#32 in the paper)
Let \tau_0 = [14, 12, 3] = -\frac{3}{7} + \frac{1}{7}i\sqrt{\frac{3}{2}} with h(D) = 2. Then
t=t_P(\tau_0)=560+384\sqrt{2}-320\sqrt{3}-224\sqrt{6} has minimal polynomial T^4-2240T^3+75264T^2-573440T+65536.
We have \mu(t) = \frac{4\sqrt{6}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (24m + 14n) q^{24m^2 + 24mn + 7n^2}
                =8q^{7}-24q^{15}+40q^{31}-48q^{55}-24q^{63}+40q^{79}+72q^{87}-56q^{103}-88q^{127}+72q^{135}+\cdots
is in S_2(\Gamma_0(192), (\frac{384}{.}))
\mu\text{-}\mathrm{T}4\text{-}5\text{-}2 (#32 in the paper)
Let \tau_0 = [10, 4, 1] = -\frac{1}{5} + \frac{1}{5}i\sqrt{\frac{3}{2}} with h(D) = 2. Then
t = t_P(\tau_0) = 560 - 384\sqrt{2} - 320\sqrt{3} + 224\sqrt{6}
has minimal polynomial T^4 - 2240T^3 + 75264T^2 - 573440T + 65536. \\
We have \mu(t) = \frac{4\sqrt{6}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (8m + 10n) q^{8m^2 + 8mn + 5n^2}
                =24q^{5}+24q^{21}-72q^{29}-72q^{45}+24q^{53}+48q^{77}+120q^{93}+24q^{101}+48q^{125}-168q^{149}+\cdots
is in S_2\left(\Gamma_0(192), \left(\frac{384}{\cdot}\right)\right).
\mu-T4-5-3 (#32 in the paper)
Let \tau_0 = [6, 0, 1] = \frac{i}{\sqrt{6}} with h(D) = 2. Then
t = t_P(\tau_0) = 560 - 384\sqrt{2} + 320\sqrt{3} - 224\sqrt{6}
has minimal polynomial T^4 - 2240T^3 + 75264T^2 - 573440T + 65536.
We have \mu(t) = \frac{4\sqrt{6}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 6nq^{8m^2 + 3n^2}
                = 12q^3 + 24q^{11} - 36q^{27} - 48q^{35} - 72q^{59} + 84q^{75} + 120q^{83} - 72q^{99} + 120q^{107} + 24q^{131} + \cdots
is in S_2(\Gamma_0(192), (\frac{384}{.})).
\mu\text{-}\mathrm{T}4\text{-}5\text{-}4 (#32 in the paper)
Let \tau_0 = [2, 0, 3] = i\sqrt{\frac{3}{2}} with h(D) = 2. Then
t = t_P(\tau_0) = 560 + 384\sqrt{2} + 320\sqrt{3} + 224\sqrt{6}
has minimal polynomial T^4 - 2240T^3 + 75264T^2 - 573440T + 65536.
We have \mu(t) = \frac{4\sqrt{6}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 2nq^{24m^2 + n^2}
                =4q-12q^9+28q^{25}-24q^{33}+12q^{49}-56q^{73}+36q^{81}+8q^{97}+48q^{105}-4q^{121}+\cdots
is in S_2\left(\Gamma_0(192), \left(\frac{384}{\cdot}\right)\right).
\frac{\mu\text{-T4-6-1 (#33 in the paper)}}{\text{Let }\tau_0 = [4, -4, 13] = \frac{1}{2} + i\sqrt{3} \text{ with } h(D) = 4. \text{ Then } t = t_P(\tau_0) = -13312 - 9408\sqrt{2} - 7680\sqrt{3} - 5440\sqrt{6}
has minimal polynomial T^4 + 53248T^3 + 196608T^2 - 33554432T + 268435456.
We have \mu(t) = \frac{4\sqrt{3}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-8m + 4n) q^{52m^2 - 4mn + n^2}
                = 8q - 24q^9 + 40q^{25} - 72q^{49} + 48q^{57} - 80q^{73} + 72q^{81} + 112q^{97} - 88q^{121} - 144q^{129} + \cdots
is in S_2\left(\Gamma_0(768), \left(\frac{768}{\cdot}\right)\right).

\overline{\mu\text{-T4-6-2}}
 (#33 in the paper)

Let \tau_0 = [12, -12, 7] = \frac{1}{2} + \frac{i}{\sqrt{3}} with h(D) = 4. Then
t = t_P(\tau_0) = -13312 + 9408\sqrt{2} + 7680\sqrt{3} - 5440\sqrt{6}
has minimal polynomial T^4 + 53248T^3 + 196608T^2 - 33554432T + 268435456.
We have \mu(t) = \frac{4\sqrt{3}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-24m + 12n)q^{28m^2 - 12mn + 3n^2}
                =24q^3-48q^{19}-72q^{27}+144q^{43}+48q^{67}+120q^{75}-384q^{91}+240q^{139}-216q^{147}+336q^{163}+\cdots
is in S_2\left(\Gamma_0(768), \left(\frac{768}{\cdot}\right)\right)
\mu-T4-6-3 (#33 in the paper)
Let \tau_0 = [28, 12, 3] = -\frac{3}{14} + \frac{i\sqrt{3}}{7} with h(D) = 4. Then
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 $t = t_P(\tau_0) = -13312 - 9408\sqrt{2} + 7680\sqrt{3} + 5440\sqrt{6}$

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has minimal polynomial T^4 + 53248T^3 + 196608T^2 - 33554432T + 268435456. \\
We have \mu(t) = \frac{4\sqrt{3}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(24m + 28n)q^{12m^2 + 12mn + 7n^2}
                 =64q^{7}+64q^{31}-192q^{39}-192q^{63}+64q^{79}+320q^{103}-192q^{111}+320q^{127}+64q^{151}+320q^{175}+\cdots
is in S_2\left(\Gamma_0(768), \left(\frac{768}{\cdot}\right)\right).
\mu-T4-6-4 (#33 in the paper)
Let \tau_0 = [52, 4, 1] = -\frac{1}{26} + \frac{i\sqrt{3}}{13} with h(D) = 4. Then
t = t_P(\tau_0) = -13312 + 9408\sqrt{2} - 7680\sqrt{3} + 5440\sqrt{6}
has minimal polynomial T^4 + 53248T^3 + 196608T^2 - 33554432T + 268435456.
We have \mu(t) = \frac{4\sqrt{3}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 52n)q^{4m^2 + 4mn + 13n^2}
                 = 192q^{13} + 192q^{21} + 192q^{37} + 192q^{61} + 192q^{93} - 576q^{109} - 576q^{117} - 384q^{133} - 576q^{157} + 192q^{181} + \cdots
is in S_2\left(\Gamma_0(768), \left(\frac{768}{\cdot}\right)\right).
\mu-T4-7-1 (#34 in the paper)
Let \tau_0 = [13, 12, 3] = -\frac{6}{13} + \frac{i\sqrt{3}}{13} with h(D) = 1. Then
t = t_P(\tau_0) = 13328 - 9408\sqrt{2} + 7680\sqrt{3} - 5440\sqrt{6} has minimal polynomial T^4 - 53312T^3 + 2754048T^2 - 13647872T + 65536.
We have \mu(t) = \frac{16\sqrt{3}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (24m + 13n) q^{48m^2 + 48mn + 13n^2}
                =4q^{13}-12q^{21}+20q^{37}-28q^{61}+36q^{93}+4q^{109}-12q^{117}-24q^{133}-28q^{157}+52q^{181}+\cdots
is in S_2\left(\Gamma_0(192), \left(\frac{768}{\cdot}\right)\right).
\mu-T4-7-2 (#34 in the paper)
Let \tau_0 = [7, 4, 1] = -\frac{2}{7} + \frac{i\sqrt{3}}{7} with h(D) = 1. Then
t = t_P(\tau_0) = 13328 + 9408\sqrt{2} - 7680\sqrt{3} - 5440\sqrt{6}
has minimal polynomial T^4 - 53312T^3 + 2754048T^2 - 13647872T + 65536.
We have \mu(t) = \frac{16\sqrt{3}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m+7n)q^{16m^2+16mn+7n^2}
                =12q^{7}-36q^{31}+12q^{39}-36q^{63}+60q^{79}+12q^{103}+60q^{111}-36q^{127}-84q^{151}+60q^{175}+\cdots
is in S_2\left(\Gamma_0(192), \left(\frac{768}{\cdot}\right)\right).

\overline{\mu\text{-T4-7-3 (#34 in the paper)}}

Let 
\tau_0 = [3, 0, 1] = \frac{i}{\sqrt{3}}
 with 
h(D) = 1
. Then
t = t_P(\tau_0) = 13328 - 9408\sqrt{2} - 7680\sqrt{3} + 5440\sqrt{6} has minimal polynomial T^4 - 53312T^3 + 2754048T^2 - 13647872T + 65536.
We have \mu(t) = \frac{16\sqrt{3}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(	au) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 3nq^{16m^2 + 3n^2}
                 = 6q^3 + 12q^{19} - 18q^{27} - 36q^{43} + 12q^{67} + 30q^{75} + 24q^{91} + 60q^{139} - 30q^{147} - 84q^{163} + \cdots
is in S_2(\Gamma_0(192), (\frac{768}{.})).
\mu\text{-}\mathrm{T}4\text{-}7\text{-}4 (#34 in the paper)
Let \tau_0 = [1, 0, 3] = i\sqrt{3} with h(D) = 1. Then
t = t_P(\tau_0) = 13328 + 9408\sqrt{2} + 7680\sqrt{3} + 5440\sqrt{6} has minimal polynomial T^4 - 53312T^3 + 2754048T^2 - 13647872T + 65536.
We have \mu(t) = \frac{16\sqrt{3}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) n q^{48m^2 + n^2}
                 = 2q - 6q^9 + 10q^{25} - 10q^{49} - 12q^{57} + 20q^{73} + 18q^{81} - 28q^{97} - 22q^{121} + 36q^{129} + \cdots
is in S_2(\Gamma_0(192), (\frac{768}{4})).
\mu-T4-8-1 (#35 in the paper)
Let \tau_0 = [4, -4, 19] = \frac{1}{2} + \frac{3i}{\sqrt{2}} with h(D) = 4. Then
t = t_P(\tau_0) = -153632 - 108640\sqrt{2} - 88704\sqrt{3} - 62720\sqrt{6} has minimal polynomial T^4 + 614528T^3 - 9828352T^2 - 131072T + 1048576.
We have \mu(t) = \frac{6\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-8m+4n) q^{76m^2-4mn+n^2}
                = 8q - 24q^9 + 40q^{25} - 56q^{49} - 16q^{73} + 120q^{81} - 80q^{97} + 24q^{121} - 144q^{153} + 104q^{169} + \cdots
is in S_2\left(\Gamma_0(1152), \left(\frac{1152}{\cdot}\right)\right).
\overline{\mu\text{-T4-8-2}} (#35 in the paper)
Let \tau_0 = [36, 36, 11] = -\frac{1}{2} + \frac{i}{3\sqrt{2}} with h(D) = 4. Then
t = t_P(\tau_0) = -153632 - 108640\sqrt{2} + 88704\sqrt{3} + 62720\sqrt{6}
has minimal polynomial T^4 + 614528T^3 - 9828352T^2 - 131072T + 1048576.
We have \mu(t) = \frac{6\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(72m + 36n)q^{44m^2 + 36mn + 9n^2}
                 = 72q^9 - 144q^{17} + 144q^{41} - 360q^{81} + 432q^{89} - 432q^{113} + 144q^{137} + 432q^{153} - 576q^{209} + 360q^{225} + \cdots
is in S_2(\Gamma_0(1152), (\frac{1152}{.})).
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\mu-T4-8-3 (#35 in the paper)
Let \tau_0 = [44, 36, 9] = -\frac{9}{22} + \frac{3i}{11\sqrt{2}} with h(D) = 4. Then
t = t_P(\tau_0) = -153632 + 108640\sqrt{2} - 88704\sqrt{3} + 62720\sqrt{6}
has minimal polynomial T^4 + 614528T^3 - 9828352T^2 - 131072T + 1048576.
We have \mu(t) = \frac{6\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(72m + 44n)q^{36m^2 + 36mn + 11n^2}
               =32q^{11}-96q^{27}+160q^{59}+32q^{83}-96q^{99}-224q^{107}+160q^{131}+288q^{171}-224q^{179}+32q^{227}+\cdots
is in S_2\left(\Gamma_0(1152), \left(\frac{1152}{\cdot}\right)\right).
\mu-T4-8-4 (#35 in the paper)
Let \tau_0 = [76, 4, 1] = -\frac{1}{38} + \frac{3i}{19\sqrt{2}} with h(D) = 4. Then
t = t_P(\tau_0) = -153632 + 108640\sqrt{2} + 88704\sqrt{3} - 62720\sqrt{6}
has minimal polynomial T^4 + 614528T^3 - 9828352T^2 - 131072T + 1048576.
We have \mu(t) = \frac{6\sqrt{2}}{\pi^2}L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(8m+76n)q^{4m^2+4mn+19n^2}
               =288q^{19}+288q^{27}+288q^{43}+288q^{67}+288q^{99}+288q^{139}-864q^{163}-864q^{171}-576q^{187}-864q^{211}+\cdots
is in S_2\left(\Gamma_0(1152), \left(\frac{1152}{\cdot}\right)\right).
\mu-T4-9-1 (#36 in the paper)

Let \tau_0 = [38, 36, 9] = -\frac{9}{19} + \frac{3i}{19\sqrt{2}} with h(D) = 2. Then t = t_P(\tau_0) = 153648 - 108640\sqrt{2} - 88704\sqrt{3} + 62720\sqrt{6} has minimal polynomial T^4 - 614592T^3 + 19670528T^2 - 157335552T + 65536.
We have \mu(t) = \frac{12\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (72m + 38n) q^{72m^2 + 72mn + 19n^2}
               = 8q^{19} - 24q^{27} + 40q^{43} - 56q^{67} + 72q^{99} - 88q^{139} + 8q^{163} - 24q^{171} + 144q^{187} - 56q^{211} + \cdots
is in S_2(\Gamma_0(576), (\frac{1152}{2}))
\mu-T4-9-2 (#36 in the paper)
Let \tau_0 = [22, 4, 1] = -\frac{1}{11} + \frac{3i}{11\sqrt{2}} with h(D) = 2. Then
t = t_P(\tau_0) = 153648 - 108640\sqrt{2} + 88704\sqrt{3} - 62720\sqrt{6}
has minimal polynomial T^4 - 614592T^3 + 19670528T^2 - 157335552T + 65536.
We have \mu(t) = \frac{12\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 22n)q^{8m^2 + 8mn + 11n^2}
               =72q^{11}+72q^{27}+72q^{59}-216q^{83}-216q^{99}+72q^{107}-216q^{131}+72q^{171}-216q^{179}+360q^{227}+\cdots
is in S_2\left(\Gamma_0(576), \left(\frac{1152}{\cdot}\right)\right).
\mu\text{-}\mathrm{T}4\text{-}9\text{-}3 (#36 in the paper)
Let \tau_0 = [18, 0, 1] = \frac{i}{3\sqrt{2}} with h(D) = 2. Then
t = t_P(\tau_0) = 153648 + 108640\sqrt{2} - 88704\sqrt{3} - 62720\sqrt{6}
has minimal polynomial T^4 - 614592T^3 + 19670528T^2 - 157335552T + 65536.
We have \mu(t) = \frac{12\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 18nq^{8m^2 + 9n^2}
               =36q^9+72q^{17}+72q^{41}-36q^{81}-216q^{89}-216q^{113}+72q^{137}-216q^{153}-144q^{209}+180q^{225}+\cdots
is in S_2(\Gamma_0(576), (\frac{1152}{.})).
\mu-T4-9-4 (#36 in the paper)
Let \tau_0 = [2, 0, 9] = \frac{3i}{\sqrt{2}} with h(D) = 2. Then
t = t_P(\tau_0) = 153648 + 108640\sqrt{2} + 88704\sqrt{3} + 62720\sqrt{6}
has minimal polynomial T^4-614592T^3+19670528T^2-157335552T+65536. \\
We have \mu(t) = \frac{12\sqrt{2}}{\pi^2}L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)2nq^{72m^2+n^2}
               =4q-12q^9+20q^{25}-28q^{49}+8q^{73}+12q^{81}+40q^{97}-100q^{121}+72q^{153}+52q^{169}+\cdots
is in S_2(\Gamma_0(576), (\frac{1152}{...})).
\mu\text{-}\mathrm{T}4\text{-}10\text{-}1 (#37 in the paper)
Let \tau_0 = [8, -8, 7] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{5}{2}} with h(D) = 4. Then
t = t_P(\tau_0) = 8 - 48\sqrt{2} - 24\sqrt{10} has minimal polynomial T^4 - 32T^3 - 20352T^2 + 329728T + 4096.
We have \mu(t) = \frac{\sqrt{\frac{5}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-16m + 8n) q^{14m^2 - 4mn + n^2}
               =16q-48q^9-32q^{11}+96q^{19}+80q^{25}-160q^{35}+32q^{41}-208q^{49}+224q^{59}+160q^{65}+\cdots
is in S_2\left(\Gamma_0(320), \left(\frac{160}{\cdot}\right)\right).
\mu-T4-10-2 (#37 in the paper)
Let \tau_0 = [40, 40, 11] = -\frac{1}{2} + \frac{i}{2\sqrt{10}} with h(D) = 4. Then
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 $t = t_P(\tau_0) = 8 + 48\sqrt{2} - 24\sqrt{10}$

has minimal polynomial $T^4 - 32T^3 - 20352T^2 + 329728T + 4096$.

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We have \mu(t) = \frac{\sqrt{\frac{5}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \ddot{\chi}_{-4}(n)(80m + 40n)q^{22m^2 + 20mn + 5n^2}
                =80q^{5}-160q^{7}+160q^{13}-160q^{23}+160q^{37}-240q^{45}+480q^{47}-480q^{53}-160q^{55}+480q^{63}+\cdots
is in S_2\left(\Gamma_0(320), \left(\frac{160}{\cdot}\right)\right).
\mu-T4-10-3 (#37 in the paper)
Let \tau_0 = [40, 0, 1] = \frac{i}{2\sqrt{10}} with h(D) = 4. Then
t=t_P(\tau_0)=8-48\sqrt{2}+24\sqrt{10} has minimal polynomial T^4-32T^3-20352T^2+329728T+4096.
We have \mu(t) = \frac{\sqrt{\frac{5}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 40 n q^{2m^2 + 5n^2}
                =80q^{5}+160q^{7}+160q^{13}+160q^{23}+160q^{23}+160q^{37}-240q^{45}-480q^{47}-480q^{53}+160q^{55}-480q^{63}+\cdots
is in S_2(\Gamma_0(320), (\frac{160}{2})).
\mu-T4-10-4 (#37 in the paper)
Let \tau_0 = [8, 0, 5] = \frac{1}{2}i\sqrt{\frac{5}{2}} with h(D) = 4. Then
t = t_P(\tau_0) = 8 + 48\sqrt{2} + 24\sqrt{10}
has minimal polynomial T^4 - 32T^3 - 20352T^2 + 329728T + 4096.
We have \mu(t) = \frac{\sqrt{\frac{5}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 8nq^{10m^2 + n^2}
               = 16q - 48q^9 + 32q^{11} - 96q^{19} + 80q^{25} + 160q^{35} + 32q^{41} - 208q^{49} - 224q^{59} + 160q^{65} + \cdots
is in S_2(\Gamma_0(320), (\frac{160}{.})).
\mu\text{-}\mathrm{T}4\text{-}11\text{-}1 (#38 in the paper)
Let \tau_0 = [4, -4, 11] = \frac{1}{2} + i\sqrt{\frac{5}{2}} with h(D) = 4. Then
t = t_P(\tau_0) = -5152 - 3648\sqrt{2} - 2304\sqrt{5} - 1632\sqrt{10}
has minimal polynomial T^4 + 20608T^3 - 325632T^2 - 131072T + 1048576.
We have \mu(t) = \frac{2\sqrt{10}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-8m+4n) q^{44m^2-4mn+n^2}
               =8q-24q^9+40q^{25}-16q^{41}-8q^{49}-80q^{65}+72q^{81}+112q^{89}-232q^{121}+192q^{161}+\cdots
is in S_2(\Gamma_0(640), (\frac{640}{2})).
\overline{\mu\text{-T4-11-2}} (#38 in the paper)
Let \tau_0 = [20, 20, 7] = -\frac{1}{2} + \frac{i}{\sqrt{10}} with h(D) = 4. Then t = t_P(\tau_0) = -5152 + 3648\sqrt{2} + 2304\sqrt{5} - 1632\sqrt{10}
has minimal polynomial T^4 + 20608T^3 - 325632T^2 - 131072T + 1048576.
We have \mu(t) = \frac{2\sqrt{10}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(40m + 20n)q^{28m^2 + 20mn + 5n^2}
               =40q^{5}-80q^{13}+80q^{37}-120q^{45}+240q^{53}-320q^{77}+240q^{117}+200q^{125}-320q^{133}+400q^{157}+\cdots
is in S_2\left(\Gamma_0(640), \left(\frac{640}{\cdot}\right)\right).
\mu-T4-11-3 (#38 in the paper)
Let \tau_0 = [28, 20, 5] = -\frac{5}{14} + \frac{1}{7}i\sqrt{\frac{5}{2}} with h(D) = 4. Then
t = t_P(\tau_0) = -5152 - 3648\sqrt{2} + 2304\sqrt{5} + 1632\sqrt{10} has minimal polynomial T^4 + 20608T^3 - 325632T^2 - 131072T + 1048576.
We have \mu(t) = \frac{2\sqrt{10}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(40m + 28n)q^{20m^2 + 20mn + 7n^2}
                =32q^{7}-96q^{23}+32q^{47}+160q^{55}-96q^{63}+160q^{95}-224q^{103}+32q^{127}-320q^{143}+288q^{167}+\cdots
is in S_2(\Gamma_0(640), (\frac{640}{2})).
\mu\text{-}\mathrm{T}4\text{-}11\text{-}4 (#38 in the paper)
Let \tau_0 = [44, 4, 1] = -\frac{1}{22} + \frac{1}{11}i\sqrt{\frac{5}{2}} with h(D) = 4. Then
t = t_P(\tau_0) = -5152 + 3648\sqrt{2} - 2304\sqrt{5} + 1632\sqrt{10} has minimal polynomial T^4 + 20608T^3 - 325632T^2 - 131072T + 1048576.
We have \mu(t) = \frac{2\sqrt{10}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 44n)q^{4m^2 + 4mn + 11n^2}
                = 160q^{11} + 160q^{19} + 160q^{35} + 160q^{59} - 320q^{91} - 480q^{99} - 480q^{115} + 160q^{131} - 480q^{139} - 480q^{171} + \cdots
is in S_2(\Gamma_0(640), (\frac{640}{4})).
\mu\text{-}\mathrm{T}4\text{-}12\text{-}1 (#39 in the paper)
Let \tau_0 = [22, 20, 5] = -\frac{5}{11} + \frac{1}{11}i\sqrt{\frac{5}{2}} with h(D) = 2. Then
t=t_P(\tau_0)=5168-3648\sqrt{2}+2304\sqrt{5}-1632\sqrt{10} has minimal polynomial T^4-20672T^3+665088T^2-5292032T+65536.
We have \mu(t) = \frac{4\sqrt{10}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(40m + 22n)q^{40m^2 + 40mn + 11n^2}
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=8q^{11}-24q^{19}+40q^{35}-56q^{59}+80q^{91}-24q^{99}+40q^{115}-88q^{131}-56q^{139}+72q^{171}+\cdots
is in S_2(\Gamma_0(320), (\frac{640}{.})).
\mu-T4-12-2 (#39 in the paper)
Let \tau_0 = [14, 4, 1] = -\frac{1}{7} + \frac{1}{7}i\sqrt{\frac{5}{2}} with h(D) = 2. Then
t = t_P(\tau_0) = 5168 + 3648\sqrt{2} - 2304\sqrt{5} - 1632\sqrt{10} has minimal polynomial T^4 - 20672T^3 + 665088T^2 - 5292032T + 65536.
We have \mu(t) = \frac{4\sqrt{10}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 14n)q^{8m^2 + 8mn + 7n^2}
                =40q^{7}+40q^{23}-120q^{47}+40q^{55}-120q^{63}-120q^{95}+40q^{103}+200q^{127}+80q^{143}+40q^{167}+\cdots
is in S_2\left(\Gamma_0(320), \left(\frac{640}{\cdot}\right)\right).
\mu-T4-12-3 (#39 in the paper)
Let \tau_0 = [10, 0, 1] = \frac{i}{\sqrt{10}} with h(D) = 2. Then
t = t_P(\tau_0) = 5168 - 3648\sqrt{2} - 2304\sqrt{5} + 1632\sqrt{10}
has minimal polynomial T^4 - 20672T^3 + 665088T^2 - 5292032T + 65536.
We have \mu(t) = \frac{4\sqrt{10}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 10 n q^{8m^2 + 5n^2}
                =20q^5+40q^{13}+40q^{37}-60q^{45}-120q^{53}-80q^{77}-120q^{117}+100q^{125}+240q^{133}+200q^{157}+\cdots
is in S_2\left(\Gamma_0(320), \left(\frac{640}{\cdot}\right)\right).
\mu\text{-}\mathrm{T}4\text{-}12\text{-}4 (#39 in the paper)
Let \tau_0 = [2, 0, 5] = i\sqrt{\frac{5}{2}} with h(D) = 2. Then
t=t_P(\tau_0)=5168+3648\sqrt{2}+2304\sqrt{5}+1632\sqrt{10} has minimal polynomial T^4-20672T^3+665088T^2-5292032T+65536.
We have \mu(t) = \frac{4\sqrt{10}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 2nq^{40m^2 + n^2}
                =4q-12q^9+20q^{25}+8q^{41}-52q^{49}+40q^{65}+36q^{81}-56q^{89}+28q^{121}-80q^{161}+\cdots
is in S_2(\Gamma_0(320), (\frac{640}{.})).
\mu-T4-13-1 (#40 in the paper)
Let \tau_0 = [64, 60, 15] = -\frac{15}{32} + \frac{i\sqrt{15}}{32} with h(D) = 4. Then
t = t_P(\tau_0) = 8 - \frac{7\sqrt{3}}{2} - \frac{\sqrt{15}}{2}
has minimal polynomial T^4 - 32T^3 + 303T^2 - 752T + 1. We have \mu(t) = \frac{\sqrt{15}}{8\pi^2}L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(120m + 64n)q^{15m^2 + 15mn + 4n^2}
                = 16q^4 - 48q^6 + 80q^{10} - 112q^{16} + 144q^{24} - 160q^{34} - 48q^{36} + 80q^{40} + 96q^{46} + 144q^{54} + \cdots
is in S_2\left(\Gamma_0(240), \left(\frac{60}{\cdot}\right)\right).
\mu\text{-}\mathrm{T}4\text{-}13\text{-}2\ (\#40\ \mathrm{in\ the\ paper})
Let \tau_0 = [32, 20, 5] = -\frac{5}{16} + \frac{i\sqrt{15}}{16} with h(D) = 4. Then
t = t_P(\tau_0) = 8 - \frac{7\sqrt{3}}{2} + \frac{\sqrt{15}}{2}
has minimal polynomial T^4 - 32T^3 + 303T^2 - 752T + 1.
We have \mu(t) = \frac{\sqrt{15}}{8\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(40m + 32n)q^{5m^2 + 5mn + 2n^2}
               =48q^2-144q^8+48q^{12}-144q^{18}+240q^{20}+240q^{30}+48q^{32}-480q^{38}-336q^{48}+240q^{50}+\cdots
is in S_2\left(\Gamma_0(240), \left(\frac{60}{\cdot}\right)\right).
\mu-T4-13-3 (#40 in the paper)
Let \tau_0 = [32, 12, 3] = -\frac{3}{16} + \frac{i\sqrt{15}}{16} with h(D) = 4. Then t = t_P(\tau_0) = 8 + \frac{7\sqrt{3}}{2} - \frac{\sqrt{15}}{2} has minimal polynomial T^4 - 32T^3 + 303T^2 - 752T + 1.
We have \mu(t) = \frac{\sqrt{15}}{8\pi^2}L(\Theta_{P,\tau_0},2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-4}(n)(24m+32n)q^{3m^2+3mn+2n^2}
                =80q^2+80q^8-240q^{12}-240q^{18}+80q^{20}-240q^{30}+400q^{32}+480q^{38}-240q^{48}+400q^{50}+\cdots
is in S_2\left(\Gamma_0(240), \left(\frac{60}{\cdot}\right)\right).
\mu-T4-13-4 (#40 in the paper)
Let \tau_0 = [64, 4, 1] = -\frac{1}{32} + \frac{i\sqrt{15}}{32} with h(D) = 4. Then
t = t_P(\tau_0) = 8 + \frac{7\sqrt{3}}{2} + \frac{\sqrt{15}}{2}
has minimal polynomial T^4 - 32T^3 + 303T^2 - 752T + 1.
We have \mu(t) = \frac{\sqrt{15}}{8\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 64n)q^{m^2 + mn + 4n^2}
                =240q^4+240q^6+240q^{10}+240q^{16}+240q^{24}-480q^{34}-720q^{36}-720q^{40}-480q^{46}-720q^{54}+\cdots
is in S_2\left(\Gamma_0(240), \left(\frac{60}{\cdot}\right)\right).
\mu-T4-14-1 (#41 in the paper)
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Let \tau_0 = [60, 0, 1] = \frac{i}{2\sqrt{15}} with h(D) = 4. Then
t=t_P(\tau_0)=48128-27776\sqrt{3}+21504\sqrt{5}-12416\sqrt{15} has minimal polynomial T^4-192512T^3+19857408T^2-536870912T+4294967296.
We have \mu(t) = \frac{2\sqrt{15}}{\pi^2}L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)60nq^{4m^2+15n^2}
              =120q^{15}+240q^{19}+240q^{31}+240q^{51}+240q^{79}+240q^{115}-360q^{135}-720q^{139}-720q^{151}+240q^{159}+\cdots
is in S_2\left(\Gamma_0(960), \left(\frac{960}{\cdot}\right)\right).
\mu-T4-14-2 (#41 in the paper)
Let \tau_0 = [20, 0, 3] = \frac{1}{2}i\sqrt{\frac{3}{5}} with h(D) = 4. Then
t = t_P(\tau_0) = 48128 - 27776\sqrt{3} - 21504\sqrt{5} + 12416\sqrt{15}
has minimal polynomial T^4 - 192512T^3 + 19857408T^2 - 536870912T + 4294967296.
We have \mu(t) = \frac{2\sqrt{15}}{\pi^2}L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)20nq^{12m^2+5n^2}
               =40q^5+80q^{17}-120q^{45}+80q^{53}-240q^{57}-240q^{93}+80q^{113}+200q^{125}+400q^{137}-240q^{153}+\cdots
is in S_2(\Gamma_0(960), (\frac{960}{.})).
\mu-T4-14-3 (#41 in the paper)
Let \tau_0 = [12, 0, 5] = \frac{1}{2}i\sqrt{\frac{5}{3}} with h(D) = 4. Then
t=t_P(\tau_0)=48128+27776\sqrt{3}-21504\sqrt{5}-12416\sqrt{15} has minimal polynomial T^4-192512T^3+19857408T^2-536870912T+4294967296.
We have \mu(t) = \frac{2\sqrt{15}}{\pi^2}L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)12nq^{20m^2+3n^2}
               =24q^3+48q^{23}-72q^{27}-144q^{47}+120q^{75}+48q^{83}+240q^{95}-144q^{107}-168q^{147}+240q^{155}+\cdots
is in S_2(\Gamma_0(960), (\frac{960}{.})).
\mu-T4-14-4 (#41 in the paper)
Let \tau_0 = [4, 0, 15] = \frac{i\sqrt{15}}{2} with h(D) = 4. Then
t=t_P(\tau_0)=48128+27776\sqrt{3}+21504\sqrt{5}+12416\sqrt{15} has minimal polynomial T^4-192512T^3+19857408T^2-536870912T+4294967296.
We have \mu(t) = \frac{2\sqrt{15}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 4nq^{60m^2 + n^2}
              =8q-24q^9+40q^{25}-56q^{49}+16q^{61}-48q^{69}+72q^{81}+80q^{85}-112q^{109}-88q^{121}+\cdots
is in S_2\left(\Gamma_0(960), \left(\frac{960}{\cdot}\right)\right).
\mu-T4-15-1 (#42 in the paper)
Let \tau_0 = [1, -1, 4] = \frac{1}{2} + \frac{i\sqrt{15}}{2} with h(D) = 2. Then
t = t_P(\tau_0) = -48112 - 27776\sqrt{3} - 21504\sqrt{5} - 12416\sqrt{15}
has minimal polynomial T^4 + 192448T^3 + 10618368T^2 + 49266688T + 65536.
We have \mu(t) = \frac{8\sqrt{15}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-2m+n)q^{64m^2-4mn+n^2}
               =2q-6q^9+10q^{25}-14q^{49}-4q^{61}+12q^{69}+18q^{81}-20q^{85}+28q^{109}-22q^{121}+\cdots
is in S_2(\Gamma_0(240), (\frac{960}{.})).
\mu-T4-15-2 (#42 in the paper)
Let \tau_0 = [3, -3, 2] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{5}{3}} with h(D) = 2. Then
t = t_P(\tau_0) = -48112 - 27776\sqrt{3} + 21504\sqrt{5} + 12416\sqrt{15}
has minimal polynomial T^4 + 192448T^3 + 10618368T^2 + 49266688T + 65536.
We have \mu(t) = \frac{8\sqrt{15}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-6m+3n)q^{32m^2-12mn+3n^2}
               = 6q^3 - 12q^{23} - 18q^{27} + 36q^{47} + 30q^{75} + 12q^{83} - 60q^{95} - 36q^{107} - 42q^{147} + 60q^{155} + \cdots
is in S_2\left(\Gamma_0(240), \left(\frac{960}{\cdot}\right)\right).
\mu-T4-15-3 (#42 in the paper)
Let \tau_0 = [5, 5, 2] = -\frac{1}{2} + \frac{1}{2}i\sqrt{\frac{3}{5}} with h(D) = 2. Then
t = t_P(\tau_0) = -48112 + 27776\sqrt{3} + 21504\sqrt{5} - 12416\sqrt{15}
has minimal polynomial T^4 + 192448T^3 + 10618368T^2 + 49266688T + 65536.
We have \mu(t) = \frac{8\sqrt{15}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (10m + 5n) q^{32m^2 + 20mn + 5n^2}
              = 10q^5 - 20q^{17} - 30q^{45} + 20q^{53} + 60q^{57} - 60q^{93} - 20q^{113} + 50q^{125} - 100q^{137} + 60q^{153} + \cdots
is in S_2\left(\Gamma_0(240), \left(\frac{960}{\cdot}\right)\right)
\overline{\mu\text{-T4-15-4}} (#42 in the paper)
Let \tau_0 = [15, 15, 4] = -\frac{1}{2} + \frac{i}{2\sqrt{15}} with h(D) = 2. Then
t = t_P(\tau_0) = -48112 + 27776\sqrt{3} - 21504\sqrt{5} + 12416\sqrt{15}
has minimal polynomial T^4 + 192448T^3 + 10618368T^2 + 49266688T + 65536.
We have \mu(t) = \frac{8\sqrt{15}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
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\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(30m + 15n)q^{64m^2 + 60mn + 15n^2}
                =30q^{15}-60q^{19}+60q^{31}-60q^{51}+60q^{79}-60q^{115}-90q^{135}+180q^{139}-180q^{151}+60q^{159}+\cdots
is in S_2(\Gamma_0(240), (\frac{960}{.})).
\mu-T4-16-1 (#43 in the paper)
Let \tau_0 = [16, -16, 11] = \frac{1}{2} + \frac{i\sqrt{7}}{4} with h(D) = 4. Then
t = t_P(\tau_0) = 8 - \frac{51}{\sqrt{2}} - 15\sqrt{\frac{7}{2}}
We have \mu(t)=\frac{\sqrt{7}}{4\pi^2}L(\Theta_{P,\tau_0},2), where \Theta_{P,\tau_0}(\tau)=\sum_{m,n\in\mathbb{Z}}\chi_{-4}(n)(-32m+16n)q^{11m^2-4mn+n^2}
has minimal polynomial T^4 - 32T^3 - 3792T^2 + 64768T + 1.
               =32q-64q^8-96q^9+192q^{16}+160q^{25}+64q^{29}-320q^{32}-192q^{37}-224q^{49}+320q^{53}+\cdots
is in S_2(\Gamma_0(448), (\frac{112}{.})).

\overline{\mu\text{-T4-16-2}}
 (#43 in the paper)

Let \tau_0 = [112, 112, 29] = -\frac{1}{2} + \frac{i}{4\sqrt{7}} with h(D) = 4. Then
t=t_P(\tau_0)=8-\frac{51}{\sqrt{2}}+15\sqrt{\frac{7}{2}} has minimal polynomial T^4-32T^3-3792T^2+64768T+1.
We have \mu(t) = \frac{\sqrt{7}}{4\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(224m + 112n)q^{29m^2 + 28mn + 7n^2}
               =224q^{7}-448q^{8}+448q^{11}-448q^{16}+448q^{23}-448q^{32}+448q^{43}-448q^{56}-672q^{63}+1344q^{64}+\cdots
is in S_2(\Gamma_0(448), (\frac{112}{.})).
\mu\text{-}\mathrm{T}4\text{-}16\text{-}3 (#43 in the paper)
Let \tau_0 = [112, 0, 1] = \frac{i}{4\sqrt{7}} with h(D) = 4. Then
t = t_P(\tau_0) = 8 + \frac{51}{\sqrt{2}} - 15\sqrt{\frac{7}{2}}
has minimal polynomial T^4 - 32T^3 - 3792T^2 + 64768T + 1.
We have \mu(t) = \frac{\sqrt{7}}{4\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 112 n q^{m^2 + 7n^2}
               =224q^{7}+448q^{8}+448q^{11}+448q^{16}+448q^{23}+448q^{32}+448q^{43}+448q^{56}-672q^{63}-1344q^{64}+\cdots
is in S_2(\Gamma_0(448), (\frac{112}{\cdot})).
\mu\text{-}\mathrm{T}4\text{-}16\text{-}4 (#43 in the paper)
Let \tau_0 = [16, 0, 7] = \frac{i\sqrt{7}}{4} with h(D) = 4. Then
t=t_P(\tau_0)=8+\frac{51}{\sqrt{2}}+15\sqrt{\frac{7}{2}} has minimal polynomial T^4-32T^3-3792T^2+64768T+1.
We have \mu(t) = \frac{\sqrt{7}}{4\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 16nq^{7m^2+n^2}
               =32q+64q^8-96q^9-192q^{16}+160q^{25}+64q^{29}+320q^{32}-192q^{37}-224q^{49}+320q^{53}+\cdots
is in S_2(\Gamma_0(448), (\frac{112}{.})).
\mu-T4-17-1 (#44 in the paper)
Let \tau_0 = [4, -4, 29] = \frac{1}{2} + i\sqrt{7} with h(D) = 4. Then
t = t_P(\tau_0) = -4145152 - 2931072\sqrt{2} - 1566720\sqrt{7} - 1107840\sqrt{14}
has minimal polynomial T^4 + 16580608T^3 - 248512512T^2 - 536870912T + 4294967296.
We have \mu(t) = \frac{4\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n) (-8m + 4n) q^{116m^2 - 4mn + n^2}
               = 8q - 24q^9 + 40q^{25} - 56q^{49} + 72q^{81} - 16q^{113} - 40q^{121} - 80q^{137} + 112q^{161} + 104q^{169} + \cdots
is in S_2\left(\Gamma_0(1792), \left(\frac{1792}{\cdot}\right)\right).
\mu\text{-}\mathrm{T}4\text{-}17\text{-}2 (#44 in the paper)
Let \tau_0 = [28, 28, 11] = -\frac{1}{2} + \frac{i}{\sqrt{7}} with h(D) = 4. Then
t = t_P(\tau_0) = -4145152 - 2931072\sqrt{2} + 1566720\sqrt{7} + 1107840\sqrt{14}
has minimal polynomial T^4 + 16580608T^3 - 248512512T^2 - 536870912T + 4294967296.
We have \mu(t) = \frac{4\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (56m + 28n) q^{44m^2 + 28mn + 7n^2}
               =56q^{7}-112q^{23}-168q^{63}+112q^{71}+336q^{79}-336q^{127}-112q^{151}+280q^{175}-560q^{191}+336q^{207}+\cdots
is in S_2(\Gamma_0(1792), (\frac{1792}{.})).
\mu-T4-17-3 (#44 in the paper)
Let \tau_0 = [44, 28, 7] = -\frac{7}{22} + \frac{i\sqrt{7}}{11} with h(D) = 4. Then
t = t_P(\tau_0) = -4145152 + 2931072\sqrt{2} + 1566720\sqrt{7} - 1107840\sqrt{14} has minimal polynomial T^4 + 16580608T^3 - 248512512T^2 - 536870912T + 4294967296.
We have \mu(t) = \frac{4\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(56m + 44n)q^{28m^2 + 28mn + 11n^2}
                =64q^{11}-192q^{43}+64q^{67}-192q^{99}+320q^{107}+320q^{163}+64q^{179}-448q^{203}-192q^{211}-448q^{259}+\cdots
is in S_2(\Gamma_0(1792), (\frac{1792}{.})).
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\mu-T4-17-4 (#44 in the paper)
Let \tau_0 = [116, 4, 1] = -\frac{1}{58} + \frac{i\sqrt{7}}{29} with h(D) = 4. Then
t = t_P(\tau_0) = -4145152 + 2931072\sqrt{2} - 1566720\sqrt{7} + 1107840\sqrt{14}
has minimal polynomial T^4 + 16580608T^3 - 248512512T^2 - 536870912T + 4294967296.
We have \mu(t) = \frac{4\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 116n)q^{4m^2 + 4mn + 29n^2}
               =448q^{29}+448q^{37}+448q^{53}+448q^{77}+448q^{109}+448q^{149}+448q^{197}-896q^{253}-1344q^{261}-1344q^{277}+\cdots
is in S_2(\Gamma_0(1792), (\frac{1792}{.})).
\mu-T4-18-1 (#45 in the paper)
Let \tau_0 = [29, 28, 7] = -\frac{14}{29} + \frac{i\sqrt{7}}{29} with h(D) = 1. Then t = t_P(\tau_0) = 4145168 - 2931072\sqrt{2} + 1566720\sqrt{7} - 1107840\sqrt{14}
has minimal polynomial T^4 - 16580672T^3 + 547358208T^2 - 4244652032T + 65536.
We have \mu(t) = \frac{16\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (56m + 29n) q^{112m^2 + 112mn + 29n^2}
               =4q^{29}-12q^{37}+20q^{53}-28q^{77}+36q^{109}-44q^{149}+52q^{197}-56q^{253}-12q^{261}+20q^{277}+\cdots
is in S_2(\Gamma_0(448), (\frac{1792}{.})).
\mu-T4-18-2 (#45 in the paper)
Let \tau_0 = [11, 4, 1] = -\frac{2}{11} + \frac{i\sqrt{7}}{11} with h(D) = 1. Then
t = t_P(\tau_0) = 4145168 - 2931072\sqrt{2} - 1566720\sqrt{7} + 1107840\sqrt{14} has minimal polynomial T^4 - 16580672T^3 + 547358208T^2 - 4244652032T + 65536.
We have \mu(t) = \frac{16\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(8m+11n)q^{16m^2+16mn+11n^2}
               =28q^{11} + 28q^{43} - 84q^{67} - 84q^{99} + 28q^{107} - 84q^{163} + 140q^{179} + 28q^{203} + 140q^{211} - 84q^{259} + \cdots
is in S_2(\Gamma_0(448), (\frac{1792}{.})).
\mu\text{-}\mathrm{T}4\text{-}18\text{-}3 (#45 in the paper)
Let \tau_0 = [7, 0, 1] = \frac{i}{\sqrt{7}} with h(D) = 1. Then
t = t_P(\tau_0) = 4145168 + 2931072\sqrt{2} - 1566720\sqrt{7} - 1107840\sqrt{14}
has minimal polynomial T^4-16580672T^3+547358208T^2-4244652032T+65536.
We have \mu(t) = \frac{16\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 7nq^{16m^2 + 7n^2}
               =14q^{7}+28q^{23}-42q^{63}+28q^{71}-84q^{79}-84q^{127}+28q^{151}+70q^{175}+140q^{191}-84q^{207}+\cdots
is in S_2(\Gamma_0(448), (\frac{1792}{.})).
\mu-T4-18-4 (#45 in the paper)
Let \tau_0 = [1, 0, 7] = i\sqrt{7} with h(D) = 1. Then
t = t_P(\tau_0) = 4145168 + 2931072\sqrt{2} + 1566720\sqrt{7} + 1107840\sqrt{14}
has minimal polynomial T^4-16580672T^3+547358208T^2-4244652032T+65536.
We have \mu(t) = \frac{16\sqrt{7}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) n q^{112m^2 + n^2}
               =2q-6q^9+10q^{25}-14q^{49}+18q^{81}+4q^{113}-34q^{121}+20q^{137}-28q^{161}+26q^{169}+\cdots
is in S_2\left(\Gamma_0(448), \left(\frac{1792}{\cdot}\right)\right)
\mu-T4-19-1 (#46 in the paper)
Let \tau_0 = [8, -8, 13] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{11}{2}} with h(D) = 4. Then
t = t_P(\tau_0) = 8 - 240\sqrt{11} - 168\sqrt{22}
has minimal polynomial T^4 - 32T^3 - 2508672T^2 + 40142848T + 4096.
We have \mu(t) = \frac{\sqrt{\frac{11}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-16m + 8n) q^{26m^2 - 4mn + n^2}
               =16q-48q^9-32q^{23}+80q^{25}+96q^{31}-160q^{47}-112q^{49}+224q^{71}+144q^{81}+32q^{89}+\cdots
is in S_2\left(\Gamma_0(704), \left(\frac{352}{\cdot}\right)\right).
μ-T4-19-2 (#46 in the paper)

