



# Robotics

## Legged Locomotion

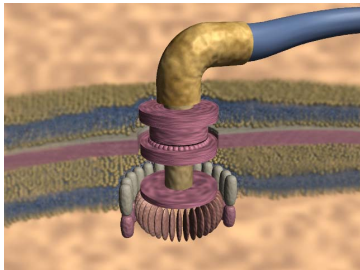
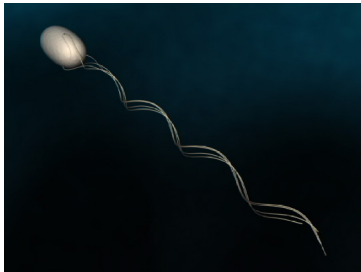
*Why legs, Raibert hopper, statically stable walking, zero moment point, human walking, compass gait, passive walker*

Marc Toussaint  
FU Berlin

# Legged Locomotion

- Why legs?

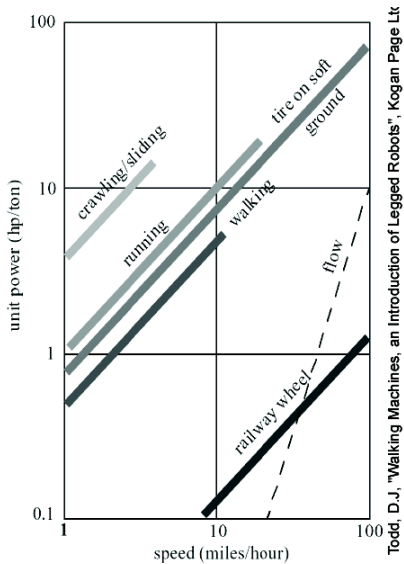
Bacterial Flagellum: (rotational “motors” in Biology?)



# Legged Locomotion

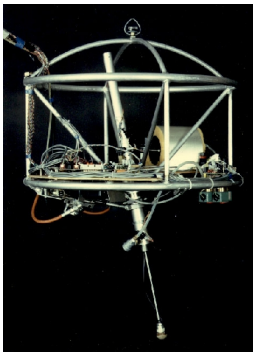
- Why legs?
  - Human/Animal Locomotion Research
  - Potentially less weight
  - Better handling of rough terrains  
(climbing, isolated footholds, ladders, stairs)

# Rolling vs walking



Todd, D.J., "Walking Machines, an Introduction of Legged Robots", Kogan Page Ltd

# One-legged locomotion



- Three separate controllers for:
  - hopping height
  - horizontal velocity (foot placement)
  - attitude (hip torques during stance)
- Each a simple (PD-like) controller

Raibert et al.: *Dynamically Stable Legged Locomotion*. 1985

<http://dspace.mit.edu/handle/1721.1/6820>

Tedrake: *Applied Optimal Control for Dynamically Stable Legged Locomotion*. PhD thesis (2004).

[http://groups.csail.mit.edu/robotics-center/public\\_papers/Tedrake04b.pdf](http://groups.csail.mit.edu/robotics-center/public_papers/Tedrake04b.pdf)

# Biped locomotion

- Walking vs Running

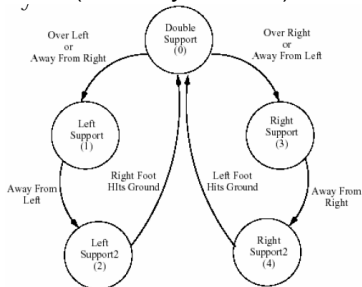
Walking := in all instances at least one foot is on ground

Running := otherwise

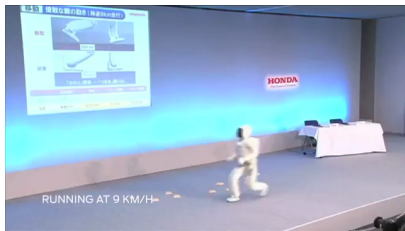
- 2 phases of Walking

- double-support phase (in Robotics often statically stable)

- single-support phase (statically instable)



# Asimo



# Statically Stable Walk

- You could rest (hold pose) at any point in time and not fall over

$\iff$  *CoG projected on ground is within **support polygon***

CoG = center of gravity of all body masses

support polygon = hull of foot contact points

- Try yourself: Move as slow as you can but make normal length steps...



# Zero Moment Point

- Vukobratovic's view on walking, leading to ZMP ideas:  
“Basic characteristics of all biped locomotion systems are:  
  
(i) the possibility of rotation of the overall system about one of the foot edges caused by strong disturbances, which is equivalent to the appearance of an unpowered (passive) DOF,  
  
(ii) gait repeatability (symmetry), which is related to regular gait only  
  
(iii) regular interchangeability of single- and double-support phases”

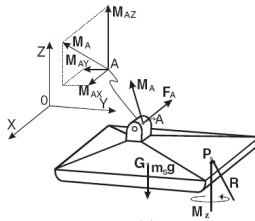
Vukobratovic & Borovac: *Zero-moment point—Thirty five years of its life*. International