Let \tau_0 = [88, 88, 23] = -\frac{1}{2} + \frac{i}{2\sqrt{22}} with h(D) = 4. Then t = t_P(\tau_0) = 8 - 240\sqrt{11} + 168\sqrt{22} has minimal polynomial T^4 - 32T^3 - 2508672T^2 + 40142848T + 4096.
We have \mu(t) = \frac{\sqrt{\frac{11}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (176m + 88n) q^{46m^2 + 44mn + 11n^2}
               = 176q^{11} - 352q^{13} + 352q^{19} - 352q^{29} + 352q^{43} - 352q^{61} + 352q^{83} - 528q^{99} + 1056q^{101} - 1056q^{107} + \cdots
is in S_2\left(\Gamma_0(704), \left(\frac{352}{\cdot}\right)\right).
\mu\text{-}\mathrm{T}4\text{-}19\text{-}3 (#46 in the paper)
Let \tau_0 = [88, 0, 1] = \frac{i}{2\sqrt{22}} with h(D) = 4. Then
t = t_P(\tau_0) = 8 + 240\sqrt{11} - 168\sqrt{22}
has minimal polynomial T^4 - 32T^3 - 2508672T^2 + 40142848T + 4096.
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We have \mu(t) = \frac{\sqrt{\frac{11}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 88nq^{2m^2+11n^2}
                = 176q^{11} + 352q^{13} + 352q^{19} + 352q^{29} + 352q^{43} + 352q^{61} + 352q^{83} - 528q^{99} - 1056q^{101} - 1056q^{107} + \cdots
is in S_2\left(\Gamma_0(704), \left(\frac{352}{\cdot}\right)\right).
\mu-T4-19-4 (#46 in the paper)
Let \tau_0 = [8, 0, 11] = \frac{1}{2}i\sqrt{\frac{11}{2}} with h(D) = 4. Then
t = t_P(\tau_0) = 8 + 240\sqrt{11} + 168\sqrt{22} has minimal polynomial T^4 - 32T^3 - 2508672T^2 + 40142848T + 4096.
We have \mu(t) = \frac{\sqrt{\frac{11}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 8nq^{22m^2 + n^2}
               = 16q - 48q^9 + 32q^{23} + 80q^{25} - 96q^{31} + 160q^{47} - 112q^{49} - 224q^{71} + 144q^{81} + 32q^{89} + \cdots
is in S_2\left(\Gamma_0(704), \left(\frac{352}{\cdot}\right)\right).
\mu-T4-20-1 (#47 in the paper)
Let \tau_0 = [4, -4, 23] = \frac{1}{2} + i\sqrt{\frac{11}{2}} with h(D) = 4. Then
t=t_P(\tau_0)=-627232-443520\sqrt{2}-189120\sqrt{11}-133728\sqrt{22} has minimal polynomial T^4+2508928T^3-40138752T^2-131072T+1048576.
We have \mu(t) = \frac{2\sqrt{22}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-8m+4n)q^{92m^2-4mn+n^2}
               =8q-24q^9+40q^{25}-56q^{49}+72q^{81}-16q^{89}+48q^{97}-80q^{113}-88q^{121}+112q^{137}+\cdots
is in S_2(\Gamma_0(1408), (\frac{1408}{.})).
\mu-T4-20-2 (#47 in the paper)
Let \tau_0 = [44, 44, 13] = -\frac{1}{2} + \frac{i}{\sqrt{22}} with h(D) = 4. Then
t = t_P(\tau_0) = -627232 + 443520\sqrt{2} - 189120\sqrt{11} + 133728\sqrt{22}
has minimal polynomial T^4 + 2508928T^3 - 40138752T^2 - 131072T + 1048576.
We have \mu(t) = \frac{2\sqrt{22}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(88m + 44n)q^{52m^2 + 44mn + 11n^2}
                =88q^{11}-176q^{19}+176q^{43}-176q^{83}-264q^{99}+528q^{107}-528q^{131}+176q^{139}+528q^{171}-176q^{211}+\cdots
is in S_2(\Gamma_0(1408), (\frac{1408}{.})).
\mu-T4-20-3 (#47 in the paper)
Let \tau_0 = [52, 44, 11] = -\frac{11}{26} + \frac{1}{13}i\sqrt{\frac{11}{2}} with h(D) = 4. Then
t=t_P(\tau_0)=-627232+443520\sqrt{2}+189120\sqrt{11}-133728\sqrt{22} has minimal polynomial T^4+2508928T^3-40138752T^2-131072T+1048576.
We have \mu(t) = \frac{2\sqrt{22}}{\pi^2}L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(88m + 52n)q^{44m^2 + 44mn + 13n^2}
                =32q^{13}-96q^{29}+160q^{61}+32q^{101}-224q^{109}-96q^{117}+160q^{149}+288q^{173}-224q^{197}-352q^{253}+\cdots
is in S_2(\Gamma_0(1408), (\frac{1408}{.})).
\mu-T4-20-4 (#47 in the paper)
Let \tau_0 = [92, 4, 1] = -\frac{1}{46} + \frac{1}{23}i\sqrt{\frac{11}{2}} with h(D) = 4. Then
t=t_P(\tau_0)=-627232-443520\sqrt{2}+189120\sqrt{11}+133728\sqrt{22} has minimal polynomial T^4+2508928T^3-40138752T^2-131072T+1048576.
We have \mu(t) = \frac{2\sqrt{22}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum\limits_{m,n \in \mathbb{Z}} \chi_{-4}(n) (8m + 92n) q^{4m^2 + 4mn + 23n^2}
               =352q^{23}+352q^{31}+352q^{47}+352q^{71}+352q^{103}+352q^{143}+352q^{191}-1056q^{199}-1056q^{207}-1056q^{223}+\cdots
is in S_2(\Gamma_0(1408), (\frac{1408}{.})).
\mu\text{-}\mathrm{T}4\text{-}21\text{-}1 (#48 in the paper)
Let \tau_0 = [46, 44, 11] = -\frac{11}{23} + \frac{1}{23}i\sqrt{\frac{11}{2}} with h(D) = 2. Then
t = t_P(\tau_0) = 627248 + 443520\sqrt{2} - 189120\sqrt{11} - 133728\sqrt{22}
has minimal polynomial T^4 - 2508992T^3 + 80291328T^2 - 642301952T + 65536.
We have \mu(t) = \frac{4\sqrt{22}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(88m + 46n)q^{88m^2 + 88mn + 23n^2}
                =8q^{23} - 24q^{31} + 40q^{47} - 56q^{71} + 72q^{103} - 88q^{143} + 104q^{191} + 8q^{199} - 24q^{207} + 40q^{223} + \cdots
is in S_2(\Gamma_0(704), (\frac{1408}{.})).
\mu\text{-}\mathrm{T}4\text{-}21\text{-}2 (#48 in the paper)
Let \tau_0 = [26, 4, 1] = -\frac{1}{13} + \frac{1}{13}i\sqrt{\frac{11}{2}} with h(D) = 2. Then
t = t_P(\tau_0) = 627248 - 443520\sqrt{2} - 189120\sqrt{11} + 133728\sqrt{22} has minimal polynomial T^4 - 2508992T^3 + 80291328T^2 - 642301952T + 65536.
We have \mu(t) = \frac{4\sqrt{22}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 26n)q^{8m^2 + 8mn + 13n^2}
                =88q^{13} + 88q^{29} + 88q^{61} - 264q^{101} + 88q^{109} - 264q^{117} - 264q^{149} + 88q^{173} - 264q^{197} + 88q^{253} + \cdots
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is in S_2(\Gamma_0(704), (\frac{1408}{.})).
 \mu-T4-21-3 (#48 in the paper)
 Let \tau_0 = [22, 0, 1] = \frac{i}{\sqrt{22}} with h(D) = 2. Then
t = t_P(\tau_0) = 627248 - 443520\sqrt{2} + 189120\sqrt{11} - 133728\sqrt{22} has minimal polynomial T^4 - 2508992T^3 + 80291328T^2 - 642301952T + 65536.
We have \mu(t) = \frac{4\sqrt{22}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 22nq^{8m^2+11n^2}
                =44q^{11} + 88q^{19} + 88q^{43} + 88q^{83} - 132q^{99} - 264q^{107} - 264q^{131} + 88q^{139} - 264q^{171} + 88q^{211} + \cdots
is in S_2(\Gamma_0(704), (\frac{1408}{...})).
\mu\text{-}\mathrm{T}4\text{-}21\text{-}4 (#48 in the paper)
Let \tau_0 = [2, 0, 11] = i\sqrt{\frac{11}{2}} with h(D) = 2. Then
t = t_P(\tau_0) = 627248 + 443520\sqrt{2} + 189120\sqrt{11} + 133728\sqrt{22}
has minimal polynomial T^4 - 2508992T^3 + 80291328T^2 - 642301952T + 65536.
We have \mu(t) = \frac{4\sqrt{22}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 2nq^{88m^2 + n^2}
                =4q-12q^9+20q^{25}-28q^{49}+36q^{81}+8q^{89}-24q^{97}+40q^{113}-44q^{121}-56q^{137}+\cdots
is in S_2(\Gamma_0(704), (\frac{1408}{.})).
\mu\text{-}\mathrm{T}4\text{-}22\text{-}1 (#49 in the paper)
Let \tau_0 = [8, -8, 31] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{29}{2}} with h(D) = 4. Then
t=t_P(\tau_0)=8-55440\sqrt{2}-10296\sqrt{58} has minimal polynomial T^4-32T^3-24591257472T^2+393460123648T+4096.
We have \mu(t) = \frac{\sqrt{\frac{29}{\pi^2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-16m + 8n) q^{62m^2 - 4mn + n^2}
                = 16q - 48q^9 + 80q^{25} - 112q^{49} - 32q^{59} + 96q^{67} + 144q^{81} - 160q^{83} + 224q^{107} - 176q^{121} + \cdots
is in S_2(\Gamma_0(1856), (\frac{928}{.})).
\mu\text{-}\mathrm{T}4\text{-}22\text{-}2 (#49 in the paper)
Let \tau_0 = [232, 232, 59] = -\frac{1}{2} + \frac{i}{2\sqrt{58}} with h(D) = 4. Then t = t_P(\tau_0) = 8 + 55440\sqrt{2} - 10296\sqrt{58} has minimal polynomial T^4 - 32T^3 - 24591257472T^2 + 393460123648T + 4096.
We have \mu(t) = \frac{\sqrt{\frac{29}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (464m + 232n) q^{118m^2 + 116mn + 29n^2}
                =464q^{29}-928q^{31}+928q^{37}-928q^{47}+928q^{61}-928q^{79}+928q^{101}-928q^{127}+928q^{157}-928q^{191}+\cdots
is in S_2\left(\Gamma_0(1856), \left(\frac{928}{\cdot}\right)\right).
\mu-T4-22-3 (#49 in the paper)
Let \tau_0 = [232, 0, 1] = \frac{i}{2\sqrt{58}} with h(D) = 4. Then
t = t_P(\tau_0) = 8 - 55440\sqrt{2} + 10296\sqrt{58} has minimal polynomial T^4 - 32T^3 - 24591257472T^2 + 393460123648T + 4096.
We have \mu(t)=\frac{\sqrt{\frac{29}{2}}}{\pi^2}L(\Theta_{P,\tau_0},2), where \Theta_{P,\tau_0}(\tau)=\sum_{m,n\in\mathbb{Z}}\chi_{-4}(n)232nq^{2m^2+29n^2}
                =464q^{29}+928q^{31}+928q^{37}+928q^{47}+928q^{47}+928q^{61}+928q^{79}+928q^{101}+928q^{127}+928q^{157}+928q^{191}+\cdots
is in S_2(\Gamma_0(1856), (\frac{928}{.})).
\mu-T4-22-4 (#49 in the paper)
Let \tau_0 = [8, 0, 29] = \frac{1}{2}i\sqrt{\frac{29}{2}} with h(D) = 4. Then
t=t_P(\tau_0)=8+55440\sqrt{2}+10296\sqrt{58} has minimal polynomial T^4-32T^3-24591257472T^2+393460123648T+4096.
We have \mu(t) = \frac{\sqrt{\frac{29}{2}}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 8nq^{58m^2 + n^2}
                =16q-48q^9+80q^{25}-112q^{49}+32q^{59}-96q^{67}+144q^{81}+160q^{83}-224q^{107}-176q^{121}+\cdots
is in S_2(\Gamma_0(1856), (\frac{928}{.})).
\mu\text{-}\mathrm{T}4\text{-}23\text{-}1 (#50 in the paper)
Let \tau_0 = [4, -4, 59] = \frac{1}{2} + i\sqrt{\frac{29}{2}} with h(D) = 4. Then
t = t_P(\tau_0) = -6147814432 - 4347161280\sqrt{2} - 1141620480\sqrt{29} - 807247584\sqrt{58}
has minimal polynomial T^4 + 24591257728T^3 - 393460119552T^2 - 131072T + 1048576.
We have \mu(t) = \frac{2\sqrt{58}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-8m+4n)q^{236m^2-4mn+n^2}
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 $=8q-24q^9+40q^{25}-56q^{49}+72q^{81}-88q^{121}+104q^{169}-120q^{225}-16q^{233}+48q^{241}+\cdots$

is in $S_2(\Gamma_0(3712), (\frac{3712}{2}))$.

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Let \tau_0 = [116, 116, 31] = -\frac{1}{2} + \frac{i}{\sqrt{58}} with h(D) = 4. Then
t = t_P(\tau_0) = -6147814432 + 4347161280\sqrt{2} + 1141620480\sqrt{29} - 807247584\sqrt{58} has minimal polynomial T^4 + 24591257728T^3 - 393460119552T^2 - 131072T + 1048576.
We have \mu(t) = \frac{2\sqrt{58}}{\pi^2}L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(	au) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (232m + 116n) q^{124m^2 + 116mn + 29n^2}
                =232q^{29}-464q^{37}+464q^{61}-464q^{101}+464q^{157}-464q^{229}-696q^{261}+1392q^{269}-1392q^{293}+464q^{317}+\cdots
is in S_2(\Gamma_0(3712), (\frac{3712}{.})).
\mu-T4-23-3 (#50 in the paper)
Let \tau_0 = [124, 116, 29] = -\frac{29}{62} + \frac{1}{31}i\sqrt{\frac{29}{2}} with h(D) = 4. Then
t = t_P(\tau_0) = -6147814432 - 4347161280\sqrt{2} + 1141620480\sqrt{29} + 807247584\sqrt{58} has minimal polynomial T^4 + 24591257728T^3 - 393460119552T^2 - 131072T + 1048576.
We have \mu(t) = \frac{2\sqrt{58}}{\pi^2}L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-4}(n)(232m+124n)q^{116m^2+116mn+31n^2}
                = 32q^{31} - 96q^{47} + 160q^{79} - 224q^{127} + 288q^{191} + 32q^{263} - 352q^{271} - 96q^{279} + 160q^{311} - 224q^{359} + \cdots
is in S_2(\Gamma_0(3712), (\frac{3712}{.})).
\mu-T4-23-4 (#50 in the paper)
Let \tau_0 = [236,4,1] = -\frac{1}{118} + \frac{1}{59}i\sqrt{\frac{29}{2}} with h(D) = 4. Then t = t_P(\tau_0) = -6147814432 + 4347161280\sqrt{2} - 1141620480\sqrt{29} + 807247584\sqrt{58} has minimal polynomial T^4 + 24591257728T^3 - 393460119552T^2 - 131072T + 1048576.
We have \mu(t) = \frac{2\sqrt{58}}{\pi^2}L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(8m + 236n)q^{4m^2 + 4mn + 59n^2}
                =928q^{59} + 928q^{67} + 928q^{83} + 928q^{107} + 928q^{139} + 928q^{179} + 928q^{227} + 928q^{283} + 928q^{347} + 928q^{419} + \cdots
is in S_2(\Gamma_0(3712), (\frac{3712}{.})).
\mu-T4-24-1 (#51 in the paper)
Let \tau_0 = [118, 116, 29] = -\frac{29}{59} + \frac{1}{59}i\sqrt{\frac{29}{2}} with h(D) = 2. Then
t = t_P(\tau_0) = 6147814448 - 4347161280\sqrt{2} + 1141620480\sqrt{29} - 807247584\sqrt{58} has minimal polynomial T^4 - 24591257792T^3 + 786920252928T^2 - 6295361994752T + 65536.
We have \mu(t) = \frac{4\sqrt{58}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(232m + 118n)q^{232m^2 + 232mn + 59n^2}
                =8q^{59}-24q^{67}+40q^{83}-56q^{107}+72q^{139}-88q^{179}+104q^{227}-120q^{283}+136q^{347}-152q^{419}+\cdots
is in S_2(\Gamma_0(1856), (\frac{3712}{.})).
\mu-T4-24-2 (#51 in the paper)
Let \tau_0 = [62, 4, 1] = -\frac{1}{31} + \frac{1}{31}i\sqrt{\frac{29}{2}} with h(D) = 2. Then
t = t_P(\tau_0) = 6147814448 + 4347161280\sqrt{2} - 1141620480\sqrt{29} - 807247584\sqrt{58} has minimal polynomial T^4 - 24591257792T^3 + 786920252928T^2 - 6295361994752T + 65536.
We have \mu(t) = \frac{4\sqrt{58}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(	au) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 62n)q^{8m^2 + 8mn + 31n^2}
                =232q^{31}+232q^{47}+232q^{79}+232q^{127}+232q^{191}-696q^{263}+232q^{271}-696q^{279}-696q^{311}-696q^{359}+\cdots
is in S_2(\Gamma_0(1856), (\frac{3712}{.})).
\mu-T4-24-3 (#51 in the paper)
Let \tau_0 = [58, 0, 1] = \frac{i}{\sqrt{58}} with h(D) = 2. Then
t = t_P(\tau_0) = 6147814448 - 4347161280\sqrt{2} - 1141620480\sqrt{29} + 807247584\sqrt{58}
has minimal polynomial T^4 - 24591257792T^3 + 786920252928T^2 - 6295361994752T + 65536.
We have \mu(t) = \frac{4\sqrt{58}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 58nq^{8m^2 + 29n^2}
                = 116q^{29} + 232q^{37} + 232q^{61} + 232q^{101} + 232q^{157} + 232q^{229} - 348q^{261} - 696q^{269} - 696q^{293} + 232q^{317} + \cdots
is in S_2(\Gamma_0(1856), (\frac{3712}{.})).
\mu\text{-}\mathrm{T}4\text{-}24\text{-}4 (#51 in the paper)
Let \tau_0 = [2, 0, 29] = i\sqrt{\frac{29}{2}} with h(D) = 2. Then
t=t_P(\tau_0)=6147814448+4347161280\sqrt{2}+1141620480\sqrt{29}+807247584\sqrt{58} has minimal polynomial T^4-24591257792T^3+786920252928T^2-6295361994752T+65536.
We have \mu(t) = \frac{4\sqrt{58}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 2nq^{232m^2 + n^2}
                =4q-12q^9+20q^{25}-28q^{49}+36q^{81}-44q^{121}+52q^{169}-60q^{225}+8q^{233}-24q^{241}+\cdots
is in S_2(\Gamma_0(1856), (\frac{3712}{.})).
Let \tau_0 = [8,7,2] = -\frac{7}{16} + \frac{i\sqrt{15}}{16} with h(D) = 2. Then t = t_P(\tau_0) = \frac{47}{4} + \frac{17i\sqrt{3}}{4} - \frac{21\sqrt{5}}{4} - \frac{7i\sqrt{15}}{4} has minimal polynomial T^4 - 47T^3 + 753T^2 - 32T + 256.
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We have $\mu(t) = \frac{\sqrt{15}}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

$$\begin{split} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (14m + 8n) q^{8m^2 + 7mn + 2n^2} \\ &= 16q^2 - 12q^3 - 20q^5 + 8q^8 + 24q^{12} + 40q^{17} - 48q^{18} - 40q^{20} + 24q^{23} + 36q^{27} + \cdots \\ \text{is in } \mathcal{S}_2\left(\Gamma_0(60), \left(\frac{60}{\cdot}\right)\right). \end{split}$$

μ -N4-1-2

Let
$$\tau_0 = [8, -7, 2] = \frac{7}{16} + \frac{i\sqrt{15}}{16}$$
 with $h(D) = 2$. Then $t = t_P(\tau_0) = \frac{47}{4} - \frac{17i\sqrt{3}}{4} - \frac{21\sqrt{5}}{4} + \frac{7i\sqrt{15}}{4}$ has minimal polynomial $T^4 - 47T^3 + 753T^2 - 32T + 256$.

$$t = t_P(\tau_0) = \frac{47}{4} - \frac{17i\sqrt{3}}{4} - \frac{21\sqrt{5}}{4} + \frac{7i\sqrt{15}}{4}$$

We have $\mu(t) = \frac{\sqrt{15}}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-14m+8n)q^{8m^2-7mn+2n^2}$$

$$= 16q^2 - 12q^3 - 20q^5 + 8q^8 + 24q^{12} + 40q^{17} - 48q^{18} - 40q^{20} + 24q^{23} + 36q^{27} + \cdots$$

is in $S_2\left(\Gamma_0(60), \left(\frac{60}{\cdot}\right)\right)$.

μ -N4-1-3

Let
$$\tau_0 = [4, 1, 1] = -\frac{1}{8} + \frac{i\sqrt{15}}{8}$$
 with $h(D) = 2$. Then $t = t_P(\tau_0) = \frac{47}{4} + \frac{17i\sqrt{3}}{4} + \frac{21\sqrt{5}}{4} + \frac{7i\sqrt{15}}{4}$

$$t = t_P(\tau_0) = \frac{47}{4} + \frac{17i\sqrt{3}}{12i\sqrt{3}} + \frac{21\sqrt{5}}{12i\sqrt{5}} + \frac{7i\sqrt{15}}{12i\sqrt{5}}$$

has minimal polynomial $T^4 - 47T^3 + 753T^2 - 32T + 256$.

We have $\mu(t) = \frac{\sqrt{15}}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(2m+4n)q^{4m^2+mn+n^2}$$

$$= 8q + 4q^4 + 12q^6 - 24q^9 - 20q^{10} - 28q^{16} + 36q^{24} + 40q^{25} + 40q^{34} - 12q^{36} + \cdots$$

is in $S_2\left(\Gamma_0(60), \left(\frac{60}{\cdot}\right)\right)$.

μ -N4-1-4

Let
$$\tau_0 = [4, -1, 1] = \frac{1}{8} + \frac{i\sqrt{15}}{8}$$
 with $h(D) = 2$. Then

$$t = t_P(\tau_0) = \frac{47}{4} - \frac{17i\sqrt{3}}{4} + \frac{21\sqrt{5}}{4} - \frac{7i\sqrt{15}}{4}$$

 $t = t_P(\tau_0) = \frac{47}{4} - \frac{17i\sqrt{3}}{4} + \frac{21\sqrt{5}}{4} - \frac{7i\sqrt{15}}{4}$ has minimal polynomial $T^4 - 47T^3 + 753T^2 - 32T + 256$.

We have $\mu(t) = \frac{\sqrt{15}}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

We have
$$\mu(t) = \frac{\sqrt{15}}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$$
, where
$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-2m+4n) q^{4m^2-mn+n^2}$$
$$= 8q + 4q^4 + 12q^6 - 24q^9 - 20q^{10} - 28q^{16} + 36q^{24} + 40q^{25} + 40q^{34} - 12q^{36} + \cdots$$

is in $S_2\left(\Gamma_0(60), \left(\frac{60}{\cdot}\right)\right)$.

μ -N4-2-1

Let
$$\tau_0 = [2, 1, 2] = -\frac{1}{4} + \frac{i\sqrt{15}}{4}$$
 with $h(D) = 2$. Then

$$t = t_P(\tau_0) = 8 + 128i\sqrt{3} + 56i\sqrt{15}$$

has minimal polynomial $T^4 - 32T^3 + 192768T^2 - 3080192T + 16777216$.

We have $\mu(t) = \frac{2\sqrt{15}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(2m+2n)q^{16m^2+2mn+n^2}$$

 $=4q-12q^9+20q^{25}-28q^{49}-8q^{61}+24q^{69}+36q^{81}-40q^{85}+56q^{109}-44q^{121}+\cdots$

is in $S_2\left(\Gamma_0(120), \left(\frac{240}{\cdot}\right)\right)$.

μ-N4-2-2

Let
$$\tau_0 = [2, -1, 2] = \frac{1}{4} + \frac{i\sqrt{15}}{4}$$
 with $h(D) = 2$. Then

$$t = t_P(\tau_0) = 8 - 128i\sqrt{3} - 56i\sqrt{15}$$

 $t=t_P(\tau_0)=8-128i\sqrt{3}-56i\sqrt{15}$ has minimal polynomial $T^4-32T^3+192768T^2-3080192T+16777216.$

We have $\mu(t) = \frac{2\sqrt{15}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-2m+2n)q^{16m^2-2mn+n^2}
= 4q - 12q^9 + 20q^{25} - 28q^{49} - 8q^{61} + 24q^{69} + 36q^{81} - 40q^{85} + 56q^{109} - 44q^{121} + \cdots$$

is in $S_2\left(\Gamma_0(120), \left(\frac{240}{\cdot}\right)\right)$.

Let
$$\tau_0 = [6, 3, 1] = -\frac{1}{4} + \frac{1}{4}i\sqrt{\frac{5}{3}}$$
 with $h(D) = 2$. Then

$$t = t_P(\tau_0) = 8 + 128i\sqrt{3} - 56i\sqrt{15}$$

has minimal polynomial $T^4 - 32T^3 + 192768T^2 - 3080192T + 16777216.$

We have $\mu(t) = \frac{2\sqrt{15}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(6m+6n)q^{8m^2+6mn+3n^2}$$

$$= 12q^3 - 24q^{23} - 36q^{27} + 72q^{47} + 60q^{75} + 24q^{83} - 120q^{95} - 72q^{107} - 84q^{147} + 120q^{155} + \cdots$$

is in $S_2\left(\Gamma_0(120), \left(\frac{240}{\cdot}\right)\right)$.

μ -N4-2-4

Let
$$\tau_0 = [6, -3, 1] = \frac{1}{4} + \frac{1}{4}i\sqrt{\frac{5}{3}}$$
 with $h(D) = 2$. Then

$$t = t_P(\tau_0) = 8 - 128i\sqrt{3} + 56i\sqrt{15}$$

has minimal polynomial $T^4 - 32T^3 + 192768T^2 - 3080192T + 16777216. \\$

We have $\mu(t) = \frac{2\sqrt{15}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-6m+6n)q^{8m^2-6mn+3n^2}$$

$$= 12q^3 - 24q^{23} - 36q^{27} + 72q^{47} + 60q^{75} + 24q^{83} - 120q^{95} - 72q^{107} - 84q^{147} + 120q^{155} + \cdots$$

is in $S_2(\Gamma_0(120), (\frac{240}{.}))$.

μ -N4-3-1

Let
$$\tau_0 = [8, 6, 3] = -\frac{3}{8} + \frac{i\sqrt{15}}{8}$$
 with $h(D) = 2$. Then

$$t = t_P(\tau_0) = \frac{17}{4} + \frac{17i\sqrt{3}}{4} - \frac{21\sqrt{5}}{4} + \frac{7i\sqrt{15}}{4}$$

Let $\tau_0 = [8,6,3] = -\frac{3}{8} + \frac{i\sqrt{15}}{8}$ with h(D) = 2. Then $t = t_P(\tau_0) = \frac{17}{4} + \frac{17i\sqrt{3}}{4} - \frac{21\sqrt{5}}{4} + \frac{7i\sqrt{15}}{4}$ has minimal polynomial $T^4 - 17T^3 + 33T^2 - 4352T + 65536$.

We have
$$\mu(t) = \frac{\sqrt{15}}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$$
, where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(12m+8n)q^{6m^2+3mn+n^2}$$

$$= 16q - 8q^4 - 24q^6 - 48q^9 + 40q^{10} + 56q^{16} - 72q^{24} + 80q^{25} - 80q^{34} + 24q^{36} + \cdots$$

is in $S_2\left(\Gamma_0(120), \left(\frac{60}{\cdot}\right)\right)$.

μ -N4-3-2

Let
$$\tau_0 = [8, -6, 3] = \frac{3}{8} + \frac{i\sqrt{15}}{8}$$
 with $h(D) = 2$. Then $t = t_P(\tau_0) = \frac{17}{4} - \frac{17i\sqrt{3}}{4} - \frac{21\sqrt{5}}{4} - \frac{7i\sqrt{15}}{4}$ has minimal polynomial $T^4 - 17T^3 + 33T^2 - 4352T + 65536$.

$$t = t_P(\tau_0) = \frac{17}{4} - \frac{17i\sqrt{3}}{4} - \frac{21\sqrt{5}}{4} - \frac{7i\sqrt{15}}{4}$$

We have
$$\mu(t) = \frac{\sqrt{15}}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$$
, where
$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-12m + 8n) q^{6m^2 - 3mn + n^2}$$
$$= 16q - 8q^4 - 24q^6 - 48q^9 + 40q^{10} + 56q^{16} - 72q^{24} + 80q^{25} - 80q^{34} + 24q^{36} + \cdots$$

is in $S_2\left(\Gamma_0(120), \left(\frac{60}{\cdot}\right)\right)$.

μ -N4-3-3

Let
$$\tau_0 = [16, 2, 1] = -\frac{1}{16} + \frac{i\sqrt{15}}{16}$$
 with $h(D) = 2$. Then

$$t = t_P(\tau_0) = \frac{17}{17} + \frac{17i\sqrt{3}}{17i\sqrt{3}} + \frac{21\sqrt{5}}{17i\sqrt{5}} - \frac{7i\sqrt{15}}{17i\sqrt{15}}$$

Let $\tau_0 = [16, 2, 1] = -\frac{1}{16} + \frac{i\sqrt{15}}{16}$ with h(D) = 2. Then $t = t_P(\tau_0) = \frac{17}{4} + \frac{17i\sqrt{3}}{4} + \frac{21\sqrt{5}}{4} - \frac{7i\sqrt{15}}{4}$ has minimal polynomial $T^4 - 17T^3 + 33T^2 - 4352T + 65536$.

We have $\mu(t) = \frac{\sqrt{15}}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(4m+16n)q^{2m^2+mn+2n^2}$$

$$= 32q^2 + 24q^3 + 40q^5 + 16q^8 + 48q^{12} - 80q^{17} - 96q^{18} - 80q^{20} - 48q^{23} - 72q^{27} + \cdots$$

is in $S_2\left(\Gamma_0(120), \left(\frac{60}{\cdot}\right)\right)$.

μ -N4-3-4

Let
$$\tau_0 = [16, -2, 1] = \frac{1}{16} + \frac{i\sqrt{15}}{16}$$
 with $h(D) = 2$. Then $t = t_P(\tau_0) = \frac{17}{4} - \frac{17i\sqrt{3}}{4} + \frac{21\sqrt{5}}{4} + \frac{7i\sqrt{15}}{4}$ has minimal polynomial $T^4 - 17T^3 + 33T^2 - 4352T + 65536$.

$$t = t_P(\tau_0) = \frac{17}{100} - \frac{17i\sqrt{3}}{1000} + \frac{21\sqrt{5}}{1000} + \frac{7i\sqrt{15}}{1000}$$

We have $\mu(t) = \frac{\sqrt{15}}{4\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

We have
$$\mu(t) = \frac{1}{4\pi^2} L(OP, \tau_0, 2)$$
, where $\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-4m + 16n) q^{2m^2 - mn + 2n^2}$
 $= 32q^2 + 24q^3 + 40q^5 + 16q^8 + 48q^{12} - 80q^{17} - 96q^{18} - 80q^{20} - 48q^{23} - 72q^{27} + \cdots$

is in $S_2\left(\Gamma_0(120), \left(\frac{60}{\cdot}\right)\right)$.

μ-N4-4-1

Let
$$\tau_0 = [9, 8, 2] = -\frac{4}{9} + \frac{i\sqrt{2}}{9}$$
 with $h(D) = 1$. Then

$$t = t_P(\tau_0) = 1808 + 1280\sqrt{2} - 64\sqrt{2\left(799 + 565\sqrt{2}\right)}$$

has minimal polynomial $T^4 - 7232T^3 - 31232T^2 - 1851392T + 65536$.

We have $\mu(t) = \frac{16\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(16m + 9n)q^{32m^2 + 32mn + 9n^2}$$

$$= 4q^9 - 12q^{17} + 20q^{33} - 28q^{57} + 4q^{73} - 12q^{81} + 36q^{89} + 20q^{97} - 28q^{121} - 44q^{129} + \cdots$$

is in $S_2\left(\Gamma_0(128), \left(\frac{512}{\cdot}\right)\right)$.

μ -N4-4-2

Let
$$\tau_0 = [1, 0, 2] = i\sqrt{2}$$
 with $h(D) = 1$. Then

Let
$$\tau_0 = [1, 0, 2] = i\sqrt{2}$$
 with $h(D) = 1$. Then $t = t_P(\tau_0) = 1808 + 1280\sqrt{2} + 64\sqrt{2(799 + 565\sqrt{2})}$

has minimal polynomial $T^4 - 7232T^3 - 31232T^2 - 1851392T + 65536$.

We have $\mu(t) = \frac{16\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

$$\begin{split} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) n q^{32m^2 + n^2} \\ &= 2q - 6q^9 + 10q^{25} + 4q^{33} - 12q^{41} - 14q^{49} + 20q^{57} - 10q^{81} + 36q^{113} - 22q^{121} + \cdots \\ \text{is in } \mathcal{S}_2\left(\Gamma_0(128), \left(\frac{512}{\cdot}\right)\right). \end{split}$$

μ -N4-4-3

Let
$$\tau_0 = [3, 2, 1] = -\frac{1}{3} + \frac{i\sqrt{2}}{3}$$
 with $h(D) = 1$. Then

$$t = t_P(\tau_0) = 1808 - 1280\sqrt{2} + 64i\sqrt{2\left(-799 + 565\sqrt{2}\right)}$$

has minimal polynomial $T^4 - 7232T^3 - 31232T^2 - 1851392T + 65536$.

We have $\mu(t) = \frac{16\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

$$\begin{split} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (4m+3n) q^{16m^2+8mn+3n^2} \\ &= 6q^3 - 2q^{11} - 10q^{19} - 4q^{27} - 2q^{43} + 12q^{51} + 14q^{59} - 26q^{67} + 30q^{75} + 22q^{83} + \cdots \\ \text{is in } \mathcal{S}_2\left(\Gamma_0(128), \left(\frac{512}{5}\right)\right). \end{split}$$

μ-N4-4-4

Let
$$\tau_0 = [3, -2, 1] = \frac{1}{3} + \frac{i\sqrt{2}}{3}$$
 with $h(D) = 1$. Then

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t = t_P(\tau_0) = 1808 - 1280\sqrt{2} - 64i\sqrt{2\left(-799 + 565\sqrt{2}\right)}
has minimal polynomial T^4 - 7232T^3 - 31232T^2 - 1851392T + 65536.
We have \mu(t) = \frac{16\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-4m+3n)q^{16m^2-8mn+3n^2}
               = 6q^{3} - 2q^{11} - 10q^{19} - 4q^{27} - 2q^{43} + 12q^{51} + 14q^{59} - 26q^{67} + 30q^{75} + 22q^{83} + \cdots
is in S_2(\Gamma_0(128), (\frac{512}{...})).
\mu-N4-5-1
Let \tau_0 = [17, 16, 4] = -\frac{8}{17} + \frac{2i}{17} with h(D) = 1. Then
t = t_P(\tau_0) = 71696 - 60288 \sqrt[4]{2} + 50688 \sqrt{2} - 42624 \sqrt[4]{8} has minimal polynomial T^4 - 286784T^3 + 7079424T^2 - 73416704T + 65536.
We have \mu(t) = \frac{32}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(32m + 17n)q^{64m^2 + 64mn + 17n^2}
               =4q^{17}-12q^{25}+20q^{41}-28q^{65}+36q^{97}-44q^{137}+4q^{145}-12q^{153}+20q^{169}+52q^{185}+\cdots
is in S_2(\Gamma_0(256), (\frac{1024}{})).
Let \tau_0 = [1, 0, 4] = 2i with h(D) = 1. Then
t = t_P(\tau_0) = 71696 + 60288\sqrt[4]{2} + 50688\sqrt{2} + 42624\sqrt[4]{8}
has minimal polynomial T^4 - 286784T^3 + 7079424T^2 - 73416704T + 65536
We have \mu(t) = \frac{32}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(	au) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) n q^{64m^2 + n^2}
              =2q-6q^9+10q^{25}-14q^{49}+4q^{65}-12q^{73}+18q^{81}+20q^{89}-28q^{113}-22q^{121}+\cdots
is in S_2(\Gamma_0(256), (\frac{1024}{.})).
Let \tau_0 = [5, 2, 1] = -\frac{1}{5} + \frac{2i}{5} with h(D) = 1. Then
t = t_P(\tau_0) = 71696 + 60288i\sqrt[4]{2} - 50688\sqrt{2} - 42624i\sqrt[4]{8}
has minimal polynomial T^4 - 286784T^3 + 7079424T^2 - 73416704T + 65536.
We have \mu(t) = \frac{32}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (4m + 5n) q^{16m^2 + 8mn + 5n^2}
               = 10q^5 + 2q^{13} + 18q^{29} - 22q^{37} - 30q^{45} - 6q^{53} - 14q^{61} - 12q^{85} + 42q^{101} + 34q^{109} + \cdots
is in S_2(\Gamma_0(256), (\frac{1024}{.})).
\mu-N4-5-4
Let \tau_0 = [5, -2, 1] = \frac{1}{5} + \frac{2i}{5} with h(D) = 1. Then
t = t_P(\tau_0) = 71696 - 60288i\sqrt[4]{2} - 50688\sqrt{2} + 42624i\sqrt[4]{8}
has minimal polynomial T^4 - 286784T^3 + 7079424T^2 - 73416704T + 65536.
We have \mu(t) = \frac{32}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-4m+5n)q^{16m^2-8mn+5n^2}
               = 10q^{5} + 2q^{13} + 18q^{29} - 22q^{37} - 30q^{45} - 6q^{53} - 14q^{61} - 12q^{85} + 42q^{101} + 34q^{109} + \cdots
is in S_2\left(\Gamma_0(256), \left(\frac{1024}{\cdot}\right)\right)
Let \tau_0 = [32, 32, 9] = -\frac{1}{2} + \frac{i}{4\sqrt{2}} with h(D) = 4. Then
```

Let
$$\tau_0 = [32, 32, 9] = -\frac{1}{2} + \frac{i}{4\sqrt{2}}$$
 with $h(D) = 4$. Then $t = t_P(\tau_0) = 8 - 20\sqrt{799 + 565\sqrt{2}} + 14\sqrt{2(799 + 565\sqrt{2})}$ has minimal polynomial $T^4 - 32T^3 + 368T^2 - 1792T - 64$.

We have $\mu(t) = \frac{1}{2\sqrt{2}\pi^2}L(\Theta_{P,\tau_0}, 2)$, where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(64m + 32n)q^{9m^2 + 8mn + 2n^2}$$

$$= 64q^2 - 128q^3 + 128q^6 - 128q^{11} - 64q^{18} + 384q^{19} - 384q^{22} + 256q^{27} - 384q^{34} + 128q^{38} + \cdots$$
is in $S_2\left(\Gamma_0(128), \left(\frac{32}{2}\right)\right)$.

Let
$$\tau_0 = [32, 0, 1] = \frac{i}{4\sqrt{2}}$$
 with $h(D) = 4$. Then

$$t = t_P(\tau_0) = 8 + 20\sqrt{799 + 565\sqrt{2}} - 14\sqrt{2\left(799 + 565\sqrt{2}\right)}$$

has minimal polynomial $T^4 - 32T^3 + 368T^2 - 1792T - 648T^3 + 368T^2 - 1792T - 648T^3 + 368T^2 - 1792T - 648T^3 + 368T^3 + 368T^3 - 1792T - 648T^3 + 368T^3 + 3675T^3 + 367$

We have $\mu(t) = \frac{1}{2\sqrt{2}\pi^2}L(\Theta_{P,\tau_0}, 2)$, where

$$\begin{split} \Theta_{P,\tau_0}(\tau) &= \sum_{m,n\in\mathbb{Z}}^{2\sqrt{2N}} \chi_{-4}(n) 32nq^{m^2+2n^2} \\ &= 64q^2 + 128q^3 + 128q^6 + 128q^{11} - 64q^{18} - 384q^{19} - 384q^{22} - 256q^{27} - 384q^{34} + 128q^{38} + \cdots \\ &\text{is in } \mathcal{S}_2\left(\Gamma_0(128),\left(\frac{32}{\cdot}\right)\right). \end{split}$$

Let
$$\tau_0 = [16, 8, 3] = -\frac{1}{4} + \frac{i}{2\sqrt{2}}$$
 with $h(D) = 4$. Then

$$t = t_P(\tau_0) = 8 + 20i\sqrt{-799 + 565\sqrt{2}} + 14i\sqrt{2\left(-799 + 565\sqrt{2}\right)}$$

has minimal polynomial $T^4 - 32T^3 + 368T^2 - 1792T - 64$.

We have $\mu(t) = \frac{1}{2\sqrt{2}\pi^2}L(\Theta_{P,\tau_0}, 2)$, where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(16m + 16n)q^{3m^2 + 2mn + n^2}$$

```
=32q-160q^9+192q^{17}+160q^{25}-256q^{33}-192q^{41}-224q^{49}+768q^{57}-64q^{73}+32q^{81}+\cdots
is in S_2\left(\Gamma_0(128), \left(\frac{32}{\cdot}\right)\right).
\mu-N4-6-4
Let \tau_0 = [16, -8, 3] = \frac{1}{4} + \frac{i}{2\sqrt{2}} with h(D) = 4. Then
t = t_P(\tau_0) = 8 - 20i\sqrt{-799 + 565\sqrt{2}} - 14i\sqrt{2\left(-799 + 565\sqrt{2}\right)}
has minimal polynomial T^4 - 32T^3 + 368T^2 - 1792T - 64.
We have \mu(t) = \frac{1}{2\sqrt{2}\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-16m + 16n)q^{3m^2 - 2mn + n^2}
              =32q-160q^9+192q^{17}+160q^{25}-256q^{33}-192q^{41}-224q^{49}+768q^{57}-64q^{73}+32q^{81}+\cdots
is in S_2\left(\Gamma_0(128), \left(\frac{32}{\cdot}\right)\right).
Let \tau_0 = [64, 64, 17] = -\frac{1}{2} + \frac{i}{8} with h(D) = 4. Then
t = t_P(\tau_0) = 8 + 6\sqrt[4]{2} - 9\sqrt[4]{8}
has minimal polynomial T^4 - 32T^3 + 816T^2 - 8960T - 8.
We have \mu(t) = \frac{1}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (128m + 64n) q^{17m^2 + 16mn + 4n^2}
              =128q^{4}-256q^{5}+256q^{8}-256q^{13}+256q^{20}-256q^{29}-384q^{36}+768q^{37}-512q^{40}+768q^{45}+\cdots
is in S_2\left(\Gamma_0(256), \left(\frac{64}{\cdot}\right)\right).
\mu-N4-7-2
Let \tau_0 = [64, 0, 1] = \frac{i}{8} with h(D) = 4. Then
t = t_P(\tau_0) = 8 - 6\sqrt[4]{2} + 9\sqrt[4]{8}
has minimal polynomial T^4 - 32T^3 + 816T^2 - 8960T - 8.
We have \mu(t) = \frac{1}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 64nq^{m^2 + 4n^2}
              = 128q^4 + 256q^5 + 256q^8 + 256q^{13} + 256q^{20} + 256q^{29} - 384q^{36} - 768q^{37} - 512q^{40} - 768q^{45} + \cdots
is in S_2\left(\Gamma_0(256), \left(\frac{64}{\cdot}\right)\right).
Let \tau_0 = [16, 8, 5] = -\frac{1}{4} + \frac{i}{2} with h(D) = 4. Then
t = t_P(\tau_0) = 8 + 6i\sqrt[4]{2} + 9i\sqrt[4]{8}
has minimal polynomial T^4 - 32T^3 + 816T^2 - 8960T - 8.
We have \mu(t) = \frac{1}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (16m + 16n) q^{5m^2 + 2mn + n^2}
              =32q-96q^9-64q^{17}+352q^{25}-320q^{41}-224q^{49}+512q^{65}-192q^{73}+288q^{81}+320q^{89}+\cdots
is in S_2\left(\Gamma_0(256), \left(\frac{64}{\cdot}\right)\right).
\mu-N4-7-4
Let \tau_0 = [16, -8, 5] = \frac{1}{4} + \frac{i}{2} with h(D) = 4. Then
t = t_P(\tau_0) = 8 - 6i\sqrt[4]{2} - 9i\sqrt[4]{8}
has minimal polynomial T^4 - 32T^3 + 816T^2 - 8960T - 8.
We have \mu(t) = \frac{1}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-16m + 16n) q^{5m^2 - 2mn + n^2}
              =32q-96q^9-64q^{17}+352q^{25}-320q^{41}-224q^{49}+512q^{65}-192q^{73}+288q^{81}+320q^{89}+\cdots
is in S_2(\Gamma_0(256), (\frac{64}{.})).
\mu-N4-8-1
Let \tau_0 = [2, -2, 3] = \frac{1}{2} + \frac{i\sqrt{5}}{2} with h(D) = 2. Then
t = t_P(\tau_0) = -272 - 128\sqrt{5} - 64\sqrt{38 + 17\sqrt{5}}
has minimal polynomial T^4 + 1088T^3 - 31232T^2 + 278528T + 65536.
We have \mu(t) = \frac{4\sqrt{5}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-4m+2n)q^{24m^2-4mn+n^2}
              = 4q - 12q^9 - 8q^{21} + 20q^{25} + 24q^{29} - 40q^{45} - 28q^{49} + 56q^{69} + 44q^{81} - 24q^{89} + \cdots
is in S_2(\Gamma_0(160), (\frac{320}{.})).
Let \tau_0 = [10, 10, 3] = -\frac{1}{2} + \frac{i}{2\sqrt{5}} with h(D) = 2. Then
t = t_P(\tau_0) = -272 - 128\sqrt{5} + 64\sqrt{38 + 17\sqrt{5}}
has minimal polynomial T^4 + 1088T^3 - 31232T^2 + 278528T + 65536.
We have \mu(t) = \frac{4\sqrt{5}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(20m+10n)q^{24m^2+20mn+5n^2}
              =20q^{5}-40q^{9}+40q^{21}-40q^{41}-60q^{45}+120q^{49}-120q^{61}+40q^{69}+120q^{81}-40q^{105}+\cdots
is in S_2\left(\Gamma_0(160), \left(\frac{320}{\cdot}\right)\right).
```

 μ -N4-8-3

Let $\tau_0 = [6, 2, 1] = -\frac{1}{6} + \frac{i\sqrt{5}}{6}$ with h(D) = 2. Then $t = t_P(\tau_0) = -272 + 128\sqrt{5} + 64i\sqrt{-38 + 17\sqrt{5}}$

```
has minimal polynomial T^4 + 1088T^3 - 31232T^2 + 278528T + 65536.
We have \mu(t) = \frac{4\sqrt{5}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(4m+6n)q^{8m^2+4mn+3n^2}
                = 12q^{3} + 4q^{7} + 20q^{15} - 28q^{23} - 40q^{27} - 20q^{35} + 28q^{43} - 44q^{47} + 28q^{63} + 44q^{67} + \cdots
is in S_2\left(\Gamma_0(160), \left(\frac{320}{\cdot}\right)\right).
μ-N4-8-4
Let \tau_0 = [6, -2, 1] = \frac{1}{6} + \frac{i\sqrt{5}}{6} with h(D) = 2. Then t = t_P(\tau_0) = -272 + 128\sqrt{5} - 64i\sqrt{-38 + 17\sqrt{5}} has minimal polynomial T^4 + 1088T^3 - 31232T^2 + 278528T + 65536.
We have \mu(t) = \frac{4\sqrt{5}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-4m+6n)q^{8m^2-4mn+3n^2}
               = 12q^3 + 4q^7 + 20q^{15} - 28q^{23} - 40q^{27} - 20q^{35} + 28q^{43} - 44q^{47} + 28q^{63} + 44q^{67} + \cdots
is in S_2\left(\Gamma_0(160), \left(\frac{320}{\cdot}\right)\right).
\mu-N4-9-1
Let \tau_0 = [2, -2, 5] = \frac{1}{2} + \frac{3i}{2} with h(D) = 2. Then
t = t_P(\tau_0) = -3088 - 1664\sqrt[4]{12} - 1792\sqrt{3} - 960\sqrt[4]{108}
has minimal polynomial T^4 + 12352T^3 - 391680T^2 + 3162112T + 65536.
We have \mu(t) = \frac{12}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-4m+2n)q^{40m^2-4mn+n^2}
               =4q-12q^9+20q^{25}-8q^{37}+24q^{45}-28q^{49}-40q^{61}+36q^{81}+56q^{85}-72q^{117}+\cdots
is in S_2\left(\Gamma_0(288), \left(\frac{576}{\cdot}\right)\right).
μ-N4-9-2
Let \tau_0 = [18, 18, 5] = -\frac{1}{2} + \frac{i}{6} with h(D) = 2. Then t = t_P(\tau_0) = -3088 + 1664 \sqrt[4]{12} - 1792 \sqrt{3} + 960 \sqrt[4]{108}
has minimal polynomial T^4 + 12352T^3 - 391680T^2 + 3162112T + 65536.
We have \mu(t) = \frac{12}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(36m+18n)q^{40m^2+36mn+9n^2}
                = 36q^9 - 72q^{13} + 72q^{25} - 72q^{45} + 72q^{73} - 108q^{81} + 216q^{85} - 216q^{97} - 72q^{109} + 216q^{117} + \cdots
is in S_2\left(\Gamma_0(288), \left(\frac{576}{\cdot}\right)\right).
Let \tau_0 = [10, 2, 1] = -\frac{1}{10} + \frac{3i}{10} with h(D) = 2. Then
t = t_P(\tau_0) = -3088 + 1664i\sqrt[4]{12} + 1792\sqrt{3} - 960i\sqrt[4]{108}
has minimal polynomial T^4 + 12352T^3 - 391680T^2 + 3162112T + 65536.
We have \mu(t) = \frac{12}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(4m+10n)q^{8m^2+4mn+5n^2}
                =20q^{5}+12q^{9}+28q^{17}+4q^{29}-52q^{41}-24q^{45}-44q^{53}-72q^{65}-36q^{81}+44q^{89}+\cdots
is in S_2\left(\Gamma_0(288), \left(\frac{576}{\cdot}\right)\right).
\mu-N4-9-4
Let \tau_0 = [10, -2, 1] = \frac{1}{10} + \frac{3i}{10} with h(D) = 2. Then t = t_P(\tau_0) = -3088 - 1664i\sqrt[4]{12} + 1792\sqrt{3} + 960i\sqrt[4]{108}
has minimal polynomial T^4 + 12352T^3 - 391680T^2 + 3162112T + 65536.
We have \mu(t) = \frac{12}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \tilde{\chi}_{-4}(n)(-4m+10n)q^{8m^2-4mn+5n^2}
               =20q^{5}+12q^{9}+28q^{17}+4q^{29}-52q^{41}-24q^{45}-44q^{53}-72q^{65}-36q^{81}+44q^{89}+\cdots
is in S_2(\Gamma_0(288), (\frac{576}{.})).
\mu-N4-10-1
Let \tau_0 = [2, -2, 7] = \frac{1}{2} + \frac{i\sqrt{13}}{2} with h(D) = 2. Then
t = t_P(\tau_0) = -20752 - 5760\sqrt{13} - 192\sqrt{23382 + 6485\sqrt{13}} has minimal polynomial T^4 + 83008T^3 - 2652672T^2 + 21250048T + 65536.
We have \mu(t) = \frac{4\sqrt{13}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(	au) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-4m+2n)q^{56m^2-4mn+n^2}
                =4q-12q^9+20q^{25}-28q^{49}-8q^{53}+24q^{61}-40q^{77}+36q^{81}+56q^{101}-44q^{121}+\cdots
is in S_2\left(\Gamma_0(416), \left(\frac{832}{\cdot}\right)\right).
\mu-N4-10-2
Let \tau_0 = [26, 26, 7] = -\frac{1}{2} + \frac{i}{2\sqrt{13}} with h(D) = 2. Then
t = t_P(\tau_0) = -20752 - 5760\sqrt{13} + 192\sqrt{23382 + 6485\sqrt{13}}
has minimal polynomial T^4 + 83008T^3 - 2652672T^2 + 21250048T + 65536.
We have \mu(t) = \frac{4\sqrt{13}}{\pi^2}L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-4}(n)(52m+26n)q^{56m^2+52mn+13n^2}
                = 52q^{\overline{13}} - 104q^{\overline{17}} + 104q^{\overline{29}} - 104q^{\overline{49}} + 104q^{\overline{77}} - 104q^{\overline{113}} - 156q^{\overline{117}} + 312q^{\overline{121}} - 312q^{\overline{133}} + 312q^{\overline{153}} + \cdots
is in S_2(\Gamma_0(416), (\frac{832}{})).
```

μ -N4-10-3

Let
$$\tau_0 = [14, 2, 1] = -\frac{1}{14} + \frac{i\sqrt{13}}{14}$$
 with $h(D) = 2$. Then

$$t = t_P(\tau_0) = -20752 + 5760\sqrt{13} + 192i\sqrt{-23382 + 6485\sqrt{13}}$$

has minimal polynomial $T^4 + 83008T^3 - 2652672T^2 + 21250048T + 65536$.

We have $\mu(t) = \frac{4\sqrt{13}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(4m+14n)q^{8m^2+4mn+7n^2}$$

$$= 28q^7 + 20q^{11} + 36q^{19} + 12q^{31} + 44q^{47} - 76q^{59} - 84q^{63} + 4q^{67} - 68q^{71} - 92q^{83} + \cdots$$

is in $S_2\left(\Gamma_0(416), \left(\frac{832}{\cdot}\right)\right)$

μ-N4-10-4

Let
$$\tau_0 = [14, -2, 1] = \frac{1}{14} + \frac{i\sqrt{13}}{14}$$
 with $h(D) = 2$. Then

$$t = t_P(\tau_0) = -20752 + 5760\sqrt{13} - 192i\sqrt{-23382 + 6485\sqrt{13}}$$

Let $\tau_0=[14,-2,1]=\frac{1}{14}+\frac{i\sqrt{13}}{14}$ with h(D)=2. Then $t=t_P(\tau_0)=-20752+5760\sqrt{13}-192i\sqrt{-23382+6485\sqrt{13}}$ has minimal polynomial $T^4+83008T^3-2652672T^2+21250048T+65536$.

We have $\mu(t) = \frac{4\sqrt{13}}{\pi^2}L(\Theta_{P,\tau_0}, 2)$, where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-4m+14n)q^{8m^2-4mn+7n^2}$$

$$= 28q^7 + 20q^{11} + 36q^{19} + 12q^{31} + 44q^{47} - 76q^{59} - 84q^{63} + 4q^{67} - 68q^{71} - 92q^{83} + \cdots$$
is in $S_2\left(\Gamma_0(416), \left(\frac{832}{2}\right)\right)$.

μ-N4-11-1

Let
$$\tau_0 = [20, 0, 1] = \frac{i}{2\sqrt{5}}$$
 with $h(D) = 4$. Then

$$t = t_P(\tau_0) = 288 + 128\sqrt{5} - 576\sqrt{-2 + \sqrt{5}} - 256\sqrt{5(-2 + \sqrt{5})}$$

has minimal polynomial $T^4 - 1152T^3 + 22528T^2 - 131072T + 1048576$.

We have $\mu(t) = \frac{2\sqrt{5}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)20nq^{4m^2+5n^2}$$

$$= 40q^5 + 80q^9 + 80q^{21} + 80q^{41} - 120q^{45} - 240q^{49} - 240q^{61} + 80q^{69} - 240q^{81} + 80q^{105} + \cdots$$

$$\vdots : C_{p,r_0}(n) (320)$$

is in $S_2\left(\Gamma_0(320), \left(\frac{320}{\cdot}\right)\right)$.

μ -N4-11-2

Let
$$\tau_0 = [4, 0, 5] = \frac{i\sqrt{5}}{2}$$
 with $h(D) = 4$. Then

$$t = t_P(\tau_0) = 288 + 128\sqrt{5} + 576\sqrt{-2 + \sqrt{5}} + 256\sqrt{5(-2 + \sqrt{5})}$$

has minimal polynomial $T^4 - 1152T^3 + 22528T^2 - 131072T + 1048576$.

We have $\mu(t) = \frac{2\sqrt{5}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)4nq^{20m^2+n^2}$$

$$= 8q - 24q^9 + 16q^{21} + 40q^{25} - 48q^{29} + 80q^{45} - 56q^{49} - 112q^{69} + 88q^{81} - 48q^{89} + \cdots$$
is in S. (F. (220), (320))

is in $S_2\left(\Gamma_0(320), \left(\frac{320}{\cdot}\right)\right)$.

μ -N4-11-3

Let
$$\tau_0 = [12, 8, 3] = -\frac{1}{3} + \frac{i\sqrt{5}}{6}$$
 with $h(D) = 4$. Then

$$t = t_P(\tau_0) = 288 - 128\sqrt{5} + 576i\sqrt{2 + \sqrt{5}} - 256i\sqrt{5\left(2 + \sqrt{5}\right)}$$

has minimal polynomial $T^4 - 1152T^3 + 22528T^2 - 131072T + 1048576$.

We have
$$\mu(t) = \frac{2\sqrt{5}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$$
, where
$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (16m + 12n) q^{12m^2 + 8mn + 3n^2}$$
$$= 24q^3 - 8q^7 - 40q^{15} + 56q^{23} - 80q^{27} - 40q^{35} + 56q^{43} + 88q^{47} - 56q^{63} + 88q^{67} + \cdots$$
is in So (Fe(320) (320))

is in $S_2(\Gamma_0(320), (\frac{320}{2}))$.

μ -N4-11-4

Let
$$\tau_0 = [12, -8, 3] = \frac{1}{3} + \frac{i\sqrt{5}}{6}$$
 with $h(D) = 4$. Then

$$t = t_P(\tau_0) = 288 - 128\sqrt{5} - 576i\sqrt{2 + \sqrt{5}} + 256i\sqrt{5\left(2 + \sqrt{5}\right)}$$

has minimal polynomial $T^4 - 1152T^3 + 22528T^2 - 131072T + 1048576$.

We have $\mu(t) = \frac{2\sqrt{5}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

We have
$$\mu(t) = \frac{1}{\pi^2} E(OP_{\tau_0}, 2)$$
, where
$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-16m + 12n)q^{12m^2 - 8mn + 3n^2}$$

$$= 24q^3 - 8q^7 - 40q^{15} + 56q^{23} - 80q^{27} - 40q^{35} + 56q^{43} + 88q^{47} - 56q^{63} + 88q^{67} + \cdots$$
is in $S_2\left(\Gamma_0(320), \left(\frac{320}{2}\right)\right)$.

μ -N4-12-1

Let
$$\tau_0 = [24, 20, 5] = -\frac{5}{12} + \frac{i\sqrt{5}}{12}$$
 with $h(D) = 4$. Then

$$t = t_{\rm B}(\tau_0) = 8 = 144\sqrt{38 + 17\sqrt{5} + 64\sqrt{190 + 85\sqrt{5}}}$$

 $t = t_P(\tau_0) = 8 - 144\sqrt{38 + 17\sqrt{5}} + 64\sqrt{190 + 85\sqrt{5}}$ has minimal polynomial $T^4 - 32T^3 + 1408T^2 - 18432T + 4096$.

We have
$$\mu(t) = \frac{\sqrt{5}}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$$
, where $\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(40m + 24n)q^{10m^2 + 10mn + 3n^2}$
 $= 16q^3 - 48q^7 + 80q^{15} + 16q^{23} - 160q^{27} + 80q^{35} + 144q^{43} - 112q^{47} - 16q^{63} - 48q^{67} + \cdots$ is in $S_2\left(\Gamma_0(160), \left(\frac{80}{2}\right)\right)$.

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Let \tau_0 = [24, 4, 1] = -\frac{1}{12} + \frac{i\sqrt{5}}{12} with h(D) = 4. Then t = t_P(\tau_0) = 8 + 144\sqrt{38 + 17\sqrt{5}} - 64\sqrt{190 + 85\sqrt{5}} has minimal polynomial T^4 - 32T^3 + 1408T^2 - 18432T + 4096.
We have \mu(t) = \frac{\sqrt{5}}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 24n)q^{2m^2 + 2mn + 3n^2}
                 =80q^3+80q^7+80q^{15}-240q^{23}-160q^{27}-240q^{35}+80q^{43}-240q^{47}+240q^{63}+400q^{67}+\cdots
is in S_2\left(\Gamma_0(160), \left(\frac{80}{\cdot}\right)\right).
μ-N4-12-3
Let \tau_0 = [8,4,3] = -\frac{1}{4} + \frac{i\sqrt{5}}{4} with h(D) = 4. Then t = t_P(\tau_0) = 8 + 144i\sqrt{-38 + 17\sqrt{5}} + 64i\sqrt{-190 + 85\sqrt{5}} has minimal polynomial T^4 - 32T^3 + 1408T^2 - 18432T + 4096.
We have \mu(t) = \frac{\sqrt{5}}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 8n)q^{6m^2 + 2mn + n^2}
                 =16q-48q^9-32q^{21}+80q^{25}+96q^{29}-160q^{45}-112q^{49}+224q^{69}+176q^{81}-96q^{89}+\cdots
is in S_2\left(\Gamma_0(160), \left(\frac{80}{\cdot}\right)\right).
\mu-N4-12-4
Let \tau_0 = [8, -4, 3] = \frac{1}{4} + \frac{i\sqrt{5}}{4} with h(D) = 4. Then
t=t_P(\tau_0)=8-144i\sqrt{-38+17\sqrt{5}}-64i\sqrt{-190+85\sqrt{5}} has minimal polynomial T^4-32T^3+1408T^2-18432T+4096.
We have \mu(t) = \frac{\sqrt{5}}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-8m + 8n) q^{6m^2 - 2mn + n^2}
                 =16q-48q^9-32q^{21}+80q^{25}+96q^{29}-160q^{45}-112q^{49}+224q^{69}+176q^{81}-96q^{89}+\cdots
is in S_2\left(\Gamma_0(160), \left(\frac{80}{\cdot}\right)\right).
\mu-N4-13-1
Let \tau_0 = [2, -2, 13] = \frac{1}{2} + \frac{5i}{2} with h(D) = 2. Then
t = t_P(\tau_0) = -1658896 - 1109376\sqrt[4]{5} - 741888\sqrt{5} - 496128\sqrt[4]{125}
has minimal polynomial T^4 + 6635584T^3 - 212335104T^2 + 1698709504T + 65536.
We have \mu(t) = \frac{20}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-4m + 2n) q^{104m^2 - 4mn + n^2}
                 =4q-12q^9+20q^{25}-28q^{49}+36q^{81}-8q^{101}+24q^{109}-44q^{121}-40q^{125}+56q^{149}+\cdots
is in S_2(\Gamma_0(800), (\frac{1600}{.})).
μ-N4-13-2
Let \tau_0 = [50, 50, 13] = -\frac{1}{2} + \frac{i}{10} with h(D) = 2. Then
t = t_P(\tau_0) = -165896 + 1109376\sqrt[4]{5} - 741888\sqrt{5} + 496128\sqrt[4]{125} has minimal polynomial T^4 + 6635584T^3 - 212335104T^2 + 1698709504T + 65536.
We have \mu(t) = \frac{20}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(100m + 50n)q^{104m^2 + 100mn + 25n^2}
                 =100q^{25}-200q^{29}+200q^{41}-200q^{61}+200q^{89}-200q^{125}+200q^{169}-200q^{221}-300q^{225}+600q^{229}+\cdots
is in S_2(\Gamma_0(800), (\frac{1600}{.})).
μ-N4-13-3
Let \tau_0 = [26, 2, 1] = -\frac{1}{26} + \frac{5i}{26} with h(D) = 2. Then
t = t_P(\tau_0) = -1658896 + 1109376i\sqrt[4]{5} + 741888\sqrt{5} - 496128i\sqrt[4]{125}
has minimal polynomial T^4 + 6635584T^3 - 212335104T^2 + 1698709504T + 65536.
We have \mu(t) = \frac{20}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (4m + 26n) q^{8m^2 + 4mn + 13n^2}
                 = 52q^{13} + 44q^{17} + 60q^{25} + 36q^{37} + 68q^{53} + 28q^{73} + 76q^{97} - 148q^{113} - 156q^{117} - 120q^{125} + \cdots
is in S_2(\Gamma_0(800), (\frac{1600}{.})).
\mu-N4-13-4
Let \tau_0 = [26, -2, 1] = \frac{1}{26} + \frac{5i}{26} with h(D) = 2. Then
has minimal polynomial T^4+6635584T^3-212335104T^2+1698709504T+65536. We have \mu(t)=\frac{20}{\pi^2}L(\Theta_{P,\tau_0},2), where \Theta_{P,\tau_0}(\tau)=\sum_{m,n\in\mathbb{Z}}\chi_{-4}(n)(-4m+26n)q^{8m^2-4mn+13n^2}
t = t_P(\tau_0) = -1658896 - 1109376i\sqrt[4]{5} + 741888\sqrt{5} + 496128i\sqrt[4]{125}
                 = 52q^{13} + 44q^{17} + 60q^{25} + 36q^{37} + 68q^{53} + 28q^{73} + 76q^{97} - 148q^{113} - 156q^{117} - 120q^{125} + \cdots
is in S_2\left(\Gamma_0(800), \left(\frac{1600}{\cdot}\right)\right).
\mu-N4-14-1
 Let \tau_0 = [4, -4, 9] = \frac{1}{2} + i\sqrt{2} with h(D) = 4. Then
t=t_P(\tau_0)=-1792-1280\sqrt{2}-640\sqrt{1+5\sqrt{2}}-448\sqrt{2+10\sqrt{2}} has minimal polynomial T^4+7168T^3-376832T^2+8388608T-67108864.
We have \mu(t) = \frac{4\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-8m+4n)q^{36m^2-4mn+n^2}
                 = 8q - 24q^9 + 40q^{25} - 16q^{33} + 48q^{41} - 56q^{49} - 80q^{57} + 184q^{81} - 144q^{113} - 88q^{121} + \cdots
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is in S_2\left(\Gamma_0(512), \left(\frac{512}{\cdot}\right)\right).
\mu-N4-14-2
Let \tau_0 = [36, 4, 1] = -\frac{1}{18} + \frac{i\sqrt{2}}{9} with h(D) = 4. Then
t = t_P(\tau_0) = -1792 - 1280\sqrt{2} + 640\sqrt{1 + 5\sqrt{2}} + 448\sqrt{2 + 10\sqrt{2}}
has minimal polynomial T^4 + 7168T^3 - 376832T^2 + 8388608T - 67108864.
We have \mu(t) = \frac{4\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 36n)q^{4m^2 + 4mn + 9n^2}
                = 128q^9 + 128q^{17} + 128q^{33} + 128q^{57} - 384q^{73} - 384q^{81} + 128q^{89} - 384q^{97} - 384q^{121} + 128q^{129} + \cdots
is in S_2(\Gamma_0(512), (\frac{512}{.})).
\mu-N4-14-3
Let \tau_0 = [12, 4, 3] = -\frac{1}{6} + \frac{i\sqrt{2}}{3} with h(D) = 4. Then
t = t_P(\tau_0) = -1792 + 1280\sqrt{2} + 640i\sqrt{-1 + 5\sqrt{2}} - 448i\sqrt{-2 + 10\sqrt{2}}
has minimal polynomial T^4 + 7168T^3 - 376832T^2 + 8388608T - 67108864.
We have \mu(t) = \frac{4\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m+12n)q^{12m^2+4mn+3n^2}
                =24q^3+8q^{11}+40q^{19}-128q^{27}-8q^{43}-128q^{51}+56q^{59}+104q^{67}+120q^{75}+88q^{83}+\cdots
is in S_2\left(\Gamma_0(512), \left(\frac{512}{\cdot}\right)\right).
Let \tau_0 = [12, -4, 3] = \frac{1}{6} + \frac{i\sqrt{2}}{3} with h(D) = 4. Then
t=t_P(\tau_0)=-1792+1280\sqrt{2}-640i\sqrt{-1+5\sqrt{2}}+448i\sqrt{-2+10\sqrt{2}} has minimal polynomial T^4+7168T^3-376832T^2+8388608T-67108864.
We have \mu(t) = \frac{4\sqrt{2}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-8m+12n)q^{12m^2-4mn+3n^2}
               =24q^3+8q^{11}+40q^{19}-128q^{27}-8q^{43}-128q^{51}+56q^{59}+104q^{67}+120q^{75}+88q^{83}+\cdots
is in S_2\left(\Gamma_0(512), \left(\frac{512}{\cdot}\right)\right).
μ-N4-15-1
Let \tau_0 = [36, 0, 1] = \frac{i}{6} with h(D) = 4. Then
t = t_P(\tau_0) = 3104 - 1664\sqrt[4]{12} + 1792\sqrt{3} - 960\sqrt[4]{108}
has minimal polynomial T^4 - 12416T^3 + 202752T^2 - 131072T + 1048576.
We have \mu(t) = \frac{6}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n) 36nq^{4m^2+9n^2}
                = 72q^9 + 144q^{13} + 144q^{25} + 144q^{45} + 144q^{73} - 216q^{81} - 432q^{85} - 432q^{97} + 144q^{109} - 432q^{117} + \cdots
is in S_2\left(\Gamma_0(576), \left(\frac{576}{\cdot}\right)\right).
Let \tau_0 = [4, 0, 9] = \frac{3i}{2} with h(D) = 4. Then
t = t_P(\tau_0) = 3104 + 1664\sqrt[4]{12} + 1792\sqrt{3} + 960\sqrt[4]{108}
has minimal polynomial T^4 - 12416T^3 + 202752T^2 - 131072T + 1048576.
We have \mu(t) = \frac{6}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 4nq^{36m^2 + n^2}
               = 8q - 24q^9 + 40q^{25} + 16q^{37} - 48q^{45} - 56q^{49} + 80q^{61} + 72q^{81} - 112q^{85} + 144q^{117} + \cdots
is in S_2(\Gamma_0(576), (\frac{576}{6})).
\mu-N4-15-3
Let \tau_0 = [20, 16, 5] = -\frac{2}{5} + \frac{3i}{10} with h(D) = 4. Then t = t_P(\tau_0) = 3104 + 1664i \sqrt[4]{12} - 1792\sqrt{3} - 960i \sqrt[4]{108}
has minimal polynomial T^4 - 12416T^3 + 202752T^2 - 131072T + 1048576.
We have \mu(t) = \frac{6}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (32m + 20n) q^{20m^2 + 16mn + 5n^2}
                = 40q^{5} - 24q^{9} - 56q^{17} + 8q^{29} + 104q^{41} - 48q^{45} - 88q^{53} + 144q^{65} + 72q^{81} - 88q^{89} + \cdots
is in S_2\left(\Gamma_0(576), \left(\frac{576}{\cdot}\right)\right).
Let \tau_0 = [20, -16, 5] = \frac{2}{5} + \frac{3i}{10} with h(D) = 4. Then t = t_P(\tau_0) = 3104 - 1664i \sqrt[4]{12} - 1792\sqrt{3} + 960i \sqrt[4]{108}
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has minimal polynomial $T^4 - 12416T^3 + 202752T^2 - 131072T + 1048576$.

We have $\mu(t) = \frac{6}{\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

 $\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-32m + 20n) q^{20m^2 - 16mn + 5n^2}$ $= 40q^{5} - 24q^{9} - 56q^{17} + 8q^{29} + 104q^{41} - 48q^{45} - 88q^{53} + 144q^{65} + 72q^{81} - 88q^{89} + \cdots$ is in $S_2(\Gamma_0(576), (\frac{576}{.}))$

Let $\tau_0 = [40, 36, 9] = -\frac{9}{20} + \frac{3i}{20}$ with h(D) = 4. Then

 $t = t_P(\tau_0) = 8 - 32\sqrt[4]{12} + 16\sqrt[4]{108}$

has minimal polynomial $T^4 - 32T^3 + 12672T^2 - 198656T + 4096$.

We have $\mu(t) = \frac{3}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

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\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (72m + 40n) q^{18m^2 + 18mn + 5n^2}
               = 16q^5 - 48q^9 + 80q^{17} - 112q^{29} + 16q^{41} + 96q^{45} + 80q^{53} - 288q^{65} + 144q^{81} + 208q^{89} + \cdots
is in S_2\left(\Gamma_0(288), \left(\frac{144}{\cdot}\right)\right).
Let \tau_0 = [40, 4, 1] = -\frac{1}{20} + \frac{3i}{20} with h(D) = 4. Then t = t_P(\tau_0) = 8 + 32\sqrt[4]{12} - 16\sqrt[4]{108}
has minimal polynomial T^4 - 32T^3 + 12672T^2 - 198656T + 4096.
We have \mu(t) = \frac{3}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(8m+40n)q^{2m^2+2mn+5n^2}
               = 144q^5 + 144q^9 + 144q^{17} + 144q^{29} - 432q^{41} - 288q^{45} - 432q^{53} - 288q^{65} - 432q^{81} + 144q^{89} + \cdots
is in S_2(\Gamma_0(288), (\frac{144}{.})).
\mu-N4-16-3
Let \tau_0 = [8, 4, 5] = -\frac{1}{4} + \frac{3i}{4} with h(D) = 4. Then
t = t_P(\tau_0) = 8 + 32i\sqrt[4]{12} + 16i\sqrt[4]{108}
has minimal polynomial T^4 - 32T^3 + 12672T^2 - 198656T + 4096.
We have \mu(t) = \frac{3}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(8m+8n)q^{10m^2+2mn+n^2}
               = 16q - 48q^9 + 80q^{25} - 32q^{37} + 96q^{45} - 112q^{49} - 160q^{61} + 144q^{81} + 224q^{85} - 288q^{117} + \cdots
is in S_2\left(\Gamma_0(288), \left(\frac{144}{\cdot}\right)\right).
\mu-N4-16-4
Let \tau_0 = [8, -4, 5] = \frac{1}{4} + \frac{3i}{4} with h(D) = 4. Then
t = t_P(\tau_0) = 8 - 32i \sqrt[4]{12} - 16i \sqrt[4]{108} has minimal polynomial T^4 - 32T^3 + 12672T^2 - 198656T + 4096.
We have \mu(t) = \frac{3}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-8m + 8n)q^{10m^2 - 2mn + n^2}
               = 16q - 48q^9 + 80q^{25} - 32q^{37} + 96q^{45} - 112q^{49} - 160q^{61} + 144q^{81} + 224q^{85} - 288q^{117} + \cdots
is in S_2(\Gamma_0(288), (\frac{144}{.})).
Let \tau_0 = [100, 0, 1] = \frac{i}{10} with h(D) = 4. Then
t = t_P(\tau_0) = 1658912 - 1109376\sqrt[4]{5} + 741888\sqrt{5} - 496128\sqrt[4]{125}
has minimal polynomial T^4 - 6635648T^3 + 106174464T^2 - 131072T + 1048576.
We have \mu(t) = \frac{10}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 100 n q^{4m^2 + 25n^2}
               =200q^{25}+400q^{29}+400q^{41}+400q^{61}+400q^{89}+400q^{125}+400q^{169}+400q^{221}-600q^{225}-1200q^{229}+\cdots
is in S_2(\Gamma_0(1600), (\frac{1600}{.})).
\mu-N4-17-2
Let \tau_0 = [4, 0, 25] = \frac{5i}{2} with h(D) = 4. Then
t = t_P(\tau_0) = 1658912 + 1109376\sqrt[4]{5} + 741888\sqrt{5} + 496128\sqrt[4]{125}
has minimal polynomial T^4 - 6635648T^3 + 106174464T^2 - 131072T + 1048576.
We have \mu(t) = \frac{10}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 4nq^{100m^2 + n^2}
               =8q-24q^9+40q^{25}-56q^{49}+72q^{81}+16q^{101}-48q^{109}-88q^{121}+80q^{125}-112q^{149}+\cdots
is in S_2\left(\Gamma_0(1600), \left(\frac{1600}{\cdot}\right)\right).
Let \tau_0 = [52, 48, 13] = -\frac{6}{13} + \frac{5i}{26} with h(D) = 4. Then
t = t_P(\tau_0) = 1658912 + 1109376i\sqrt[4]{5} - 741888\sqrt{5} - 496128i\sqrt[4]{125} has minimal polynomial T^4 - 6635648T^3 + 106174464T^2 - 131072T + 1048576.
We have \mu(t) = \frac{10}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(96m + 52n)q^{52m^2 + 48mn + 13n^2}
               = 104q^{13} - 88q^{17} - 120q^{25} + 72q^{37} + 136q^{53} - 56q^{73} - 152q^{97} + 296q^{113} - 312q^{117} - 240q^{125} + \cdots
is in S_2(\Gamma_0(1600), (\frac{1600}{.})).
\mu-N4-17-4
Let \tau_0 = [52, -48, 13] = \frac{6}{13} + \frac{5i}{26} with h(D) = 4. Then
t = t_P(\tau_0) = 1658912 - 1109376i\sqrt[4]{5} - 741888\sqrt{5} + 496128i\sqrt[4]{125}
has minimal polynomial T^4 - 6635648T^3 + 106174464T^2 - 131072T + 1048576.
We have \mu(t) = \frac{10}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-96m + 52n)q^{52m^2 - 48mn + 13n^2}
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 $\mu\text{-N4-18-1}$

is in $S_2\left(\Gamma_0(1600), \left(\frac{1600}{\cdot}\right)\right)$.

Let $\tau_0 = [2, -2, 19] = \frac{1}{2} + \frac{i\sqrt{37}}{2}$ with h(D) = 2. Then $t = t_0(\tau_0) = -49787152 = 8184960\sqrt{37} = 13444\sqrt{27445185}$

 $t = t_P(\tau_0) = -49787152 - 8184960\sqrt{37} - 1344\sqrt{2744518518 + 451196065\sqrt{37}}$

has minimal polynomial $T^4 + 199148608T^3 - 6372751872T^2 + 50982043648T + 65536$.

 $= 104q^{13} - 88q^{17} - 120q^{25} + 72q^{37} + 136q^{53} - 56q^{73} - 152q^{97} + 296q^{113} - 312q^{117} - 240q^{125} + \cdots$

We have
$$\mu(t) = \frac{4\sqrt{2}}{12} L(\theta_{T,n_1}, t_2)$$
, where $\theta_{T,n_1}(\tau) = \sum_{m \neq 1/2} \chi_m(t)(-4m + 2n)q^{1/2} e^{-4m + 2m} e^{-2m} e^{-4m + 2m} e^{-2m} e^{-4m + 2m} e^{-2m} e^{-4m} e^{-4m} e^{-2m} e^{-4m} e^{-4m} e^{-2m} e^{-4m} e^{-4m} e^{-2m} e^{-4m} e^{$

 $= 56q^7 - 40q^{11} - 72q^{19} + 24q^{31} + 88q^{47} + 152q^{59} - 168q^{63} - 8q^{67} - 136q^{71} + 184q^{83} + \cdots$

is in $S_2(\Gamma_0(832), (\frac{832}{}))$.

Let $\tau_0 = [28, -24, 7] = \frac{3}{7} + \frac{i\sqrt{13}}{14}$ with h(D) = 4. Then

We have $\mu(t) = \frac{2\sqrt{13}}{\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

 $t = t_P(\tau_0) = 20768 - 5760\sqrt{13} - 124608i\sqrt{18 + 5\sqrt{13}} + 34560i\sqrt{13}\left(18 + 5\sqrt{13}\right)$ has minimal polynomial $T^4 - 83072T^3 + 1333248T^2 - 131072T + 1048576$.

 μ -N4-19-4

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\Theta_{P,\tau_0}(	au) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-48m + 28n) q^{28m^2 - 24mn + 7n^2}
                 = 56q^7 - 40q^{11} - 72q^{19} + 24q^{31} + 88q^{47} + 152q^{59} - 168q^{63} - 8q^{67} - 136q^{71} + 184q^{83} + \cdots
is in S_2\left(\Gamma_0(832), \left(\frac{832}{\cdot}\right)\right).
\mu-N4-20-1
Let \tau_0 = [56, 52, 13] = -\frac{13}{28} + \frac{i\sqrt{13}}{28} with h(D) = 4. Then
t=t_P(\tau_0)=8-31152\sqrt{23382+6485\sqrt{13}}+8640\sqrt{303966+84305\sqrt{13}} has minimal polynomial T^4-32T^3+83328T^2-1329152T+4096.
We have \mu(t) = \frac{\sqrt{13}}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(104m + 56n)q^{26m^2 + 26mn + 7n^2}
                =16q^{7}-48q^{11}+80q^{19}-112q^{31}+144q^{47}+16q^{59}-48q^{63}-176q^{67}+80q^{71}-112q^{83}+\cdots
is in S_2\left(\Gamma_0(416), \left(\frac{208}{\cdot}\right)\right).
\mu-N4-20-2
Let \tau_0 = [56, 4, 1] = -\frac{1}{28} + \frac{i\sqrt{13}}{28} with h(D) = 4. Then
t = t_P(\tau_0) = 8 + 31152\sqrt{23382 + 6485\sqrt{13}} - 8640\sqrt{303966 + 84305\sqrt{13}} has minimal polynomial T^4 - 32T^3 + 83328T^2 - 1329152T + 4096.
We have \mu(t) = \frac{\sqrt{13}}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (8m + 56n) q^{2m^2 + 2mn + 7n^2}
                =208q^{7}+208q^{11}+208q^{19}+208q^{31}+208q^{47}-624q^{59}-624q^{63}+208q^{67}-624q^{71}-624q^{83}+\cdots
is in S_2(\Gamma_0(416), (\frac{208}{\cdot})).
Let \tau_0 = [8,4,7] = -\frac{1}{4} + \frac{i\sqrt{13}}{4} with h(D) = 4. Then t = t_P(\tau_0) = 8 + 31152i\sqrt{-23382 + 6485\sqrt{13}} + 8640i\sqrt{-303966 + 84305\sqrt{13}} has minimal polynomial T^4 - 32T^3 + 83328T^2 - 1329152T + 4096.
We have \mu(t) = \frac{\sqrt{13}}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (8m + 8n) q^{14m^2 + 2mn + n^2}
                 = 16q - 48q^9 + 80q^{25} - 112q^{49} - 32q^{53} + 96q^{61} - 160q^{77} + 144q^{81} + 224q^{101} - 176q^{121} + \cdots
is in S_2\left(\Gamma_0(416), \left(\frac{208}{\cdot}\right)\right).
\mu-N4-20-4
Let \tau_0 = [8, -4, 7] = \frac{1}{4} + \frac{i\sqrt{13}}{4} with h(D) = 4. Then t = t_P(\tau_0) = 8 - 31152i\sqrt{-23382 + 6485\sqrt{13}} - 8640i\sqrt{-303966 + 84305\sqrt{13}} has minimal polynomial T^4 - 32T^3 + 83328T^2 - 1329152T + 4096.
We have \mu(t) = \frac{\sqrt{13}}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-8m+8n)q^{14m^2-2mn+n^2}
                 = 16q - 48q^9 + 80q^{25} - 112q^{49} - 32q^{53} + 96q^{61} - 160q^{77} + 144q^{81} + 224q^{101} - 176q^{121} + \cdots
is in S_2\left(\Gamma_0(416), \left(\frac{208}{\cdot}\right)\right).
\mu-N4-21-1
Let \tau_0 = [104, 100, 25] = -\frac{25}{52} + \frac{5i}{52} with h(D) = 4. Then
t = t_P(\tau_0) = 8 - 864\sqrt[4]{5} + 384\sqrt[4]{125}
has minimal polynomial T^4 - 32T^3 + 6635904T^2 - 106170368T + 4096.
We have \mu(t) = \frac{5}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(200m + 104n)q^{50m^2 + 50mn + 13n^2}
                 = 16q^{13} - 48q^{17} + 80q^{25} - 112q^{37} + 144q^{53} - 176q^{73} + 208q^{97} + 16q^{113} - 48q^{117} - 160q^{125} + \cdots
is in S_2(\Gamma_0(800), (\frac{400}{.})).
\mu-N4-21-2
Let \tau_0 = [104, 4, 1] = -\frac{1}{52} + \frac{5i}{52} with h(D) = 4. Then t = t_P(\tau_0) = 8 + 864 \sqrt[4]{5} - 384 \sqrt[4]{125}
has minimal polynomial T^4 - 32T^3 + 6635904T^2 - 106170368T + 4096.
We have \mu(t) = \frac{5}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(8m+104n)q^{2m^2+2mn+13n^2}
                 =400q^{13}+400q^{17}+400q^{25}+400q^{37}+400q^{53}+400q^{73}+400q^{97}-1200q^{113}-1200q^{117}-800q^{125}+\cdots
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is in $S_2(\Gamma_0(800), (\frac{400}{.}))$.

Let $\tau_0 = [8, 4, 13] = -\frac{1}{4} + \frac{5i}{4}$ with h(D) = 4. Then

 $t = t_P(\tau_0) = 8 + 864i\sqrt[4]{5} + 384i\sqrt[4]{125}$ has minimal polynomial $T^4 - 32T^3 + 6635904T^2 - 106170368T + 4096$.

We have $\mu(t) = \frac{5}{2\pi^2} L(\Theta_{P,\tau_0}, 2)$, where

$$\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(8m+8n)q^{26m^2+2mn+n^2}$$

$$= 16q - 48q^9 + 80q^{25} - 112q^{49} + 144q^{81} - 32q^{101} + 96q^{109} - 176q^{121} - 160q^{125} + 224q^{149} + \cdots$$
is in $S_2\left(\Gamma_0(800), \left(\frac{400}{100}\right)\right)$.

μ-N4-21-4

Let $\tau_0 = [8, -4, 13] = \frac{1}{4} + \frac{5i}{4}$ with h(D) = 4. Then

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t = t_P(\tau_0) = 8 - 864i\sqrt[4]{5} - 384i\sqrt[4]{125}
has minimal polynomial T^4 - 32T^3 + 6635904T^2 - 106170368T + 4096.
We have \mu(t) = \frac{5}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (-8m + 8n) q^{26m^2 - 2mn + n^2}
              = 16q - 48q^9 + 80q^{25} - 112q^{49} + 144q^{81} - 32q^{101} + 96q^{109} - 176q^{121} - 160q^{125} + 224q^{149} + \cdots
is in S_2(\Gamma_0(800), (\frac{400}{.})).
\mu-N4-22-1
Let \tau_0 = [152, 148, 37] = -\frac{37}{76} + \frac{i\sqrt{37}}{76} with h(D) = 4. Then
t = t_P(\tau_0) = 8 - 522765264\sqrt{2744518518 + 451196065\sqrt{37}} + 85942080\sqrt{37}\left(2744518518 + 451196065\sqrt{37}\right)
has minimal polynomial T^4 - 32T^3 + 199148928T^2 - 3186378752T + 4096.
We have \mu(t) = \frac{\sqrt{37}}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(296m + 152n)q^{74m^2 + 74mn + 19n^2}
              = 16q^{19} - 48q^{23} + 80q^{31} - 112q^{43} + 144q^{59} - 176q^{79} + 208q^{103} - 240q^{131} + 272q^{163} + 16q^{167} + \cdots
is in S_2(\Gamma_0(1184), (\frac{592}{.})).
Let \tau_0 = [152, 4, 1] = -\frac{1}{76} + \frac{i\sqrt{37}}{76} with h(D) = 4. Then
t = t_P(\tau_0) = 8 + 522765264\sqrt{2744518518 + 451196065\sqrt{37}} - 85942080\sqrt{37}\left(2744518518 + 451196065\sqrt{37}\right)
has minimal polynomial T^4 - 32T^3 + 199148928T^2 - 3186378752T + 4096.
We have \mu(t) = \frac{\sqrt{37}}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 152n)q^{2m^2 + 2mn + 19n^2}
              = 592q^{19} + 592q^{23} + 592q^{31} + 592q^{43} + 592q^{59} + 592q^{79} + 592q^{103} + 592q^{131} + 592q^{163} - 1776q^{167} + \cdots
is in S_2(\Gamma_0(1184), (\frac{592}{.})).
\mu-N4-22-3
Let \tau_0 = [8, 4, 19] = -\frac{1}{4} + \frac{i\sqrt{37}}{4} with h(D) = 4. Then
t = t_P(\tau_0) = 8 + 522765264i\sqrt{-2744518518 + 451196065\sqrt{37}} + 85942080i\sqrt{37}\left(-2744518518 + 451196065\sqrt{37}\right)
has minimal polynomial T^4 - 32T^3 + 199148928T^2 - 3186378752T + 4096.
We have \mu(t) = \frac{\sqrt{37}}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(8m + 8n)q^{38m^2 + 2mn + n^2}
              = 16q - 48q^9 + 80q^{25} - 112q^{49} + 144q^{81} - 176q^{121} - 32q^{149} + 96q^{157} + 208q^{169} - 160q^{173} + \cdots
is in S_2(\Gamma_0(1184), (\frac{592}{.})).
\mu-N4-22-4
Let \tau_0 = [8, -4, 19] = \frac{1}{4} + \frac{i\sqrt{37}}{4} with h(D) = 4. Then
t = t_P(\tau_0) = 8 - 522765264i\sqrt{-2744518518 + 451196065\sqrt{37}} - 85942080i\sqrt{37}\left(-2744518518 + 451196065\sqrt{37}\right)
has minimal polynomial T^4 - 32T^3 + 199148928T^2 - 3186378752T + 4096.
We have \mu(t) = \frac{\sqrt{37}}{2\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-8m+8n)q^{38m^2-2mn+n^2}
              = 16q - 48q^9 + 80q^{25} - 112q^{49} + 144q^{81} - 176q^{121} - 32q^{149} + 96q^{157} + 208q^{169} - 160q^{173} + \cdots
is in S_2(\Gamma_0(1184), (\frac{592}{\cdot})).
Let \tau_0 = [148, 0, 1] = \frac{i}{2\sqrt{37}} with h(D) = 4. Then
t = t_P(\tau_0) = 49787168 + 8184960\sqrt{37} - 2091061056\sqrt{-882 + 145\sqrt{37}} - 343768320\sqrt{37} \left(-882 + 145\sqrt{37}\right)
has minimal polynomial T^4 - 199148672T^3 + 3186382848T^2 - 131072T + 1048576.
We have \mu(t) = \frac{2\sqrt{37}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 148nq^{4m^2+37n^2}
              =296q^{37}+592q^{41}+592q^{53}+592q^{73}+592q^{101}+592q^{137}+592q^{181}+592q^{233}+592q^{293}-888q^{333}+\cdots
is in S_2\left(\Gamma_0(2368), \left(\frac{2368}{\cdot}\right)\right).
\mu-N4-23-2
Let \tau_0 = [4, 0, 37] = \frac{i\sqrt{37}}{2} with h(D) = 4. Then
t = t_P(\tau_0) = 49787168 + 8184960\sqrt{37} + 2091061056\sqrt{-882 + 145\sqrt{37}} + 343768320\sqrt{37}\left(-882 + 145\sqrt{37}\right)
has minimal polynomial T^4 - 199148672T^3 + 3186382848T^2 - 131072T + 1048576.
We have \mu(t) = \frac{2\sqrt{37}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where \Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) 4nq^{148m^2 + n^2}
              = 8q - 24q^9 + 40q^{25} - 56q^{49} + 72q^{81} - 88q^{121} + 16q^{149} - 48q^{157} + 104q^{169} + 80q^{173} + \cdots
is in S_2(\Gamma_0(2368), (\frac{2368}{.})).
\mu-N4-23-3
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Let $\tau_0 = [76, 72, 19] = -\frac{9}{19} + \frac{i\sqrt{37}}{38}$ with h(D) = 4. Then

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t = t_P(\tau_0) = 49787168 - 8184960\sqrt{37} + 2091061056i\sqrt{882 + 145\sqrt{37}} - 343768320i\sqrt{37}\left(882 + 145\sqrt{37}\right)
has minimal polynomial T^4-199148672T^3+3186382848T^2-131072T+1048576
We have \mu(t) = \frac{2\sqrt{37}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (144m + 76n) q^{76m^2 + 72mn + 19n^2}
             = 152q^{19} - 136q^{23} - 168q^{31} + 120q^{43} + 184q^{59} - 104q^{79} - 200q^{103} + 88q^{131} + 216q^{163} + 440q^{167} + \cdots
is in S_2(\Gamma_0(2368), (\frac{2368}{...})).
\mu-N4-23-4
Let \tau_0 = [76, -72, 19] = \frac{9}{19} + \frac{i\sqrt{37}}{38} with h(D) = 4. Then
t = t_P(\tau_0) = 49787168 - 8184960\sqrt{37} - 2091061056i\sqrt{882 + 145\sqrt{37}} + 343768320i\sqrt{37}\left(882 + 145\sqrt{37}\right)
has minimal polynomial T^4-199148672T^3+3186382848T^2-131072T+1048576
We have \mu(t) = \frac{2\sqrt{37}}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-144m + 76n)q^{76m^2 - 72mn + 19n^2}
             = 152q^{19} - 136q^{23} - 168q^{31} + 120q^{43} + 184q^{59} - 104q^{79} - 200q^{103} + 88q^{131} + 216q^{163} + 440q^{167} + \cdots
is in S_2(\Gamma_0(2368), (\frac{2368}{.})).
\mu-N4-24-1
Let \tau_0 = [4, -4, 17] = \frac{1}{2} + 2i with h(D) = 4. Then
t = t_P(\tau_0) = -71680 - 60288\sqrt[4]{2} - 50688\sqrt{2} - 42624\sqrt[4]{8}
has minimal polynomial T^4 + 286720T^3 - 6684672T^2 + 67108864T - 536870912.
We have \mu(t) = \frac{8}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n)(-8m+4n)q^{68m^2-4mn+n^2}
             = 8q - 24q^9 + 40q^{25} - 56q^{49} - 16q^{65} + 48q^{73} + 72q^{81} - 80q^{89} + 112q^{113} - 88q^{121} + \cdots
is in S_2(\Gamma_0(1024), (\frac{1024}{.})).
Let \tau_0 = [68, 4, 1] = -\frac{1}{34} + \frac{2i}{17} with h(D) = 4. Then
t = t_P(\tau_0) = -71680 + 60288\sqrt{2} - 50688\sqrt{2} + 42624\sqrt[4]{8}
has minimal polynomial T^4 + 286720T^3 - 6684672T^2 + 67108864T - 536870912.
We have \mu(t) = \frac{8}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(8m + 68n)q^{4m^2 + 4mn + 17n^2}
             =256q^{17}+256q^{25}+256q^{41}+256q^{65}+256q^{97}+256q^{137}-768q^{145}-768q^{153}-768q^{169}+256q^{185}+\cdots
is in S_2\left(\Gamma_0(1024), \left(\frac{1024}{\cdot}\right)\right).
\mu-N4-24-3
Let \tau_0 = [20, 12, 5] = -\frac{3}{10} + \frac{2i}{5} with h(D) = 4. Then
t = t_P(\tau_0) = -71680 + 60288i\sqrt[4]{2} + 50688\sqrt{2} - 42624i\sqrt[4]{8}
has minimal polynomial T^4 + 286720T^3 - 6684672T^2 + 67108864T - 536870912.
We have \mu(t) = \frac{8}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-4}(n) (24m + 20n) q^{20m^2 + 12mn + 5n^2}
             =40q^5-8q^{13}-72q^{29}+88q^{37}-120q^{45}-24q^{53}-56q^{61}+256q^{85}-168q^{101}+136q^{109}+\cdots
is in S_2\left(\Gamma_0(1024), \left(\frac{1024}{\cdot}\right)\right).
Let \tau_0 = [20, -12, 5] = \frac{3}{10} + \frac{2i}{5} with h(D) = 4. Then
t = t_P(\tau_0) = -71680 - 60288i\sqrt[4]{2} + 50688\sqrt{2} + 42624i\sqrt[4]{8}
has minimal polynomial T^4 + 286720T^3 - 6684672T^2 + 67108864T - 536870912.
We have \mu(t) = \frac{8}{\pi^2} L(\Theta_{P,\tau_0}, 2), where
\Theta_{P,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-4}(n)(-24m + 20n)q^{20m^2 - 12mn + 5n^2}
             =40q^{5}-8q^{13}-72q^{29}+88q^{37}-120q^{45}-24q^{53}-56q^{61}+256q^{85}-168q^{101}+136q^{109}+\cdots
is in S_2(\Gamma_0(1024), (\frac{1024}{.})).
\nu-R-1
Let \tau_0 = [1, 1, 1] = -\frac{1}{2} + \frac{i\sqrt{3}}{2} with h(D) = 1. Then
t = t_Q(\tau_0) = -216
has minimal polynomial T+216 and
We have \nu(t) = \frac{81}{16\pi^2} L(\Theta_{Q,\tau_0}, 2), where
\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(3m+2n)q^{9m^2+3mn+n^2}
             =4q-8q^4-4q^7+20q^{13}+16q^{16}-28q^{19}-20q^{25}+8q^{28}-16q^{31}+44q^{37}+\cdots
is in S_2\left(\Gamma_0(27), \left(\frac{81}{\cdot}\right)\right).
Let \tau_0 = [3, 0, 1] = \frac{i}{\sqrt{3}} with h(D) = 1. Then
t = t_Q(\tau_0) = 54
has minimal polynomial T-54 and
k = \sqrt[3]{t} \approx 3.779763149684619494302 is not in \mathcal{K}_{Q}^{\circ}.
We have \nu(t) = \frac{9}{8\pi^2} L(\Theta_{Q,\tau_0}, 2), where
\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 6nq^{3m^2 + n^2}
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$$=12q-48q^7+24q^{13}+96q^{19}-60q^{25}-48q^{31}-120q^{37}+96q^{43}+108q^{49}+168q^{61}+\cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(36),\left(\frac{36}{\cdot}\right)\right)$.

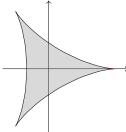
 ν -R-3

Let
$$\tau_0 = [9, 3, 1] = -\frac{1}{6} + \frac{i}{2\sqrt{3}}$$
 with $h(D) = 1$. Then

 $t = t_Q(\tau_0) = 24$

has minimal polynomial T-24 and

 $k = \sqrt[3]{t} \approx 2.884499140614816764643$ is **IN** $\mathcal{K}_{\mathcal{O}}^{\circ}$.



We have
$$\nu(t) = -\frac{3}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(9m+18n)q^{m^2+mn+n^2}
= 54q - 108q^4 - 54q^7 + 270q^{13} + 216q^{16} - 378q^{19} - 270q^{25} + 108q^{28} - 216q^{31} + 594q^{37} + \cdots$$

is in $S_2\left(\Gamma_0(27), \left(\frac{9}{\cdot}\right)\right)$.

ν -R2-1-1 (#13 in the paper)

Let
$$\tau_0 = [3, 0, 2] = i\sqrt{\frac{2}{3}}$$
 with $h(D) = 2$. Then

 $t = t_Q(\tau_0) = 108 + 54\sqrt{2}$

has minimal polynomial $T^2 - 216T + 5832$ and

 $k = \sqrt[3]{t} \approx 5.691518445260100656540$ is not in \mathcal{K}_{Q}° .

We have
$$\nu(t) = \frac{9}{4\sqrt{2}\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 6nq^{6m^2 + n^2}$$

$$= 12q - 24q^4 + 24q^7 - 48q^{10} + 48q^{16} + 96q^{22} - 36q^{25} - 48q^{28} - 120q^{31} + 96q^{40} + \cdots$$

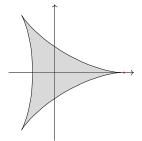
is in $S_2(\Gamma_0(72), (\frac{72}{-}))$.

$$\overline{\nu\text{-R2-1-2}}$$
 (#13 in the paper)
Let $\tau_0 = [6, 0, 1] = \frac{i}{\sqrt{6}}$ with $h(D) = 2$. Then

$$t = t_Q(\tau_0) = 108 - 54\sqrt{2}$$

has minimal polynomial $T^2 - 216T + 5832$ and

 $k = \sqrt[3]{t} \approx 3.162600661514223642574$ is not in \mathcal{K}_O° .



We have
$$\nu(t) = \frac{9}{4\sqrt{2}\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 12nq^{3m^2 + 2n^2}$$

$$= 24q^{2} + 48q^{5} - 48q^{8} - 96q^{11} + 48q^{14} - 96q^{20} + 48q^{29} + 96q^{32} + 96q^{35} + 192q^{44} + \cdots$$

is in $S_2\left(\Gamma_0(72), \left(\frac{72}{\cdot}\right)\right)$.

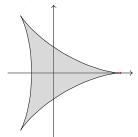
ν -R2-2-1 (#14 in the paper)

Let
$$\tau_0 = [9, 0, 1] = \frac{i}{3}$$
 with $h(D) = 2$. Then

$$t = t_O(\tau_0) = 18 + 6\sqrt{3}$$

has minimal polynomial $T^2 - 36T + 216$ and

 $k = \sqrt[3]{t} \approx 3.050705018258479587421$ is not in $\mathcal{K}_{\mathcal{O}}^{\circ}$.



We have
$$\nu(t) = \frac{\sqrt{3}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{\sqrt{3}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)18nq^{m^2+n^2}$

$$=36q+72q^2-72q^4-72q^5-144q^8+72q^{10}-144q^{13}+144q^{16}+360q^{17}+144q^{20}+\cdots$$
 is in $S_2\left(\Gamma_0(36),\left(\frac{12}{\cdot}\right)\right)$.

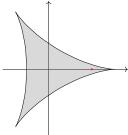
$\nu\text{-R2-2-2}$ (#14 in the paper)

Let $\tau_0 = [9, 6, 2] = -\frac{1}{3} + \frac{i}{3}$ with h(D) = 2. Then

 $t = t_Q(\tau_0) = 18 - 6\sqrt{3}$

has minimal polynomial $T^2 - 36T + 216$ and

 $k = \sqrt[3]{t} \approx 1.966758491591281434257$ is **IN** \mathcal{K}_{O}° .



We have
$$\nu(t) = -\frac{\sqrt{3}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(18m + 18n)q^{2m^2 + 2mn + n^2}
= 36q - 36q^2 - 72q^4 + 36q^5 + 72q^8 + 72q^{10} - 144q^{13} + 144q^{16} - 180q^{17} - 72q^{20} + \cdots$$

is in $S_2\left(\Gamma_0(36), \left(\frac{12}{\cdot}\right)\right)$.

ν -R2-3-1 (#15 in the paper)

Let $\tau_0 = [1, 0, 1] = i$ with h(D) = 1. Then

 $t = t_{\mathcal{O}}(\tau_0) = 270 + 162\sqrt{3}$

has minimal polynomial $T^2 - 540T - 5832$ and

 $k = \sqrt[3]{t} \approx 8.196152422706631880582$ is not in \mathcal{K}_O° .

We have
$$\nu(t) = \frac{27\sqrt{3}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n) 2nq^{9m^2+n^2}$

 $= 4q - 8q^4 + 8q^{10} - 16q^{13} + 16q^{16} + 12q^{25} - 40q^{34} + 8q^{37} - 16q^{40} + 28q^{49} + \cdots$

is in $S_2\left(\Gamma_0(36), \left(\frac{108}{\cdot}\right)\right)$.

 $\overline{\nu\text{-R2-3-2}}$ (#15 in the paper) Let $τ_0 = [2, 2, 1] = -\frac{1}{2} + \frac{i}{2}$ with h(D) = 1. Then

 $t = t_O(\tau_0) = 270 - 162\sqrt{3}$

has minimal polynomial $T^2 - 540T - 5832$ and

 $k = \sqrt[3]{t} \approx -2.196152422706631880582$ is not in $\mathcal{K}_{\Omega}^{\circ}$.

We have $\nu(t) = \frac{27\sqrt{3}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \sum_{n=1}^{\infty} \chi_{-3}(n)(6m+4n)q^{9m^2+6mn+2n^2}$$

 $=8q^{2}-8q^{5}-16q^{8}+40q^{17}+16q^{20}-32q^{26}-56q^{29}+32q^{32}-8q^{41}+24q^{50}+\cdots$

is in $S_2\left(\Gamma_0(36), \left(\frac{108}{\cdot}\right)\right)$.

 $\overline{\nu}$ -R2-4-1 (#16 in the paper) Let $\tau_0 = [3, 0, 4] = \frac{2i}{\sqrt{3}}$ with h(D) = 2. Then

 $t = t_Q(\tau_0) = 729 + 405\sqrt{3}$

has minimal polynomial $T^2 - 1458T + 39366$ and

 $k = \sqrt[3]{t} \approx 11.26749364485270551553$ is not in \mathcal{K}_{Q}° .

We have
$$\nu(t) = \frac{9}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 6nq^{12m^2+n^2}$

 $=12q-24q^4+24q^{13}-60q^{25}+96q^{28}-120q^{37}+108q^{49}-48q^{52}+168q^{61}-120q^{73}+\cdots$

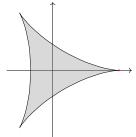
is in $S_2(\Gamma_0(144), (\frac{144}{.}))$.

$$\nu$$
-R2-4-2 (#16 in the paper)
Let $\tau_0 = [12, 0, 1] = \frac{i}{2\sqrt{3}}$ with $h(D) = 2$. Then

$$t = t_Q(\tau_0) = 729 - 405\sqrt{3}$$

has minimal polynomial $T^2 - 1458T + 39366$ and

 $k=\sqrt[3]{t}\approx 3.019115822861089757232$ is not in \mathcal{K}_Q°



We have
$$\nu(t) = \frac{9}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 24nq^{3m^2 + 4n^2}$$

 $=48q^{4}+96q^{7}-192q^{19}-192q^{28}+96q^{31}-192q^{43}+96q^{52}+384q^{67}+384q^{76}+96q^{79}+\cdots$

ν -R2-5-1 (#17 in the paper)

Let
$$\tau_0 = [3, -3, 2] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{5}{3}}$$
 with $h(D) = 2$. Then

$$t = t_Q(\tau_0) = -\frac{27}{2} - \frac{27\sqrt{5}}{2}$$

has minimal polynomial $T^2 + 27T - 729$ and

 $k = \sqrt[3]{t} \approx -3.521954990115985529900$ is not in \mathcal{K}_{O}° .

We have
$$\nu(t) = \frac{9\sqrt{5}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-9m+6n)q^{6m^2-3mn+n^2}$$
$$= 12q - 36q^4 + 60q^{10} - 12q^{16} - 48q^{19} - 60q^{25} + 96q^{31} + 120q^{34} - 60q^{40} - 240q^{46} + \cdots$$

is in $S_2\left(\Gamma_0(45), \left(\frac{45}{\cdot}\right)\right)$.

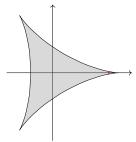
ν -R2-5-2 (#17 in the paper)

Let
$$\tau_0 = [6, 3, 1] = -\frac{1}{4} + \frac{1}{4}i\sqrt{\frac{5}{3}}$$
 with $h(D) = 2$. Then

$$t = t_Q(\tau_0) = -\frac{27}{2} + \frac{27\sqrt{5}}{2}$$

has minimal polynomial $T^2 + 27T - 729$ and

 $k = \sqrt[3]{t} \approx 2.555398926237728751166$ is **IN** \mathcal{K}_{Q}° .



We have
$$\nu(t) = -\frac{9\sqrt{5}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m+12n)q^{3m^2+3mn+2n^2}$$

$$=30q^{2}-30q^{5}-30q^{8}-60q^{17}+90q^{20}+120q^{23}-90q^{32}-120q^{38}+120q^{47}-150q^{50}+\cdots$$

is in $S_2\left(\Gamma_0(45), \left(\frac{45}{\cdot}\right)\right)$.

ν -R2-6-1 (#18 in the paper)

Let
$$\tau_0 = [3, 0, 5] = i\sqrt{\frac{5}{3}}$$
 with $h(D) = 2$. Then

$$t = t_Q(\tau_0) = \frac{3375}{2} + \frac{1485\sqrt{5}}{2}$$

has minimal polynomial $T^2-3375T+91125$ and $k=\sqrt[3]{t}\approx 14.95956587864062048278$ is not in \mathcal{K}_Q° .

We have $\nu(t) = \frac{9\sqrt{5}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 6nq^{15m^2 + n^2}$$

$$=12q-24q^4+72q^{16}-48q^{19}-60q^{25}+96q^{31}-120q^{40}+84q^{49}+24q^{61}+24q^{64}+\cdots$$

is in $S_2(\Gamma_0(180), (\frac{180}{.}))$.

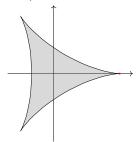
ν -R2-6-2 (#18 in the paper)

Let
$$\tau_0 = [15, 0, 1] = \frac{1}{\sqrt{15}}$$
 with $h(D) = 2$. Then $t = t_Q(\tau_0) = \frac{3375}{2} - \frac{1485\sqrt{5}}{2}$

$$t = t_{\rm O}(\tau_0) = \frac{3375}{2} - \frac{1485\sqrt{5}}{2}$$

has minimal polynomial $T^2 - 3375T + 91125$ and

 $k = \sqrt[3]{t} \approx 3.008108682100951536972$ is not in \mathcal{K}_O° .



We have
$$\nu(t) = \frac{9\sqrt{5}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 30 n q^{3m^2 + 5n^2}$$

$$= 60q^5 + 120q^8 + 120q^{17} - 120q^{20} - 240q^{23} - 120q^{32} - 240q^{47} + 120q^{53} - 240q^{68} + 360q^{80} + \cdots$$

is in $S_2(\Gamma_0(180), (\frac{180}{\cdot}))$.

$$\overline{\nu\text{-R2-7-1} \ (\#19 \text{ in the paper})}$$
Let $\tau_0 = [3, -3, 7] = \frac{1}{2} + \frac{5i}{2\sqrt{3}}$ with $h(D) = 2$. Then
$$t = t_0(\tau_0) = -4320 - 1944\sqrt{5}$$

$$t = t_Q(\tau_0) = -4320 - 1944\sqrt{5}$$

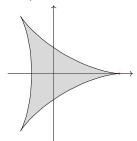
has minimal polynomial $T^2 + 8640T - 233280$ and $k = \sqrt[3]{t} \approx -20.54099758874507351758$ is not in \mathcal{K}_Q° .

We have $\nu(t) = \frac{45}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$, where

$$\begin{split} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-9m+6n) q^{21m^2-3mn+n^2} \\ &= 12q - 24q^4 + 48q^{16} - 12q^{19} - 84q^{31} + 216q^{49} - 156q^{61} - 96q^{64} + 24q^{76} - 48q^{79} + \cdots \\ \text{is in } \mathcal{S}_2\left(\Gamma_0(225),\left(\frac{225}{\cdot}\right)\right). \end{split}$$

 $\overline{\nu}$ -R2-7-2 (#19 in the paper) Let $\tau_0 = [21, 3, 1] = -\frac{1}{14} + \frac{5i}{14\sqrt{3}}$ with h(D) = 2. Then $t = t_Q(\tau_0) = -4320 + 1944\sqrt{5}$

has minimal polynomial $T^2 + 8640T - 233280$ and $k = \sqrt[3]{t} \approx 2.996891159467877124543$ is **IN** \mathcal{K}_O° .



We have
$$\nu(t) = -\frac{45}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(9m+42n)q^{3m^2+3mn+7n^2}$$

$$= 150q^7+150q^{13}-300q^{28}-300q^{37}+150q^{43}-300q^{52}+150q^{67}-300q^{73}+150q^{97}+600q^{103}+\cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(225),\left(\frac{225}{2}\right)\right)$.

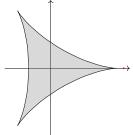
$\nu\text{-R2-8-1}$ (#20 in the paper)

Let $\tau_0 = [9, 0, 2] = \frac{i\sqrt{2}}{3}$ with h(D) = 2. Then

 $t = t_Q(\tau_0) = 32 + 2\sqrt{6}$

has minimal polynomial $T^2 - 64T + 1000$ and

 $k = \sqrt[3]{t} \approx 3.329186449936013683413$ is not in \mathcal{K}_{O}° .



We have
$$\nu(t) = \frac{\sqrt{\frac{3}{2}}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)18nq^{2m^2+n^2}$$

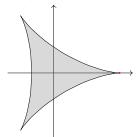
$$= 36q + 72q^3 - 72q^4 - 144q^6 + 72q^9 - 144q^{12} + 144q^{16} + 288q^{18} + 72q^{19} - 144q^{22} + \cdots$$
 is in $S_2\left(\Gamma_0(72), \left(\frac{24}{\cdot}\right)\right)$.

ν-R2-8-2 (#20 in the paper) Let $τ_0 = [18, 0, 1] = \frac{i}{3\sqrt{2}}$ with h(D) = 2. Then

 $t = t_Q(\tau_0) = 32 - 2\sqrt{6}$

has minimal polynomial $T^2 - 64T + 1000$ and

 $k=\sqrt[3]{t}\approx 3.003736843934408677123$ is not in \mathcal{K}_Q°



We have
$$\nu(t) = \frac{\sqrt{\frac{3}{2}}}{4\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)36nq^{m^2+2n^2}$

$$= 72q^2 + 144q^3 + 144q^6 - 144q^8 - 288q^9 + 144q^{11} - 288q^{12} - 288q^{17} + 144q^{18} - 288q^{24} + \cdots$$
 is in $S_2\left(\Gamma_0(72),\left(\frac{24}{\cdot}\right)\right)$.

 $\nu\text{-R2-9-1}$ (#21 in the paper) Let $\tau_0=[1,0,2]=i\sqrt{2}$ with h(D)=1. Then

 $t = t_Q(\tau_0) = 3672 + 1458\sqrt{6}$

has minimal polynomial $T^2 - 7344T + 729000$ and

 $k = \sqrt[3]{t} \approx 19.34846922834953429459$ is not in $\mathcal{K}_{\mathcal{O}}^{\circ}$.

We have $\nu(t) = \frac{27\sqrt{\frac{3}{2}}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$, where

$$\begin{split} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 2nq^{18m^2 + n^2} \\ &= 4q - 8q^4 + 16q^{16} + 8q^{19} - 16q^{22} - 20q^{25} + 32q^{34} - 40q^{43} + 28q^{49} - 32q^{64} + \cdots \\ &\text{is in } \mathcal{S}_2\left(\Gamma_0(72), \left(\frac{216}{\cdot}\right)\right). \end{split}$$

ν -R2-9-2 (#21 in the paper)

Let
$$\tau_0 = [2, 0, 1] = \frac{i}{\sqrt{2}}$$
 with $h(D) = 1$. Then $t = t_Q(\tau_0) = 3672 - 1458\sqrt{6}$

$$t = t_O(\tau_0) = 3672 - 1458\sqrt{6}$$

has minimal polynomial $T^2 - 7344T + 729000$ and

 $k = \sqrt[3]{t} \approx 4.651530771650465705408$ is not in \mathcal{K}_O° .

We have
$$\nu(t) = \frac{27\sqrt{\frac{3}{2}}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)4nq^{9m^2+2n^2}$$

$$= 8q^2 - 16q^8 + 16q^{11} - 32q^{17} + 32q^{32} + 16q^{38} + 64q^{41} - 32q^{44} - 40q^{50} - 80q^{59} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(72), \left(\frac{216}{21}\right)\right)$.

ν -R2-10-1 (#22 in the paper)

Let
$$\tau_0 = [3, -3, 5] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{17}{3}}$$
 with $h(D) = 2$. Then

$$t = t_Q(\tau_0) = -864 - 216\sqrt{17}$$

has minimal polynomial $T^2 + 1728T - 46656$ and

 $k = \sqrt[3]{t} \approx -12.06123975553198228290$ is not in \mathcal{K}_{O}° .

We have
$$\nu(t) = \frac{9\sqrt{17}}{16\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-9m+6n)q^{15m^2-3mn+n^2}$$

$$= 12q - 24q^4 - 12q^{13} + 48q^{16} + 60q^{19} - 144q^{25} + 132q^{43} + 84q^{49} + 24q^{52} - 204q^{55} + \cdots$$
is in $S_2\left(\Gamma_0(153), \left(\frac{153}{2}\right)\right)$.

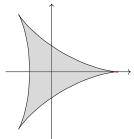
ν -R2-10-2 (#22 in the paper)

Let
$$\tau_0 = [15, 3, 1] = -\frac{1}{10} + \frac{1}{10}i\sqrt{\frac{17}{3}}$$
 with $h(D) = 2$. Then

$$t = t_O(\tau_0) = -864 + 216\sqrt{17}$$

has minimal polynomial $T^2 + 1728T - 46656$ and

 $k = \sqrt[3]{t} \approx 2.984767795822010538755$ is **IN** \mathcal{K}_{O}° .



We have
$$\nu(t) = -\frac{9\sqrt{17}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where

We have
$$\nu(t) = -\frac{1}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 30n)q^{3m^2 + 3mn + 5n^2}$$

$$= 102q^5 + 102q^{11} - 102q^{17} - 204q^{20} + 102q^{23} - 204q^{29} + 102q^{41} - 204q^{44} - 102q^{65} + 204q^{68} + \cdots$$

is in
$$S_2\left(\Gamma_0(153), \left(\frac{153}{\cdot}\right)\right)$$
.

$$\overline{\nu}$$
-R2-11-1 (#23 in the paper)
Let $\tau_0 = [3, -3, 13] = \frac{1}{2} + \frac{7i}{2\sqrt{3}}$ with $h(D) = 2$. Then

$$t = t_Q(\tau_0) = -163296 - 35640\sqrt{21}$$

has minimal polynomial $T^2 + 326592T - 8817984$ and

 $k = \sqrt[3]{t} \approx -68.86742011984472704913$ is not in \mathcal{K}_{Q}° .

We have
$$\nu(t)=\frac{63}{16\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau)=\sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(-9m+6n)q^{39m^2-3mn+n^2}$$

$$=12q-24q^4+48q^{16}-60q^{25}-12q^{37}+60q^{43}-96q^{64}+132q^{67}-156q^{79}+120q^{100}+\cdots$$

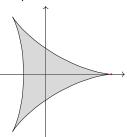
is in $S_2(\Gamma_0(441), (\frac{441}{\cdot}))$.

P-R2-11-2 (#23 in the paper)
Let
$$\tau_0 = [39, 3, 1] = -\frac{1}{26} + \frac{7i}{26\sqrt{3}}$$
 with $h(D) = 2$. Then

$$t = t_Q(\tau_0) = -163296 + 35640\sqrt{21}$$

has minimal polynomial $T^2 + 326592T - 8817984$ and

 $k = \sqrt[3]{t} \approx 2.999917339431239738841$ is **IN** $\mathcal{K}_{\mathcal{O}}^{\circ}$.



We have $\nu(t) = -\frac{63}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(9m+78n)q^{3m^2+3mn+13n^2}$ $= 294q^{13} + 294q^{19} + 294q^{31} - 588q^{52} - 588q^{61} + 294q^{73} - 588q^{76} - 588q^{97} + 294q^{103} - 588q^{124} + \cdots$ is in $\mathcal{S}_2\left(\Gamma_0(441), \left(\frac{441}{2}\right)\right)$.

ν -R2-12-1 (#24 in the paper)

Let
$$\tau_0 = [9, -9, 5] = \frac{1}{2} + \frac{i\sqrt{11}}{6}$$
 with $h(D) = 2$. Then

$$t = t_Q(\tau_0) = 4 - 4\sqrt{33}$$

has minimal polynomial $T^2 - 8T - 512$ and

 $k = \sqrt[3]{t} \approx -2.667383081517582729725$ is not in \mathcal{K}_{O}° .

We have $\nu(t) = \frac{\sqrt{33}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-27m+18n)q^{5m^2-3mn+n^2}$$

$$= 36q - 36q^3 - 72q^4 + 180q^9 + 72q^{12} - 396q^{15} + 144q^{16} - 216q^{25} + 288q^{27} + 180q^{31} + \cdots$$

is in $S_2\left(\Gamma_0(99), \left(\frac{33}{\cdot}\right)\right)$.

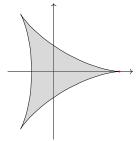
$\nu\text{-R2-12-2}$ (#24 in the paper)

Let
$$\tau_0 = [27, 3, 1] = -\frac{1}{18} + \frac{i\sqrt{11}}{18}$$
 with $h(D) = 2$. Then

$$t = t_Q(\tau_0) = 4 + 4\sqrt{33}$$

has minimal polynomial $T^2 - 8T - 512$ and

 $k = \sqrt[3]{t} \approx 2.999194249762008163998$ is **IN** \mathcal{K}_{O}° .



We have $\nu(t) = -\frac{\sqrt{33}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 54n)q^{m^2 + mn + 3n^2}$$

 $=198q^{3}+198q^{5}+198q^{9}-198q^{11}-396q^{12}-198q^{15}-396q^{20}+198q^{23}-396q^{27}+198q^{33}+\cdots$

is in $S_2\left(\Gamma_0(99), \left(\frac{33}{\cdot}\right)\right)$.

ν -R2-13-1 (#25 in the paper)

Let
$$\tau_0 = [1, -1, 3] = \frac{1}{2} + \frac{i\sqrt{11}}{2}$$
 with $h(D) = 1$. Then

$$t = t_Q(\tau_0) = -16740 - 2916\sqrt{33}$$

has minimal polynomial $T^2 + 33480T - 373248$ and

 $k = \sqrt[3]{t} \approx -32.23368793961408597955$ is not in \mathcal{K}_{Q}° .

We have $\nu(t) = \frac{27\sqrt{33}}{16\pi^2}L(\Theta_{Q,\tau_0},2)$, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-3m+2n)q^{27m^2-3mn+n^2}$$

$$= 4q - 8q^4 + 16q^{16} - 24q^{25} + 20q^{31} - 28q^{37} + 28q^{49} + 44q^{55} - 32q^{64} - 52q^{67} + \cdots$$

is in $S_2\left(\Gamma_0(99), \left(\frac{297}{\cdot}\right)\right)$.

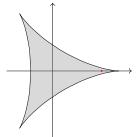
ν -R2-13-2 (#25 in the paper)

Let
$$\tau_0 = [5, 3, 1] = -\frac{3}{10} + \frac{i\sqrt{11}}{10}$$
 with $h(D) = 1$. Then

$$t = t_Q(\tau_0) = -16740 + 2916\sqrt{33}$$

has minimal polynomial $T^2 + 33480T - 373248$ and

 $k = \sqrt[3]{t} \approx 2.233687939614085979552$ is **IN** \mathcal{K}_Q° .



We have
$$\nu(t) = -\frac{27\sqrt{33}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m+10n)q^{9m^2+9mn+5n^2}$$

$$=22q^{5}-22q^{11}-44q^{20}+22q^{23}+44q^{44}-44q^{47}+88q^{53}+22q^{59}-110q^{71}+88q^{80}+\cdots$$

is in $S_2\left(\Gamma_0(99), \left(\frac{297}{\cdot}\right)\right)$.

ν -R2-14-1 (#26 in the paper)

Let
$$\tau_0 = [3, -3, 11] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{41}{3}}$$
 with $h(D) = 2$. Then

$$t = t_Q(\tau_0) = -55296 - 8640\sqrt{41}$$

has minimal polynomial $T^2 + 110592T - 2985984$ and

 $k = \sqrt[3]{t} \approx -48.00390497909782603051$ is not in \mathcal{K}_{Q}° .

We have $\nu(t) = \frac{9\sqrt{41}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m+6n)q^{33m^2-3mn+n^2}$ $=12q-24q^4+48q^{16}-60q^{25}-12q^{31}+60q^{37}-84q^{43}+84q^{49}+132q^{61}-96q^{64}+\cdots$ is in $S_2\left(\Gamma_0(369), \left(\frac{369}{\cdot}\right)\right)$.

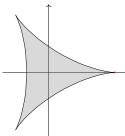
ν -R2-14-2 (#26 in the paper)

Let
$$\tau_0 = [33, 3, 1] = -\frac{1}{22} + \frac{1}{22}i\sqrt{\frac{41}{3}}$$
 with $h(D) = 2$. Then

$$t = t_O(\tau_0) = -55296 + 8640\sqrt{41}$$

 $t=t_Q(\tau_0)=-55296+8640\sqrt{41}$ has minimal polynomial $T^2+110592T-2985984$ and

 $k = \sqrt[3]{t} \approx 2.999755958660059445454$ is **IN** \mathcal{K}_{O}° .



We have
$$\nu(t) = -\frac{9\sqrt{41}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where

We have
$$\nu(t) = -\frac{9\sqrt{41}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(9m + 66n)q^{3m^2 + 3mn + 11n^2}$$
$$= 246q^{11} + 246q^{17} + 246q^{29} - 246q^{41} - 492q^{44} + 246q^{47} - 492q^{53} - 492q^{68} + 246q^{71} - 492q^{89} + \cdots$$

is in $S_2(\Gamma_0(369), (\frac{369}{2}))$.

ν -R2-15-1 (#27 in the paper)

Let
$$\tau_0 = [3, -3, 23] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{89}{3}}$$
 with $h(D) = 2$. Then

$$t = t_Q(\tau_0) = -13500000 - 1431000\sqrt{89}$$

has minimal polynomial $T^2 + 27000000T - 729000000$ and

 $k = \sqrt[3]{t} \approx -300.0000999998666669519$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{9\sqrt{89}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-9m+6n)q^{69m^2-3mn+n^2}$$
$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} - 12q^{67} + 60q^{73} - 84q^{79} + 132q^{97} + \cdots$$

is in $S_2(\Gamma_0(801), (\frac{801}{.}))$.

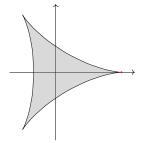
$\nu\text{-R2-15-2}$ (#27 in the paper)

Let
$$\tau_0 = [69, 3, 1] = -\frac{1}{46} + \frac{1}{46}i\sqrt{\frac{89}{3}}$$
 with $h(D) = 2$. Then

$$t = t_Q(\tau_0) = -13500000 + 1431000\sqrt{89}$$

has minimal polynomial $T^2 + 27000000T - 729000000$ and

 $k = \sqrt[3]{t} \approx 2.999999000001666662815$ is **IN** \mathcal{K}_{O}° .



We have
$$\nu(t) = -\frac{9\sqrt{89}}{5}L(\Theta_{O,\tau_0}, 2)$$
, where

We have
$$\nu(t) = -\frac{9\sqrt{89}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(9m+138n)q^{3m^2+3mn+23n^2}$

 $=534q^{23}+534q^{29}+534q^{41}+534q^{59}+534q^{83}-534q^{89}-1068q^{92}-1068q^{101}+534q^{113}-1068q^{116}+\cdots$

is in $S_2\left(\Gamma_0(801), \left(\frac{801}{\cdot}\right)\right)$.

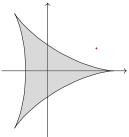
ν -I2-1-1

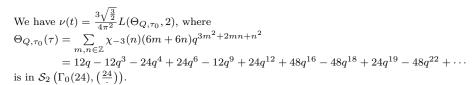
Let
$$\tau_0 = [3, 2, 1] = -\frac{1}{3} + \frac{i\sqrt{2}}{3}$$
 with $h(D) = 1$. Then

$$t = t_Q(\tau_0) = 4 + 10i\sqrt{2}$$

has minimal polynomial $T^2 - 8T + 216$ and

 $k = \sqrt[3]{t} \approx 2.224744871391589049099 + 1.024944026382329769127i$ is not in \mathcal{K}_O° .





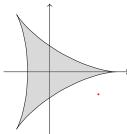
ν -I2-1-2

Let $\tau_0 = [3, -2, 1] = \frac{1}{3} + \frac{i\sqrt{2}}{3}$ with h(D) = 1. Then

 $t = t_Q(\tau_0) = 4 - 10i\sqrt{2}$

has minimal polynomial $T^2 - 8T + 216$ and

 $k = \sqrt[3]{t} \approx 2.224744871391589049099 - 1.024944026382329769127i$ is not in \mathcal{K}_O° .



We have
$$\nu(t) = \frac{3\sqrt{\frac{3}{2}}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-6m+6n)q^{3m^2-2mn+n^2}$$

$$= 12q-12q^3-24q^4+24q^6-12q^9+24q^{12}+48q^{16}-48q^{18}+24q^{19}-48q^{22}+\cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(24),\left(\frac{24}{\cdot}\right)\right)$.

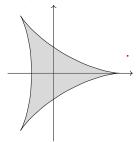
ν -I2-2-1

Let $\tau_0 = [3, 1, 1] = -\frac{1}{6} + \frac{i\sqrt{11}}{6}$ with h(D) = 1. Then

 $t = t_Q(\tau_0) = 32 + 8i\sqrt{11}$

has minimal polynomial $T^2 - 64T + 1728$ and

 $k = \sqrt[3]{t} \approx 3.372281323269014329925 + 0.792286991393261277794i$ is not in \mathcal{K}_{O}° .



We have
$$\nu(t) = \frac{3\sqrt{33}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(3m+6n)q^{3m^2+mn+n^2}$
 $= 12q+6q^3-24q^4-30q^9-12q^{12}+66q^{15}+48q^{16}-72q^{25}-48q^{27}+60q^{31}+\cdots$

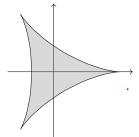
is in $S_2\left(\Gamma_0(33), \left(\frac{33}{\cdot}\right)\right)$.

Let $\tau_0 = [3, -1, 1] = \frac{1}{6} + \frac{i\sqrt{11}}{6}$ with h(D) = 1. Then

 $t = t_Q(\tau_0) = 32 - 8i\sqrt{11}$

has minimal polynomial $T^2 - 64T + 1728$ and

 $k = \sqrt[3]{t} \approx 3.372281323269014329925 - 0.792286991393261277794i$ is not in \mathcal{K}_{O}° .



We have
$$\nu(t) = \frac{3\sqrt{33}}{16-2}L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{3\sqrt{33}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-3m+6n)q^{3m^2-mn+n^2}$$
$$= 12q + 6q^3 - 24q^4 - 30q^9 - 12q^{12} + 66q^{15} + 48q^{16} - 72q^{25} - 48q^{27} + 60q^{31} + \cdots$$

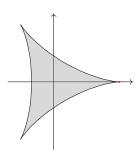
is in $S_2(\Gamma_0(33), (\frac{33}{2}))$.

Let $\tau_0 = [27, 0, 1] = \frac{i}{3\sqrt{3}}$ with h(D) = 3. Then

$$t = t_Q(\tau_0) = 6 - 6\sqrt[3]{2} + 18\sqrt[3]{4}$$

has minimal polynomial $T^3 - 18T^2 + 756T - 27000$ and

 $k = \sqrt[3]{t} \approx 3.000507048964448844951$ is not in \mathcal{K}_O° .



We have
$$\nu(t) = \frac{3}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n) 54nq^{m^2+3n^2}$$

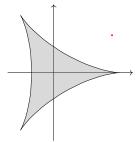
$$= 108q^3 + 216q^4 + 216q^7 - 432q^{13} - 432q^{16} + 216q^{19} - 432q^{21} - 216q^{28} - 432q^{37} + 216q^{39} + \cdots$$
is in $S_2\left(\Gamma_0(108), \left(\frac{36}{9}\right)\right)$.

Let
$$\tau_0 = [9, 6, 4] = -\frac{1}{3} + \frac{i}{\sqrt{3}}$$
 with $h(D) = 3$. Then $t = t_Q(\tau_0) = 6 + 3\sqrt[3]{2} - 9\sqrt[3]{4} + 3i\sqrt[3]{2}\sqrt{3} + 9i\sqrt[3]{4}\sqrt{3}$

$$t = t_O(\tau_0) = 6 + 3\sqrt[3]{2} - 9\sqrt[3]{4} + 3i\sqrt[3]{2}\sqrt{3} + 9i\sqrt[3]{4}\sqrt{3}$$

has minimal polynomial $T^3 - 18T^2 + 756T - 27000$ and

 $k = \sqrt[3]{t} \approx 2.659914121621583806585 + 1.709727167708387368149i$ is not in \mathcal{K}_{O}° .



We have
$$\nu(t) = \frac{3}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (18m + 18n) q^{4m^2 + 2mn + n^2}$$

$$=36q - 108q^4 + 72q^7 - 36q^{13} + 216q^{16} - 144q^{19} - 180q^{25} + 108q^{28} - 144q^{31} + 180q^{37} + \cdots$$

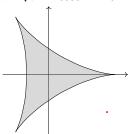
is in
$$S_2\left(\Gamma_0(108), \left(\frac{36}{\cdot}\right)\right)$$
.

Let
$$\tau_0 = [9, -6, 4] = \frac{1}{2} + \frac{i}{\sqrt{2}}$$
 with $h(D) = 3$. Then

Let
$$\tau_0 = [9, -6, 4] = \frac{1}{3} + \frac{i}{\sqrt{3}}$$
 with $h(D) = 3$. Then $t = t_Q(\tau_0) = 6 + 3\sqrt[3]{2} - 9\sqrt[3]{4} - 3i\sqrt[3]{2}\sqrt{3} - 9i\sqrt[3]{4}\sqrt{3}$

has minimal polynomial $T^3 - 18T^2 + 756T - 27000$ and

 $k = \sqrt[3]{t} \approx 2.659914121621583806585 - 1.709727167708387368149i$ is not in \mathcal{K}_{O}° .



We have
$$\nu(t) = \frac{3}{2}L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t)=\frac{3}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau)=\sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(-18m+18n)q^{4m^2-2mn+n^2}$$

$$=36q-108q^4+72q^7-36q^{13}+216q^{16}-144q^{19}-180q^{25}+108q^{28}-144q^{31}+180q^{37}+\cdots$$

is in $S_2\left(\Gamma_0(108), \left(\frac{36}{\cdot}\right)\right)$.

ν -C-2-1

Let
$$\tau_0 = [1, 0, 3] = i\sqrt{3}$$
 with $h(D) = 1$. Then

$$t = t_Q(\tau_0) = 17766 + 14094\sqrt[3]{2} + 11178\sqrt[3]{4}$$

 $t=t_Q(\tau_0)=17766+14094\sqrt[3]{2}+11178\sqrt[3]{4}$ has minimal polynomial $T^3-53298T^2+1635876T-19683000$ and

 $k = \sqrt[3]{t} \approx 37.62589891676765375567$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{81}{9-2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{81}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 2nq^{27m^2 + n^2}$

$$= 4q - 8q^4 + 16q^{16} - 20q^{25} + 8q^{28} - 16q^{31} + 32q^{43} + 28q^{49} - 40q^{52} - 32q^{64} + \cdots$$

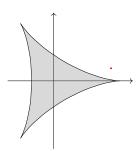
is in $S_2\left(\Gamma_0(108), \left(\frac{324}{\cdot}\right)\right)$.

ν -C-2-2

Let
$$\tau_0 = [4 \ 2 \ 1] = -\frac{1}{4} + \frac{i\sqrt{3}}{4}$$
 with $h(D) = 1$. Then

Let
$$\tau_0 = [4, 2, 1] = -\frac{1}{4} + \frac{i\sqrt{3}}{4}$$
 with $h(D) = 1$. Then $t = t_Q(\tau_0) = 17766 - 7047\sqrt[3]{2} - 5589\sqrt[3]{4} + 7047i\sqrt[3]{2}\sqrt{3} - 5589i\sqrt[3]{4}\sqrt{3}$ has minimal polynomial $T^3 - 53298T^2 + 1635876T - 19683000$ and

 $k = \sqrt[3]{t} \approx 2.616964743186866031313 + 0.572192121667771129124i$ is not in \mathcal{K}_Q° .



We have
$$\nu(t) = \frac{81}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

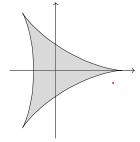
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(6m+8n)q^{9m^2+6mn+4n^2}$$

$$= 16q^4 + 4q^7 - 20q^{13} - 32q^{16} + 28q^{19} - 16q^{28} - 44q^{37} + 52q^{49} + 80q^{52} + 4q^{61} + \cdots$$
is in $S_2\left(\Gamma_0(108), \left(\frac{324}{4}\right)\right)$.



Let
$$\tau_0 = [4, -2, 1] = \frac{1}{4} + \frac{i\sqrt{3}}{4}$$
 with $h(D) = 1$. Then $t = t_Q(\tau_0) = 17766 - 7047\sqrt[3]{2} - 5589\sqrt[3]{4} - 7047i\sqrt[3]{2}\sqrt{3} + 5589i\sqrt[3]{4}\sqrt{3}$ has minimal polynomial $T^3 - 53298T^2 + 1635876T - 19683000$ and

 $k = \sqrt[3]{t} \approx 2.616964743186866031313 - 0.572192121667771129124i$ is not in \mathcal{K}_O° .



We have
$$\nu(t)=\frac{81}{8\pi^2}L(\Theta_{Q,\tau_0},2),$$
 where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-6m+8n)q^{9m^2-6mn+4n^2}$$

$$= 16q^4 + 4q^7 - 20q^{13} - 32q^{16} + 28q^{19} - 16q^{28} - 44q^{37} + 52q^{49} + 80q^{52} + 4q^{61} + \cdots$$

is in $S_2(\Gamma_0(108), (\frac{324}{2}))$.

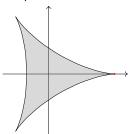
ν-C-3-1

Let
$$\tau_0 = [63, 3, 1] = -\frac{1}{42} + \frac{i\sqrt{3}}{14}$$
 with $h(D) = 3$. Then $t = t_Q(\tau_0) = 96 + 56\sqrt[3]{3} - 72\sqrt[3]{9}$

$$t = t_O(\tau_0) = 96 + 56\sqrt[3]{3} - 72\sqrt[3]{9}$$

has minimal polynomial $T^3 - 288T^2 + 63936T - 1536000$ and

 $k = \sqrt[3]{t} \approx 2.999997802867938024843$ is **IN** $\mathcal{K}_{\mathcal{O}}^{\circ}$.



We have
$$\nu(t) = -\frac{9}{8-2}L(\Theta_{Q,\tau_0},2)$$
, where

We have
$$\nu(t) = -\frac{9}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(9m+126n)q^{m^2+mn+7n^2}$$
$$= 486q^7 + 486q^9 + 486q^{13} + 486q^{19} - 972q^{28} - 972q^{31} - 972q^{36} + 486q^{37} - 972q^{43} + 486q^{49} + \cdots$$

is in $S_2\left(\Gamma_0(243), \left(\frac{81}{\cdot}\right)\right)$.

Let
$$\tau_0 = [9, 3, 7] = -\frac{1}{4} + \frac{i\sqrt{3}}{4}$$
 with $h(D) = 3$. Then

$$t = t_O(\tau_0) = 96 + 108i\sqrt[6]{3} - 28\sqrt[3]{3} + 36\sqrt[3]{9} + 28i\sqrt[6]{243}$$

Let $\tau_0 = [9, 3, 7] = -\frac{1}{6} + \frac{i\sqrt{3}}{2}$ with h(D) = 3. Then $t = t_Q(\tau_0) = 96 + 108i\sqrt[6]{3} - 28\sqrt[3]{3} + 36\sqrt[3]{9} + 28i\sqrt[6]{243}$ has minimal polynomial $T^3 - 288T^2 + 63936T - 1536000$ and

 $k = \sqrt[3]{t} \approx 5.865745215438474359242 + 2.013218760179034262415i$ is not in \mathcal{K}_O° .

We have
$$\nu(t) = \frac{9}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(9m+18n)q^{7m^2+mn+n^2}$$
$$= 36q - 72q^4 + 18q^7 - 90q^{13} + 144q^{16} + 126q^{19} - 180q^{25} - 36q^{28} + 72q^{31} - 198q^{37} + \cdots$$

is in $S_2(\Gamma_0(243), (\frac{81}{2}))$.

Let
$$\tau_0 = [9, -3, 7] = \frac{1}{6} + \frac{i\sqrt{3}}{2}$$
 with $h(D) = 3$. Then

$$t = t_O(\tau_0) = 96 - 108i\sqrt[6]{3} - 28\sqrt[3]{3} + 36\sqrt[3]{9} - 28i\sqrt[6]{243}$$

has minimal polynomial $T^3 - 288T^2 + 63936T - 1536000$ and

 $k = \sqrt[3]{t} \approx 5.865745215438474359242 - 2.013218760179034262415i$ is not in $\mathcal{K}_{\Omega}^{\circ}$.

We have $\nu(t) = \frac{9}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$, where

$$\begin{split} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-9m+18n) q^{7m^2-mn+n^2} \\ &= 36q - 72q^4 + 18q^7 - 90q^{13} + 144q^{16} + 126q^{19} - 180q^{25} - 36q^{28} + 72q^{31} - 198q^{37} + \cdots \\ &\text{is in } \mathcal{S}_2\left(\Gamma_0(243), \left(\frac{81}{\cdot}\right)\right). \end{split}$$

Let
$$\tau_0 = [1, -1, 7] = \frac{1}{2} + \frac{3i\sqrt{3}}{2}$$
 with $h(D) = 1$. Then

$$t = t_Q(\tau_0) = -4096224 - 2840184\sqrt[3]{3} - 1969272\sqrt[3]{9}$$

has minimal polynomial $T^3 + 12288672T^2 - 700259904T + 10077696000$ and

 $k = \sqrt[3]{t} \approx -230.7644944264686007685$ is not in \mathcal{K}_O° .

We have
$$\nu(t) = \frac{243}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-3m+2n) q^{63m^2-3mn+n^2}$$
$$-4a-8a^4+16a^{16}-20a^{25}+28a^{49}-4a^{61}-$$

$$=4q-8q^4+16q^{16}-20q^{25}+28q^{49}-4q^{61}-32q^{64}+20q^{67}-28q^{73}+44q^{91}+\cdots$$

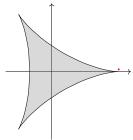
is in $S_2\left(\Gamma_0(243), \left(\frac{729}{\cdot}\right)\right)$.

ν -C-4-2

Let
$$\tau_0 = [7, 1, 1] = -\frac{1}{14} + \frac{3i\sqrt{3}}{14}$$
 with $h(D) = 1$. Then

Let
$$\tau_0 = [7, 1, 1] = -\frac{1}{14} + \frac{3i\sqrt{3}}{14}$$
 with $h(D) = 1$. Then $t = t_Q(\tau_0) = -4096224 - 2953908i\sqrt[6]{3} + 1420092\sqrt[3]{3} + 984636\sqrt[3]{9} + 1420092i\sqrt[6]{243}$ has minimal polynomial $T^3 + 12288672T^2 - 700259904T + 10077696000$ and

 $k = \sqrt[3]{t} \approx 3.057720768431478199602 + 0.1026441050822645592587i$ is not in \mathcal{K}_Q° .



We have
$$\nu(t) = \frac{243}{1600} L(\Theta_{O,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{243}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (3m+14n) q^{9m^2+3mn+7n^2}$$
$$= 28q^7 + 22q^{13} + 34q^{19} - 56q^{28} - 50q^{31} + 16q^{37} - 62q^{43} + 40q^{49} - 44q^{52} - 68q^{76} + \cdots$$

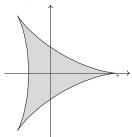
is in $S_2\left(\Gamma_0(243), \left(\frac{729}{\cdot}\right)\right)$.

ν-C-4-3

Let
$$\tau_0 = [7, -1, 1] = \frac{1}{14} + \frac{3i\sqrt{3}}{14}$$
 with $h(D) = 1$. Then

$$t = t_Q(\tau_0) = -4096224 + 2953908i \sqrt[6]{3} + 1420092 \sqrt[3]{3} + 984636 \sqrt[3]{9} - 1420092i \sqrt[6]{243}$$
 has minimal polynomial $T^3 + 12288672T^2 - 700259904T + 10077696000$ and

 $k = \sqrt[3]{t} \approx 3.057720768431478199602 - 0.1026441050822645592587i$ is not in \mathcal{K}_{O}°



We have
$$\nu(t) = \frac{243}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{1}{16\pi^2} L(QQ, \tau_0, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-3m + 14n) q^{9m^2 - 3mn + 7n^2}$$

$$= 28q^7 + 22q^{13} + 34q^{19} - 56q^{28} - 50q^{31} + 16q^{37} - 62q^{43} + 40q^{49} - 44q^{52} - 68q^{76} + \cdots$$

is in $S_2(\Gamma_0(243), (\frac{729}{.}))$.

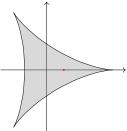
ν -T4-1-1 (#52 in the paper)

Let
$$\tau_0 = [27, 24, 8] = -\frac{4}{9} + \frac{2i\sqrt{2}}{9}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 109 - 70\sqrt{2} - 55\sqrt{3} + 35\sqrt{6}$$

has minimal polynomial $T^4 - 436T^3 + 18836T^2 - 214016T + 97336$ and

 $k = \sqrt[3]{t} \approx 0.7799151863765605325094$ is **IN** \mathcal{K}_Q° .



We have
$$\nu(t) = -\frac{\sqrt{\frac{3}{2}}}{\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where

$$\begin{split} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (72m + 54n) q^{8m^2 + 8mn + 3n^2} \\ &= 72q^3 - 72q^4 - 144q^{12} + 144q^{16} + 72q^{19} + 288q^{24} - 360q^{27} - 144q^{36} - 360q^{43} + 288q^{48} + \cdots \\ \text{is in } \mathcal{S}_2\left(\Gamma_0(288), \left(\frac{96}{\cdot}\right)\right). \end{split}$$

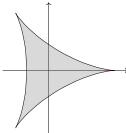
ν -T4-1-2 (#52 in the paper)

Let
$$\tau_0=[27,12,4]=-\frac{2}{9}+\frac{2i\sqrt{2}}{9}$$
 with $h(D)=4$. Then $t=t_Q(\tau_0)=109-70\sqrt{2}+55\sqrt{3}-35\sqrt{6}$

$$t = t_O(\tau_0) = 109 - 70\sqrt{2} + 55\sqrt{3} - 35\sqrt{6}$$

has minimal polynomial $T^4 - 436T^3 + 18836T^2 - 214016T + 97336$ and

 $k = \sqrt[3]{t} \approx 2.693248060916926080203$ is **IN** \mathcal{K}_{O}° .



We have
$$\nu(t) = -\frac{\sqrt{\frac{3}{2}}}{\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(36m + 54n)q^{4m^2 + 4mn + 3n^2}$$

$$= 144q^3 - 144q^8 + 144q^{11} - 288q^{12} - 288q^{24} + 144q^{27} + 288q^{32} + 576q^{36} - 288q^{44} + 576q^{48} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(288), \left(\frac{96}{2}\right)\right)$.

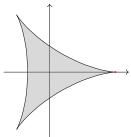
ν -T4-1-3 (#52 in the paper)

Let
$$\tau_0 = [72, 0, 1] = \frac{i}{6\sqrt{2}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 109 + 70\sqrt{2} - 55\sqrt{3} - 35\sqrt{6}$$

has minimal polynomial $T^4 - 436T^3 + 18836T^2 - 214016T + 97336$ and

 $k = \sqrt[3]{t} \approx 3.000000516756102426046$ is not in \mathcal{K}_O° .



We have
$$\nu(t) = \frac{\sqrt{\frac{3}{2}}}{2\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)144nq^{m^2+8n^2}$$

$$= 288q^8 + 576q^9 + 576q^{12} + 576q^{17} + 576q^{24} - 576q^{32} - 576q^{33} - 1152q^{36} - 1152q^{41} + 576q^{44} + \cdots$$
is in $S_2\left(\Gamma_0(288), \left(\frac{96}{10}\right)\right)$.

ν-T4-1-4 (#52 in the paper)
Let
$$τ_0 = [9, 0, 8] = \frac{2i\sqrt{2}}{3}$$
 with $h(D) = 4$. Then

$$t = t_O(\tau_0) = 109 + 70\sqrt{2} + 55\sqrt{3} + 35\sqrt{6}$$

 $t=t_Q(\tau_0)=109+70\sqrt{2}+55\sqrt{3}+35\sqrt{6}$ has minimal polynomial $T^4-436T^3+18836T^2-214016T+97336$ and

 $k = \sqrt[3]{t} \approx 7.299830388126588534035$ is not in \mathcal{K}_O° .

We have
$$\nu(t) = \frac{\sqrt{\frac{3}{2}}}{2\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)18nq^{8m^2+n^2}$
 $= 36q - 72q^4 + 72q^9 - 144q^{12} + 144q^{16} + 288q^{24} - 180q^{25} - 288q^{33} - 144q^{36} + 288q^{48} + \cdots$ is in $S_2\left(\Gamma_0(288), \left(\frac{96}{2}\right)\right)$.

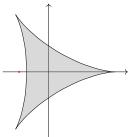
ν -T4-2-1 (#53 in the paper)

Let
$$\tau_0 = [12, 12, 5] = -\frac{1}{2} + \frac{i}{\sqrt{6}}$$
 with $h(D) = 4$. Then

$$t = t_O(\tau_0) = 7155 + 5049\sqrt{2} - 4131\sqrt{3} - 2916\sqrt{6}$$

 $t=t_Q(\tau_0)=7155+5049\sqrt{2}-4131\sqrt{3}-2916\sqrt{6}$ has minimal polynomial $T^4-28620T^3+766908T^2+314928T-4251528$ and

 $k = \sqrt[3]{t} \approx -1.348044649977884605914$ is not in \mathcal{K}_{Q}° .



We have $\nu(t) = \frac{9}{2\sqrt{2}\pi^2}L(\Theta_{Q,\tau_0},2)$, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (36m + 24n) q^{15m^2 + 12mn + 4n^2}$ $=48q^{4}-48q^{7}-96q^{16}+96q^{28}+240q^{31}-192q^{40}-384q^{55}+192q^{64}+240q^{79}+384q^{88}+\cdots$ is in $S_2\left(\Gamma_0(288), \left(\frac{288}{\cdot}\right)\right)$.

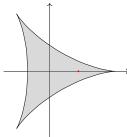
ν -T4-2-2 (#53 in the paper)

Let
$$\tau_0 = [15, 12, 4] = -\frac{2}{5} + \frac{2}{5}i\sqrt{\frac{2}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 7155 - 5049\sqrt{2} - 4131\sqrt{3} + 2916\sqrt{6}$$

 $t=t_Q(\tau_0)=7155-5049\sqrt{2}-4131\sqrt{3}+2916\sqrt{6}$ has minimal polynomial $T^4-28620T^3+766908T^2+314928T-4251528$ and

 $k = \sqrt[3]{t} \approx 1.309579622892052421771$ is **IN** \mathcal{K}_{O}° .



We have
$$\nu(t) = -\frac{9}{\sqrt{2}\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\begin{split} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (36m + 30n) q^{12m^2 + 12mn + 5n^2} \\ &= 48q^5 - 48q^8 - 96q^{20} + 48q^{29} + 96q^{32} + 192q^{44} - 240q^{53} - 96q^{56} - 192q^{77} + 192q^{80} + \cdots \\ \text{is in } \mathcal{S}_2\left(\Gamma_0(288), \left(\frac{288}{\cdot}\right)\right). \end{split}$$

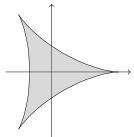
$\nu\text{-}\mathrm{T}4\text{-}2\text{-}3$ (#53 in the paper)

Let
$$\tau_0 = [24, 0, 1] = \frac{i}{2\sqrt{6}}$$
 with $h(D) = 4$. Then

$$t = t_O(\tau_0) = 7155 - 5049\sqrt{2} + 4131\sqrt{3} - 2916\sqrt{6}$$

 $t=t_Q(\tau_0)=7155-5049\sqrt{2}+4131\sqrt{3}-2916\sqrt{6}$ has minimal polynomial $T^4-28620T^3+766908T^2+314928T-4251528$ and

 $k = \sqrt[3]{t} \approx 3.000944876058051906767$ is not in \mathcal{K}_{Q}° .



We have
$$\nu(t) = \frac{9}{2\sqrt{2}\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 48nq^{3m^2 + 8n^2}$$

$$= 96q^8 + 192q^{11} + 192q^{20} - 192q^{32} - 192q^{35} - 384q^{44} + 192q^{56} - 384q^{59} - 384q^{80} + 192q^{83} + \cdots$$

is in $S_2(\Gamma_0(288), (\frac{288}{\cdot}))$.

ν -T4-2-4 (#53 in the paper)

Let
$$\tau_0 = [3, 0, 8] = 2i\sqrt{\frac{2}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 7155 + 5049\sqrt{2} + 4131\sqrt{3} + 2916\sqrt{6}$$

 $t=t_Q(\tau_0)=7155+5049\sqrt{2}+4131\sqrt{3}+2916\sqrt{6}$ has minimal polynomial $T^4-28620T^3+766908T^2+314928T-4251528$ and

 $k = \sqrt[3]{t} \approx 30.57882620119961435692$ is not in \mathcal{K}_{Q}° .

We have $\nu(t) = \frac{9}{2\sqrt{2}\pi^2}L(\Theta_{Q,\tau_0},2)$, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \sum_{m=0}^{\infty} (n)6nq^{24m^2 + n^2}$$

 $=12q-24q^4+48q^{16}-36q^{25}-48q^{28}+96q^{40}-36q^{49}-96q^{64}+168q^{73}-192q^{88}+\cdots$

is in $S_2(\Gamma_0(288), (\frac{288}{\cdot}))$.

ν -T4-3-1 (#54 in the paper)

Let
$$\tau_0 = [12, -12, 7] = \frac{1}{2} + \frac{i}{\sqrt{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 500877 - 408969\sqrt{\frac{3}{2}} - \frac{708345}{\sqrt{2}} + 289170\sqrt{3}$$

has minimal polynomial $T^4 - 2003508T^3 + 31313466T^2 + 1230187500T - 16607531250$ and

 $k = \sqrt[3]{t} \approx -2.888747638054569328812$ is not in $\mathcal{K}_{\Omega}^{\circ}$.

We have $\nu(t) = \frac{9}{2\pi^2} L(\Theta_{Q,\tau_0}, 2)$, where

We have
$$\nu(\epsilon) = \frac{1}{2\pi^2} L(3Q, \tau_0, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-36m + 24n) q^{21m^2 - 12mn + 4n^2}$$

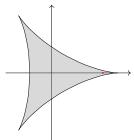
$$= 48q^4 - 48q^{13} - 96q^{16} + 240q^{37} + 96q^{52} - 336q^{61} - 240q^{100} - 48q^{109} + 384q^{112} + 768q^{133} + \cdots$$

is in $S_2(\Gamma_0(576), (\frac{576}{}))$.

$$\overline{\nu\text{-T4-3-2}}$$
 (#54 in the paper)
Let $\tau_0 = [21, 12, 4] = -\frac{2}{7} + \frac{4i}{7\sqrt{3}}$ with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 500877 - 408969\sqrt{\frac{3}{2}} + \frac{708345}{\sqrt{2}} - 289170\sqrt{3}$$

has minimal polynomial $T^4 - 2003508T^3 + 31313466T^2 + 1230187500T - 16607531250$ and $k = \sqrt[3]{t} \approx 2.335283725576904820241$ is **IN** \mathcal{K}_{O}° .



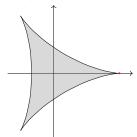
We have $\nu(t) = -\frac{9}{\pi^2} L(\Theta_{Q,\tau_0}, 2)$, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(36m + 42n)q^{12m^2 + 12mn + 7n^2}$ $=96q^{7}-96q^{16}-192q^{28}+96q^{31}+384q^{76}+96q^{79}-480q^{103}+384q^{112}-192q^{124}-480q^{127}+\cdots$ is in $S_2(\Gamma_0(576), (\frac{576}{}))$.

ν -T4-3-3 (#54 in the paper)

Let $\tau_0 = [48, 0, 1] = \frac{i}{4\sqrt{3}}$ with h(D) = 4. Then

$$t = t_Q(\tau_0) = 500877 + 408969\sqrt{\frac{3}{2}} - \frac{708345}{\sqrt{2}} - 289170\sqrt{3}$$

has minimal polynomial $T^4 - 2003508T^3 + 31313466T^2 + 1230187500T - 16607531250$ and $k = \sqrt[3]{t} \approx 3.000013476588670790643$ is not in \mathcal{K}_{Q}° .



We have $\nu(t) = \frac{9}{2\pi^2} L(\Theta_{Q,\tau_0}, 2)$, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 96nq^{3m^2 + 16n^2}$ $= 192q^{16} + 384q^{19} + 384q^{28} + 384q^{43} - 768q^{67} - 768q^{76} - 384q^{91} - 768q^{112} + 384q^{124} - 768q^{139} + \cdots$ is in $S_2(\Gamma_0(576), (\frac{576}{4}))$.

 $\overline{\nu}$ -T4-3-4 (#54 in the paper) Let $\tau_0 = [3, 0, 16] = \frac{4i}{\sqrt{3}}$ with h(D) = 4. Then

$$t = t_Q(\tau_0) = 500877 + 408969\sqrt{\frac{3}{2}} + \frac{708345}{\sqrt{3}} + 289170\sqrt{3}$$

 $t=t_Q(\tau_0)=500877+408969\sqrt{\tfrac{3}{2}}+\tfrac{708345}{\sqrt{2}}+289170\sqrt{3}$ has minimal polynomial $T^4-2003508T^3+31313466T^2+1230187500T-16607531250$ and

 $k = \sqrt[3]{t} \approx 126.0653975251660565242$ is not in \mathcal{K}_{O}°

We have
$$\nu(t) = \frac{9}{2\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 6nq^{48m^2 + n^2}$$
$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 108q^{49} - 48q^{52} - 120q^{73} + 168q^{97} + 120q^{100} - 192q^{112} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(576), \left(\frac{576}{5}\right)\right)$.

Let $\tau_0 = [4, -4, 3] = \frac{1}{2} + \frac{i}{\sqrt{2}}$ with h(D) = 2. Then

$$t = t_{\rm O}(\tau_0) = 13062249 - 9236430\sqrt{2} - 7541505\sqrt{3} + 5332635\sqrt{6}$$

 $t=t_Q(\tau_0)=13062249-9236430\sqrt{2}-7541505\sqrt{3}+5332635\sqrt{6}$ has minimal polynomial $T^4-52248996T^3-2201460444T^2+106073419104T+51728341176$ and

 $k = \sqrt[3]{t} \approx -4.134293714534110962198$ is not in \mathcal{K}_{O}° .

We have
$$\nu(t) = \frac{27\sqrt{\frac{3}{2}}}{2\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(-12m+8n)q^{27m^2-12mn+4n^2}$$
$$= 16q^4 - 32q^{16} - 16q^{19} + 80q^{43} + 64q^{64} - 112q^{67} + 32q^{76} - 64q^{88} - 80q^{100} + 128q^{136} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(288), \left(\frac{864}{\cdot}\right)\right)$.

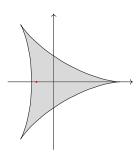
$$\nu$$
-T4-4-2 (#55 in the paper)

$$\overline{\nu}$$
-T4-4-2 (#55 in the paper)
Let $\tau_0 = [8, 8, 3] = -\frac{1}{2} + \frac{i}{2\sqrt{2}}$ with $h(D) = 2$. Then

$$t = t_Q(\tau_0) = 13062249 - 9236430\sqrt{2} + 7541505\sqrt{3} - 5332635\sqrt{6}$$

has minimal polynomial $T^4 - 52248996T^3 - 2201460444T^2 + 106073419104T + 51728341176$ and

 $k = \sqrt[3]{t} \approx -0.7845372058337206260300$ is **IN** \mathcal{K}_O° .



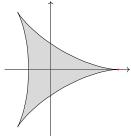
We have
$$\nu(t) = \frac{27\sqrt{\frac{3}{2}}}{2\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(24m+16n)q^{27m^2+24mn+8n^2}$$

$$= 32q^8 - 32q^{11} - 64q^{32} + 64q^{44} + 160q^{59} - 128q^{68} - 32q^{83} - 224q^{107} + 128q^{128} + 160q^{131} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(288),\left(\frac{864}{2}\right)\right)$.

 $\overline{\nu\text{-T4-4-3}}$ (#55 in the paper) Let $\tau_0 = [8, 0, 1] = \frac{i}{2\sqrt{2}}$ with h(D) = 2. Then

 $t = t_Q(\tau_0) = 13062249 + 9236430\sqrt{2} - 7541505\sqrt{3} - 5332635\sqrt{6}$

has minimal polynomial $T^4 - 52248996T^3 - 2201460444T^2 + 106073419104T + 51728341176$ and $k = \sqrt[3]{t} \approx 3.072806497095363261234$ is not in \mathcal{K}_Q° .



We have
$$\nu(t) = \frac{27\sqrt{\frac{3}{2}}}{2\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)16nq^{9m^2+8n^2}$$

$$= 32q^8 + 64q^{17} - 64q^{32} - 128q^{41} + 64q^{44} - 128q^{68} + 64q^{89} - 128q^{113} + 128q^{128} + 256q^{137} + \cdots$$
is in $S_2\left(\Gamma_0(288), \left(\frac{864}{2}\right)\right)$.

 $\nu\text{-}\mathrm{T}4\text{-}4\text{-}4$ (#55 in the paper)

Let $\tau_0=[1,0,8]=2i\sqrt{2}$ with h(D)=2. Then $t=t_Q(\tau_0)=13062249+9236430\sqrt{2}+7541505\sqrt{3}+5332635\sqrt{6}$ has minimal polynomial $T^4-52248996T^3-2201460444T^2+106073419104T+51728341176$ and

 $k = \sqrt[3]{t} \approx 373.8460244232724683270$ is not in \mathcal{K}_{O}° .

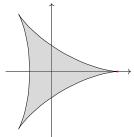
$$\begin{split} \kappa &= \sqrt{t} \approx 313.6400244232124083210 \text{ is not in } \mathcal{K}_Q. \\ \text{We have } \nu(t) &= \frac{27\sqrt{\frac{3}{2}}}{2\pi^2}L(\Theta_{Q,\tau_0},2), \text{ where} \\ \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)2nq^{72m^2+n^2} \\ &= 4q - 8q^4 + 16q^{16} - 20q^{25} + 28q^{49} - 32q^{64} + 8q^{73} - 16q^{76} + 32q^{88} - 40q^{97} + \cdots \\ \text{is in } \mathcal{S}_2\left(\Gamma_0(288), \left(\frac{864}{2}\right)\right). \end{split}$$

Let
$$\tau_0 = [30, 0, 1] = \frac{i}{\sqrt{20}}$$
 with $h(D) = 4$. Then

u-T4-5-1 (#56 in the paper) Let $\tau_0 = [30, 0, 1] = \frac{i}{\sqrt{30}}$ with h(D) = 4. Then $t = t_Q(\tau_0) = 24084 - 17010\sqrt{2} - 10692\sqrt{5} + 7560\sqrt{10}$

has minimal polynomial $T^4 - 96336T^3 + 36613296T^2 - 1836660096T + 24794911296$ and

 $k = \sqrt[3]{t} \approx 3.000281435776754682265$ is not in \mathcal{K}_O° .



We have
$$\nu(t) = \frac{9\sqrt{\frac{5}{2}}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)60nq^{3m^2+10n^2}$$

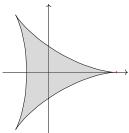
$$= 120q^{10} + 240q^{13} + 240q^{22} + 240q^{37} - 240q^{40} - 480q^{43} - 480q^{52} + 240q^{58} - 480q^{67} + 240q^{85} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(360), \left(\frac{360}{5}\right)\right)$.

 $\nu\text{-}\mathrm{T}4\text{-}5\text{-}2$ (#56 in the paper)

Let
$$\tau_0 = [15, 0, 2] = i\sqrt{\frac{2}{15}}$$
 with $h(D) = 4$. Then

 $t=t_Q(\tau_0)=24084-17010\sqrt{2}+10692\sqrt{5}-7560\sqrt{10}$ has minimal polynomial $T^4-96336T^3+36613296T^2-1836660096T+24794911296$ and

 $k = \sqrt[3]{t} \approx 3.088022061272761611574$ is not in \mathcal{K}_O° .



We have
$$\nu(t) = \frac{9\sqrt{\frac{5}{2}}}{4\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)30nq^{6m^2+5n^2}$$

$$= 60q^5 + 120q^{11} - 120q^{20} - 240q^{26} + 120q^{29} - 240q^{44} + 120q^{59} - 240q^{74} + 240q^{80} + 480q^{86} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(360), \left(\frac{360}{2}\right)\right)$.

$\nu\text{-T}4\text{-}5\text{-}3$ (#56 in the paper)

Let
$$\tau_0 = [6, 0, 5] = i\sqrt{\frac{5}{6}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 24084 + 17010\sqrt{2} - 10692\sqrt{5} - 7560\sqrt{10}$$

has minimal polynomial $T^4 - 96336T^3 + 36613296T^2 - 1836660096T + 24794911296$ and

 $k = \sqrt[3]{t} \approx 6.874743269597869965463$ is not in \mathcal{K}_O° .

We have
$$\nu(t) = \frac{9\sqrt{\frac{5}{2}}}{4\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)12nq^{15m^2+2n^2}$$

$$= 24q^2 - 48q^8 + 48q^{17} - 96q^{23} + 96q^{32} + 192q^{47} - 120q^{50} + 48q^{62} - 240q^{65} - 96q^{68} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(360), \left(\frac{360}{2}\right)\right)$.

ν -T4-5-4 (#56 in the paper)

Let
$$\tau_0 = [3, 0, 10] = i\sqrt{\frac{10}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 24084 + 17010\sqrt{2} + 10692\sqrt{5} + 7560\sqrt{10}$$

 $t=t_Q(\tau_0)=24084+17010\sqrt{2}+10692\sqrt{5}+7560\sqrt{10}$ has minimal polynomial $T^4-96336T^3+36613296T^2-1836660096T+24794911296$ and

 $k = \sqrt[3]{t} \approx 45.78135537484650292819$ is not in \mathcal{K}_{Q}° .

We have
$$\nu(t) = \frac{9\sqrt{\frac{5}{2}}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)6nq^{30m^2+n^2}$$

$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 24q^{31} - 48q^{34} + 96q^{46} + 84q^{49} - 120q^{55} - 96q^{64} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(360), \left(\frac{360}{2}\right)\right)$.

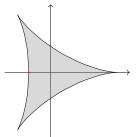
$\nu\text{-}\mathrm{T}4\text{-}6\text{-}1$ (#57 in the paper)

Let
$$\tau_0=[18,18,7]=-\frac{1}{2}+\frac{i\sqrt{5}}{6}$$
 with $h(D)=4$. Then $t=t_Q(\tau_0)=44+10\sqrt{3}-14\sqrt{5}-8\sqrt{15}$

$$t = t_O(\tau_0) = 44 + 10\sqrt{3} - 14\sqrt{5} - 8\sqrt{15}$$

has minimal polynomial $T^4 - 176T^3 + 7136T^2 - 80896T - 85184$ and

 $k = \sqrt[3]{t} \approx -0.9893232047286543709252$ is **IN** \mathcal{K}_O° .



We have
$$\nu(t) = \frac{\sqrt{15}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(54m+36n)q^{7m^2+6mn+2n^2}$$

$$= 72q^2 - 72q^3 - 144q^8 + 144q^{12} + 360q^{15} - 288q^{18} - 72q^{23} - 504q^{27} + 288q^{32} + 360q^{35} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(180), \left(\frac{60}{10}\right)\right)$.

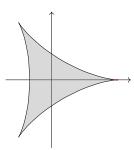
ν -T4-6-2 (#57 in the paper)

Let
$$\tau_0 = [27, 6, 2] = -\frac{1}{9} + \frac{i\sqrt{5}}{9}$$
 with $h(D) = 4$. Then

$$t = t_O(\tau_0) = 44 - 10\sqrt{3} - 14\sqrt{5} + 8\sqrt{15}$$

 $t=t_Q(\tau_0)=44-10\sqrt{3}-14\sqrt{5}+8\sqrt{15}$ has minimal polynomial $T^4-176T^3+7136T^2-80896T-85184$ and

 $k = \sqrt[3]{t} \approx 2.976046549728605636849$ is **IN** \mathcal{K}_Q° .



We have
$$\nu(t) = -\frac{\sqrt{15}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(18m + 54n)q^{2m^2 + 2mn + 3n^2}$$

$$= 180q^3 + 180q^7 - 180q^{10} - 360q^{12} + 180q^{15} - 360q^{18} + 180q^{27} - 360q^{28} + 360q^{40} + 360q^{42} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(180), \left(\frac{60}{\cdot}\right)\right)$.

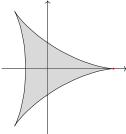
ν -T4-6-3 (#57 in the paper)

Let $\tau_0 = [45, 0, 1] = \frac{i}{3\sqrt{5}}$ with h(D) = 4. Then

 $t = t_Q(\tau_0) = 44 - 10\sqrt{3} + 14\sqrt{5} - 8\sqrt{15}$

has minimal polynomial $T^4 - 176T^3 + 7136T^2 - 80896T - 85184$ and

 $k = \sqrt[3]{t} \approx 3.000021364279298004162$ is not in \mathcal{K}_{O}° .



We have
$$\nu(t) = \frac{\sqrt{15}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)90nq^{m^2+5n^2}$
 $= 180q^5 + 360q^6 + 360q^9 + 360q^{14} - 360q^{20} - 360q^{21} - 720q^{24} - 720q^{29} + 360q^{30} - 720q^{36} + \cdots$ is in $S_2\left(\Gamma_0(180), \left(\frac{60}{2}\right)\right)$.

ν -T4-6-4 (#57 in the paper)

Let $\tau_0=[9,0,5]=\frac{i\sqrt{5}}{3}$ with h(D)=4. Then $t=t_Q(\tau_0)=44+10\sqrt{3}+14\sqrt{5}+8\sqrt{15}$ has minimal polynomial $T^4-176T^3+7136T^2-80896T-85184$ and

 $k = \sqrt[3]{t} \approx 4.981388495408017008014$ is not in \mathcal{K}_O° .

We have
$$\nu(t) = \frac{\sqrt{15}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n) 18nq^{5m^2+n^2}$$
$$= 36q - 72q^4 + 72q^6 - 144q^9 + 144q^{16} + 360q^{21} - 144q^{24} - 180q^{25} - 360q^{30} + 288q^{36} + \cdots$$
is in $S_2\left(\Gamma_0(180), \left(\frac{60}{2}\right)\right)$.

 $\nu\text{-}\mathrm{T}4\text{-}7\text{-}1$ (#58 in the paper)

Let $\tau_0 = [2, -2, 3] = \frac{1}{2} + \frac{i\sqrt{5}}{2}$ with h(D) = 2. Then

 $t = t_Q(\tau_0) = 315684 + 182250\sqrt{3} - 141426\sqrt{5} - 81648\sqrt{15}$ has minimal polynomial $T^4 - 1262736T^3 - 1357059744T^2 + 49698157824T - 45270270144$ and $k = \sqrt[3]{t} \approx -10.35160689062294407775$ is not in \mathcal{K}_Q° .

We have $\nu(t)=\frac{27\sqrt{15}}{8\pi^2}L(\Theta_{Q,\tau_0},2),$ where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-6m+4n)q^{27m^2-6mn+2n^2}$$

$$= 8q^2 - 16q^8 - 8q^{23} + 32q^{32} + 40q^{35} - 56q^{47} - 40q^{50} + 88q^{83} + 16q^{92} + 24q^{98} + \cdots$$

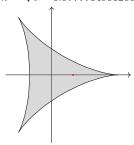
is in $S_2(\Gamma_0(180), (\frac{540}{.}))$.

ν -T4-7-2 (#58 in the paper)

Let $\tau_0 = [7, 6, 2] = -\frac{3}{7} + \frac{i\sqrt{5}}{7}$ with h(D) = 2. Then

 $t = t_Q(\tau_0) = 315684 - 182250\sqrt{3} - 141426\sqrt{5} + 81648\sqrt{15}$

has minimal polynomial $T^4 - 1262736T^3 - 1357059744T^2 + 49698157824T - 45270270144$ and $k = \sqrt[3]{t} \approx 0.9777714981286222747001$ is **IN** $\mathcal{K}_{\Omega}^{\circ}$.



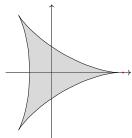
We have
$$\nu(t) = -\frac{27\sqrt{15}}{4\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where

$$\begin{array}{l} \Theta_{Q,\tau_0}(\tau) = \sum\limits_{m,n \in \mathbb{Z}} \chi_{-3}(n) (18m+14n) q^{18m^2+18mn+7n^2} \\ = 20q^7 - 20q^{10} - 40q^{28} + 40q^{40} + 20q^{43} + 80q^{58} - 100q^{67} - 40q^{82} - 100q^{103} + 80q^{112} + \cdots \\ \text{is in } \mathcal{S}_2\left(\Gamma_0(180),\left(\frac{540}{\cdot}\right)\right). \end{array}$$

ν -T4-7-3 (#58 in the paper)

Let $\tau_0 = [5, 0, 1] = \frac{i}{\sqrt{5}}$ with h(D) = 2. Then $t = t_Q(\tau_0) = 315684 - 182250\sqrt{3} + 141426\sqrt{5} - 81648\sqrt{15}$

has minimal polynomial $T^4 - 1262736T^3 - 1357059744T^2 + 49698157824T - 45270270144$ and $k = \sqrt[3]{t} \approx 3.256856697511690800059$ is not in \mathcal{K}_Q° .



We have
$$\nu(t) = \frac{27\sqrt{15}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)10nq^{9m^2+5n^2}$$

$$= 20q^5 + 40q^{14} - 40q^{20} - 80q^{29} + 40q^{41} - 80q^{56} + 80q^{80} + 40q^{86} + 160q^{89} - 80q^{101} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(180), \left(\frac{540}{2}\right)\right)$.

$\nu\text{-}\mathrm{T}4\text{-}7\text{-}4$ (#58 in the paper)

Let $\tau_0 = [1, 0, 5] = i\sqrt{5}$ with h(D) = 2. Then

 $t = t_Q(\tau_0) = 315684 + 182250\sqrt{3} + 141426\sqrt{5} + 81648\sqrt{15}$ has minimal polynomial $T^4 - 1262736T^3 - 1357059744T^2 + 49698157824T - 45270270144$ and

 $k = \sqrt[3]{t} \approx 108.1169786949826310030$ is not in \mathcal{K}_O° .

We have
$$\nu(t) = \frac{27\sqrt{15}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

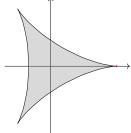
We have
$$\nu(t) = \frac{27\sqrt{15}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)2nq^{45m^2+n^2}$$

$$= 4q - 8q^4 + 16q^{16} - 20q^{25} + 8q^{46} + 12q^{49} + 32q^{61} - 32q^{64} - 40q^{70} + 56q^{94} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(180), \left(\frac{540}{2}\right)\right)$.

ν -T4-8-1 (#59 in the paper)

Let $\tau_0 = [60, 0, 1] = \frac{i}{2\sqrt{15}}$ with h(D) = 4. Then

 $t=t_Q(\tau_0)=2776869+\frac{3206385\sqrt{3}}{2}-1241838\sqrt{5}-\frac{1433943\sqrt{15}}{2}$ has minimal polynomial $T^4-11107476T^3+1591369821T^2-69739270326T+941480149401$ and $k = \sqrt[3]{t} \approx 3.000002430830552643351$ is not in \mathcal{K}_O° .



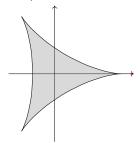
We have
$$\nu(t) = \frac{9\sqrt{5}}{4\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)120nq^{3m^2+20n^2}$$

$$= 240q^{20} + 480q^{23} + 480q^{32} + 480q^{47} + 480q^{68} - 480q^{80} - 960q^{83} - 960q^{92} + 480q^{95} - 960q^{107} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(720), \left(\frac{720}{2}\right)\right)$.

Let $\tau_0 = [15, 0, 4] = \frac{2i}{\sqrt{15}}$ with h(D) = 4. Then

 $t = t_Q(\tau_0) = 2776869 - \frac{3206385\sqrt{3}}{2} - 1241838\sqrt{5} + \frac{1433943\sqrt{15}}{2}$

has minimal polynomial $T^4 - 11107476T^3 + 1591369821T^2 - 69739270326T + 941480149401$ and $k = \sqrt[3]{t} \approx 3.493316486262842119150$ is not in $\mathcal{K}_{\mathcal{O}}^{\circ}$.



We have
$$\nu(t) = \frac{9\sqrt{5}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\begin{array}{l} \Theta_{Q,\tau_0}(\tau) = \sum\limits_{m,n\in\mathbb{Z}} \chi_{-3}(n) 30 n q^{12m^2+5n^2} \\ = 60 q^5 + 120 q^{17} - 120 q^{20} - 240 q^{32} + 120 q^{53} - 240 q^{68} + 240 q^{80} + 480 q^{92} + 120 q^{113} - 300 q^{125} + \cdots \\ \text{is in } \mathcal{S}_2\left(\Gamma_0(720),\left(\frac{720}{\epsilon}\right)\right). \end{array}$$

ν -T4-8-3 (#59 in the paper)

Let
$$\tau_0 = [12, 0, 5] = \frac{1}{2}i\sqrt{\frac{5}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 2776869 - \frac{3206385\sqrt{3}}{2} + 1241838\sqrt{5} - \frac{1433943\sqrt{15}}{2}$$

has minimal polynomial $T^4 - 11107476T^3 + 1591369821T^2 - 69739270326T + 941480149401$ and

 $k = \sqrt[3]{t} \approx 4.191546178842404673881$ is not in $\mathcal{K}_{\mathcal{O}}^{\circ}$.

We have
$$\nu(t) = \frac{9\sqrt{5}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 24nq^{15m^2 + 4n^2}$$

$$= 48q^4 - 96q^{16} + 96q^{19} - 192q^{31} + 288q^{64} - 192q^{76} + 384q^{79} - 240q^{100} - 480q^{115} + 384q^{124} + \cdots$$

is in
$$S_2\left(\Gamma_0(720), \left(\frac{720}{\cdot}\right)\right)$$
.

ν -T4-8-4 (#59 in the paper)

Let
$$\tau_0 = [3, 0, 20] = 2i\sqrt{\frac{5}{3}}$$
 with $h(D) = 4$. Then

$$t = t_O(\tau_0) = 2776869 + \frac{3206385\sqrt{3}}{2} + 1241838\sqrt{5} + \frac{1433943\sqrt{15}}{2}$$

 $t=t_Q(\tau_0)=2776869+\frac{3206385\sqrt{3}}{2}+1241838\sqrt{5}+\frac{1433943\sqrt{15}}{2}$ has minimal polynomial $T^4-11107476T^3+1591369821T^2-69739270326T+941480149401$ and

 $k = \sqrt[3]{t} \approx 223.1190200886623003702$ is not in $\mathcal{K}_{\mathcal{O}}^{\circ}$.

We have
$$\nu(t) = \frac{9\sqrt{5}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{9\sqrt{5}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)6nq^{60m^2+n^2}$$
$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} + 24q^{61} - 144q^{64} + 96q^{76} - 120q^{85} + 120q^{100} + \cdots$$

is in
$$S_2\left(\Gamma_0(720), \left(\frac{720}{\cdot}\right)\right)$$
.

ν -T4-9-1 (#60 in the paper)

Let
$$\tau_0 = [6, -6, 5] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{7}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 3672 + 2106\sqrt{3} - 1404\sqrt{7} - 810\sqrt{21}$$

 $t=t_Q(\tau_0)=3672+2106\sqrt{3}-1404\sqrt{7}-810\sqrt{21}$ has minimal polynomial $T^4-14688T^3-863136T^2+68024448T-918330048$ and

 $k = \sqrt[3]{t} \approx -4.744827653771052283184$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{9\sqrt{7}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-18m + 12n)q^{15m^2 - 6mn + 2n^2}$$

$$= 24q^{2} - 48q^{8} - 24q^{11} + 120q^{23} + 96q^{32} - 168q^{35} + 48q^{44} - 216q^{50} + 264q^{71} + 192q^{74} + \cdots$$

is in $S_2\left(\Gamma_0(252), \left(\frac{252}{\cdot}\right)\right)$.

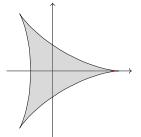
ν -T4-9-2 (#60 in the paper)

Let
$$\tau_0 = [15, 6, 2] = -\frac{1}{5} + \frac{1}{5}i\sqrt{\frac{7}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 3672 - 2106\sqrt{3} - 1404\sqrt{7} + 810\sqrt{21}$$

 $t=t_Q(\tau_0)=3672-2106\sqrt{3}-1404\sqrt{7}+810\sqrt{21}$ has minimal polynomial $T^4-14688T^3-863136T^2+68024448T-918330048$ and

 $k = \sqrt[3]{t} \approx 2.782909133110217016772$ is **IN** \mathcal{K}_Q° .



We have
$$\nu(t) = -\frac{9\sqrt{7}}{4\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (18m + 30n) q^{6m^2 + 6mn + 5n^2}$$

$$=84q^{5}-84q^{14}+84q^{17}-168q^{20}-168q^{38}+84q^{41}+168q^{56}+336q^{62}-168q^{68}+84q^{77}+\cdots$$

is in $S_2\left(\Gamma_0(252), \left(\frac{252}{\cdot}\right)\right)$.

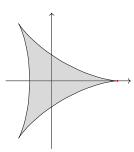
$\nu\text{-}\mathrm{T}4\text{-}9\text{-}3$ (#60 in the paper)

Let
$$\tau_0 = [21, 0, 1] = \frac{i}{\sqrt{21}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 3672 - 2106\sqrt{3} + 1404\sqrt{7} - 810\sqrt{21}$$

has minimal polynomial $T^4 - 14688T^3 - 863136T^2 + 68024448T - 918330048$ and

 $k = \sqrt[3]{t} \approx 3.001833215856110191252$ is not in \mathcal{K}_{O}° .



We have
$$\nu(t) = \frac{9\sqrt{7}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)42nq^{3m^2+7n^2}$$

$$= 84q^7 + 168q^{10} + 168q^{19} - 168q^{28} - 336q^{31} + 168q^{34} - 336q^{40} - 168q^{55} - 336q^{76} + 168q^{82} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(252), \left(\frac{252}{\cdot}\right)\right)$.

$\nu\text{-}\mathrm{T}4\text{-}9\text{-}4$ (#60 in the paper)

Let
$$\tau_0 = [3, 0, 7] = i\sqrt{\frac{7}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 3672 + 2106\sqrt{3} + 1404\sqrt{7} + 810\sqrt{21}$$

has minimal polynomial $T^4 - 14688T^3 - 863136T^2 + 68024448T - 918330048$ and

 $k=\sqrt[3]{t}\approx 24.52224560359002972316$ is not in $\mathcal{K}_Q^\circ.$

We have
$$\nu(t) = \frac{9\sqrt{7}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)6nq^{21m^2+n^2}$$
$$= 12q - 24q^4 + 48q^{16} + 24q^{22} - 108q^{25} + 96q^{37} - 120q^{46} + 84q^{49} - 96q^{64} + 168q^{70} + \cdots$$
is in $S_2\left(\Gamma_0(252), \left(\frac{252}{3}\right)\right)$.

ν -T4-10-1 (#61 in the paper)

Let
$$\tau_0 = [6, -6, 7] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{11}{3}}$$
 with $h(D) = 4$. Then

$$t = t_O(\tau_0) = 41904 - 24300\sqrt{3} - 12690\sqrt{11} + 7290\sqrt{33}$$

 $t = t_Q(\tau_0) = 41904 - 24300\sqrt{3} - 12690\sqrt{11} + 7290\sqrt{33}$ has minimal polynomial $T^4 - 167616T^3 - 57573504T^2 + 3353353344T - 45270270144$ and $k = \sqrt[3]{t} \approx -7.336871811011252320169$ is not in \mathcal{K}_Q° .

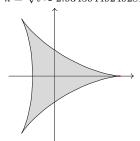
We have
$$\nu(t) = \frac{9\sqrt{11}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-18m+12n)q^{21m^2-6mn+2n^2}
= 24q^2 - 48q^8 - 24q^{17} + 120q^{29} + 96q^{32} - 168q^{41} - 120q^{50} + 48q^{68} - 96q^{74} + 264q^{77} + \cdots
\text{is in } \mathcal{S}_2\left(\Gamma_0(396), \left(\frac{396}{9}\right)\right).$$

Let
$$\tau_0 = [21, 6, 2] = -\frac{1}{7} + \frac{1}{7}i\sqrt{\frac{11}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 41904 - 24300\sqrt{3} + 12690\sqrt{11} - 7290\sqrt{33}$$

 $t=t_Q(\tau_0)=41904-24300\sqrt{3}+12690\sqrt{11}-7290\sqrt{33}$ has minimal polynomial $T^4-167616T^3-57573504T^2+3353353344T-45270270144$ and $k = \sqrt[3]{t} \approx 2.934594492492810551258$ is **IN** \mathcal{K}_{O}° .



We have
$$\nu(t) = -\frac{9\sqrt{11}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(18m + 42n)q^{6m^2 + 6mn + 7n^2}$$

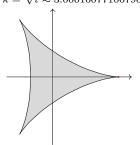
$$= 132q^7 + 132q^{19} - 132q^{22} - 264q^{28} + 132q^{43} - 264q^{46} - 264q^{76} + 132q^{79} + 264q^{88} + 528q^{94} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(396), \left(\frac{396}{2}\right)\right)$.

ν -T4-10-3 (#61 in the paper)

Let
$$\tau_0 = [33, 0, 1] = \frac{i}{\sqrt{33}}$$
 with $h(D) = 4$. Then

Let
$$\tau_0 = [33, 0, 1] = \frac{i}{\sqrt{33}}$$
 with $h(D) = 4$. Then $t = t_Q(\tau_0) = 41904 + 24300\sqrt{3} - 12690\sqrt{11} - 7290\sqrt{33}$

has minimal polynomial $T^4 - 167616T^3 - 57573504T^2 + 3353353344T - 45270270144$ and $k = \sqrt[3]{t} \approx 3.000160771067964258678$ is not in $\mathcal{K}_{\mathcal{O}}^{\circ}$.



We have
$$\nu(t) = \frac{9\sqrt{11}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)66nq^{3m^2+11n^2}$$

$$= 132q^{11} + 264q^{14} + 264q^{23} + 264q^{38} - 264q^{44} - 528q^{47} - 528q^{56} + 264q^{59} - 528q^{71} + 264q^{86} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(396), \left(\frac{396}{9}\right)\right)$.

ν -T4-10-4 (#61 in the paper)

Let
$$\tau_0 = [3, 0, 11] = i\sqrt{\frac{11}{3}}$$
 with $h(D) = 4$. Then

$$t = t_O(\tau_0) = 41904 + 24300\sqrt{3} + 12690\sqrt{11} + 7290\sqrt{33}$$

 $t=t_Q(\tau_0)=41904+24300\sqrt{3}+12690\sqrt{11}+7290\sqrt{33}$ has minimal polynomial $T^4-167616T^3-57573504T^2+3353353344T-45270270144$ and

 $k = \sqrt[3]{t} \approx 55.17395774420300781742$ is not in \mathcal{K}_{O}°

We have
$$\nu(t) = \frac{9\sqrt{11}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where

We have
$$\nu(t)=\frac{9\sqrt{11}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where $\Theta_{Q,\tau_0}(\tau)=\sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)6nq^{33m^2+n^2}$

$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 24q^{34} - 48q^{37} + 180q^{49} - 120q^{58} - 96q^{64} + 168q^{82} + \cdots$$

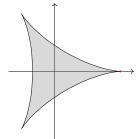
is in $S_2\left(\Gamma_0(396), \left(\frac{396}{\cdot}\right)\right)$.

$\nu\text{-T}4\text{-}11\text{-}1$ (#62 in the paper)

Let
$$\tau_0 = [78, 0, 1] = \frac{i}{\sqrt{78}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 26990604 - 19085220\sqrt{2} + 7484400\sqrt{13} - 5292270\sqrt{26}$$

has minimal polynomial $T^4 - 107962416T^3 + 1129077404496T^2 - 60812770640256T + 820972403643456$ and $k = \sqrt[3]{t} \approx 3.000000250111302002348$ is not in \mathcal{K}_Q° .



We have
$$\nu(t) = \frac{9\sqrt{\frac{13}{2}}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)156nq^{3m^2+26n^2}$

 $= 312q^{26} + 624q^{29} + 624q^{38} + 624q^{53} + 624q^{74} + 624q^{101} - 624q^{104} - 1248q^{107} - 1248q^{116} - 1248q^{131} + \cdots$

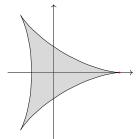
is in $S_2(\Gamma_0(936), (\frac{936}{.}))$.

$\nu\text{-}\mathrm{T}4\text{-}11\text{-}2\ (\#62\ \mathrm{in\ the\ paper})$

Let
$$\tau_0 = [39, 0, 2] = i\sqrt{\frac{2}{39}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 26990604 - 19085220\sqrt{2} - 7484400\sqrt{13} + 5292270\sqrt{26}$$

 $t = t_Q(\tau_0) = 26990604 - 19085220\sqrt{2} - 7484400\sqrt{13} + 5292270\sqrt{26}$ has minimal polynomial $T^4 - 107962416T^3 + 1129077404496T^2 - 60812770640256T + 820972403643456$ and $k = \sqrt[3]{t} \approx 3.002599405003644741088$ is not in \mathcal{K}_Q° .



We have
$$\nu(t) = \frac{9\sqrt{\frac{13}{2}}}{4\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where

We have
$$\nu(t) = \frac{9\sqrt{\frac{13}{2}}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)78nq^{6m^2+13n^2}$

$$= 156q^{13} + 312q^{19} + 312q^{37} - 312q^{52} - 624q^{58} + 312q^{67} - 624q^{76} - 624q^{106} + 312q^{109} - 624q^{148} + \cdots$$

is in $S_2\left(\Gamma_0(936), \left(\frac{936}{\cdot}\right)\right)$.

$\nu\text{-T}4\text{-}11\text{-}3~(\#62~\text{in the paper})$

Let
$$\tau_0 = [6, 0, 13] = i\sqrt{\frac{13}{6}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 26990604 + 19085220\sqrt{2} - 7484400\sqrt{13} - 5292270\sqrt{26}$$

has minimal polynomial $T^4 - 107962416T^3 + 1129077404496T^2 - 60812770640256T + 820972403643456$ and $k = \sqrt[3]{t} \approx 21.83135346928720715226$ is not in \mathcal{K}_Q° .

We have
$$\nu(t)=\frac{9\sqrt{\frac{13}{2}}}{4\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau)=\sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)12nq^{39m^2+2n^2}$$

$$=24q^2-48q^8+96q^{32}+48q^{41}-96q^{47}-120q^{50}+192q^{71}-240q^{89}+168q^{98}-192q^{128}+\cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(936),\left(\frac{936}{2}\right)\right)$.

ν -T4-11-4 (#62 in the paper)

Let
$$\tau_0 = [3, 0, 26] = i\sqrt{\frac{26}{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = 26990604 + 19085220\sqrt{2} + 7484400\sqrt{13} + 5292270\sqrt{26}$

has minimal polynomial $T^4 - 107962416T^3 + 1129077404496T^2 - 60812770640256T + 820972403643456$ and

 $k = \sqrt[3]{t} \approx 476.1496906574249660186$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{9\sqrt{\frac{13}{2}}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)6nq^{78m^2+n^2}$

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)6nq^{78m^2 + n^2}$$

$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 24q^{79} - 48q^{82} + 96q^{94} + 120q^{100} + \cdots$$

is in $S_2(\Gamma_0(936), (\frac{936}{2}))$.

$\nu\text{-}\mathrm{T}4\text{-}12\text{-}1$ (#63 in the paper)

Let
$$\tau_0 = [3, -3, 17] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{65}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -560736 - 250776\sqrt{5} - 155520\sqrt{13} - 69552\sqrt{65}$$

has minimal polynomial $T^4 + 2242944T^3 - 57573504T^2 - 161243136T + 2176782336$ and

 $k = \sqrt[3]{t} \approx -130.9004481970993878129$ is not in \mathcal{K}_{O}° .

We have
$$\nu(t) = \frac{9\sqrt{65}}{16-2}L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{9\sqrt{65}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-9m+6n)q^{51m^2-3mn+n^2}$

$$=12q-24q^4+48q^{16}-60q^{25}+72q^{49}+60q^{55}-84q^{61}-96q^{64}+132q^{79}-156q^{91}+\cdots$$

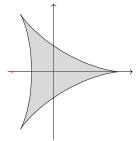
is in $S_2(\Gamma_0(585), (\frac{585}{5}))$.

ν -T4-12-2 (#63 in the paper)

Let
$$\tau_0 = [15, 15, 7] = -\frac{1}{2} + \frac{1}{2}i\sqrt{\frac{13}{15}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -560736 - 250776\sqrt{5} + 155520\sqrt{13} + 69552\sqrt{65}$$

has minimal polynomial $T^4+2242944T^3-57573504T^2-161243136T+2176782336$ and $k=\sqrt[3]{t}\approx -1.885002489933845045468$ is not in \mathcal{K}_Q° .



We have
$$\nu(t) = \frac{9\sqrt{65}}{16\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(45m + 30n)q^{21m^2 + 15mn + 5n^2}$$

$$= 60q^{5} - 60q^{11} - 120q^{20} + 300q^{41} + 120q^{44} - 240q^{59} - 420q^{71} + 240q^{80} - 60q^{89} + 780q^{119} + \cdots$$

is in $S_2(\Gamma_0(585), (\frac{585}{4}))$.

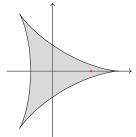
ν -T4-12-3 (#63 in the paper)

Let
$$\tau_0 = [21, 15, 5] = -\frac{5}{14} + \frac{1}{14}i\sqrt{\frac{65}{3}}$$
 with $h(D) = 4$. Then

$$t - t_0(\tau_0) = -560736 \pm 250776\sqrt{5} \pm 155520\sqrt{13} = 60552\sqrt{65}$$

 $t = t_Q(\tau_0) = -560736 + 250776\sqrt{5} + 155520\sqrt{13} - 69552\sqrt{65}$ has minimal polynomial $T^4 + 2242944T^3 - 57573504T^2 - 161243136T + 2176782336$ and

 $k = \sqrt[3]{t} \approx 1.750783430031650074781$ is **IN** \mathcal{K}_O° .



We have
$$\nu(t) = -\frac{9\sqrt{65}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = -\frac{9\sqrt{90}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(45m + 42n)q^{15m^2 + 15mn + 7n^2}$

$$=78q^{7} - 78q^{13} - 156q^{28} + 78q^{37} + 156q^{52} + 312q^{67} - 156q^{73} - 390q^{85} + 78q^{97} + 312q^{112} + \cdots$$

is in $S_2(\Gamma_0(585), (\frac{585}{.}))$.

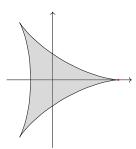
ν -T4-12-4 (#63 in the paper)

Let
$$\tau_0 = [51, 3, 1] = -\frac{1}{34} + \frac{1}{34}i\sqrt{\frac{65}{3}}$$
 with $h(D) = 4$. Then

$$t = t_{\rm O}(\tau_0) = -560736 + 250776\sqrt{5} - 155520\sqrt{13} + 69552\sqrt{65}$$

 $t=t_Q(\tau_0)=-560736+250776\sqrt{5}-155520\sqrt{13}+69552\sqrt{65}$ has minimal polynomial $T^4+2242944T^3-57573504T^2-161243136T+2176782336$ and

 $k = \sqrt[3]{t} \approx 2.999987962483975845344$ is **IN** \mathcal{K}_{O}° .



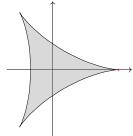
We have
$$\nu(t) = -\frac{9\sqrt{65}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(9m+102n)q^{3m^2+3mn+17n^2}$$

$$= 390q^{17} + 390q^{23} + 390q^{35} + 390q^{53} - 390q^{65} - 780q^{68} - 390q^{77} - 780q^{92} + 390q^{107} - 780q^{113} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(585), \left(\frac{585}{5}\right)\right)$.

$\nu\text{-}\mathrm{T}4\text{-}13\text{-}1$ (#64 in the paper)

Let $\tau_0 = [102, 0, 1] = \frac{i}{\sqrt{102}}$ with h(D) = 4. Then

 $t = t_Q(\tau_0) = 383973804 - 271496610\sqrt{2} - 93127320\sqrt{17} + 65847600\sqrt{34}$ has minimal polynomial $T^4 - 1535895216T^3 + 60296198732496T^2 - 3253755396329856T + 43925697850453056$ and $k = \sqrt[3]{t} \approx 3.000000017579773802155$ is not in \mathcal{K}_O° .



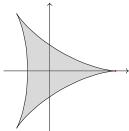
We have
$$\nu(t) = \frac{9\sqrt{\frac{17}{2}}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)204nq^{3m^2+34n^2}$$

$$= 408q^{34} + 816q^{37} + 816q^{46} + 816q^{61} + 816q^{82} + 816q^{109} - 816q^{136} - 1632q^{139} + 816q^{142} - 1632q^{148} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(1224),\left(\frac{1224}{2}\right)\right)$.

ν -T4-13-2 (#64 in the paper)

Let $\tau_0 = [51, 0, 2] = i\sqrt{\frac{2}{51}}$ with h(D) = 4. Then

 $t = t_Q(\tau_0) = 383973804 + 271496610\sqrt{2} - 93127320\sqrt{17} - 65847600\sqrt{34}$ has minimal polynomial $T^4 - 1535895216T^3 + 60296198732496T^2 - 3253755396329856T + 43925697850453056$ and $k = \sqrt[3]{t} \approx 3.000689004041877768997$ is not in $\mathcal{K}_{\mathcal{O}}^{\circ}$.



We have
$$\nu(t) = \frac{9\sqrt{\frac{17}{4\pi^2}}}{4\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)102nq^{6m^2+17n^2}$$

$$= 204q^{17} + 408q^{23} + 408q^{41} - 408q^{68} + 408q^{71} - 816q^{74} - 816q^{92} + 408q^{113} - 816q^{122} - 816q^{164} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(1224), (\frac{1224}{3})\right)$.

$\nu\text{-T}4\text{-}13\text{-}3$ (#64 in the paper)

Let
$$\tau_0 = [6, 0, 17] = i\sqrt{\frac{17}{6}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = 383973804 - 271496610\sqrt{2} + 93127320\sqrt{17} - 65847600\sqrt{34}$ has minimal polynomial $T^4 - 1535895216T^3 + 60296198732496T^2 - 3253755396329856T + 43925697850453056$ and $k = \sqrt[3]{t} \approx 33.97142910118434756295$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{9\sqrt{\frac{17}{4\pi^2}}}{4\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)12nq^{51m^2+2n^2}$$

$$= 24q^2 - 48q^8 + 96q^{32} - 120q^{50} + 48q^{53} - 96q^{59} + 192q^{83} + 168q^{98} - 240q^{101} - 192q^{128} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(1224),\left(\frac{1224}{2}\right)\right)$.

ν -T4-13-4 (#64 in the paper)

Let $\tau_0 = [3, 0, 34] = i\sqrt{\frac{34}{3}}$ with h(D) = 4. Then

 $t = t_Q(\tau_0) = 383973804 + 271496610\sqrt{2} + 93127320\sqrt{17} + 65847600\sqrt{34}$

has minimal polynomial $T^4 - 1535895216T^3 + 60296198732496T^2 - 3253755396329856T + 43925697850453056$ and

 $k = \sqrt[3]{t} \approx 1153.763588107886395599$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{9\sqrt{\frac{17}{4\pi^2}}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)6nq^{102m^2+n^2}$
 $= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 120q^{100} + 24q^{103} - 48q^{106} + 96q^{118} + \cdots$ is in $S_2\left(\Gamma_0(1224), \left(\frac{1224}{2}\right)\right)$.

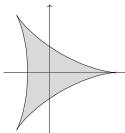
ν -T4-14-1 (#65 in the paper)

Let
$$\tau_0 = [42, 0, 1] = \frac{i}{\sqrt{42}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 196452 - 80190\sqrt{6} - 52380\sqrt{14} + 42768\sqrt{21}$$

has minimal polynomial $T^4 - 785808T^3 + 750216816T^2 - 39366000000T + 531441000000$ and

 $k = \sqrt[3]{t} \approx 3.000034402170826129903$ is not in \mathcal{K}_O° .



We have
$$\nu(t) = \frac{9\sqrt{\frac{7}{2}}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)84nq^{3m^2+14n^2}$$

$$= 168q^{14} + 336q^{17} + 336q^{26} + 336q^{41} - 336q^{56} - 672q^{59} + 336q^{62} - 672q^{68} - 672q^{83} + 336q^{89} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(504), \left(\frac{504}{2}\right)\right)$.

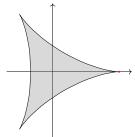
$\nu\text{-}\mathrm{T}4\text{-}14\text{-}2$ (#65 in the paper)

Let
$$\tau_0 = [21, 0, 2] = i\sqrt{\frac{2}{21}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 196452 - 80190\sqrt{6} + 52380\sqrt{14} - 42768\sqrt{21}$$

 $t = t_Q(\tau_0) = 196452 - 80190\sqrt{6} + 52380\sqrt{14} - 42768\sqrt{21}$ has minimal polynomial $T^4 - 785808T^3 + 750216816T^2 - 39366000000T + 531441000000$ and

 $k = \sqrt[3]{t} \approx 3.030580680157893721226$ is not in $\mathcal{K}_{\mathcal{O}}^{\circ}$.



We have
$$\nu(t) = \frac{9\sqrt{\frac{7}{2}}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)42nq^{6m^2+7n^2}$
 $= 84q^7 + 168q^{13} - 168q^{28} + 168q^{31} - 336q^{34} - 336q^{52} + 168q^{61} - 336q^{82} + 168q^{103} + 336q^{112} + \cdots$ is in $\mathcal{S}_2\left(\Gamma_0(504), \left(\frac{504}{\cdot}\right)\right)$.

ν -T4-14-3 (#65 in the paper)

Let
$$\tau_0 = [6, 0, 7] = i\sqrt{\frac{7}{6}}$$
 with $h(D) = 4$. Then

$$t = t_O(\tau_0) = 196452 + 80190\sqrt{6} - 52380\sqrt{14} - 42768\sqrt{21}$$

 $t=t_Q(\tau_0)=196452+80190\sqrt{6}-52380\sqrt{14}-42768\sqrt{21}$ has minimal polynomial $T^4-785808T^3+750216816T^2-39366000000T+531441000000$ and

 $k = \sqrt[3]{t} \approx 9.658365615485717586681$ is not in \mathcal{K}_{Q}° .

We have
$$\nu(t) = \frac{9\sqrt{\frac{7}{2}}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)12nq^{21m^2+2n^2}$$
$$= 24q^2 - 48q^8 + 48q^{23} - 96q^{29} + 96q^{32} - 120q^{50} + 192q^{53} - 240q^{71} + 48q^{86} - 96q^{92} + \cdots$$

is in $S_2\left(\Gamma_0(504), \left(\frac{504}{\cdot}\right)\right)$.

$\nu\text{-}\mathrm{T}4\text{-}14\text{-}4$ (#65 in the paper)

Let
$$\tau_0 = [3, 0, 14] = i\sqrt{\frac{14}{3}}$$
 with $h(D) = 4$. Then

$$t = t_O(\tau_0) = 196452 + 80190\sqrt{6} + 52380\sqrt{14} + 42768\sqrt{21}$$

 $t = t_Q(\tau_0) = 196452 + 80190\sqrt{6} + 52380\sqrt{14} + 42768\sqrt{21}$ has minimal polynomial $T^4 - 785808T^3 + 750216816T^2 - 39366000000T + 531441000000$ and $k = \sqrt[3]{t} \approx 92.24212347817280636972$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{9\sqrt{\frac{t}{2}}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t)=\frac{9\sqrt{\frac{7}{2}}}{4\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where $\Theta_{Q,\tau_0}(\tau)=\sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)6nq^{42m^2+n^2}$

$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 24q^{43} - 48q^{46} + 84q^{49} + 96q^{58} - 96q^{64} - 120q^{67} + \cdots$$

is in $S_2(\Gamma_0(504), (\frac{504}{\cdot}))$.

$\nu\text{-}\mathrm{T}4\text{-}15\text{-}1$ (#66 in the paper)

Let
$$\tau_0 = [6, -6, 11] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{19}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 1840752 - 1063530\sqrt{3} - 422604\sqrt{19} + 243810\sqrt{57}$$

has minimal polynomial $T^4 - 7363008T^3 - 19484198784T^2 + 1062882000000T - 14348907000000$ and $k = \sqrt[3]{t} \approx -13.92303144171231222418$ is not in \mathcal{K}_{Q}° .

We have $\nu(t) = \frac{9\sqrt{19}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$, where

We have
$$\nu(t) = \frac{1}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-18m+12n)q^{33m^2-6mn+2n^2}$$
$$= 24q^2 - 48q^8 - 24q^{29} + 96q^{32} + 120q^{41} - 120q^{50} - 168q^{53} + 264q^{89} + 168q^{98} - 312q^{113} + \cdots$$

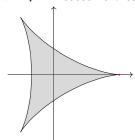
is in $S_2(\Gamma_0(684), (\frac{684}{.}))$.

$\overline{\nu}$ -T4-15-2 (#66 in the paper)

Let
$$\tau_0 = [33, 6, 2] = -\frac{1}{11} + \frac{1}{11}i\sqrt{\frac{19}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 1840752 + 1063530\sqrt{3} - 422604\sqrt{19} - 243810\sqrt{57}$$

has minimal polynomial $T^4 - 7363008T^3 - 19484198784T^2 + 1062882000000T - 14348907000000$ and $k = \sqrt[3]{t} \approx 2.990062462715962105781$ is **IN** \mathcal{K}_Q° .



We have
$$\nu(t) = -\frac{9\sqrt{19}}{4\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(18m + 66n)q^{6m^2 + 6mn + 11n^2}$$

$$= 228q^{11} + 228q^{23} - 228q^{38} - 456q^{44} + 228q^{47} - 456q^{62} + 228q^{83} - 456q^{92} + 228q^{131} - 456q^{134} + \cdots$$

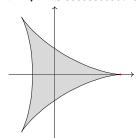
is in $S_2\left(\Gamma_0(684), \left(\frac{684}{\cdot}\right)\right)$.

ν -T4-15-3 (#66 in the paper)

Let
$$\tau_0 = [57, 0, 1] = \frac{i}{\sqrt{57}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 1840752 - 1063530\sqrt{3} + 422604\sqrt{19} - 243810\sqrt{57}$$

has minimal polynomial $T^4 - 7363008T^3 - 19484198784T^2 + 1062882000000T - 14348907000000$ and $k = \sqrt[3]{t} \approx 3.000003665671602468610$ is not in \mathcal{K}_{O}° .



We have
$$\nu(t) = \frac{9\sqrt{19}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 114nq^{3m^2 + 19n^2}$$

 $=228q^{19}+456q^{22}+456q^{31}+456q^{46}+456q^{67}-456q^{76}-912q^{79}-912q^{88}+456q^{94}-912q^{103}+\cdots$ is in $S_2(\Gamma_0(684), (\frac{684}{.}))$

ν -T4-15-4 (#66 in the paper)

Let
$$\tau_0 = [3, 0, 19] = i\sqrt{\frac{19}{3}}$$
 with $h(D) = 4$. Then

$$t = t_{\rm O}(\tau_0) = 1840752 + 1063530\sqrt{3} + 422604\sqrt{19} + 243810\sqrt{57}$$

 $t = t_Q(\tau_0) = 1840752 + 1063530\sqrt{3} + 422604\sqrt{19} + 243810\sqrt{57}$ has minimal polynomial $T^4 - 7363008T^3 - 19484198784T^2 + 1062882000000T - 14348907000000$ and

 $k = \sqrt[3]{t} \approx 194.5675559011432169189$ is not in \mathcal{K}_{Q}° .

We have
$$\nu(t) = \frac{9\sqrt{19}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{9\sqrt{19}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)6nq^{57m^2+n^2}$$
$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} + 24q^{58} - 48q^{61} - 96q^{64} + 96q^{73} - 120q^{82} + \cdots$$

is in $S_2(\Gamma_0(684), (\frac{684}{}))$.

$\nu\text{-}\mathrm{T}4\text{-}16\text{-}1$ (#67 in the paper)

Let
$$\tau_0 = [9, -9, 11] = \frac{1}{2} + \frac{i\sqrt{35}}{6}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -108 - 52\sqrt{5} - 28\sqrt{21} - 12\sqrt{105}$$

has minimal polynomial $T^4 + 432T^3 - 20224T^2 + 230400T - 512000$ and

 $k = \sqrt[3]{t} \approx -7.805469883833490659350$ is not in \mathcal{K}_{Q}° .

We have
$$\nu(t) = \frac{\sqrt{105}}{16-2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{\sqrt{105}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n) (-27m + 18n) q^{11m^2 - 3mn + n^2}$$
$$= 36q - 72q^4 - 36q^9 + 180q^{15} + 144q^{16} - 252q^{21} - 180q^{25} + 72q^{36} + 252q^{39} + 252q^{49} + \cdots$$

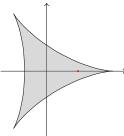
$\nu\text{-}\mathrm{T}4\text{-}16\text{-}2$ (#67 in the paper)

Let
$$\tau_0 = [27, 21, 7] = -\frac{7}{18} + \frac{i\sqrt{35}}{18}$$
 with $h(D) = 4$. Then

$$t = t_O(\tau_0) = -108 + 52\sqrt{5} - 28\sqrt{21} + 12\sqrt{105}$$

 $t=t_Q(\tau_0)=-108+52\sqrt{5}-28\sqrt{21}+12\sqrt{105}$ has minimal polynomial $T^4+432T^3-20224T^2+230400T-512000$ and

 $k = \sqrt[3]{t} \approx 1.430426554208694631300$ is **IN** \mathcal{K}_{O}° .



We have
$$\nu(t) = -\frac{\sqrt{105}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(63m + 54n)q^{7m^2 + 7mn + 3n^2}$$
$$= 90q^3 - 90q^5 - 180q^{12} + 90q^{17} + 180q^{20} + 360q^{27} - 630q^{33} + 90q^{45} - 450q^{47} + 360q^{48} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(315), \left(\frac{105}{5}\right)\right)$.

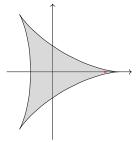
ν -T4-16-3 (#67 in the paper)

Let
$$\tau_0 = [27, 15, 5] = -\frac{5}{18} + \frac{i\sqrt{35}}{18}$$
 with $h(D) = 4$. Then

$$t = t_{\rm O}(\tau_0) = -108 + 52\sqrt{5} + 28\sqrt{21} - 12\sqrt{105}$$

 $t = t_Q(\tau_0) = -108 + 52\sqrt{5} + 28\sqrt{21} - 12\sqrt{105}$ has minimal polynomial $T^4 + 432T^3 - 20224T^2 + 230400T - 512000$ and

 $k = \sqrt[3]{t} \approx 2.388383978853552004590$ is **IN** \mathcal{K}_Q° .



We have
$$\nu(t) = -\frac{\sqrt{105}}{2}L(\Theta_{O,\tau_0},2)$$
, where

We have
$$\nu(t) = -\frac{\sqrt{105}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(45m + 54n)q^{5m^2 + 5mn + 3n^2}$$
$$= 126q^3 - 126q^7 - 252q^{12} + 126q^{13} - 252q^{27} + 252q^{28} + 630q^{33} - 630q^{45} + 504q^{48} - 252q^{52} + \cdots$$

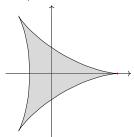
is in $S_2(\Gamma_0(315), (\frac{105}{.}))$.

$\nu\text{-}\mathrm{T}4\text{-}16\text{-}4$ (#67 in the paper)

Let
$$\tau_0 = [81, 3, 1] = -\frac{1}{54} + \frac{i\sqrt{35}}{54}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -108 - 52\sqrt{5} + 28\sqrt{21} + 12\sqrt{105}$$

has minimal polynomial $T^4+432T^3-20224T^2+230400T-512000$ and $k=\sqrt[3]{t}\approx 2.999999771121820929736$ is IN \mathcal{K}_Q° .



We have
$$\nu(t) = -\frac{\sqrt{105}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(9m+162n)q^{m^2+mn+9n^2}$$

$$=630q^9+630q^{11}+630q^{15}+630q^{21}+630q^{29}-630q^{35}-1260q^{36}-630q^{39}-1260q^{44}-630q^{51}+\cdots$$

is in $S_2(\Gamma_0(315), (\frac{105}{2}))$

ν -T4-17-1 (#68 in the paper)

Let
$$\tau_0 = [1, -1, 9] = \frac{1}{2} + \frac{i\sqrt{35}}{2}$$
 with $h(D) = 2$. Then

$$t = t_Q(\tau_0) = -29491668 - 13189068\sqrt{5} - 6435612\sqrt{21} - 2878092\sqrt{105}$$

has minimal polynomial $T^4 + 117966672T^3 + 617538816T^2 - 82133222400T - 272097792000$ and

 $k = \sqrt[3]{t} \approx -490.4406237403115043826$ is not in \mathcal{K}_{O}°

We have
$$\nu(t) = \frac{27\sqrt{105}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\begin{aligned} \kappa &= \sqrt{t} \approx -490.4406231403115043826 \text{ is not in } \mathcal{K}_{\widetilde{Q}}. \\ \text{We have } \nu(t) &= \frac{27\sqrt{105}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2), \text{ where} \\ \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-3m+2n) q^{81m^2-3mn+n^2} \\ &= 4q - 8q^4 + 16q^{16} - 20q^{25} + 28q^{49} - 32q^{64} - 4q^{79} + 20q^{85} - 28q^{91} + 40q^{100} + \cdots \end{aligned}$$

is in $S_2\left(\Gamma_0(315), \left(\frac{945}{\cdot}\right)\right)$.

ν -T4-17-2 (#68 in the paper)

Let
$$\tau_0 = [5, -5, 3] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{7}{5}}$$
 with $h(D) = 2$. Then

$$t = t_O(\tau_0) = -29491668 + 13189068\sqrt{5} - 6435612\sqrt{21} + 2878092\sqrt{105}$$

 $t=t_Q(\tau_0)=-29491668+13189068\sqrt{5}-6435612\sqrt{21}+2878092\sqrt{105}$ has minimal polynomial $T^4+117966672T^3+617538816T^2-82133222400T-272097792000$ and

 $k = \sqrt[3]{t} \approx -3.018463787303855973129$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{27\sqrt{105}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-15m + 10n) q^{27m^2 - 15mn + 5n^2}$$

$$=20q^{5}-20q^{17}-40q^{20}+100q^{47}+40q^{68}-140q^{77}+80q^{80}-80q^{83}-100q^{125}+140q^{143}+\cdots$$

is in $S_2(\Gamma_0(315), (\frac{945}{.}))$.

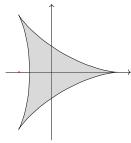
ν -T4-17-3 (#68 in the paper)

Let
$$\tau_0 = [7, 7, 3] = -\frac{1}{2} + \frac{1}{2}i\sqrt{\frac{5}{7}}$$
 with $h(D) = 2$. Then

$$t = t_Q(\tau_0) = -29491668 + 13189068\sqrt{5} + 6435612\sqrt{21} - 2878092\sqrt{105}$$

 $t=t_Q(\tau_0)=-29491668+13189068\sqrt{5}+6435612\sqrt{21}-2878092\sqrt{105}$ has minimal polynomial $T^4+117966672T^3+617538816T^2-82133222400T-272097792000$ and

 $k = \sqrt[3]{t} \approx -1.486194642718856814680$ is not in \mathcal{K}_{O}° .



We have
$$\nu(t) = \frac{27\sqrt{105}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(21m + 14n)q^{27m^2 + 21mn + 7n^2}$$

$$= 28q^7 - 28q^{13} - 56q^{28} + 56q^{52} + 140q^{55} - 112q^{73} - 196q^{97} - 28q^{103} + 112q^{112} + 140q^{145} + \cdots$$

is in $S_2\left(\Gamma_0(315), \left(\frac{945}{\cdot}\right)\right)$.

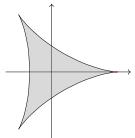
$\nu\text{-}\mathrm{T}4\text{-}17\text{-}4$ (#68 in the paper)

Let
$$\tau_0 = [11, 3, 1] = -\frac{3}{22} + \frac{i\sqrt{35}}{22}$$
 with $h(D) = 2$. Then

Let
$$\tau_0 = [11, 3, 1] = -\frac{3}{22} + \frac{i\sqrt{35}}{22}$$
 with $h(D) = 2$. Then $t = t_Q(\tau_0) = -29491668 - 13189068\sqrt{5} + 6435612\sqrt{21} + 2878092\sqrt{105}$

has minimal polynomial $T^4 + 117966672T^3 + 617538816T^2 - 82133222400T - 272097792000$ and

 $k = \sqrt[3]{t} \approx 2.945282170334217170434$ is **IN** $\mathcal{K}_{\Omega}^{\circ}$.



We have
$$\nu(t) = -\frac{27\sqrt{105}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 22n)q^{9m^2 + 9mn + 11n^2}$$

$$= 70q^{11} + 70q^{29} - 70q^{35} - 140q^{44} + 70q^{65} - 140q^{71} - 140q^{116} + 70q^{119} + 140q^{140} + 280q^{149} + \cdots$$

is in $S_2(\Gamma_0(315), (\frac{945}{.}))$.

ν -T4-18-1 (#69 in the paper)

Let
$$\tau_0 = [3, -3, 37] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{145}{3}}$$
 with $h(D) = 4$. Then

$$t = t_0(\tau_0) = -764507376 - 341898084\sqrt{5} - 141965460\sqrt{29} - 63488880\sqrt{145}$$

 $t = t_Q(\tau_0) = -764507376 - 341898084\sqrt{5} - 141965460\sqrt{29} - 63488880\sqrt{145}$ has minimal polynomial $T^4 + 3058029504T^3 + 56747272896T^2 - 7522959753216T + 101559956668416$ and

 $k = \sqrt[3]{t} \approx -1451.489469928476265374$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{9\sqrt{145}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{n,n \in \mathbb{Z}} \chi_{-3}(n)(-9m + 6n)q^{111m^2 - 3mn + n^2}$$

$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 120q^{100} - 12q^{109} + 60q^{115} - 216q^{121} + \cdots$$

is in $S_2(\Gamma_0(1305), (\frac{1305}{...}))$

ν -T4-18-2 (#69 in the paper)

Let
$$\tau_0 = [15, -15, 11] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{29}{15}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -764507376 + 341898084\sqrt{5} - 141965460\sqrt{29} + 63488880\sqrt{145}$$

has minimal polynomial $T^4 + 3058029504T^3 + 56747272896T^2 - 7522959753216T + 101559956668416$ and $k = \sqrt[3]{t} \approx -4.012422491869414401188$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{9\sqrt{145}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-45m + 30n)q^{33m^2 - 15mn + 5n^2}$$

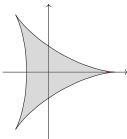
$$=60q^5-120q^{20}-60q^{23}+300q^{53}+240q^{80}-420q^{83}+120q^{92}-240q^{107}-300q^{125}+480q^{167}+\cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(1305),\left(\frac{1305}{\cdot}\right)\right)$.

ν -T4-18-3 (#69 in the paper)

Let
$$\tau_0 = [33, 15, 5] = -\frac{5}{22} + \frac{1}{22}i\sqrt{\frac{145}{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = -764507376 - 341898084\sqrt{5} + 141965460\sqrt{29} + 63488880\sqrt{145}$

has minimal polynomial $T^4 + 3058029504T^3 + 56747272896T^2 - 7522959753216T + 101559956668416$ and $k = \sqrt[3]{t} \approx 2.670334723648260851547$ is **IN** \mathcal{K}_{Q}° .



We have
$$\nu(t) = -\frac{9\sqrt{145}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

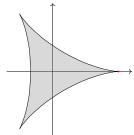
We have
$$\nu(t) = -\frac{1}{8\pi^2} L(OQ, \tau_0, 2)$$
, where $\Theta_{Q, \tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (45m + 66n) q^{15m^2 + 15mn + 11n^2}$

$$= 174q^{11} - 174q^{29} + 174q^{41} - 348q^{44} - 348q^{89} + 174q^{101} + 348q^{116} + 696q^{131} - 348q^{164} + 696q^{176} + \cdots$$
is in $S_2\left(\Gamma_0(1305), \left(\frac{1305}{\cdot}\right)\right)$.

ν -T4-18-4 (#69 in the paper)

Let
$$\tau_0 = [111, 3, 1] = -\frac{1}{74} + \frac{1}{74}i\sqrt{\frac{145}{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = -764507376 + 341898084\sqrt{5} + 141965460\sqrt{29} - 63488880\sqrt{145}$ has minimal polynomial $T^4 + 3058029504T^3 + 56747272896T^2 - 7522959753216T + 101559956668416$ and $k = \sqrt[3]{t} \approx 2.999999991170784987653$ is **IN** \mathcal{K}_{O}° .



We have
$$\nu(t) = -\frac{9\sqrt{145}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 222n)q^{3m^2 + 3mn + 37n^2}$$

$$= 870q^{37} + 870q^{43} + 870q^{55} + 870q^{73} + 870q^{97} + 870q^{127} - 870q^{145} - 1740q^{148} - 1740q^{157} + 870q^{163} + \cdots$$
is in $S_2\left(\Gamma_0(1305), \left(\frac{1305}{\cdot}\right)\right)$.

ν -T4-19-1 (#70 in the paper)

Let
$$\tau_0 = [6, -6, 17] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{31}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 147786552 + 85324590\sqrt{3} - 26545428\sqrt{31} - 15326010\sqrt{93}$$

has minimal polynomial $T^4 - 591146208T^3 - 14332946052384T^2 + 774840978000000T - 10460353203000000$ and $k = \sqrt[3]{t} \approx -28.96428380372967473288$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{9\sqrt{31}}{2}L(\Theta_{O,\tau_0}, 2)$$
, where

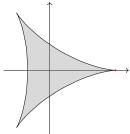
We have
$$\nu(t) = \frac{9\sqrt{31}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(-18m+12n)q^{51m^2-6mn+2n^2}$$
$$= 24q^2 - 48q^8 + 96q^{32} - 24q^{47} - 120q^{50} + 120q^{59} - 168q^{71} + 168q^{98} + 264q^{107} - 192q^{128} + \cdots$$
 is in \mathcal{S}_2 ($\Gamma_0(1116)$, ($\frac{1116}{9}$)).

ν -T4-19-2 (#70 in the paper)

Let
$$\tau_0 = [51, 6, 2] = -\frac{1}{17} + \frac{1}{17}i\sqrt{\frac{31}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 147786552 - 85324590\sqrt{3} - 26545428\sqrt{31} + 15326010\sqrt{93}$$

has minimal polynomial $T^4 - 591146208T^3 - 14332946052384T^2 + 774840978000000T - 10460353203000000$ and $k = \sqrt[3]{t} \approx 2.998889665517662550346$ is **IN** \mathcal{K}_{O}° .



We have
$$\nu(t) = -\frac{9\sqrt{31}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t)=-\frac{9\sqrt{31}}{4\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau)=\sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(18m+102n)q^{6m^2+6mn+17n^2}$$

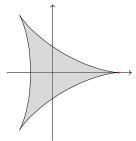
$$=372q^{17}+372q^{29}+372q^{53}-372q^{62}-744q^{68}-744q^{86}+372q^{89}-744q^{116}+372q^{137}-744q^{158}+\cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(1116),\left(\frac{1116}{2}\right)\right)$.

$\nu\text{-T}4\text{-}19\text{-}3$ (#70 in the paper)

Let $\tau_0 = [93, 0, 1] = \frac{i}{\sqrt{93}}$ with h(D) = 4. Then

 $t = t_Q(\tau_0) = 147786552 - 85324590\sqrt{3} + 26545428\sqrt{31} - 15326010\sqrt{93}$

has minimal polynomial $T^4 - 591146208T^3 - 14332946052384T^2 + 774840978000000T - 10460353203000000$ and $k = \sqrt[3]{t} \approx 3.000000045672108076038$ is not in \mathcal{K}_O° .



We have
$$\nu(t) = \frac{9\sqrt{31}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)186nq^{3m^2+31n^2}$$

$$= 372q^{31} + 744q^{34} + 744q^{43} + 744q^{58} + 744q^{79} + 744q^{106} - 744q^{124} - 1488q^{127} - 1488q^{136} + 744q^{139} + \cdots$$
is in $S_2\left(\Gamma_0(1116), \left(\frac{1116}{9}\right)\right)$.

ν -T4-19-4 (#70 in the paper)

Let
$$\tau_0 = [3, 0, 31] = i\sqrt{\frac{31}{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = 147786552 + 85324590\sqrt{3} + 26545428\sqrt{31} + 15326010\sqrt{93}$ has minimal polynomial $T^4 - 591146208T^3 - 14332946052384T^2 + 774840978000000T - 10460353203000000$ and $k = \sqrt[3]{t} \approx 839.2749095354222491264$ is not in \mathcal{K}_{Q}° .

We have
$$\nu(t) = \frac{9\sqrt{31}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)6nq^{93m^2+n^2}$$
$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 24q^{94} - 48q^{97} + 120q^{100} + 96q^{109} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(1116), \left(\frac{1116}{t}\right)\right)$.

$\nu\text{-}\mathrm{T}4\text{-}20\text{-}1$ (#71 in the paper)

Let
$$\tau_0 = [3, -3, 47] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{185}{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = -12945890016 - 5789578032\sqrt{5} - 2128291200\sqrt{37} - 951800760\sqrt{185}$

has minimal polynomial $T^4 + 51783560064T^3 + 3891696679296T^2 - 285652051255296T + 3856302691946496$ and $k = \sqrt[3]{t} \approx -3727.325334919065611177$ is not in \mathcal{K}_{O}° .

We have
$$\nu(t) = \frac{9\sqrt{185}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-9m+6n)q^{141m^2-3mn+n^2}$$
$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 120q^{100} - 132q^{121} - 12q^{139} + 60q^{145} + \cdots$$
is in $S_2\left(\Gamma_0(1665), \left(\frac{1665}{1665}\right)\right)$.

is in $S_2(\Gamma_0(1665), (\frac{1665}{.}))$.

ν -T4-20-2 (#71 in the paper)

Let
$$\tau_0 = [15, -15, 13] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{37}{15}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = -12945890016 - 5789578032\sqrt{5} + 2128291200\sqrt{37} + 951800760\sqrt{185}$ has minimal polynomial $T^4 + 51783560064T^3 + 3891696679296T^2 - 285652051255296T + 3856302691946496$ and $k = \sqrt[3]{t} \approx -4.991131435766682204562$ is not in \mathcal{K}_{O}° .

We have $\nu(t) = \frac{9\sqrt{185}}{16\pi^2}L(\Theta_{Q,\tau_0},2)$, where

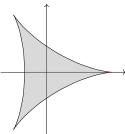
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-45m + 30n)q^{39m^2 - 15mn + 5n^2}$$

$$= 60q^5 - 120q^{20} - 60q^{29} + 300q^{59} + 240q^{80} - 420q^{89} + 120q^{116} - 300q^{125} - 240q^{131} + 660q^{179} + \cdots$$
is in $S_2\left(\Gamma_0(1665), \left(\frac{1665}{1}\right)\right)$.

$\nu\text{-}\mathrm{T}4\text{-}20\text{-}3$ (#71 in the paper)

Let
$$\tau_0 = [39, 15, 5] = -\frac{5}{26} + \frac{1}{26}i\sqrt{\frac{185}{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = -12945890016 + 5789578032\sqrt{5} + 2128291200\sqrt{37} - 951800760\sqrt{185}$ has minimal polynomial $T^4 + 51783560064T^3 + 3891696679296T^2 - 285652051255296T + 3856302691946496$ and $k = \sqrt[3]{t} \approx 2.809783185779822031690$ is **IN** \mathcal{K}_{O}° .



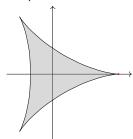
We have
$$\nu(t) = -\frac{9\sqrt{185}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\begin{split} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (45m + 78n) q^{15m^2 + 15mn + 13n^2} \\ &= 222q^{13} - 222q^{37} + 222q^{43} - 444q^{52} - 444q^{97} + 222q^{103} + 444q^{148} + 888q^{163} - 444q^{172} + 222q^{193} + \cdots \\ \text{is in } \mathcal{S}_2\left(\Gamma_0(1665), \left(\frac{1665}{\cdot}\right)\right). \end{split}$$

ν -T4-20-4 (#71 in the paper)

Let
$$\tau_0 = [141, 3, 1] = -\frac{1}{94} + \frac{1}{94}i\sqrt{\frac{185}{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = -12945890016 + 5789578032\sqrt{5} - 2128291200\sqrt{37} + 951800760\sqrt{185}$ has minimal polynomial $T^4 + 51783560064T^3 + 3891696679296T^2 - 285652051255296T + 3856302691946496$ and $k = \sqrt[3]{t} \approx 2.99999999478598999442$ is **IN** \mathcal{K}_{Q}° .



We have
$$\nu(t) = -\frac{9\sqrt{185}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(9m+282n)q^{3m^2+3mn+47n^2}$$

$$= 1110q^{47}+1110q^{53}+1110q^{65}+1110q^{83}+1110q^{107}+1110q^{137}+1110q^{173}-1110q^{185}-2220q^{188}-2220q^{197}+\cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(1665),\left(\frac{1665}{\cdot}\right)\right)$.

$-\frac{1}{\nu - T_{4-21-1}}$ (#72 in the paper)

Let
$$\tau_0 = [3, -3, 67] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{265}{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = -1663957782000 - 744144542460\sqrt{5} - 228562179300\sqrt{53} - 102216114000\sqrt{265}$ has minimal polynomial $T^4 + 6655831128000T^3 + 1997074895544000T^2 - 117546246144000000T + 1586874322944000000$ and $k = \sqrt[3]{t} \approx -18810.51841576643192570$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{9\sqrt{265}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-9m+6n)q^{201m^2-3mn+n^2}$$

$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 120q^{100} - 132q^{121} + 156q^{169} - 168q^{196} + \cdots$$

is in $S_2(\Gamma_0(2385), (\frac{2385}{.}))$.

ν -T4-21-2 (#72 in the paper)

Let
$$\tau_0 = [15, -15, 17] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{53}{15}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = -1663957782000 - 744144542460\sqrt{5} + 228562179300\sqrt{53} + 102216114000\sqrt{265}$ has minimal polynomial $T^4 + 6655831128000T^3 + 1997074895544000T^2 - 117546246144000000T + 1586874322944000000$ and $k = \sqrt[3]{t} \approx -7.061539506556527015363$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{9\sqrt{265}}{16\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-45m + 30n) q^{51m^2 - 15mn + 5n^2}$$

$$= 60q^5 - 120q^{20} - 60q^{41} + 300q^{71} + 240q^{80} - 420q^{101} - 300q^{125} + 120q^{164} - 240q^{179} + 660q^{191} + \cdots$$

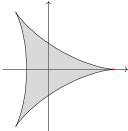
$$\vdots \quad \vdots \quad C_{-1}(n) (2085) (2385)$$

is in $S_2(\Gamma_0(2385), (\frac{2385}{.}))$.

$\nu\text{-T}4\text{-}21\text{-}3$ (#72 in the paper)

Let
$$\tau_0 = [51, 15, 5] = -\frac{5}{34} + \frac{1}{34}i\sqrt{\frac{265}{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = -1663957782000 + 744144542460\sqrt{5} - 228562179300\sqrt{53} + 102216114000\sqrt{265}$ has minimal polynomial $T^4 + 6655831128000T^3 + 1997074895544000T^2 - 117546246144000000T + 1586874322944000000$ and $k = \sqrt[3]{t} \approx 2.927022752085963462815$ is **IN** \mathcal{K}_{O}° .



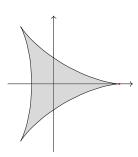
We have
$$\nu(t) = -\frac{9\sqrt{265}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = -\frac{9\sqrt{265}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(45m+102n)q^{15m^2+15mn+17n^2}$$
$$= 318q^{17} + 318q^{47} - 318q^{53} - 636q^{68} + 318q^{107} - 636q^{113} - 636q^{188} + 318q^{197} + 636q^{212} + 1272q^{227} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(2385), \left(\frac{2385}{5}\right)\right)$.

ν -T4-21-4 (#72 in the paper)

Let
$$\tau_0 = [201, 3, 1] = -\frac{1}{134} + \frac{1}{134}i\sqrt{\frac{265}{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = -1663957782000 + 744144542460\sqrt{5} + 228562179300\sqrt{53} - 102216114000\sqrt{265}$ has minimal polynomial $T^4 + 6655831128000T^3 + 1997074895544000T^2 - 117546246144000000T + 1586874322944000000$ and



We have $\nu(t) = -\frac{9\sqrt{265}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$, where

We have
$$\nu(t) = -\frac{1}{8\pi^2} L(Q_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(9m + 402n)q^{3m^2 + 3mn + 67n^2}$
 $= 1590q^{67} + 1590q^{73} + 1590q^{85} + 1590q^{103} + 1590q^{127} + 1590q^{157} + 1590q^{193} + 1590q^{235} - 1590q^{265} - 3180q^{268} + \cdots$ is in $S_2\left(\Gamma_0(2385), \left(\frac{2385}{5}\right)\right)$.

$\nu\text{-}\mathrm{T}4\text{-}22\text{-}1$ (#73 in the paper)

Let
$$\tau_0 = [6, -6, 31] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{59}{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = 315624577104 - 182226258900\sqrt{3} - 41090893110\sqrt{59} + 23723795970\sqrt{177}$ has minimal polynomial $T^4 - 1262498308416T^3 - 1418469875132085504T^2 + 76599213979666287744T - 1034089388725494884544$ and $k = \sqrt[3]{t} \approx -103.9608765556193856071$ is not in \mathcal{K}_Q° .

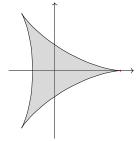
We have
$$\nu(t) = \frac{9\sqrt{59}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-18m+12n)q^{93m^2-6mn+2n^2}$$

$$= 24q^2 - 48q^8 + 96q^{32} - 120q^{50} - 24q^{89} + 168q^{98} + 120q^{101} - 168q^{113} - 192q^{128} + 264q^{149} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(2124), \left(\frac{2124}{5}\right)\right)$.

$\nu\text{-}\mathrm{T}4\text{-}22\text{-}2$ (#73 in the paper)

Let
$$\tau_0 = [93, 6, 2] = -\frac{1}{31} + \frac{1}{31}i\sqrt{\frac{59}{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = 315624577104 - 182226258900\sqrt{3} + 41090893110\sqrt{59} - 23723795970\sqrt{177}$ has minimal polynomial $T^4 - 1262498308416T^3 - 1418469875132085504T^2 + 76599213979666287744T - 1034089388725494884544$ and $k = \sqrt[3]{t} \approx 2.999975970374139770994$ is **IN** \mathcal{K}_{O}° .



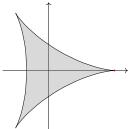
We have
$$\nu(t) = -\frac{9\sqrt{59}}{4\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(18m+186n)q^{6m^2+6mn+31n^2}$$

$$= 708q^{31} + 708q^{43} + 708q^{67} + 708q^{103} - 708q^{118} - 1416q^{124} - 1416q^{142} + 708q^{151} - 1416q^{172} + 708q^{211} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(2124), \left(\frac{2124}{\epsilon}\right)\right)$.

ν -T4-22-3 (#73 in the paper)

Let
$$\tau_0 = [177, 0, 1] = \frac{i}{\sqrt{177}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = 315624577104 + 182226258900\sqrt{3} - 41090893110\sqrt{59} - 23723795970\sqrt{177}$ has minimal polynomial $T^4 - 1262498308416T^3 - 1418469875132085504T^2 + 76599213979666287744T - 1034089388725494884544$ and $k = \sqrt[3]{t} \approx 3.000000000021386148237$ is not in \mathcal{K}_Q° .



We have
$$\nu(t) = \frac{9\sqrt{59}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)354nq^{3m^2+59n^2}$$

$$= 708q^{59} + 1416q^{62} + 1416q^{71} + 1416q^{86} + 1416q^{107} + 1416q^{134} + 1416q^{167} + 1416q^{206} - 1416q^{236} - 2832q^{239} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(2124), \left(\frac{2124}{2}\right)\right)$.

$\nu\text{-}\mathrm{T}4\text{-}22\text{-}4\ (\#73\ \mathrm{in\ the\ paper})$

Let
$$\tau_0 = [3, 0, 59] = i\sqrt{\frac{59}{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = 315624577104 + 182226258900\sqrt{3} + 41090893110\sqrt{59} + 23723795970\sqrt{177}$ has minimal polynomial $T^4 - 1262498308416T^3 - 1418469875132085504T^2 + 76599213979666287744T - 1034089388725494884544$ and $k = \sqrt[3]{t} \approx 10807.96004376491659607$ is not in \mathcal{K}_{Q}° .

We have
$$\nu(t) = \frac{9\sqrt{59}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)6nq^{177m^2+n^2}$$

$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 120q^{100} - 132q^{121} + 156q^{169} + 24q^{178} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(2124),\left(\frac{2124}{5}\right)\right)$.

$\nu\text{-}\mathrm{T}4\text{-}23\text{-}1$ (#74 in the paper)

Let
$$\tau_0 = [3, -3, 41] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{161}{3}}$$
 with $h(D) = 4$. Then

$$t = t_O(\tau_0) = -2471866848 - \frac{1}{539405568} \sqrt{21} - \frac{1}{297577800} \sqrt{69} - \frac{1}{194810400} \sqrt{161}$$

 $t = t_Q(\tau_0) = -2471866848 - 539405568\sqrt{21} - 297577800\sqrt{69} - 194810400\sqrt{161}$ has minimal polynomial $T^4 + 9887467392T^3 - 220305619584T^2 - 2519424000000T + 34012224000000$ and $k = \sqrt[3]{t} \approx -2146.322714573342218474$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{9\sqrt{161}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{9\sqrt{161}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-9m+6n)q^{123m^2-3mn+n^2}$$
$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 120q^{100} - 144q^{121} + 60q^{127} - 84q^{133} + \cdots$$

is in
$$S_2(\Gamma_0(1449), (\frac{1449}{.}))$$
.

ν -T4-23-2 (#74 in the paper)

Let
$$\tau_0 = [21, -21, 11] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{23}{21}}$$
 with $h(D) = 4$. Then

$$t = t_O(\tau_0) = -2471866848 + 539405568\sqrt{21} + 297577800\sqrt{69} - 194810400\sqrt{161}$$

 $t = t_Q(\tau_0) = -2471866848 + 539405568\sqrt{21} + 297577800\sqrt{69} - 194810400\sqrt{161}$ has minimal polynomial $T^4 + 9887467392T^3 - 220305619584T^2 - 2519424000000T + 34012224000000$ and

 $k = \sqrt[3]{t} \approx -2.403852339561353514411$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{9\sqrt{161}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{9\sqrt{161}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-63m + 42n)q^{33m^2 - 21mn + 7n^2}$$
$$= 84q^7 - 84q^{19} - 168q^{28} + 420q^{61} + 168q^{76} - 336q^{97} - 588q^{103} + 336q^{112} - 84q^{157} - 420q^{175} + \cdots$$

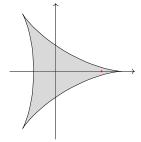
is in
$$S_2(\Gamma_0(1449), (\frac{1449}{2}))$$
.

$\nu\text{-}\mathrm{T}4\text{-}23\text{-}3$ (#74 in the paper)

Let
$$\tau_0 = [33, 21, 7] = -\frac{7}{22} + \frac{1}{22}i\sqrt{\frac{161}{3}}$$
 with $h(D) = 4$. Then

$$t = t_O(\tau_0) = -2471866848 + 539405568\sqrt{21} - 297577800\sqrt{69} + 194810400\sqrt{161}$$

 $t=t_Q(\tau_0)=-2471866848+539405568\sqrt{21}-297577800\sqrt{69}+194810400\sqrt{161}$ has minimal polynomial $T^4+9887467392T^3-220305619584T^2-2519424000000T+34012224000000$ and $k = \sqrt[3]{t} \approx 2.093249264186714483631$ is **IN** \mathcal{K}_Q° .



We have
$$\nu(t) = -\frac{9\sqrt{161}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where

We have
$$\nu(t) = -\frac{1}{8\pi^2} L(O_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (63m + 66n) q^{21m^2 + 21mn + 11n^2}$

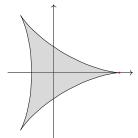
 $= 138q^{11} - 138q^{23} - 276q^{44} + 138q^{53} + 276q^{92} - 276q^{107} + 552q^{113} + 138q^{137} - 690q^{149} + 552q^{176} + \cdots$ is in $S_2(\Gamma_0(1449), (\frac{1449}{.}))$.

ν -T4-23-4 (#74 in the paper)

Let
$$\tau_0 = [123, 3, 1] = -\frac{1}{82} + \frac{1}{82}i\sqrt{\frac{161}{3}}$$
 with $h(D) = 4$. Then

$$t = t_{\mathcal{O}}(\tau_0) = -2471866848 - 539405568\sqrt{21} + 297577800\sqrt{69} + 194810400\sqrt{161}$$

 $t = t_Q(\tau_0) = -2471866848 - 539405568\sqrt{21} + 297577800\sqrt{69} + 194810400\sqrt{161}$ has minimal polynomial $T^4 + 9887467392T^3 - 220305619584T^2 - 2519424000000T + 34012224000000$ and $k = \sqrt[3]{t} \approx 2.999999997269270398621$ is **IN** \mathcal{K}_{O}° .



We have
$$\nu(t) = -\frac{9\sqrt{161}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = -\frac{9\sqrt{161}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(9m+246n)q^{3m^2+3mn+41n^2}$

 $=966q^{41}+966q^{47}+966q^{59}+966q^{77}+966q^{101}+966q^{131}-966q^{161}-1932q^{164}+966q^{167}-1932q^{173}+\cdots$

is in $S_2(\Gamma_0(1449), (\frac{1449}{.}))$.

ν -T4-24-1 (#75 in the paper)

Let
$$\tau_0 = [3, -3, 53] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{209}{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = -61082965296 - 10633179420\sqrt{33} - 8090636400\sqrt{57} - 4225197060\sqrt{209}$

has minimal polynomial $T^4 + 244331861184T^3 - 6584729661504T^2 - 660451885056T + 8916100448256$ and $k = \sqrt[3]{t} \approx -6251.631456432370137700$ is not in \mathcal{K}_{Q}° .

We have
$$\nu(t) = \frac{9\sqrt{209}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

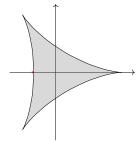
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-9m+6n)q^{159m^2-3mn+n^2}
= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 120q^{100} - 132q^{121} - 12q^{157} + 60q^{163} + \cdots
\text{is in } \mathcal{S}_2\left(\Gamma_0(1881), \left(\frac{1881}{\cdot}\right)\right).$$

 $\nu\text{-}\mathrm{T}4\text{-}24\text{-}2$ (#75 in the paper)

Let
$$\tau_0 = [33, 33, 13] = -\frac{1}{2} + \frac{1}{2}i\sqrt{\frac{19}{33}}$$
 with $h(D) = 4$. Then

$$t = t_O(\tau_0) = -61082965296 - 10633179420\sqrt{33} + 8090636400\sqrt{57} + 4225197060\sqrt{209}$$

 $t = t_Q(\tau_0) = -61082965296 - 10633179420\sqrt{33} + 8090636400\sqrt{57} + 4225197060\sqrt{209}$ has minimal polynomial $T^4 + 244331861184T^3 - 6584729661504T^2 - 660451885056T + 8916100448256$ and $k = \sqrt[3]{t} \approx -1.059062443227299151536$ is not in \mathcal{K}_{O}° .



We have
$$\nu(t) = \frac{9\sqrt{209}}{10^{-2}}L(\Theta_{O,\tau_0},2)$$
, where

We have
$$\nu(t) = \frac{9\sqrt{209}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(99m + 66n)q^{39m^2 + 33mn + 11n^2}$$

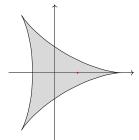
$$= 132q^{11} - 132q^{17} - 264q^{44} + 264q^{68} + 660q^{83} - 528q^{101} - 132q^{131} - 924q^{149} + 528q^{176} + 660q^{197} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(1881), \left(\frac{1881}{2}\right)\right)$.

ν -T4-24-3 (#75 in the paper)

Let
$$\tau_0 = [39, 33, 11] = -\frac{11}{26} + \frac{1}{26}i\sqrt{\frac{209}{3}}$$
 with $h(D) = 4$. Then

$$t = t_O(\tau_0) = -61082965296 + 10633179420\sqrt{33} + 8090636400\sqrt{57} - 4225197060\sqrt{209}$$

 $t=t_Q(\tau_0)=-61082965296+10633179420\sqrt{33}+8090636400\sqrt{57}-4225197060\sqrt{209}$ has minimal polynomial $T^4+244331861184T^3-6584729661504T^2-660451885056T+8916100448256$ and $k = \sqrt[3]{t} \approx 1.043971864736952165677$ is **IN** \mathcal{K}_{Q}°



We have
$$\nu(t) = -\frac{9\sqrt{209}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = -\frac{9\sqrt{209}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(99m + 78n)q^{33m^2 + 33mn + 13n^2}$$

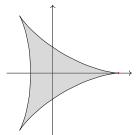
$$= 114q^{13} - 114q^{19} - 228q^{52} + 228q^{76} + 114q^{79} + 456q^{109} - 570q^{127} - 228q^{151} - 570q^{193} + 456q^{208} + \cdots$$
is in \mathcal{S}_2 $\left(\Gamma_0(1881), \left(\frac{1881}{2}\right)\right)$.

ν -T4-24-4 (#75 in the paper)

Let
$$\tau_0 = [159, 3, 1] = -\frac{1}{106} + \frac{1}{106}i\sqrt{\frac{209}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -61082965296 + 10633179420\sqrt{33} - 8090636400\sqrt{57} + 4225197060\sqrt{209}$$

 $t = t_Q(\tau_0) = -61082965296 + 10633179420\sqrt{33} - 8090636400\sqrt{57} + 4225197060\sqrt{209}$ has minimal polynomial $T^4 + 244331861184T^3 - 6584729661504T^2 - 660451885056T + 8916100448256$ and $k = \sqrt[3]{t} \approx 2.999999999889494559309$ is **IN** \mathcal{K}_{O}° .



We have
$$y(t) = -\frac{9\sqrt{209}}{2}L(\Theta_{\odot})$$
 where

We have
$$\nu(t) = -\frac{9\sqrt{209}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(9m+318n)q^{3m^2+3mn+53n^2}$$

$$= 1254q^{53}+1254q^{59}+1254q^{71}+1254q^{89}+1254q^{113}+1254q^{143}+1254q^{179}-1254q^{209}-2508q^{212}-1254q^{221}+\cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(1881),\left(\frac{1881}{\cdot}\right)\right)$.

Let
$$\tau_0 = [1, -1, 2] = \frac{1}{2} + \frac{i\sqrt{7}}{2}$$
 with $h(D) = 1$. Then

$$t = t_Q(\tau_0) = -\frac{4023}{4} - \frac{891\sqrt{21}}{4} - \frac{243\sqrt[4]{3}\sqrt{93\sqrt{3}+61\sqrt{7}}}{2\sqrt{2}}$$

has minimal polynomial $T^4 + 4023T^3 - 133407T^2 + 17458821T - 66430125$ and

 $k = \sqrt[3]{t} \approx -15.94898512139843890669$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{27\sqrt{21}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-3m+2n)q^{18m^2-3mn+n^2}$$

$$= 4q - 8q^4 + 12q^{16} + 20q^{22} - 20q^{25} - 28q^{28} + 44q^{46} + 28q^{49} - 52q^{58} - 24q^{64} + \cdots$$

is in $S_2(\Gamma_0(63), (\frac{189}{.}))$.

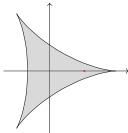
ν -N4-1-2

Let
$$\tau_0 = [4, 3, 1] = -\frac{3}{8} + \frac{i\sqrt{7}}{8}$$
 with $h(D) = 1$. Then

$$t = t_Q(\tau_0) = -\frac{4023}{4} - \frac{891\sqrt{21}}{4} + \frac{243\sqrt[4]{3}\sqrt{93\sqrt{3} + 61\sqrt{7}}}{2\sqrt{2}}$$

has minimal polynomial $T^4 + 4023T^3 - 133407T^2 + 17458821T - 66430125$ and

 $k = \sqrt[3]{t} \approx 1.575121578964678896813$ is **IN** \mathcal{K}_{Q}° .



We have
$$\nu(t) = -\frac{27\sqrt{21}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 8n)q^{9m^2 + 9mn + 4n^2}$$

$$=14q^{4}-14q^{7}-28q^{16}+14q^{22}+28q^{28}+56q^{37}-28q^{43}-70q^{46}+14q^{58}-14q^{64}+\cdots$$

is in $S_2(\Gamma_0(63), (\frac{189}{}))$

Let
$$\tau_0 = [2, 1, 1] = -\frac{1}{4} + \frac{i\sqrt{7}}{4}$$
 with $h(D) = 1$. Then

$$t = t_Q(\tau_0) = -\frac{4023}{4} + \frac{891\sqrt{21}}{4} + \frac{243i\sqrt[4]{3}\sqrt{-93\sqrt{3}+61\sqrt{7}}}{2\sqrt{2}}$$

has minimal polynomial $T^4 + 4023T^3 - 133407T^2 + 17458821T - 66430125$ and $k = \sqrt[3]{t} \approx 3.623187281867491681756 + 1.730348100185317455035i$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{27\sqrt{21}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t)=\frac{27\sqrt{21}}{16\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau)=\sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(3m+4n)q^{9m^2+3mn+2n^2}$$

$$= 8q^{2} - 14q^{8} - 10q^{11} + 14q^{14} - 22q^{23} + 26q^{29} + 24q^{32} + 6q^{44} - 40q^{50} + 38q^{53} + \cdots$$

is in $S_2\left(\Gamma_0(63), \left(\frac{189}{\cdot}\right)\right)$.

Let
$$\tau_0 = [2, -1, 1] = \frac{1}{4} + \frac{i\sqrt{7}}{4}$$
 with $h(D) = 1$. Then

$$t = t_0(\tau_0) = -\frac{4023}{4023} + \frac{891\sqrt{21}}{4023} - \frac{243i\sqrt[4]{3}\sqrt{-93\sqrt{3}+61\sqrt{7}}}{4023}$$

 $t=t_Q(\tau_0)=-\frac{4023}{4}+\frac{891\sqrt{21}}{4}-\frac{243i\sqrt[4]{3}\sqrt{-93\sqrt{3}+61\sqrt{7}}}{2\sqrt{2}}$ has minimal polynomial $T^4+4023T^3-133407T^2+17458821T-66430125$ and

 $k = \sqrt[3]{t} \approx 3.623187281867491681756 - 1.730348100185317455035i$ is not in \mathcal{K}_{O}°

We have
$$v(t) = \frac{27\sqrt{21}}{2}L(\Theta_{O_{1}}, z_{1}, z_{2})$$
, where

We have
$$\nu(t) = \frac{27\sqrt{21}}{16\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-3m+4n)q^{9m^2-3mn+2n^2}$

$$= 8q^{2} - 14q^{8} - 10q^{11} + 14q^{14} - 22q^{23} + 26q^{29} + 24q^{32} + 6q^{44} - 40q^{50} + 38q^{53} + \cdots$$

is in $S_2\left(\Gamma_0(63), \left(\frac{189}{\cdot}\right)\right)$.

ν -N4-2-1

Let
$$\tau_0 = [1, -1, 5] = \frac{1}{2} + \frac{i\sqrt{19}}{2}$$
 with $h(D) = 1$. Then

$$t = t_Q(\tau_0) = -221346 - 29322\sqrt{57} - 486\sqrt[4]{3}\sqrt{138318\sqrt{3} + 54962\sqrt{19}}$$

has minimal polynomial $T^4 + 885384T^3 - 71523648T^2 + 1944995328T - 17414258688$ and

 $k = \sqrt[3]{t} \approx -96.02635191438447995375$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{27\sqrt{57}}{16.2}L(\Theta_{Q,\tau_0},2)$$
, where

We have
$$\nu(t) = \frac{27\sqrt{57}}{16\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-3m+2n)q^{45m^2-3mn+n^2}$

$$= 4q - 8q^4 + 16q^{16} - 20q^{25} - 4q^{43} + 48q^{49} - 28q^{55} - 32q^{64} + 44q^{73} - 52q^{85} + \cdots$$

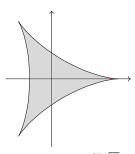
is in $S_2(\Gamma_0(171), (\frac{513}{.}))$.

Let
$$\tau_0 = [7, 3, 1] = -\frac{3}{14} + \frac{i\sqrt{19}}{14}$$
 with $h(D) = 1$. Then

$$t = t_O(\tau_0) = -221346 - 29322\sqrt{57} + 486\sqrt[4]{3}\sqrt{138318\sqrt{3} + 54962\sqrt{19}}$$

has minimal polynomial $T^4 + 885384T^3 - 71523648T^2 + 1944995328T - 17414258688$ and

 $k = \sqrt[3]{t} \approx 2.727345302759981770335$ is **IN** \mathcal{K}_{O}° .



We have $\nu(t) = -\frac{27\sqrt{57}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(9m+14n)q^{9m^2+9mn+7n^2}$$

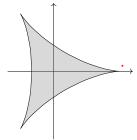
$$= 38q^7 - 38q^{19} + 38q^{25} - 76q^{28} - 76q^{55} + 38q^{61} + 76q^{76} + 152q^{85} - 76q^{100} + 152q^{112} + \cdots$$
is in $S_2\left(\Gamma_0(171), \left(\frac{513}{2}\right)\right)$.

 ν -N4-2-3

Let
$$\tau_0 = [5, 1, 1] = -\frac{1}{10} + \frac{i\sqrt{19}}{10}$$
 with $h(D) = 1$. Then

$$t = t_{O}(\tau_{0}) = -221346 + 29322\sqrt{57} + 486i\sqrt[4]{3}\sqrt{-138318\sqrt{3} + 54962\sqrt{19}}$$

Let $au_0 = [5,1,1] = -\frac{1}{10} + \frac{i\sqrt{19}}{10}$ with h(D) = 1. Then $t = t_Q(au_0) = -221346 + 29322\sqrt{57} + 486i\sqrt[4]{3}\sqrt{-138318\sqrt{3} + 54962\sqrt{19}}$ has minimal polynomial $T^4 + 885384T^3 - 71523648T^2 + 1944995328T - 17414258688$ and $k = \sqrt[3]{t} \approx 3.135915712951375547117 + 0.251102521170458487535i$ is not in \mathcal{K}_{O}° .



We have
$$\nu(t) = \frac{27\sqrt{57}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(3m+10n)q^{9m^2+3mn+5n^2}$$
$$= 20q^5 + 14q^{11} + 26q^{17} - 40q^{20} - 34q^{23} - 38q^{35} - 28q^{44} + 32q^{47} - 52q^{68} + 76q^{77} + \cdots$$

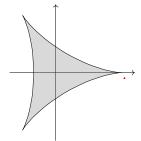
is in $S_2(\Gamma_0(171), (\frac{513}{\cdot}))$.

ν-N4-2-4

Let
$$\tau_0 = [5, -1, 1] = \frac{1}{10} + \frac{i\sqrt{19}}{10}$$
 with $h(D) = 1$. Then $t = t_Q(\tau_0) = -221346 + 29322\sqrt{57} - 486i\sqrt[4]{3}\sqrt{-138318\sqrt{3} + 54962\sqrt{19}}$

$$t = t_0(\tau_0) = -221346 + 29322\sqrt{57} - 486i\sqrt[4]{3}\sqrt{-138318\sqrt{3} + 54962\sqrt{19}}$$

has minimal polynomial $T^4 + 885384T^3 - 71523648T^2 + 1944995328T - 17414258688$ and $k = \sqrt[3]{t} \approx 3.135915712951375547117 - 0.251102521170458487535i$ is not in \mathcal{K}_{O}° .



We have
$$\nu(t) = \frac{27\sqrt{57}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-3m+10n)q^{9m^2-3mn+5n^2}$$
$$-20e^5 + 14e^{11} + 26e^{17} - 40e^{20} - 34e^{23} - 38e^{23}$$

$$= 20q^{5} + 14q^{11} + 26q^{17} - 40q^{20} - 34q^{23} - 38q^{35} - 28q^{44} + 32q^{47} - 52q^{68} + 76q^{77} + \cdots$$

is in $S_2\left(\Gamma_0(171), \left(\frac{513}{\cdot}\right)\right)$.

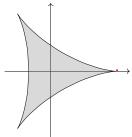
 ν -N4-3-1

Let
$$\tau_0 = [9, 1, 1] = -\frac{1}{18} + \frac{i\sqrt{35}}{18}$$
 with $h(D) = 2$. Then

$$t = t_Q(\tau_0) = 144 - 52\sqrt{5} - 80i\sqrt{7} + 36i\sqrt{35}$$

Let $\tau_0 = [9,1,1] = -\frac{1}{18} + \frac{i\sqrt{35}}{18}$ with h(D) = 2. Then $t = t_Q(\tau_0) = 144 - 52\sqrt{5} - 80i\sqrt{7} + 36i\sqrt{35}$ has minimal polynomial $T^4 - 576T^3 + 277696T^2 - 14155776T + 191102976$ and

 $k = \sqrt[3]{t} \approx 3.027355824977709699703 + 0.047968520678194350363i$ is not in \mathcal{K}_O° .



We have $\nu(t) = \frac{3\sqrt{105}}{16\pi^2}L(\Theta_{Q,\tau_0},2)$, where

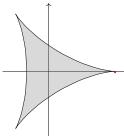
$$\begin{split} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (3m+18n) q^{3m^2+mn+3n^2} \\ &= 36q^3 + 30q^5 + 42q^7 - 72q^{12} - 42q^{13} - 30q^{17} - 60q^{20} + 18q^{27} - 84q^{28} - 90q^{45} + \cdots \\ \text{is in } \mathcal{S}_2\left(\Gamma_0(105), \left(\frac{105}{5}\right)\right). \end{split}$$

ν -N4-3-2

Let
$$\tau_0 = [9, -1, 1] = \frac{1}{18} + \frac{i\sqrt{35}}{18}$$
 with $h(D) = 2$. Then

$$t = t_O(\tau_0) = 144 - 52\sqrt{5} + 80i\sqrt{7} - 36i\sqrt{35}$$

Let $\tau_0 = [9, -1, 1] = \frac{1}{18} + \frac{i\sqrt{35}}{18}$ with h(D) = 2. Then $t = t_Q(\tau_0) = 144 - 52\sqrt{5} + 80i\sqrt{7} - 36i\sqrt{35}$ has minimal polynomial $T^4 - 576T^3 + 277696T^2 - 14155776T + 191102976$ and $k = \sqrt[3]{t} \approx 3.027355824977709699703 - 0.047968520678194350363i$ is not in \mathcal{K}_{O}° .



We have
$$\nu(t) = \frac{3\sqrt{105}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{3\sqrt{105}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-3m+18n)q^{3m^2-mn+3n^2}$$

$$= 36q^3 + 30q^5 + 42q^7 - 72q^{12} - 42q^{13} - 30q^{17} - 60q^{20} + 18q^{27} - 84q^{28} - 90q^{45} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(105), \left(\frac{105}{105}\right)\right)$.

ν-N4-3-3

Let
$$\tau_0 = [3, 1, 3] = -\frac{1}{6} + \frac{i\sqrt{35}}{6}$$
 with $h(D) = 2$. Then

$$t = t_O(\tau_0) = 144 + 52\sqrt{5} + 80i\sqrt{7} + 36i\sqrt{35}$$

 $t=t_Q(\tau_0)=144+52\sqrt{5}+80i\sqrt{7}+36i\sqrt{35}$ has minimal polynomial $T^4-576T^3+277696T^2-14155776T+191102976$ and

 $k = \sqrt[3]{t} \approx 7.472135954999579392818 + 2.645751311064590590502i$ is not in \mathcal{K}_O° .

We have
$$\nu(t) = \frac{3\sqrt{105}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$V(t) = \frac{16\pi^2}{16\pi^2} \frac{B(Q_Q, \tau_0, 2)}{B(Q_Q, \tau_0, 2)}$$
, where $\Theta_{Q, \tau_0}(\tau) = \sum_{m, n \in \mathbb{Z}} \chi_{-3}(n)(3m + 6n)q^{9m^2 + mn + n^2}$

$$= 12q - 24q^4 + 6q^9 - 30q^{15} + 48q^{16} + 42q^{21} - 60q^{25} - 12q^{36} - 42q^{39} + 84q^{49} + \cdots$$
is in $S_2\left(\Gamma_0(105), \left(\frac{105}{...}\right)\right)$.

ν -N4-3-4

Let
$$\tau_0 = [3, -1, 3] = \frac{1}{6} + \frac{i\sqrt{35}}{6}$$
 with $h(D) = 2$. Then

$$t = t_Q(\tau_0) = 144 + 52\sqrt{5} - 80i\sqrt{7} - 36i\sqrt{35}$$

Let $t_0 = [5, -1, 5] = \frac{1}{6} + \frac{1}{6} = \text{with } h(D) = 2$. Then $t = t_Q(\tau_0) = 144 + 52\sqrt{5} - 80i\sqrt{7} - 36i\sqrt{35}$ has minimal polynomial $T^4 - 576T^3 + 277696T^2 - 14155776T + 191102976$ and $k = \sqrt[3]{t} \approx 7.472135954999579392818 - 2.645751311064590590502i$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{3\sqrt{105}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{3\sqrt{105}}{16\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(-3m+6n)q^{9m^2-mn+n^2}$$
$$= 12q - 24q^4 + 6q^9 - 30q^{15} + 48q^{16} + 42q^{21} - 60q^{25} - 12q^{36} - 42q^{39} + 84q^{49} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(105), \left(\frac{105}{5}\right)\right)$.

ν-N4-4-1

Let
$$\tau_0 = [1, -1, 11] = \frac{1}{2} + \frac{i\sqrt{43}}{2}$$
 with $h(D) = 1$. Then

$$t = t_{\rm O}(\tau_0) = -221184162 - 19474182\sqrt{129} - 1458\sqrt{46028106018 + 4052548766\sqrt{129}}$$

 $t = t_Q(\tau_0) = -221184162 - 19474182\sqrt{129} - 1458\sqrt{46028106018 + 4052548766\sqrt{129}}$ has minimal polynomial $T^4 + 884736648T^3 - 71663476032T^2 + 1934927709696T - 17414258688000$ and

 $k = \sqrt[3]{t} \approx -960.0002636717221142411$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{27\sqrt{129}}{120}L(\Theta_{O,\tau_0}, 2)$$
, where

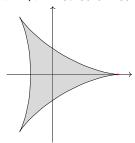
We have
$$\nu(t) = \frac{27\sqrt{129}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(-3m+2n)q^{99m^2-3mn+n^2}$
 $= 4q - 8q^4 + 16q^{16} - 20q^{25} + 28q^{49} - 32q^{64} - 4q^{97} + 40q^{100} + 20q^{103} - 28q^{109} + \cdots$ is in $\mathcal{S}_2\left(\Gamma_0(387), \left(\frac{1161}{2}\right)\right)$.

ν -N4-4-2

Let
$$\tau_0 = [13, 3, 1] = -\frac{3}{26} + \frac{i\sqrt{43}}{26}$$
 with $h(D) = 1$. Then

$$t = t_Q(\tau_0) = -221184162 - 19474182\sqrt{129} + 1458\sqrt{46028106018 + 4052548766\sqrt{129}}$$

 $t=t_Q(\tau_0)=-221184162-19474182\sqrt{129}+1458\sqrt{46028106018+4052548766\sqrt{129}}$ has minimal polynomial $T^4+884736648T^3-71663476032T^2+1934927709696T-17414258688000$ and $k = \sqrt[3]{t} \approx 2.971962624499130926145$ is **IN** \mathcal{K}_{O}° .



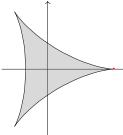
We have
$$\nu(t) = -\frac{27\sqrt{129}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\begin{array}{l} \Theta_{Q,\tau_0}(\tau) = \sum\limits_{m,n\in\mathbb{Z}} \chi_{-3}(n)(9m+26n)q^{9m^2+9mn+13n^2} \\ = 86q^{13} + 86q^{31} - 86q^{43} - 172q^{52} + 86q^{67} - 172q^{79} + 86q^{121} - 172q^{124} + 172q^{172} + 344q^{181} + \cdots \\ \text{is in } \mathcal{S}_2\left(\Gamma_0(387),\left(\frac{1161}{\cdot}\right)\right). \end{array}$$

ν -N4-4-3

Let
$$\tau_0 = [11, 1, 1] = -\frac{1}{22} + \frac{i\sqrt{43}}{22}$$
 with $h(D) = 1$. Then

Let $\tau_0 = [11, 1, 1] = -\frac{1}{22} + \frac{i\sqrt{43}}{22}$ with h(D) = 1. Then $t = t_Q(\tau_0) = -221184162 + 19474182\sqrt{129} + 1458i\sqrt{-46028106018 + 4052548766\sqrt{129}}$ has minimal polynomial $T^4 + 884736648T^3 - 71663476032T^2 + 1934927709696T - 17414258688000$ and $k = \sqrt[3]{t} \approx 3.014018275763481888956 + 0.024433072518158402813i$ is not in \mathcal{K}_Q° .

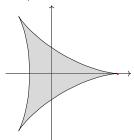


We have
$$\nu(t) = \frac{27\sqrt{129}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(3m+22n)q^{9m^2+3mn+11n^2}$$
$$= 44q^{11} + 38q^{17} + 50q^{23} + 32q^{41} - 88q^{44} - 82q^{47} + 56q^{53} - 94q^{59} - 76q^{68} + 26q^{83} + \cdots$$
is in $S_2\left(\Gamma_0(387), \left(\frac{1161}{\cdot}\right)\right)$.

ν-N4-4-4

Let
$$\tau_0 = [11, -1, 1] = \frac{1}{22} + \frac{i\sqrt{43}}{22}$$
 with $h(D) = 1$. Then

 $t = t_Q(\tau_0) = -221184162 + 19474182\sqrt{129} - 1458i\sqrt{-46028106018 + 4052548766\sqrt{129}}$ has minimal polynomial $T^4 + 884736648T^3 - 71663476032T^2 + 1934927709696T - 17414258688000$ and $k = \sqrt[3]{t} \approx 3.014018275763481888956 - 0.024433072518158402813i$ is not in \mathcal{K}_O°



We have
$$\nu(t) = \frac{27\sqrt{129}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-3m+22n)q^{9m^2-3mn+11n^2}$$

$$= 44q^{11} + 38q^{17} + 50q^{23} + 32q^{41} - 88q^{44} - 82q^{47} + 56q^{53} - 94q^{59} - 76q^{68} + 26q^{83} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(387), \left(\frac{1161}{110}\right)\right)$.

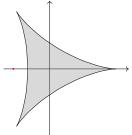
ν -N4-5-1

Let
$$\tau_0 = [9, 9, 4] = -\frac{1}{2} + \frac{i\sqrt{7}}{6}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 18 - \frac{3\sqrt{21}}{2} - \frac{9}{2}\sqrt[4]{3}\sqrt{\sqrt{3} + 2\sqrt{7}}$$

 $t=t_Q(\tau_0)=18-\frac{3\sqrt{21}}{2}-\frac{9}{2}\sqrt[4]{3}\sqrt{\sqrt{3}+2\sqrt{7}}$ has minimal polynomial $T^4-72T^3+1728T^2-10449T-91125$ and

 $k = \sqrt[3]{t} \approx -1.659386141097061711062$ is not in \mathcal{K}_{O}° .



We have
$$\nu(t) = \frac{\sqrt{21}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(27m+18n)q^{4m^2+3mn+n^2}$$

$$= 36q - 36q^2 - 72q^4 + 252q^8 - 144q^{11} - 252q^{14} + 108q^{16} + 180q^{22} + 288q^{23} - 180q^{25} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(63),\left(\frac{21}{2}\right)\right)$.

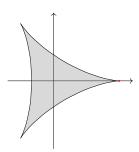
ν -N4-5-2

Let
$$\tau_0 = [18, 3, 1] = -\frac{1}{12} + \frac{i\sqrt{7}}{12}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 18 - \frac{3\sqrt{21}}{2} + \frac{9}{2}\sqrt[4]{3}\sqrt{\sqrt{3} + 2\sqrt{7}}$$

has minimal polynomial $T^{4} - 72T^{3} + 1728T^{2} - 10449T - 91125$ and

 $k = \sqrt[3]{t} \approx 2.993374122716392032259$ is **IN** $\mathcal{K}_{\mathcal{O}}^{\circ}$.



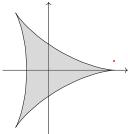
We have
$$\nu(t) = -\frac{\sqrt{21}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(9m+36n)q^{m^2+mn+2n^2}$$

$$= 126q^2 + 126q^4 - 126q^7 - 126q^8 - 252q^{11} + 126q^{14} - 252q^{16} + 126q^{22} - 252q^{23} + 252q^{28} + \cdots$$
is in $S_2\left(\Gamma_0(63), \left(\frac{21}{2}\right)\right)$.

Let
$$\tau_0 = [9,3,2] = -\frac{1}{6} + \frac{i\sqrt{7}}{6}$$
 with $h(D) = 4$. Then $t = t_Q(\tau_0) = 18 + \frac{3\sqrt{21}}{2} + \frac{9}{2}i\sqrt{-3 + 2\sqrt{21}}$ has minimal polynomial $T^4 - 72T^3 + 1728T^2 - 10449T - 91125$ and

$$t = t_Q(\tau_0) = 18 + \frac{3\sqrt{21}}{2} + \frac{9}{2}i\sqrt{-3 + 2\sqrt{21}}$$

 $k = \sqrt[3]{t} \approx 2.980143505070886883246 + 0.422186045999587410327i$ is not in \mathcal{K}_O° .



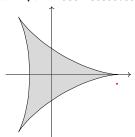
We have
$$\nu(t) = \frac{\sqrt{21}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(9m+18n)q^{2m^2+mn+n^2}$
 $= 36q+18q^2-72q^4-126q^8+72q^{11}+126q^{14}+108q^{16}+180q^{22}-144q^{23}-180q^{25}+\cdots$ is in $S_2\left(\Gamma_0(63),\left(\frac{21}{\epsilon}\right)\right)$.

Let
$$\tau_0 = [9, -3, 2] = \frac{1}{6} + \frac{i\sqrt{7}}{6}$$
 with $h(D) = 4$. Then $t = t_Q(\tau_0) = 18 + \frac{3\sqrt{21}}{2} - \frac{9}{2}i\sqrt{-3 + 2\sqrt{21}}$

$$t = t_O(\tau_0) = 18 + \frac{3\sqrt{21}}{2} - \frac{9}{2}i\sqrt{-3 + 2\sqrt{21}}$$

has minimal polynomial $T^{4} - 72T^{3} + 1728T^{2} - 10449T - 91125$ and

 $k = \sqrt[3]{t} \approx 2.980143505070886883246 - 0.422186045999587410327i$ is not in \mathcal{K}_{O}° .



We have
$$\nu(t) = \frac{\sqrt{21}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-9m+18n)q^{2m^2-mn+n^2}$$
$$= 36q+18q^2-72q^4-126q^8+72q^{11}+126q^{14}+108q^{16}+180q^{22}-144q^{23}-180q^{25}+\cdots$$

is in $S_2\left(\Gamma_0(63), \left(\frac{21}{\cdot}\right)\right)$.

Let
$$\tau_0 = [1, -1, 17] = \frac{1}{2} + \frac{i\sqrt{67}}{2}$$
 with $h(D) = 1$. Then

 $t = t_Q(\tau_0) = -36799488162 - 2595635766\sqrt{201} - 3402\sqrt{234015667197498 + 16506192489998\sqrt{201}}$ has minimal polynomial $T^4 + 147197952648T^3 - 11923033972032T^2 + 321921931101696T - 2897297289216000$ and $k=\sqrt[3]{t}\approx -\tilde{5280.000008716425589457}$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{27\sqrt{201}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-3m+2n)q^{153m^2-3mn+n^2}$$
$$= 4q - 8q^4 + 16q^{16} - 20q^{25} + 28q^{49} - 32q^{64} + 40q^{100} - 44q^{121} - 4q^{151} + 20q^{157} + \cdots$$

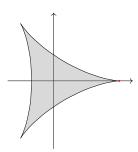
is in $S_2\left(\Gamma_0(603), \left(\frac{1809}{\cdot}\right)\right)$.

Let
$$\tau_0 = [19, 3, 1] = -\frac{3}{38} + \frac{i\sqrt{67}}{38}$$
 with $h(D) = 1$. Then

 $t = t_Q(\tau_0) = -36799488162 - 2595635766\sqrt{201} + 3402\sqrt{234015667197498 + 16506192489998\sqrt{201}}$

has minimal polynomial $T^4 + 147197952648T^3 - 11923033972032T^2 + 321921931101696T - 2897297289216000$ and

 $k = \sqrt[3]{t} \approx 2.994889267470101707326$ is **IN** \mathcal{K}_{O}° .

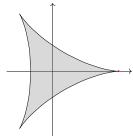


We have
$$\nu(t) = -\frac{27\sqrt{201}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = -\frac{8\pi^2 - L(Q_Q, \tau_0, 2)}{8\pi^2 - L(Q_Q, \tau_0, 2)}$$
, where $\Theta_{Q, \tau_0}(\tau) = \sum_{m, n \in \mathbb{Z}} \chi_{-3}(n)(9m + 38n)q^{9m^2 + 9mn + 19n^2}$
 $= 134q^{19} + 134q^{37} - 134q^{67} + 134q^{73} - 268q^{76} - 268q^{103} + 134q^{127} - 268q^{148} + 134q^{199} - 268q^{211} + \cdots$ is in $S_2\left(\Gamma_0(603), \left(\frac{1809}{2}\right)\right)$.

Let
$$\tau_0 = [17, 1, 1] = -\frac{1}{34} + \frac{i\sqrt{67}}{34}$$
 with $h(D) = 1$. Then

 $t=t_Q(\tau_0)=-36799488162+2595635766\sqrt{201}+3402i\sqrt{-234015667197498+16506192489998\sqrt{201}}$ has minimal polynomial $T^4+147197952648T^3-11923033972032T^2+321921931101696T-2897297289216000$ and $k = \sqrt[3]{t} \approx 3.002555363788691880693 + 0.004431055203832080052i$ is not in \mathcal{K}_O° .



We have
$$\nu(t) = \frac{27\sqrt{201}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$V(t) = \frac{1}{16\pi^2} L(O_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (3m + 34n) q^{9m^2 + 3mn + 17n^2}$$

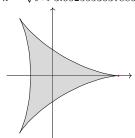
$$= 68q^{17} + 62q^{23} + 74q^{29} + 56q^{47} + 80q^{59} - 136q^{68} - 130q^{71} - 142q^{83} + 50q^{89} - 124q^{92} + \cdots$$

is in $S_2(\Gamma_0(603), (\frac{1809}{.}))$.

ν -N4-6-4

Let
$$\tau_0 = [17, -1, 1] = \frac{1}{34} + \frac{i\sqrt{67}}{34}$$
 with $h(D) = 1$. Then

 $t = t_Q(\tau_0) = -36799488162 + 2595635766\sqrt{201} - 3402i\sqrt{-234015667197498 + 16506192489998\sqrt{201}}$ has minimal polynomial $T^4 + 147197952648T^3 - 11923033972032T^2 + 321921931101696T - 2897297289216000$ and $k = \sqrt[3]{t} \approx 3.002555363788691880693 - 0.004431055203832080052i$ is not in \mathcal{K}_{O}°



We have
$$\nu(t) = \frac{27\sqrt{201}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{27\sqrt{201}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-3m+34n)q^{9m^2-3mn+17n^2}$
 $= 68q^{17} + 62q^{23} + 74q^{29} + 56q^{47} + 80q^{59} - 136q^{68} - 130q^{71} - 142q^{83} + 50q^{89} - 124q^{92} + \cdots$

is in $S_2(\Gamma_0(603), (\frac{1809}{.}))$.

Let
$$\tau_0 = [1, -1, 41] = \frac{1}{2} + \frac{i\sqrt{163}}{2}$$
 with $h(D) = 1$. Then

 $t = t_Q(\tau_0) = -65634353160192162 - 2968088047202178\sqrt{489} - 37422\sqrt{6152313052585724680358658} + 278217213315981187416074\sqrt{489}$ has minimal polynomial $T^4 + 262537412640768648T^3 - 21265530423902068032T^2 + 574169321445369693696T - 5167523893008236544000$ and $k = \sqrt[3]{t} \approx -640320.0000000005926689$ is not in \mathcal{K}_Q° .

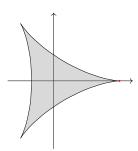
We have
$$\nu(t) = \frac{27\sqrt{489}}{10.3}L(\Theta_{O,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{27\sqrt{489}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-3m+2n)q^{369m^2-3mn+n^2}$
 $= 4q - 8q^4 + 16q^{16} - 20q^{25} + 28q^{49} - 32q^{64} + 40q^{100} - 44q^{121} + 52q^{169} - 56q^{196} + \cdots$

is in $S_2(\Gamma_0(1467), (\frac{4401}{.}))$.

Let
$$\tau_0 = [43, 3, 1] = -\frac{3}{86} + \frac{i\sqrt{163}}{86}$$
 with $h(D) = 1$. Then

 $t = t_Q(\tau_0) = -65634353160192162 - 2968088047202178\sqrt{489} + 37422\sqrt{6152313052585724680358658} + 278217213315981187416074\sqrt{489}$ has minimal polynomial $T^4 + 262537412640768648T^3 - 21265530423902068032T^2 + 574169321445369693696T - 5167523893008236544000$ and $k = \sqrt[3]{t} \approx 2.999957833780763770823$ is **IN** \mathcal{K}_Q° .



We have
$$\nu(t) = -\frac{27\sqrt{489}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where

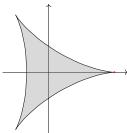
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(9m + 86n)q^{9m^2 + 9mn + 43n^2}$$

$$= 326q^{43} + 326q^{61} + 326q^{97} + 326q^{151} - 326q^{163} - 652q^{172} - 652q^{199} + 326q^{223} - 652q^{244} - 652q^{307} + \cdots$$

is in $S_2(\Gamma_0(1467), (\frac{4401}{}))$.

Let
$$\tau_0 = [41, 1, 1] = -\frac{1}{82} + \frac{i\sqrt{163}}{82}$$
 with $h(D) = 1$. Then

 $t = t_Q(\tau_0) = -65634353160192162 + 2968088047202178\sqrt{489} + 37422\sqrt{6152313052585724680358658} - 278217213315981187416074\sqrt{489}$ has minimal polynomial $T^4 + 262537412640768648T^3 - 21265530423902068032T^2 + 574169321445369693696T - 5167523893008236544000$ and $k = \sqrt[3]{t} \approx 3.000021083109616726215 + 0.000036517359216866885i$ is not in \mathcal{K}_{O}° .



We have
$$\nu(t) = \frac{27\sqrt{489}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

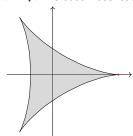
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(3m + 82n)q^{9m^2 + 3mn + 41n^2}$$

$$= 164q^{41} + 158q^{47} + 170q^{53} + 152q^{71} + 176q^{83} + 146q^{113} + 182q^{131} - 328q^{164} - 322q^{167} + 140q^{173} + \cdots$$
 is in $S_2\left(\Gamma_0(1467), \left(\frac{4401}{\cdot}\right)\right)$.

ν -N4-7-4

Let
$$\tau_0 = [41, -1, 1] = \frac{1}{82} + \frac{i\sqrt{163}}{82}$$
 with $h(D) = 1$. Then

has minimal polynomial $T^4 + 262537412640768648T^3 - 21265530423902068032T^2 + 574169321445369693696T - 5167523893008236544000$ and $k = \sqrt[3]{t} \approx 3.000021083109616726215 - 0.000036517359216866885i$ is not in $\mathcal{K}_{\Omega}^{\circ}$.



We have
$$v(t) = \frac{27\sqrt{489}}{6}L(\Theta_{O_1} - 2)$$
 where

We have
$$\nu(t)=\frac{27\sqrt{489}}{16\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau)=\sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(-3m+82n)q^{9m^2-3mn+41n^2}$$

$$= 164q^{41} + 158q^{47} + 170q^{53} + 152q^{71} + 176q^{83} + 146q^{113} + 182q^{131} - 328q^{164} - 322q^{167} + 140q^{173} + \cdots$$

is in $S_2(\Gamma_0(1467), (\frac{4401}{.}))$.

ν-N4-8-1

Let
$$\tau_0 = [9, -9, 7] = \frac{1}{2} + \frac{i\sqrt{19}}{6}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 18 - 6\sqrt{57} - 18\sqrt{2(-3 + \sqrt{57})}$$

has minimal polynomial $T^4 - 72T^3 + 1728T^2 + 870912T - 23887872$ and $k = \sqrt[3]{t} \approx -4.337355981044898204485$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{\sqrt{57}}{10^{-2}} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{\sqrt{57}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-27m + 18n) q^{7m^2 - 3mn + n^2}$

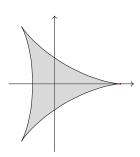
$$= 36q - 72q^4 - 36q^5 + 180q^{11} + 144q^{16} - 252q^{17} + 72q^{20} - 144q^{23} - 180q^{25} + 684q^{35} + \cdots$$

is in $S_2\left(\Gamma_0(171), \left(\frac{57}{\cdot}\right)\right)$

Let
$$\tau_0 = [45, 3, 1] = -\frac{1}{30} + \frac{i\sqrt{19}}{30}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 18 - 6\sqrt{57} + 18\sqrt{2\left(-3 + \sqrt{57}\right)}$$

has minimal polynomial $T^4 - 72T^3 + 1728T^2 + 870912T - 23887872$ and $k = \sqrt[3]{t} \approx 2.999969508159073701732$ is **IN** \mathcal{K}_Q° .



We have
$$\nu(t) = -\frac{\sqrt{57}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(9m+90n)q^{m^2+mn+5n^2}$$

$$= 342q^5 + 342q^7 + 342q^{11} + 342q^{17} - 342q^{19} - 684q^{20} - 684q^{23} + 342q^{25} - 684q^{28} - 342q^{35} + \cdots$$
is in $S_2\left(\Gamma_0(171), \left(\frac{57}{2}\right)\right)$.

Let
$$\tau_0 = [9, 3, 5] = -\frac{1}{6} + \frac{i\sqrt{19}}{6}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 18 + 6\sqrt{57} + 18i\sqrt[4]{3}\sqrt{2\left(\sqrt{3} + \sqrt{19}\right)}$$

$$\begin{split} t &= t_Q(\tau_0) = 18 + 6\sqrt{57} + 18i\sqrt[4]{3}\sqrt{2\left(\sqrt{3} + \sqrt{19}\right)} \\ \text{has minimal polynomial } T^4 - 72T^3 + 1728T^2 + 870912T - 23887872 \text{ and } \\ k &= \sqrt[3]{t} \approx 4.486362111210192137840 + 1.416361433154947478404i \text{ is not in } \mathcal{K}_Q^\circ. \end{split}$$

We have
$$\nu(t) = \frac{\sqrt{57}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{\sqrt{57}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 18n)q^{5m^2 + mn + n^2}$$
$$= 36q - 72q^4 + 18q^5 - 90q^{11} + 144q^{16} + 126q^{17} - 36q^{20} + 72q^{23} - 180q^{25} - 342q^{35} + \cdots$$

is in $S_2\left(\Gamma_0(171), \left(\frac{57}{\cdot}\right)\right)$.

ν-N4-8-4

Let
$$\tau_0 = [9, -3, 5] = \frac{1}{6} + \frac{i\sqrt{19}}{6}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 18 + 6\sqrt{57} - 18i\sqrt[4]{3}\sqrt{2\left(\sqrt{3} + \sqrt{19}\right)}$$

has minimal polynomial $T^4 - 72T^3 + 1728T^2 + 870912T - 23887872$ and

 $k = \sqrt[3]{t} \approx 4.486362111210192137840 - 1.416361433154947478404i$ is not in \mathcal{K}_O° .

We have
$$\nu(t) = \frac{\sqrt{57}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{\sqrt{57}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n) (-9m+18n) q^{5m^2-mn+n^2}$

 $=36q-72q^4+18q^5-90q^{11}+144q^{16}+126q^{17}-36q^{20}+72q^{23}-180q^{25}-342q^{35}+\cdots$

is in $S_2\left(\Gamma_0(171), \left(\frac{57}{\cdot}\right)\right)$.

Let
$$\tau_0 = [9, -9, 13] = \frac{1}{2} + \frac{i\sqrt{43}}{6}$$
 with $h(D) = 4$. Then

$$t = t_0(\tau_0) = 18 - 42\sqrt{129} - 54\sqrt[4]{3}\sqrt{-26\sqrt{3} + 14\sqrt{43}}$$

Let $\tau_0 = [9, -9, 13] = \frac{1}{2} + \frac{i\sqrt{43}}{6}$ with h(D) = 4. Then $t = t_Q(\tau_0) = 18 - 42\sqrt{129} - 54\sqrt[4]{3}\sqrt{-26\sqrt{3} + 14\sqrt{43}}$ has minimal polynomial $T^4 - 72T^3 + 1728T^2 + 884722176T - 23887872000$ and $k = \sqrt[3]{t} \approx -9.813394848885976189306$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{\sqrt{129}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-27m + 18n) q^{13m^2 - 3mn + n^2}$$

 $=36q-72q^4-36q^{11}+144q^{16}+180q^{17}-252q^{23}-180q^{25}+396q^{41}+72q^{44}-144q^{47}+\cdots$

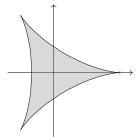
is in $S_2(\Gamma_0(387), (\frac{129}{.}))$.

ν -N4-9-2

Let
$$\tau_0 = [99, 3, 1] = -\frac{1}{66} + \frac{i\sqrt{43}}{66}$$
 with $h(D) = 4$. Then

$$t = t_O(\tau_0) = 18 - 42\sqrt{129} + 54\sqrt[4]{3}\sqrt{-26\sqrt{3} + 14\sqrt{43}}$$

 $t=t_Q(\tau_0)=18-42\sqrt{129}+54\sqrt[4]{3}\sqrt{-26\sqrt{3}+14\sqrt{43}}$ has minimal polynomial $T^4-72T^3+1728T^2+884722176T-23887872000$ and



We have
$$\nu(t) = -\frac{\sqrt{129}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = -\frac{\sqrt{129}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(9m+198n)q^{m^2+mn+11n^2}$

 $= 774q^{11} + 774q^{13} + 774q^{17} + 774q^{23} + 774q^{31} + 774q^{41} - 774q^{43} - 1548q^{44} - 1548q^{47} - 1548q^{52} + \cdots$

is in $S_2(\Gamma_0(387), (\frac{129}{\cdot}))$.

ν -N4-9-3

Let
$$\tau_0 = [9, 3, 11] = -\frac{1}{6} + \frac{i\sqrt{43}}{6}$$
 with $\underline{h(D)} = 4$. Then

$$t = t_Q(\tau_0) = 18 + 42\sqrt{129} + 54i\sqrt[4]{3}\sqrt{26\sqrt{3} + 14\sqrt{43}}$$

has minimal polynomial $T^4 - 72T^3 + 1728T^2 + 884722176T - 23887872000$ and

 $k = \sqrt[3]{t} \approx 9.309297906367732047008 + 3.341024875163763351406i \text{ is not in } \mathcal{K}_Q^\circ.$

We have
$$\nu(t) = \frac{\sqrt{129}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{\sqrt{129}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(9m+18n)q^{11m^2+mn+n^2}$$
$$= 36q - 72q^4 + 18q^{11} + 144q^{16} - 90q^{17} + 126q^{23} - 180q^{25} - 198q^{41} - 36q^{44} + 72q^{47} + \cdots$$

is in $S_2(\Gamma_0(387), (\frac{129}{.}))$.

Let
$$\tau_0 = [9, -3, 11] = \frac{1}{6} + \frac{i\sqrt{43}}{6}$$
 with $h(D) = 4$. Then

$$t = t_{\mathcal{O}}(\tau_0) = 18 + 42\sqrt{129} - 54i\sqrt[4]{3}\sqrt{26\sqrt{3} + 14\sqrt{43}}$$

Let $\tau_0 = [9, -3, 11] = \frac{1}{6} + \frac{i\sqrt{43}}{6}$ with h(D) = 4. Then $t = t_Q(\tau_0) = 18 + 42\sqrt{129} - 54i\sqrt[4]{3}\sqrt{26\sqrt{3} + 14\sqrt{43}}$ has minimal polynomial $T^4 - 72T^3 + 1728T^2 + 884722176T - 23887872000$ and

 $k = \sqrt[3]{t} \approx 9.309297906367732047008 - 3.341024875163763351406i$ is not in $\mathcal{K}_{\Omega}^{\circ}$.

We have
$$\nu(t) = \frac{\sqrt{129}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\begin{aligned} \kappa &= \sqrt{t} \approx 9.309287900307732047008 - 3.341024873103703331400t \text{ is find iff } \mathcal{K}_{\widehat{Q}}. \end{aligned}$$
 We have $\nu(t) = \frac{\sqrt{129}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-9m + 18n) q^{11m^2 - mn + n^2}$$
$$= 36q - 72q^4 + 18q^{11} + 144q^{16} - 90q^{17} + 126q^{23} - 180q^{25} - 198q^{41} - 36q^{44} + 72q^{47} + \cdots$$

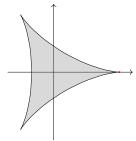
is in $S_2\left(\Gamma_0(387), \left(\frac{129}{\cdot}\right)\right)$.

Let
$$\tau_0 = [36, 0, 1] = \frac{i}{6}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 18 - \frac{45\sqrt[4]{3}}{\sqrt{2}} + 21\sqrt{3} + \frac{9\sqrt[4]{27}}{\sqrt{2}}$$

 $t=t_Q(\tau_0)=18-\frac{45\sqrt[4]{3}}{\sqrt{2}}+21\sqrt{3}+\frac{9\sqrt[4]{27}}{\sqrt{2}}$ has minimal polynomial $T^4-72T^3+1728T^2-301320T+7762392$ and

 $k = \sqrt[3]{t} \approx 3.000094159228714042426$ is not in $\mathcal{K}_{\mathcal{O}}^{\circ}$.



We have
$$\nu(t) = \frac{\sqrt{3}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 72nq^{m^2 + 4n^2}$$

$$= 144q^4 + 288q^5 + 288q^8 + 288q^{13} - 288q^{16} - 576q^{17} - 288q^{20} - 576q^{25} + 288q^{29} - 576q^{32} + \cdots$$

is in $S_2(\Gamma_0(144), (\frac{48}{\cdot}))$.

Let
$$\tau_0 = [9, 0, 4] = \frac{2i}{3}$$
 with $h(D) = 4$. Then

$$t = t_0(\tau_0) = 18 + \frac{45\sqrt[4]{3}}{5} + 21\sqrt{3} - \frac{9\sqrt[4]{27}}{5}$$

 $t = t_Q(\tau_0) = 18 + \frac{45\sqrt[4]{3}}{\sqrt{2}} + 21\sqrt{3} - \frac{9\sqrt[4]{27}}{\sqrt{2}}$ has minimal polynomial $T^4 - 72T^3 + 1728T^2 - 301320T + 7762392$ and $k = \sqrt[3]{t} \approx 4.339948457607474796175$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{\sqrt{3}}{2} L(\Theta_{O,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{\sqrt{3}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n) 18nq^{4m^2+n^2}$

$$= 36q - 72q^4 + 72q^5 - 144q^8 + 144q^{16} + 72q^{17} + 144q^{20} - 180q^{25} - 360q^{29} + 288q^{32} + \cdots$$

is in $S_2(\Gamma_0(144), (\frac{48}{\cdot}))$.

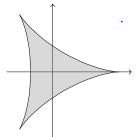
ν -N4-10-3

Let
$$\tau_0 = [9, 6, 5] = -\frac{1}{3} + \frac{2i}{3}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 18 + \frac{45i\sqrt[4]{3}}{\sqrt{2}} - 21\sqrt{3} + \frac{9i\sqrt[4]{27}}{\sqrt{2}}$$

 $t=t_Q(\tau_0)=18+\frac{45i\frac{\sqrt[4]{3}}{\sqrt{2}}-21\sqrt{3}+\frac{9i\frac{\sqrt[4]{27}}{\sqrt{2}}}{\sqrt{2}}$ has minimal polynomial $T^4-72T^3+1728T^2-301320T+7762392$ and

 $k = \sqrt[3]{t} \approx 3.154215655068299027894 + 2.293032966296685370141i$ is not in \mathcal{K}_O° .



We have
$$\nu(t) = \frac{\sqrt{3}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t)=\frac{\sqrt{3}}{4\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau)=\sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(18m+18n)q^{5m^2+2mn+n^2}$$

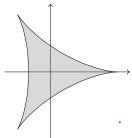
$$= 36q - 72q^4 - 36q^5 + 72q^8 + 144q^{16} - 36q^{17} - 72q^{20} - 180q^{25} + 180q^{29} - 144q^{32} + \cdots$$

is in $S_2\left(\Gamma_0(144), \left(\frac{48}{\cdot}\right)\right)$.

Let
$$\tau_0 = [9, -6, 5] = \frac{1}{3} + \frac{2i}{3}$$
 with $h(D) = 4$. Then

Let
$$\tau_0 = [9, -6, 5] = \frac{1}{3} + \frac{2i}{3}$$
 with $h(D) = 4$. Then $t = t_Q(\tau_0) = 18 - \frac{45i\sqrt[4]{3}}{\sqrt{2}} - 21\sqrt{3} - \frac{9i\sqrt[4]{27}}{\sqrt{2}}$

has minimal polynomial $T^4 - 72T^3 + 1728T^2 - 301320T + 7762392$ and $k = \sqrt[3]{t} \approx 3.154215655068299027894 - 2.293032966296685370141i$ is not in \mathcal{K}_{O}° .



We have
$$\nu(t) = \frac{\sqrt{3}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-18m+18n)q^{5m^2-2mn+n^2}$$

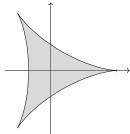
$$= 36q - 72q^4 - 36q^5 + 72q^8 + 144q^{16} - 36q^{17} - 72q^{20} - 180q^{25} + 180q^{29} - 144q^{32} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(144), \left(\frac{48}{2}\right)\right)$.

ν -N4-11-1

Let
$$\tau_0 = [63, 0, 1] = \frac{i}{3\sqrt{7}}$$
 with $h(D) = 4$. Then $t = t_Q(\tau_0) = 18 + \frac{57\sqrt{21}}{2} - \frac{27}{2}\sqrt{-93 + 38\sqrt{21}}$

has minimal polynomial $T^4 - 72T^3 + 1728T^2 - 16595199T + 447697125$ and

 $k = \sqrt[3]{t} \approx 3.000001628406355859161$ is not in \mathcal{K}_O° .



We have
$$\nu(t) = \frac{\sqrt{21}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)126nq^{m^2+7n^2}$$

$$= 252q^7 + 504q^8 + 504q^{11} + 504q^{16} + 504q^{23} - 504q^{28} - 1008q^{29} - 504q^{32} - 1008q^{37} + 504q^{43} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(252), \left(\frac{84}{\cdot}\right)\right)$.

Let $\tau_0 = [9, 0, 7] = \frac{i\sqrt{7}}{3}$ with h(D) = 4. Then

$$t = t_Q(\tau_0) = 18 + \frac{57\sqrt{21}}{2} + \frac{27}{2}\sqrt{-93 + 38\sqrt{21}}$$

has minimal polynomial $T^4 = 72T^3 + 1728T^2 - 16595199T + 447697125$ and

 $k = \sqrt[3]{t} \approx 6.464953552253781031509$ is not in $\mathcal{K}_{\mathcal{O}}^{\circ}$.

We have
$$\nu(t) = \frac{\sqrt{21}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{\sqrt{21}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n) 18nq^{7m^2+n^2}$$
$$= 36q - 72q^4 + 72q^8 - 144q^{11} - 18q^{11} + 18q^{1$$

$$= 36q - 72q^4 + 72q^8 - 144q^{11} + 144q^{16} + 288q^{23} - 180q^{25} + 72q^{29} - 504q^{32} + 288q^{44} + \cdots$$

s in S₂ (Γ_0 (252) (84))

is in $S_2\left(\Gamma_0(252), \left(\frac{84}{\cdot}\right)\right)$.

ν -N4-11-3

Let
$$\tau_0 = [9, 6, 8] = -\frac{1}{3} + \frac{i\sqrt{7}}{3}$$
 with $h(D) = 4$. Then

$$t = t_0(\tau_0) = 18 - \frac{57\sqrt{21}}{2} + \frac{27}{2}i\sqrt{93 + 38\sqrt{21}}$$

 $\begin{array}{l} t=t_Q(\tau_0)=18-\frac{57\sqrt{21}}{2}+\frac{27}{2}i\sqrt{93+38\sqrt{21}}\\ \text{has minimal polynomial } T^4-72T^3+1728T^2-16595199T+447697125 \text{ and } \\ k=\sqrt[3]{t}\approx 4.879947060114502569811+3.953420558712342562452i \text{ is not in } \mathcal{K}_Q^\circ. \end{array}$

We have
$$\nu(t) = \frac{\sqrt{21}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t)=\frac{\sqrt{21}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau)=\sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(18m+18n)q^{8m^2+2mn+n^2}$$

$$= 36q - 72q^4 - 36q^8 + 72q^{11} + 144q^{16} - 144q^{23} - 180q^{25} - 36q^{29} + 252q^{32} - 144q^{44} + \cdots$$

is in $S_2\left(\Gamma_0(252), \left(\frac{84}{\cdot}\right)\right)$.

Let
$$\tau_0 = [9, -6, 8] = \frac{1}{3} + \frac{i\sqrt{7}}{3}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 18 - \frac{57\sqrt{21}}{2} - \frac{27}{2}i\sqrt{93} + 38\sqrt{21}$$

 $t = t_Q(\tau_0) = 18 - \frac{57\sqrt{21}}{2} - \frac{27}{2}i\sqrt{93 + 38\sqrt{21}}$ has minimal polynomial $T^4 - 72T^3 + 1728T^2 - 16595199T + 447697125$ and $k = \sqrt[3]{t} \approx 4.879947060114502569811 - 3.953420558712342562452i$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{\sqrt{21}}{2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{\sqrt{21}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n) (-18m + 18n) q^{8m^2 - 2mn + n^2}$$
$$= 36q - 72q^4 - 36q^8 + 72q^{11} + 144q^{16} - 144q^{23} - 180q^{25} - 36q^{29} + 252q^{32} - 144q^{44} + \cdots$$

is in $S_2\left(\Gamma_0(252), \left(\frac{84}{\cdot}\right)\right)$.

ν-N4-12-1

Let $\tau_0 = [3, 2, 2] = -\frac{1}{3} + \frac{i\sqrt{5}}{3}$ with h(D) = 2. Then

 $t=t_Q(\tau_0)=(-8+44i)-(14-22i)\sqrt{5}$ has minimal polynomial $T^4+32T^3+7136T^2-432000T+5832000$ and $k = \sqrt[3]{t} \approx 3.690317755422442060866 + 2.844250129034582322015i$ is not in \mathcal{K}_O° . We have $\nu(t) = \frac{3\sqrt{15}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(6m+6n)q^{6m^2+2mn+n^2}$ $= 12q - 24q^4 - 12q^6 + 24q^9 + 48q^{16} - 60q^{21} + 24q^{24} - 60q^{25} + 60q^{30} - 48q^{36} + \cdots$

ν-N4-12-2

is in $S_2\left(\Gamma_0(60), \left(\frac{60}{\cdot}\right)\right)$.

Let $\tau_0 = [3, -2, 2] = \frac{1}{3} + \frac{i\sqrt{5}}{3}$ with h(D) = 2. Then $t = t_Q(\tau_0) = (-8 - 44i) - (14 + 22i)\sqrt{5}$ has minimal polynomial $T^4 + 32T^3 + 7136T^2 - 432000T + 5832000$ and

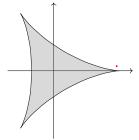
 $k = \sqrt[3]{t} \approx 3.690317755422442060866 - 2.844250129034582322015i$ is not in \mathcal{K}_O° .

We have
$$\nu(t) = \frac{3\sqrt{15}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(-6m+6n)q^{6m^2-2mn+n^2}$$
$$= 12q - 24q^4 - 12q^6 + 24q^9 + 48q^{16} - 60q^{21} + 24q^{24} - 60q^{25} + 60q^{30} - 48q^{36} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(60),\left(\frac{60}{10}\right)\right)$.

ν-N4-12-3

Let $\tau_0 = [6, 2, 1] = -\frac{1}{6} + \frac{i\sqrt{5}}{6}$ with h(D) = 2. Then $t = t_Q(\tau_0) = (-8 - 44i) + (14 + 22i)\sqrt{5}$ has minimal polynomial $T^4 + 32T^3 + 7136T^2 - 432000T + 5832000$ and

 $k = \sqrt[3]{t} \approx 2.871860071068628466481 + 0.210275532819020968779i$ is not in \mathcal{K}_O° .



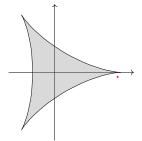
We have
$$\nu(t) = \frac{3\sqrt{15}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(6m+12n)q^{3m^2+2mn+2n^2}$$
$$= 24q^2 + 12q^3 - 48q^8 - 24q^{12} - 60q^{15} + 48q^{18} - 24q^{23} + 84q^{27} + 96q^{32} + 120q^{35} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(60), \left(\frac{60}{\cdot}\right)\right)$.

ν -N4-12-4

Let $\tau_0 = [6, -2, 1] = \frac{1}{6} + \frac{i\sqrt{5}}{6}$ with h(D) = 2. Then

 $t=t_Q(\tau_0)=(-8+44i)+(14-22i)\sqrt{5}$ has minimal polynomial $T^4+32T^3+7136T^2-432000T+5832000$ and

 $k = \sqrt[3]{t} \approx 2.871860071068628466481 - 0.210275532819020968779i$ is not in \mathcal{K}_{O}° .



We have
$$\nu(t) = \frac{3\sqrt{15}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(-6m+12n)q^{3m^2-2mn+2n^2}$$

$$= 24q^2+12q^3-48q^8-24q^{12}-60q^{15}+48q^{18}-24q^{23}+84q^{27}+96q^{32}+120q^{35}+\cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(60),\left(\frac{60}{\cdot}\right)\right)$.

ν -N4-13-1

Let $\tau_0 = [9, -9, 19] = \frac{1}{2} + \frac{i\sqrt{67}}{6}$ with h(D) = 4. Then

$$t = t_{\rm O}(\tau_0) = 18 - 186\sqrt{201} - 126\sqrt{-438 + 62\sqrt{201}}$$

 $t = t_Q(\tau_0) = 18 - 186\sqrt{201} - 126\sqrt{-438 + 62\sqrt{201}}$ has minimal polynomial $T^4 - 72T^3 + 1728T^2 + 147197938176T - 3974344704000$ and $k = \sqrt[3]{t} \approx -17.39668127009333151704$ is not in \mathcal{K}_Q° .

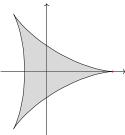
We have
$$\nu(t) = \frac{\sqrt{201}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{\sqrt{201}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n) (-27m + 18n) q^{19m^2 - 3mn + n^2}$$
$$= 36q - 72q^4 + 144q^{16} - 36q^{17} + 180q^{23} - 180q^{25} - 252q^{29} + 396q^{47} + 252q^{49} - 468q^{59} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(603), \left(\frac{201}{2}\right)\right)$.

Let $\tau_0 = [153, 3, 1] = -\frac{1}{102} + \frac{i\sqrt{67}}{102}$ with h(D) = 4. Then

 $t=t_Q(\tau_0)=18-186\sqrt{201}+126\sqrt{-438+62\sqrt{201}}$ has minimal polynomial $T^4-72T^3+1728T^2+147197938176T-3974344704000$ and

 $k = \sqrt[3]{t} \approx 2.999999999916573535867$ is **IN** \mathcal{K}_O° .



We have
$$\nu(t) = -\frac{\sqrt{201}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = -\frac{\sqrt{201}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(9m + 306n)q^{m^2 + mn + 17n^2}$$
$$= 1206q^{17} + 1206q^{19} + 1206q^{23} + 1206q^{29} + 1206q^{37} + 1206q^{47} + 1206q^{59} - 1206q^{67} - 2412q^{68} - 2412q^{71} + \cdots$$

is in $S_2(\Gamma_0(603), (\frac{201}{\cdot}))$.

ν -N4-13-3

Let
$$\tau_0 = [9, 3, 17] = -\frac{1}{6} + \frac{i\sqrt{67}}{6}$$
 with $h(D) = 4$. Then

$$t = t_{\rm O}(\tau_0) = 18 + 186\sqrt{201} + 126i\sqrt{438 + 62\sqrt{201}}$$

 $t=t_Q(\tau_0)=18+186\sqrt{201}+126i\sqrt{438+62\sqrt{201}}$ has minimal polynomial $T^4-72T^3+1728T^2+147197938176T-3974344704000$ and

 $k = \sqrt[3]{t} \approx 16.37566506023077055824 + 5.94506164250723133745i$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{\sqrt{201}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{\sqrt{201}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(9m+18n)q^{17m^2+mn+n^2}$

 $=36q-72q^4+144q^{16}+18q^{17}-90q^{23}-180q^{25}+126q^{29}-198q^{47}+252q^{49}+234q^{59}+\cdots$

is in $S_2(\Gamma_0(603), (\frac{201}{\cdot}))$.

ν -N4-13-4

Let
$$\tau_0 = [9, -3, 17] = \frac{1}{6} + \frac{i\sqrt{67}}{6}$$
 with $h(D) = 4$. Then

$$t = t_O(\tau_0) = 18 + 186\sqrt{201} - 126i\sqrt{438 + 62\sqrt{201}}$$

 $t=t_Q(\tau_0)=18+186\sqrt{201}-126i\sqrt{438+62\sqrt{201}}$ has minimal polynomial $T^4-72T^3+1728T^2+147197938176T-3974344704000$ and

 $k = \sqrt[3]{t} \approx 16.37566506023077055824 - 5.94506164250723133745i$ is not in \mathcal{K}_{O}° .

We have
$$\nu(t) = \frac{\sqrt{201}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t)=\frac{\sqrt{201}}{16\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau)=\sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(-9m+18n)q^{17m^2-mn+n^2}$$

 $= 36q - 72q^4 + 144q^{16} + 18q^{17} - 90q^{23} - 180q^{25} + 126q^{29} - 198q^{47} + 252q^{49} + 234q^{59} + \cdots$

is in $S_2(\Gamma_0(603), (\frac{201}{1000}))$.

ν -N4-14-1

Let
$$\tau_0 = [9, -9, 43] = \frac{1}{2} + \frac{i\sqrt{163}}{6}$$
 with $h(D) = 4$. Then

$$t = t_O(\tau_0) = 18 - 14478\sqrt{489} - 1386\sqrt{-53358 + 4826\sqrt{489}}$$

Let $\tau_0 = [9, -9, 43] = \frac{1}{2} + \frac{i\sqrt{163}}{6}$ with h(D) = 4. Then $t = t_Q(\tau_0) = 18 - 14478\sqrt{489} - 1386\sqrt{-53358 + 4826\sqrt{489}}$ has minimal polynomial $T^4 - 72T^3 + 1728T^2 + 262537412640754176T - 7088510141300736000$ and

 $k = \sqrt[3]{t} \approx -86.19107506807197279940$ is not in \mathcal{K}_Q° .

We have
$$\nu(t)=\frac{\sqrt{489}}{16\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau)=\sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(-27m+18n)q^{43m^2-3mn+n^2}$$

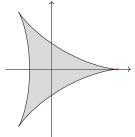
 $=36q-72q^4+144q^{16}-180q^{25}-36q^{41}+180q^{47}+252q^{49}-252q^{53}-288q^{64}+396q^{71}+\cdots$

is in $S_2(\Gamma_0(1467), (\frac{489}{\cdot}))$.

Let
$$\tau_0 = [369, 3, 1] = -\frac{1}{246} + \frac{i\sqrt{163}}{246}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = 18 - 14478\sqrt{489} + 1386\sqrt{-53358} + 4826\sqrt{489}$$

 $t=t_Q(\tau_0)=18-14478\sqrt{489}+1386\sqrt{-53358+4826\sqrt{489}}$ has minimal polynomial $T^4-72T^3+1728T^2+262537412640754176T-7088510141300736000$ and



We have
$$\nu(t) = -\frac{\sqrt{489}}{2}L(\Theta_{O_1}, 2)$$
, where

We have
$$\nu(t) = -\frac{\sqrt{489}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(9m+738n)q^{m^2+mn+41n^2}$$

 $= 2934q^{41} + 2934q^{43} + 2934q^{47} + 2934q^{53} + 2934q^{61} + 2934q^{71} + 2934q^{83} + 2934q^{97} + 2934q^{113} + 2934q^{131} + \cdots$

is in $S_2\left(\Gamma_0(1467), \left(\frac{489}{\cdot}\right)\right)$.

ν -N4-14-3

Let $\tau_0 = [9, 3, 41] = -\frac{1}{6} + \frac{i\sqrt{163}}{6}$ with h(D) = 4. Then

 $t = t_Q(\tau_0) = 18 + 14478\sqrt{489} + 1386i\sqrt{53358 + 4826\sqrt{489}}$

has minimal polynomial $T^4 - 72T^3 + 1728T^2 + 262537412640754176T - 7088510141300736000$ and

 $k = \sqrt[3]{t} \approx 80.99426524403682211188 + 29.47888142264088778219i$ is not in \mathcal{K}_{O}°

We have
$$\nu(t) = \frac{\sqrt{489}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m+18n)q^{41m^2+mn+n^2}$

 $= 36q - 72q^4 + 144q^{16} - 180q^{25} + 18q^{41} - 90q^{47} + 252q^{49} + 126q^{53} - 288q^{64} - 198q^{71} + \cdots$

is in $S_2(\Gamma_0(1467), (\frac{489}{\cdot}))$.

ν-N4-14-4

Let $\tau_0 = [9, -3, 41] = \frac{1}{6} + \frac{i\sqrt{163}}{6}$ with h(D) = 4. Then $t = t_Q(\tau_0) = 18 + 14478\sqrt{489} - 1386i\sqrt{53358} + 4826\sqrt{489}$

has minimal polynomial $T^4 - 72T^3 + 1728T^2 + 262537412640754176T - 7088510141300736000$ and

 $k = \sqrt[3]{t} \approx 80.99426524403682211188 - 29.47888142264088778219i$ is not in \mathcal{K}_O° .

We have
$$\nu(t) = \frac{\sqrt{489}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n) (-9m+18n) q^{41m^2-mn+n^2}$$

 $=36q-72q^4+144q^{16}-180q^{25}+18q^{41}-90q^{47}+252q^{49}+126q^{53}-288q^{64}-198q^{71}+\cdots$

is in $S_2\left(\Gamma_0(1467), \left(\frac{489}{\cdot}\right)\right)$.

ν -N4-15-1

Let $\tau_0=[3,2,3]=-\frac{1}{3}+\frac{2i\sqrt{2}}{3}$ with h(D)=2. Then $t=t_Q(\tau_0)=(-73+161i)-(70-115i)\sqrt{2}$ has minimal polynomial $T^4+292T^3+117116T^2-6750000T+91125000$ and

 $k = \sqrt[3]{t} \approx 5.535533905932737622004 + 4.535533905932737622004i$ is not in \mathcal{K}_O° .

We have
$$\nu(t) = \frac{3\sqrt{\frac{3}{2}}}{2\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t)=\frac{3\sqrt{\frac{3}{2}}}{2\pi^2}L(\Theta_{Q,\tau_0},2),$$
 where
$$\Theta_{Q,\tau_0}(\tau)=\sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(6m+6n)q^{9m^2+2mn+n^2}$$

$$= 12q - 24q^4 - 12q^9 + 24q^{12} + 48q^{16} - 48q^{24} - 60q^{25} + 48q^{33} + 24q^{36} - 48q^{48} + \cdots$$

is in $S_2\left(\Gamma_0(96), \left(\frac{96}{\cdot}\right)\right)$.

Let
$$\tau_0 = [3, -2, 3] = \frac{1}{3} + \frac{2i\sqrt{2}}{3}$$
 with $h(D) = 2$. Then $t = t_Q(\tau_0) = (-73 - 161i) - (70 + 115i)\sqrt{2}$

$$t = t_Q(\tau_0) = (-73 - 161i) - (70 + 115i)\sqrt{2}$$

has minimal polynomial $T^4 + 292T^3 + 117116T^2 - 6750000T + 91125000$ and

 $k = \sqrt[3]{t} \approx 5.535533905932737622004 - 4.535533905932737622004i$ is not in \mathcal{K}_O° .

We have
$$\nu(t) = \frac{3\sqrt{\frac{3}{2}}}{2\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{3\sqrt{\frac{3}{2}}}{2\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-6m+6n)q^{9m^2-2mn+n^2}$$

$$= 12q - 24q^4 - 12q^9 + 24q^{12} + 48q^{16} - 48q^{24} - 60q^{25} + 48q^{33} + 24q^{36} - 48q^{48} + \cdots$$

is in $S_2\left(\Gamma_0(96), \left(\frac{96}{\cdot}\right)\right)$.

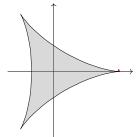
$\nu\text{-N}4\text{-}15\text{-}3$

Let
$$\tau_0 = [9, 2, 1] = -\frac{1}{9} + \frac{2i\sqrt{2}}{9}$$
 with $h(D) = 2$. Then

$$t = t_Q(\tau_0) = (-73 - 161i) + (70 + 115i)\sqrt{2}$$

has minimal polynomial $T^4 + 292T^3 + 117116T^2 - 6750000T + 91125000$ and

 $k = \sqrt[3]{t} \approx 2.963603727660902786985 + 0.062044417943726518217i$ is not in \mathcal{K}_{O}°



We have
$$\nu(t) = \frac{3\sqrt{\frac{3}{2}}}{2}L(\Theta_{0,5}, 2)$$
, where

We have
$$\nu(t) = \frac{3\sqrt{\frac{3}{2}}}{2\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{\substack{m,n \in \mathbb{Z} \\ 2C-3+2A+4+48+8}} \chi_{-3}(n)(6m+18n)q^{3m^2+2mn+3n^2}$

$$=36q^3+24q^4+48q^8-48q^{11}-72q^{12}-48q^{16}-24q^{19}-36q^{27}-96q^{32}+72q^{36}+\cdots$$

is in $S_2(\Gamma_0(96), (\frac{96}{}))$.

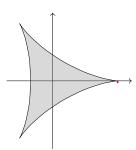
ν -N4-15-4

Let
$$\tau_0 = [9, -2, 1] = \frac{1}{9} + \frac{2i\sqrt{2}}{9}$$
 with $h(D) = 2$. Then

$$t = t_Q(\tau_0) = (-73 + 161i) + (70 - 115i)\sqrt{2}$$

 $t=t_Q(\tau_0)=(-73+161i)+(70-115i)\sqrt{2}$ has minimal polynomial $T^4+292T^3+117116T^2-6750000T+91125000$ and

 $k = \sqrt[3]{t} \approx 2.963603727660902786985 - 0.062044417943726518217i$ is not in \mathcal{K}_O°



We have
$$\nu(t) = \frac{3\sqrt{\frac{3}{2}}}{2\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(-6m+18n)q^{3m^2-2mn+3n^2}$$

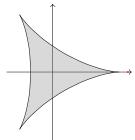
$$= 36q^3 + 24q^4 + 48q^8 - 48q^{11} - 72q^{12} - 48q^{16} - 24q^{19} - 36q^{27} - 96q^{32} + 72q^{36} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(96), \left(\frac{96}{2}\right)\right)$.

ν-N4-16-1

Let $\tau_0 = [4, 0, 1] = \frac{i}{2}$ with h(D) = 1. Then

$$t = t_Q(\tau_0) = 71712 - \frac{77031\sqrt[4]{3}}{\sqrt{2}} + 41391\sqrt{3} - \frac{44469\sqrt[4]{2}}{\sqrt{2}}$$

 $t=t_Q(\tau_0)=71712-\frac{77031\sqrt[4]{3}}{\sqrt{2}}+41391\sqrt{3}-\frac{44469\sqrt[4]{27}}{\sqrt{2}}$ has minimal polynomial $T^4-286848T^3+23427144T^2-618676056T+5658783768$ and $k = \sqrt[3]{t} \approx 3.428951936017602887126$ is not in $\mathcal{K}_{\mathcal{O}}^{\circ}$.



We have $\nu(t) = \frac{27\sqrt{3}}{4\pi^2}L(\Theta_{Q,\tau_0},2)$, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)8nq^{9m^2+4n^2}$ $=16q^4+32q^{13}-32q^{16}-64q^{25}+32q^{40}-64q^{52}+64q^{64}+128q^{73}+32q^{85}-64q^{97}+\cdots$ is in $S_2\left(\Gamma_0(144), \left(\frac{432}{\cdot}\right)\right)$.

ν -N4-16-2

Let $\tau_0 = [1, 0, 4] = 2i$ with h(D) = 1. Then

$$t = t_O(\tau_0) = 71712 + \frac{77031\sqrt[4]{3}}{5} + 41391\sqrt{3} + \frac{44469\sqrt[4]{27}}{5}$$

 $t = t_Q(\tau_0) = 71712 + \frac{77031\sqrt[4]{3}}{\sqrt{2}} + 41391\sqrt{3} + \frac{44469\sqrt[4]{27}}{\sqrt{2}}$ has minimal polynomial $T^4 - 286848T^3 + 23427144T^2 - 618676056T + 5658783768$ and $k = \sqrt[3]{t} \approx 65.94411502292882027695$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{27\sqrt{3}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)2nq^{36m^2+n^2}$$
$$= 4q - 8q^4 + 16q^{16} - 20q^{25} + 8q^{37} - 16q^{40} + 28q^{49} + 32q^{52} - 40q^{61} - 32q^{64} + \cdots$$

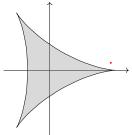
is in $S_2(\Gamma_0(144), (\frac{432}{.}))$.

ν -N4-16-3

Let $\tau_0 = [5, 2, 1] = -\frac{1}{5} + \frac{2i}{5}$ with h(D) = 1. Then

$$t = t_Q(\tau_0) = 71712 + \frac{77031i\sqrt[4]{3}}{\sqrt{2}} - 41391\sqrt{3} - \frac{44469i\sqrt[4]{27}}{\sqrt{2}}$$

has minimal polynomial $T^4 - 286848T^3 + 23427144T^2 - 618676056T + 5658783768$ and $k = \sqrt[3]{t} \approx 2.786795221476077119283 + 0.338484145446773894913i$ is not in \mathcal{K}_O°



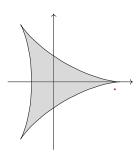
We have
$$\nu(t) = \frac{27\sqrt{3}}{4\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(6m+10n)q^{9m^2+6mn+5n^2}$$
$$= 20q^5 + 8q^8 - 28q^{17} - 8q^{20} - 4q^{29} - 16q^{32} - 52q^{41} + 44q^{53} + 64q^{65} + 40q^{68} + \cdots$$
 is in $S_2\left(\Gamma_0(144), \left(\frac{432}{2}\right)\right)$.

ν -N4-16-4

Let $\tau_0 = [5, -2, 1] = \frac{1}{5} + \frac{2i}{5}$ with h(D) = 1. Then

$$t = t_Q(\tau_0) = 71712 - \frac{77031i\sqrt[3]{3}}{\sqrt{2}} - 41391\sqrt{3} + \frac{44469i\sqrt[3]{2}}{\sqrt{2}}$$

 $t=t_Q(\tau_0)=71712-\frac{77031i}{\sqrt{2}}\frac{\sqrt[4]{3}}{\sqrt{2}}-41391\sqrt{3}+\frac{44469i}{\sqrt{2}}\frac{\sqrt[4]{27}}{\sqrt{2}}$ has minimal polynomial $T^4-286848T^3+23427144T^2-618676056T+5658783768$ and $k = \sqrt[3]{t} \approx 2.786795221476077119283 - 0.338484145446773894913i$ is not in \mathcal{K}_O° .



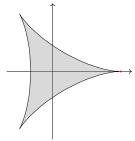
We have $\nu(t) = \frac{27\sqrt{3}}{4\pi^2} L(\Theta_{Q,\tau_0}, 2)$, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-6m+10n)q^{9m^2-6mn+5n^2}$$

$$= 20q^5 + 8q^8 - 28q^{17} - 8q^{20} - 4q^{29} - 16q^{32} - 52q^{41} + 44q^{53} + 64q^{65} + 40q^{68} + \cdots$$
is in $S_2\left(\Gamma_0(144), \left(\frac{432}{2}\right)\right)$.

$$t = t_Q(\tau_0) = \frac{16580727}{4} + \frac{3618189\sqrt{21}}{4} - \frac{729}{4}\sqrt{1034609022 + 225770198\sqrt{21}}$$

Let $\tau_0 = [7,0,1] = \frac{i}{\sqrt{7}}$ with h(D) = 1. Then $t = t_Q(\tau_0) = \frac{16580727}{4} + \frac{3618189\sqrt{21}}{4} - \frac{729}{4}\sqrt{1034609022 + 225770198\sqrt{21}}$ has minimal polynomial $T^4 - 16580727T^3 + 1343231343T^2 - 36253389429T + 326371204125$ and $k = \sqrt[3]{t} \approx 3.107144446086423167471$ is not in \mathcal{K}_Q° .



We have $\nu(t) = \frac{27\sqrt{21}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$, where

We have
$$\nu(t) = \frac{1}{8\pi^2} L(Q_Q, \tau_0, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 14nq^{9m^2 + 7n^2}$$

$$= 28q^7 + 56q^{16} - 56q^{28} - 112q^{37} + 56q^{43} - 112q^{64} + 56q^{88} - 112q^{109} + 112q^{112} + 224q^{121} + \cdots$$

is in
$$S_2(\Gamma_0(252), (\frac{756}{2}))$$
.

Let $\tau_0 = [1, 0, 7] = i\sqrt{7}$ with h(D) = 1. Then

$$t = t_Q(\tau_0) = \frac{16580727}{4} + \frac{3618189\sqrt{21}}{4} + \frac{729}{4}\sqrt{1034609022 + 225770198\sqrt{21}}$$

has minimal polynomial $T^4 - 16580727T^3 + 1343231343T^2 - 36253389429T + 326371204125$ and

 $k = \sqrt[3]{t} \approx 254.9962628601550170203$ is not in \mathcal{K}_O° .

We have
$$\nu(t) = \frac{27\sqrt{21}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)2nq^{63m^2+n^2}$

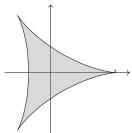
$$= 4q - 8q^{4} + 16q^{16} - 20q^{25} + 28q^{49} - 24q^{64} - 16q^{67} + 32q^{79} - 40q^{88} + 40q^{100} + \cdots$$

is in $S_2\left(\Gamma_0(252), \left(\frac{756}{\cdot}\right)\right)$.

Let
$$\tau_0 = [8, 2, 1] = -\frac{1}{8} + \frac{i\sqrt{1}}{8}$$
 with $h(D) = 1$. Then

Let
$$\tau_0 = [8, 2, 1] = -\frac{1}{8} + \frac{i\sqrt{7}}{8}$$
 with $h(D) = 1$. Then $t = t_Q(\tau_0) = \frac{16580727}{4} - \frac{3618189\sqrt{21}}{4} + \frac{729}{4}i\sqrt{-1034609022 + 225770198\sqrt{21}}$

has minimal polynomial $T^4 - {16580727} T^3 + 1343231343 T^2 - 36253389429 T + 326371204125$ and $k = \sqrt[3]{t} \approx 2.946449760454525199770 + 0.090619481311418379611i$ is not in \mathcal{K}_O°



We have
$$\nu(t) \equiv \frac{27\sqrt{21}}{L(\Theta_{O,70}, 2)}$$
, where

We have
$$\nu(t) = \frac{27\sqrt{21}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(6m+16n)q^{9m^2+6mn+8n^2}$$
$$= 32q^8 + 20q^{11} + 44q^{23} - 52q^{29} - 56q^{32} - 40q^{44} - 76q^{53} + 56q^{56} - 4q^{71} - 28q^{77} + \cdots$$

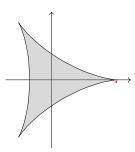
is in $S_2\left(\Gamma_0(252), \left(\frac{756}{\cdot}\right)\right)$.

Let
$$\tau_0 = [8, -2, 1] = \frac{1}{8} + \frac{i\sqrt{7}}{8}$$
 with $h(D) = 1$. Then

$$t = t_{O}(\tau_{0}) = \frac{16580727}{16580727} - \frac{3618189\sqrt{21}}{16580727} - \frac{729}{16580727}i\sqrt{-1034609022 + 225770198\sqrt{21}}$$

 $t=t_Q(\tau_0)=\frac{16580727}{4}-\frac{3618189\sqrt{21}}{4}-\frac{729}{4}i\sqrt{-1034609022+225770198\sqrt{21}}$ has minimal polynomial $T^4-16580727T^3+1343231343T^2-36253389429T+326371204125$ and

 $k = \sqrt[3]{t} \approx 2.946449760454525199770 - 0.0906194813114183796114i$ is not in \mathcal{K}_{Q}° .



We have $\nu(t) = \frac{27\sqrt{21}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$, where

We have
$$\nu(t) = \frac{1}{8\pi^2} L(O_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-6m+16n)q^{9m^2-6mn+8n^2}$
 $= 32q^8 + 20q^{11} + 44q^{23} - 52q^{29} - 56q^{32} - 40q^{44} - 76q^{53} + 56q^{56} - 4q^{71} - 28q^{77} + \cdots$
in its $S_{-1}(T_{-1}(r_0, r_0), (756))$

is in $S_2(\Gamma_0(252), (\frac{756}{2}))$.

ν -N4-18-1

Let
$$\tau_0 = [3, -3, 4] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{13}{3}}$$
 with $h(D) = 4$. Then

$$t=t_Q(\tau_0)=-\frac{621}{4}-\frac{189\sqrt{13}}{4}-\frac{81}{2}\sqrt{\frac{3}{2}\left(25+7\sqrt{13}\right)}$$
 has minimal polynomial $T^4+621T^3-36450T^2+1062882T-14348907$ and

 $k = \sqrt[3]{t} \approx -8.781527031312423193580$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{9\sqrt{13}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{9\sqrt{13}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-9m+6n)q^{12m^2-3mn+n^2}$$

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m+6n)q^{12m} - 3mn + n$$

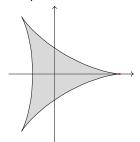
$$= 12q - 24q^4 - 12q^{10} + 108q^{16} - 84q^{22} - 60q^{25} + 156q^{40} - 48q^{43} + 84q^{49} - 156q^{52} + \cdots$$
is in $S_2\left(\Gamma_0(117), \left(\frac{117}{\cdot}\right)\right)$.

ν -N4-18-2

Let
$$\tau_0 = [12, 3, 1] = -\frac{1}{8} + \frac{1}{8}i\sqrt{\frac{13}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -\frac{621}{4} - \frac{189\sqrt{13}}{4} + \frac{81}{2}\sqrt{\frac{3}{2}\left(25 + 7\sqrt{13}\right)}$$

$$\begin{split} t &= t_Q(\tau_0) = -\frac{621}{4} - \frac{189\sqrt{13}}{4} + \frac{81}{2}\sqrt{\frac{3}{2}\left(25 + 7\sqrt{13}\right)} \\ \text{has minimal polynomial } T^4 + 621T^3 - 36450T^2 + 1062882T - 14348907 \text{ and } \\ k &= \sqrt[3]{t} \approx 2.961157292411807669795 \text{ is } \mathbf{IN} \ \mathcal{K}_Q^{\circ}. \end{split}$$



We have $\nu(t) = -\frac{9\sqrt{13}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$, where

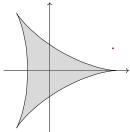
We have
$$\nu(t) = -\frac{1}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 24n)q^{3m^2 + 3mn + 4n^2}$
 $-78a^4 + 78a^{10} - 78a^{13} - 156a^{16} + 78a^{22} - 78a^{13} + 78a^{13} - 78a^{13} - 78a^{13} + 78a^{13} - 78a^{$

 $=78q^4+78q^{10}-78q^{13}-156q^{16}+78q^{22}-156q^{25}-78q^{40}+156q^{52}+312q^{55}-156q^{61}+\cdots$ is in $S_2\left(\Gamma_0(117), \left(\frac{117}{\cdot}\right)\right)$.

Let
$$\tau_0 = [6, 3, 2] = -\frac{1}{4} + \frac{1}{4}i\sqrt{\frac{13}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -\frac{621}{4} + \frac{189\sqrt{13}}{4} + \frac{81}{2}i\sqrt{\frac{3}{2}\left(-25 + 7\sqrt{13}\right)}$$

 $t=t_Q(\tau_0)=-\frac{621}{4}+\frac{189\sqrt{13}}{4}+\frac{81}{2}i\sqrt{\frac{3}{2}\left(-25+7\sqrt{13}\right)}$ has minimal polynomial $T^4+621T^3-36450T^2+1062882T-14348907$ and $k = \sqrt[3]{t} \approx 2.884198072361761680992 + 1.013066403227640218407i$ is not in \mathcal{K}_O°



We have
$$\nu(t) = \frac{9\sqrt{13}}{16\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(9m+12n)q^{6m^2+3mn+2n^2}$$

$$= 24q^2 + 6q^5 - 78q^8 + 42q^{11} - 90q^{20} + 78q^{26} + 216q^{32} - 102q^{41} - 162q^{44} - 30q^{47} + \cdots$$

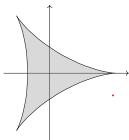
is in $S_2\left(\Gamma_0(117), \left(\frac{117}{\cdot}\right)\right)$.

ν-N4-18-4

Let
$$\tau_0 = [6, -3, 2] = \frac{1}{4} + \frac{1}{4}i\sqrt{\frac{13}{3}}$$
 with $h(D) = 4$. Then

$$t=t_Q(\tau_0)=-\tfrac{621}{4}+\tfrac{189\sqrt{13}}{4}-\tfrac{81}{2}i\sqrt{\tfrac{3}{2}\left(-25+7\sqrt{13}\right)}$$

has minimal polynomial $T^4 + 621T^3 - 36450T^2 + 1062882T - 14348907$ and $k = \sqrt[3]{t} \approx 2.884198072361761680992 - 1.013066403227640218407i$ is not in \mathcal{K}_O°



We have
$$\nu(t) = \frac{9\sqrt{13}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-9m+12n)q^{6m^2-3mn+2n^2}$$

$$= 24q^2 + 6q^5 - 78q^8 + 42q^{11} - 90q^{20} + 78q^{26} + 216q^{32} - 102q^{41} - 162q^{44} - 30q^{47} + \cdots$$

is in $S_2\left(\Gamma_0(117), \left(\frac{117}{\cdot}\right)\right)$.

ν -N4-19-1

Let
$$\tau_0 = [3, -3, 19] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{73}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -1343520 - 157248\sqrt{73} - 648\sqrt{8597643 + 1006278\sqrt{73}}$$

 $t=t_Q(\tau_0)=-1343520-157248\sqrt{73}-648\sqrt{8597643+1006278\sqrt{73}}$ has minimal polynomial $T^4+5374080T^3-225721728T^2+4353564672T-58773123072$ and

 $k = \sqrt[3]{t} \approx -175.1603645856597943265$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{9\sqrt{73}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-9m + 6n)q^{57m^2 - 3mn + n^2}$$

$$=12q-24q^4+48q^{16}-60q^{25}+84q^{49}-12q^{55}+60q^{61}-96q^{64}-84q^{67}+132q^{85}+\cdots$$

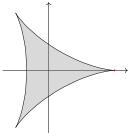
is in $S_2\left(\Gamma_0(657), \left(\frac{657}{\cdot}\right)\right)$.

Let
$$\tau_0 = [57, 3, 1] = -\frac{1}{38} + \frac{1}{38}i\sqrt{\frac{73}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -1343520 - 157248\sqrt{73} + 648\sqrt{8597643 + 1006278\sqrt{73}}$$

has minimal polynomial $T^4 + 5374080T^3 - 225721728T^2 + 4353564672T - 58773123072$ and

 $k = \sqrt[3]{t} \approx 2.999994975940338128580$ is **IN** \mathcal{K}_Q° .



We have
$$\nu(t) = -\frac{9\sqrt{73}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m+114n)q^{3m^2+3mn+19n^2}$$

 $=438q^{19}+438q^{25}+438q^{37}+438q^{55}-438q^{73}-876q^{76}+438q^{79}-876q^{85}-876q^{100}+438q^{109}+\cdots$

is in $S_2\left(\Gamma_0(657), \left(\frac{657}{\cdot}\right)\right)$.

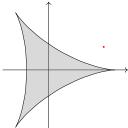
ν -N4-19-3

Let
$$\tau_0 = [15, 9, 5] = -\frac{3}{10} + \frac{1}{10}i\sqrt{\frac{73}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -1343520 + 157248\sqrt{73} + 648i\sqrt{-8597643 + 1006278\sqrt{73}}$$

has minimal polynomial $T^4 + 5374080T^3 - 225721728T^2 + 4353564672T - 58773123072$ and

 $k = \sqrt[3]{t} \approx 2.509288293114881198229 + 1.049961325166505352713i$ is not in \mathcal{K}_Q° .



We have
$$\nu(t) = \frac{9\sqrt{73}}{16\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(27m + 30n)q^{15m^2 + 9mn + 5n^2}$$

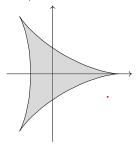
 $=60q^{5}+6q^{11}-66q^{17}-120q^{20}+114q^{29}-12q^{44}-48q^{47}-174q^{53}+186q^{59}+132q^{68}+\cdots$

is in $S_2\left(\Gamma_0(657), \left(\frac{657}{\cdot}\right)\right)$.

Let
$$\tau_0 = [15, -9, 5] = \frac{3}{10} + \frac{1}{10}i\sqrt{\frac{73}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -1343520 + 157248\sqrt{73} - 648i\sqrt{-8597643} + 1006278\sqrt{73}$$

has minimal polynomial $T^4 + 5374080T^3 - 225721728T^2 + 4353564672T - 58773123072$ and $k = \sqrt[3]{t} \approx 2.509288293114881198229 - 1.049961325166505352713i$ is not in \mathcal{K}_Q° .



We have
$$\nu(t) = \frac{9\sqrt{73}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-27m+30n)q^{15m^2-9mn+5n^2}$$

$$= 60q^5 + 6q^{11} - 66q^{17} - 120q^{20} + 114q^{29} - 12q^{44} - 48q^{47} - 174q^{53} + 186q^{59} + 132q^{68} + \cdots$$
is in $S_2\left(\Gamma_0(657), \left(\frac{657}{\cdot}\right)\right)$.

ν -N4-20-1

Let
$$\tau_0 = [3, -3, 25] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{97}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -14325552 - 1454544\sqrt{97} - 324\sqrt{6\left(651648859 + 66164917\sqrt{97}\right)}$$

has minimal polynomial $T^4 + 57302208T^3 - 6706939968T^2 + 278628139008T - 3761479876608$ and $k = \sqrt[3]{t} \approx -385.5293220589523968077$ is not in \mathcal{K}_O° .

We have
$$\nu(t) = \frac{9\sqrt{97}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = \frac{9\sqrt{97}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-9m+6n)q^{75m^2-3mn+n^2}$$
$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} - 12q^{73} + 60q^{79} - 84q^{85} + 120q^{100} + \cdots$$
is in Section (873) (873)

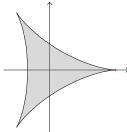
is in $S_2\left(\Gamma_0(873), \left(\frac{873}{\cdot}\right)\right)$.

ν -N4-20-2

Let
$$\tau_0 = [75, 3, 1] = -\frac{1}{50} + \frac{1}{50}i\sqrt{\frac{97}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -14325552 - 1454544\sqrt{97} + 324\sqrt{6\left(651648859 + 66164917\sqrt{97}\right)}$$

has minimal polynomial $T^4 + 57302208T^3 - 6706939968T^2 + 278628139008T - 3761479876608$ and $k = \sqrt[3]{t} \approx 2.999999528815078663627$ is **IN** \mathcal{K}_Q° .



We have
$$\nu(t) = -\frac{9\sqrt{97}}{9.2}L(\Theta_{Q,\tau_0}, 2)$$
, where

We have
$$\nu(t) = -\frac{9\sqrt{97}}{8\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(9m+150n)q^{3m^2+3mn+25n^2}$$
$$= 582q^{25} + 582q^{31} + 582q^{43} + 582q^{61} + 582q^{85} - 582q^{97} - 1164q^{100} - 1164q^{109} + 582q^{115} - 1164q^{124} + \cdots$$

is in $S_2(\Gamma_0(873), (\frac{873}{2}))$.

ν -N4-20-3

Let
$$\tau_0 = [15, 3, 5] = -\frac{1}{10} + \frac{1}{10}i\sqrt{\frac{97}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -14325552 + 1454544\sqrt{97} + 324i\sqrt{6\left(-651648859 + 66164917\sqrt{97}\right)}$$

has minimal polynomial $T^4 + 57302208T^3 - 6706939968T^2 + 278628139008T - 3761479876608$ and $k = \sqrt[3]{t} \approx 3.631066876272990679995 + 0.511667372258937333181i$ is not in \mathcal{K}_{O}°

We have
$$\nu(t) = \frac{9\sqrt{97}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m+30n)q^{15m^2+3mn+5n^2}$$

$$= 60q^5 + 42q^{17} - 120q^{20} + 78q^{23} - 102q^{29} - 138q^{41} + 24q^{59} - 84q^{68} + 96q^{71} + 240q^{80} + \cdots$$

is in $S_2\left(\Gamma_0(873), \left(\frac{873}{\cdot}\right)\right)$.

ν -N4-20-4

Let
$$\tau_0 = [15, -3, 5] = \frac{1}{10} + \frac{1}{10}i\sqrt{\frac{97}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -14325552 + 1454544\sqrt{97} - 324i\sqrt{6\left(-651648859 + 66164917\sqrt{97}\right)}$$

has minimal polynomial $T^4 + 57302208T^3 - 6706939968T^2 + 278628139008T - 3761479876608$ and $k = \sqrt[3]{t} \approx 3.631066876272990679995 - 0.511667372258937333181i$ is not in \mathcal{K}_Q° .

We have $\nu(t) = \frac{9\sqrt{97}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$, where

$$\begin{split} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-9m + 30n) q^{15m^2 - 3mn + 5n^2} \\ &= 60q^5 + 42q^{17} - 120q^{20} + 78q^{23} - 102q^{29} - 138q^{41} + 24q^{59} - 84q^{68} + 96q^{71} + 240q^{80} + \cdots \\ \text{is in } \mathcal{S}_2\left(\Gamma_0(873), \left(\frac{873}{\cdot}\right)\right). \end{split}$$

ν -N4-21-1

Let
$$\tau_0 = [3, -3, 31] = \frac{1}{2} + \frac{11i}{2\sqrt{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -115582896 - 20120400\sqrt{33} - 4860\sqrt{2\left(565606921 + 98459527\sqrt{33}\right)}$$

has minimal polynomial $T^4 + 462331584T^3 - 14585085504T^2 + 113515167744T - 1532454764544$ and $k = \sqrt[3]{t} \approx -773.2463243485238180888$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{99}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-9m+6n)q^{93m^2-3mn+n^2}$$

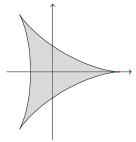
$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} - 12q^{91} + 60q^{97} + 120q^{100} - 84q^{103} + \cdots$$
is in $S_2\left(\Gamma_0(1089), \left(\frac{1089}{\cdot}\right)\right)$.

ν -N4-21-2

Let
$$\tau_0 = [93, 3, 1] = -\frac{1}{62} + \frac{11i}{62\sqrt{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -115582896 - 20120400\sqrt{33} + 4860\sqrt{2\left(565606921 + 98459527\sqrt{33}\right)}$$

has minimal polynomial $T^4 + 462331584T^3 - 14585085504T^2 + 113515167744T - 1532454764544$ and $k = \sqrt[3]{t} \approx 2.99999941600362075881$ is **IN** \mathcal{K}_O^c .



We have
$$\nu(t) = -\frac{99}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

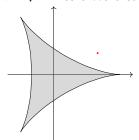
$$\begin{aligned} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(9m+186n)q^{3m^2+3mn+31n^2} \\ &= 726q^{31} + 726q^{37} + 726q^{49} + 726q^{67} + 726q^{91} - 1452q^{124} - 1452q^{133} - 1452q^{148} + 726q^{157} - 1452q^{169} + \cdots \\ &\text{is in } \mathcal{S}_2\left(\Gamma_0(1089),\left(\frac{1089}{3}\right)\right). \end{aligned}$$

ν -N4-21-3

Let
$$\tau_0 = [21, 15, 7] = -\frac{5}{14} + \frac{11i}{14\sqrt{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -115582896 + 20120400\sqrt{33} + 4860i\sqrt{2\left(-565606921 + 98459527\sqrt{33}\right)}$$

has minimal polynomial $T^4 + 462331584T^3 - 14585085504T^2 + 113515167744T - 1532454764544$ and $k = \sqrt[3]{t} \approx 2.002817764643678099769 + 0.979145264759923260540i$ is not in \mathcal{K}_O° .



We have
$$\nu(t) = \frac{99}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

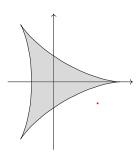
$$\begin{split} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (45m + 42n) q^{21m^2 + 15mn + 7n^2} \\ &= 84q^7 - 6q^{13} - 78q^{19} - 168q^{28} + 174q^{43} + 12q^{52} - 96q^{61} + 246q^{73} + 156q^{76} - 258q^{79} + \cdots \\ &\text{is in } \mathcal{S}_2\left(\Gamma_0(1089), \left(\frac{1089}{5}\right)\right). \end{split}$$

ν-N4-21-4

Let
$$\tau_0 = [21, -15, 7] = \frac{5}{14} + \frac{11i}{14\sqrt{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -115582896 + 20120400\sqrt{33} - 4860i\sqrt{2\left(-565606921 + 98459527\sqrt{33}\right)}$$

has minimal polynomial $T^4+462331584T^3-14585085504T^2+113515167744T-1532454764544$ and $k=\sqrt[3]{t}\approx 2.002817764643678099769-0.979145264759923260540i$ is not in \mathcal{K}_Q° .



We have $\nu(t) = \frac{99}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n) (-45m + 42n) q^{21m^2 - 15mn + 7n^2}$$

$$= 84q^7 - 6q^{13} - 78q^{19} - 168q^{28} + 174q^{43} + 12q^{52} - 96q^{61} + 246q^{73} + 156q^{76} - 258q^{79} + \cdots$$
is in $S_2(\Gamma_0(1089), (\frac{1089}{2}))$.

Let
$$\tau_0 = [3, -3, 43] = \frac{1}{2} + \frac{13i}{2\sqrt{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = -4348502496 - 1206057600\sqrt{13} - 3240\sqrt{3602628061773 + 999189246396\sqrt{13}}$ has minimal polynomial $T^4 + 17394009984T^3 - 123368869504T^2 + 41258732396544T - 556992887353344$ and $k = \sqrt[3]{t} \approx -2590.994984257351217105$ is not in \mathcal{K}_Q° .

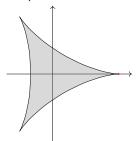
We have
$$\nu(t) = \frac{117}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) (-9m + 6n) q^{129m^2 - 3mn + n^2}$$
$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 120q^{100} - 132q^{121} - 12q^{127} + 60q^{133} + \cdots$$

is in $S_2\left(\Gamma_0(1521), \left(\frac{1521}{\cdot}\right)\right)$.

Let
$$\tau_0 = [129, 3, 1] = -\frac{1}{86} + \frac{13i}{86\sqrt{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = -4348502496 - 1206057600\sqrt{13} + 3240\sqrt{3602628061773} + 999189246396\sqrt{13}$

has minimal polynomial $T^4 + 17394009984T^3 - 1233688869504T^2 + 41258732396544T - 556992887353344$ and $k = \sqrt[3]{t} \approx 2.999999998447741499125$ is **IN** $\mathcal{K}_{\mathcal{O}}^{\circ}$.



We have $\nu(t) = -\frac{117}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$, where

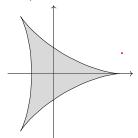
We have
$$\nu(t) = -\frac{2}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 258n)q^{3m^2 + 3mn + 43n^2}$$

$$= 1014q^{43} + 1014q^{49} + 1014q^{61} + 1014q^{79} + 1014q^{103} + 1014q^{133} - 2028q^{172} - 2028q^{181} - 2028q^{196} + 1014q^{211} + \cdots$$
is in $S_2\left(\Gamma_0(1521), \left(\frac{1521}{2}\right)\right)$.

Let
$$\tau_0 = [21, 9, 7] = -\frac{3}{14} + \frac{13i}{14\sqrt{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = -4348502496 + 1206057600\sqrt{13} + 3240i\sqrt{-3602628061773 + 999189246396\sqrt{13}}$

has minimal polynomial $T^4 + 17394009984T^3 - 1233688869504T^2 + 41258732396544T - 556992887353344$ and $k = \sqrt[3]{t} \approx 3.114748570755876067671 + 0.939921106262329843702i$ is not in \mathcal{K}_O°



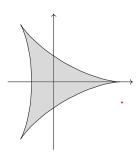
We have $\nu(t) = \frac{117}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$, where

$$\begin{split} \Theta_{Q,\tau_0}(\tau) &= \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(27m+42n)q^{21m^2+9mn+7n^2} \\ &= 84q^7 + 30q^{19} - 168q^{28} - 114q^{31} + 138q^{37} - 222q^{67} - 24q^{73} - 60q^{76} + 282q^{97} + 192q^{109} + \cdots \\ &\text{is in } \mathcal{S}_2\left(\Gamma_0(1521),\left(\frac{1521}{\cdot}\right)\right). \end{split}$$

Let
$$\tau_0 = [21, -9, 7] = \frac{3}{14} + \frac{13i}{14\sqrt{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = -4348502496 + 1206057600\sqrt{13} - 3240i\sqrt{-3602628061773 + 999189246396\sqrt{13}}$

has minimal polynomial $T^4 + 17394009984T^3 - 1233688869504T^2 + 41258732396544T - 556992887353344$ and $k = \sqrt[3]{t} \approx 3.114748570755876067671 - 0.939921106262329843702i$ is not in \mathcal{K}_O°



We have $\nu(t) = \frac{117}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$, where

We have
$$V(t) = \frac{1}{16\pi^2} 2 \log_{\tau_0, \tau_0, 2}$$
, where $\Theta_{Q, \tau_0}(\tau) = \sum_{m, n \in \mathbb{Z}} \chi_{-3}(n) (-27m + 42n) q^{21m^2 - 9mn + 7n^2}$
 $= 84q^7 + 30q^{19} - 168q^{28} - 114q^{31} + 138q^{37} - 222q^{67} - 24q^{73} - 60q^{76} + 282q^{97} + 192q^{109} + \cdots$ is in $S_2\left(\Gamma_0(1521), \left(\frac{1521}{2}\right)\right)$.

Let
$$\tau_0 = [3, -3, 61] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{241}{3}}$$
 with $h(D) = 4$. Then

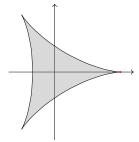
 $t=t_Q(\tau_0)=-423340998048-27269790912\sqrt{241}-3240\sqrt{34144490291088327}+2199439967608338\sqrt{241}$ has minimal polynomial $T^4+1693363992192T^3-46980539789184T^2+68024448000000T-918330048000000$ and $k=\sqrt[3]{t}\approx-11919.28232822148138230$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{9\sqrt{241}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(-9m+6n)q^{183m^2-3mn+n^2}$
$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 84q^{49} - 96q^{64} + 120q^{100} - 132q^{121} + 156q^{169} - 12q^{181} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(2169),\left(\frac{2169}{\epsilon}\right)\right)$.

 ν -N4-23-2

Let
$$\tau_0 = [183, 3, 1] = -\frac{1}{122} + \frac{1}{122}i\sqrt{\frac{241}{3}}$$
 with $h(D) = 4$. Then

has minimal polynomial $T^4 + 1693363992192T^3 - 46980539789184T^2 + 68024448000000T - 918330048000000$ and $k = \sqrt[3]{t} \approx 2.9999999999984055406797$ is **IN** $\mathcal{K}_{\mathcal{O}}^{\circ}$.

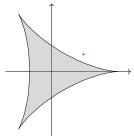


We have
$$\nu(t) = -\frac{9\sqrt{241}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(9m+366n)q^{3m^2+3mn+61n^2}$$

$$= 1446q^{61}+1446q^{67}+1446q^{79}+1446q^{97}+1446q^{121}+1446q^{151}+1446q^{187}+1446q^{229}-1446q^{241}-2892q^{244}+\cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(2169),\left(\frac{2169}{2169}\right)\right)$.

Let
$$\tau_0 = [33, 27, 11] = -\frac{9}{22} + \frac{1}{22}i\sqrt{\frac{241}{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = -423340998048 + 27269790912\sqrt{241} + 3240i\sqrt{-34144490291088327 + 2199439967608338\sqrt{241}}$ has minimal polynomial $T^4 + 1693363992192T^3 - 46980539789184T^2 + 68024448000000T - 918330048000000$ and $k=\sqrt[3]{t}\approx 1.450116699490707386282+0.784503680032144004279i$ is not in \mathcal{K}_Q°

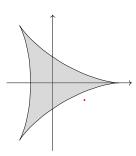


We have
$$\nu(t) = \frac{9\sqrt{241}}{16\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(81m+66n)q^{33m^2+27mn+11n^2}$$

$$= 132q^{11} - 30q^{17} - 102q^{23} - 264q^{44} + 60q^{68} + 294q^{71} - 192q^{89} + 204q^{92} + 366q^{101} - 426q^{131} + \cdots$$
 is in $\mathcal{S}_2\left(\Gamma_0(2169),\left(\frac{2169}{100}\right)\right)$.

Let
$$\tau_0 = [33, -27, 11] = \frac{9}{22} + \frac{1}{22}i\sqrt{\frac{241}{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = -423340998048 + 27269790912\sqrt{241} - 3240i\sqrt{-34144490291088327} + 2199439967608338\sqrt{241} + 219943996760838\sqrt{241} + 219943996760836\sqrt{241} + 21994396760876\sqrt{241} + 21994396760876\sqrt{241} + 21994396760876\sqrt{241} + 2199439676\sqrt{241} + 2199439676\sqrt{241} + 21994396760876\sqrt{241} + 21994396760876\sqrt{241} + 21994396760876\sqrt{241} + 21994396760876\sqrt{241} + 21994396760876\sqrt{241} + 2199439676\sqrt{241} + 2199439676\sqrt{241} + 2199476\sqrt{241} + 219$ has minimal polynomial $T^4 + 1693363992192T^3 - 46980539789184T^2 + 68024448000000T - 918330048000000$ and $k = \sqrt[3]{t} \approx 1.450116699490707386282 - 0.784503680032144004279i$ is not in \mathcal{K}_O°



We have
$$\nu(t) = \frac{9\sqrt{241}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(-81m + 66n)q^{33m^2 - 27mn + 11n^2}$$

$$= 132q^{11} - 30q^{17} - 102q^{23} - 264q^{44} + 60q^{68} + 294q^{71} - 192q^{89} + 204q^{92} + 366q^{101} - 426q^{131} + \cdots$$

is in $S_2(\Gamma_0(2169), (\frac{2169}{.}))$.

ν -N4-24-1

Let
$$\tau_0 = [3, -3, 103] = \frac{1}{2} + \frac{1}{2}i\sqrt{\frac{409}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -2131340796546048 - 105388019702784\sqrt{409} - 12960\sqrt{6\left(9015182817623475045667 + 445772100805075082311\sqrt{409}\right)}$$

has minimal polynomial $T^4 + 8525363186184192T^3 - 560410748554973184T^2 + 17832200896512000000T - 240734712102912000000$ and $k = \sqrt[3]{t} \approx -204285.5413317431374449$ is not in \mathcal{K}_Q° .

We have
$$\nu(t) = \frac{9\sqrt{409}}{16\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

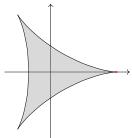
We have
$$\nu(t) = \frac{9\sqrt{409}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-9m+6n)q^{309m^2-3mn+n^2}$$

 $=12q-24q^4+48q^{16}-60q^{25}+84q^{49}-96q^{64}+120q^{100}-132q^{121}+156q^{169}-168q^{196}+\cdots$ is in $\mathcal{S}_2\left(\Gamma_0(3681),\left(\frac{3681}{\cdot}\right)\right)$.

ν -N4-24-2

Let
$$\tau_0 = [309, 3, 1] = -\frac{1}{206} + \frac{1}{206}i\sqrt{\frac{409}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = -2131340796546048 - 105388019702784\sqrt{409} + 12960\sqrt{6\left(9015182817623475045667 + 445772100805075082311\sqrt{409}\right)}$$



We have
$$\nu(t) = -\frac{9\sqrt{409}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

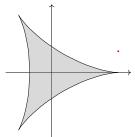
we have
$$\nu(t) = -\frac{2}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(9m + 618n)q^{3m^2 + 3mn + 103n^2}$

 $=2454q^{103}+2454q^{109}+2454q^{121}+2454q^{139}+2454q^{163}+2454q^{193}+2454q^{229}+2454q^{271}+2454q^{319}+2454q^{373}+\cdots$ is in $S_2(\Gamma_0(3681), (\frac{3681}{.}))$.

Let
$$\tau_0 = [33, 15, 11] = -\frac{5}{22} + \frac{1}{22}i\sqrt{\frac{409}{3}}$$
 with $h(D) = 4$. Then

$$t=t_Q(\tau_0)=-2131340796546048+105388019702784\sqrt{409}+12960i\sqrt{6\left(-9015182817623475045667+445772100805075082311\sqrt{409}\right)}$$

has minimal polynomial $T^4 + 8525363186184192T^3 - 560410748554973184T^2 + 17832200896512000000T - 240734712102912000000$ and $k = \sqrt[3]{t} \approx 3.034541370202187095363 + 0.970596093498807327403i$ is not in \mathcal{K}_Q° .



We have
$$\nu(t) = \frac{9\sqrt{409}}{10^{-2}}L(\Theta_{O,\tau_0},2)$$
, where

We have
$$\nu(t)=\frac{9\sqrt{409}}{16\pi^2}L(\Theta_{Q,\tau_0},2)$$
, where
$$\Theta_{Q,\tau_0}(\tau)=\sum_{m,n\in\mathbb{Z}}\chi_{-3}(n)(45m+66n)q^{33m^2+15mn+11n^2}$$

$$= 132q^{11} + 42q^{29} - 264q^{44} - 174q^{47} + 222q^{59} - 354q^{107} - 48q^{113} - 84q^{116} + 438q^{149} + 312q^{173} + \cdots$$

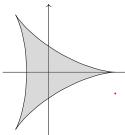
is in $S_2\left(\Gamma_0(3681), \left(\frac{3681}{\cdot}\right)\right)$.

ν -N4-24-4

Let
$$\tau_0 = [33, -15, 11] = \frac{5}{22} + \frac{1}{22}i\sqrt{\frac{409}{3}}$$
 with $h(D) = 4$. Then

 $t = t_Q(\tau_0) = -2131340796546048 + 105388019702784\sqrt{409} - 12960i\sqrt{6\left(-9015182817623475045667 + 445772100805075082311\sqrt{409}\right)}$

has minimal polynomial $T^4 + 8525363186184192T^3 - 560410748554973184T^2 + 17832200896512000000T - 240734712102912000000$ and $k = \sqrt[3]{t} \approx 3.034541370202187095363 - 0.970596093498807327403i$ is not in \mathcal{K}_O°



We have
$$\nu(t) = \frac{9\sqrt{409}}{16\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where $\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-45m + 66n)q^{33m^2 - 15mn + 11n^2}$

$$= 132q^{11} + 42q^{29} - 264q^{44} - 174q^{47} + 222q^{59} - 354q^{107} - 48q^{113} - 84q^{116} + 438q^{149} + 312q^{173} + \cdots$$

is in $S_2(\Gamma_0(3681), (\frac{3681}{.}))$.

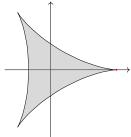
ν -N4-25-1

Let
$$\tau_0 = [39, 0, 1] = \frac{i}{\sqrt{39}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = \frac{479061}{4} + \frac{132867\sqrt{13}}{4} - \frac{243}{4}\sqrt{7771398 + 2155398\sqrt{13}}$$

 $t=t_Q(\tau_0)=\frac{479061}{4}+\frac{132867\sqrt{13}}{4}-\frac{243}{4}\sqrt{7771398+2155398\sqrt{13}}$ has minimal polynomial $T^4-479061T^3+13466088T^2-28697814T+387420489$ and

 $k = \sqrt[3]{t} \approx 3.000056365680104470817$ is not in $\mathcal{K}_{\mathcal{O}}^{\circ}$.



We have
$$\nu(t) = \frac{9\sqrt{13}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 78nq^{3m^2 + 13n^2}$$

$$= 156q^{13} + 312q^{16} + 312q^{25} + 312q^{40} - 312q^{52} - 624q^{55} + 312q^{61} - 624q^{64} - 624q^{79} + 312q^{88} + \cdots$$

is in $S_2(\Gamma_0(468), (\frac{468}{}))$.

ν -N4-25-2

Let
$$\tau_0 = [3, 0, 13] = i\sqrt{\frac{13}{3}}$$
 with $h(D) = 4$. Then

$$t = t_O(\tau_0) = \frac{479061}{1000} + \frac{132867\sqrt{13}}{1000} + \frac{243}{1000}\sqrt{7771398 + 2155398\sqrt{13}}$$

 $t=t_Q(\tau_0)=\frac{479061}{4}+\frac{132867\sqrt{13}}{4}+\frac{243}{4}\sqrt{7771398+2155398\sqrt{13}}$ has minimal polynomial $T^4-479061T^3+13466088T^2-28697814T+387420489$ and

 $k = \sqrt[3]{t} \approx 78.24473259220120996252$ is not in \mathcal{K}_{O}° .

We have $\nu(t) = \frac{9\sqrt{13}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n) 6nq^{39m^2 + n^2}$$

$$= 12q - 24q^4 + 48q^{16} - 60q^{25} + 24q^{40} - 48q^{43} + 84q^{49} + 96q^{55} - 216q^{64} + 168q^{88} + \cdots$$

is in $S_2(\Gamma_0(468), (\frac{468}{}))$.

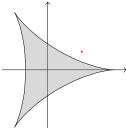
ν -N4-25-3

Let
$$\tau_0 = [15, 12, 5] = -\frac{2}{5} + \frac{1}{5}i\sqrt{\frac{13}{3}}$$
 with $h(D) = 4$. Then

$$t = t_Q(\tau_0) = \frac{479061}{4} - \frac{132867\sqrt{13}}{4} + \frac{243}{4}i\sqrt{-7771398 + 2155398\sqrt{13}}$$

has minimal polynomial $T^4 - 479061T^3 + 13466088T^2 - 28697814T + 387420489$ and

 $k = \sqrt[3]{t} \approx 1.555106354180089945361 + 0.8289911002049984849582i$ is not in \mathcal{K}_{O}° .



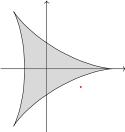
We have
$$\nu(t) = \frac{9\sqrt{13}}{8\pi^2} L(\Theta_{Q,\tau_0}, 2)$$
, where

$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n \in \mathbb{Z}} \chi_{-3}(n)(36m + 30n)q^{15m^2 + 12mn + 5n^2}$$

$$= 60q^{5} - 12q^{8} - 48q^{11} - 120q^{20} + 156q^{32} - 84q^{41} + 96q^{44} + 168q^{47} - 192q^{59} - 156q^{65} + \cdots$$

is in $S_2(\Gamma_0(468), (\frac{468}{.}))$.

Let
$$\tau_0=[15,-12,5]=\frac{2}{5}+\frac{1}{5}i\sqrt{\frac{13}{3}}$$
 with $h(D)=4$. Then
$$t=t_Q(\tau_0)=\frac{479061}{4}-\frac{132867\sqrt{13}}{4}-\frac{243}{4}i\sqrt{-7771398+2155398\sqrt{13}}$$
 has minal polynomial $T^4-479061T^3+13466088T^2-28697814T+387420489$ and $k=\sqrt[3]{t}\approx 1.555106354180089945361-0.828991100204998484958i$ is not in \mathcal{K}_Q° .



We have
$$\nu(t) = \frac{9\sqrt{13}}{8\pi^2}L(\Theta_{Q,\tau_0}, 2)$$
, where
$$\Theta_{Q,\tau_0}(\tau) = \sum_{m,n\in\mathbb{Z}} \chi_{-3}(n)(-36m + 30n)q^{15m^2 - 12mn + 5n^2}$$
$$= 60q^5 - 12q^8 - 48q^{11} - 120q^{20} + 156q^{32} - 84q^{41} + 96q^{44} + 168q^{47} - 192q^{59} - 156q^{65} + \cdots$$
is in $\mathcal{S}_2\left(\Gamma_0(468), \left(\frac{468}{\cdot}\right)\right)$.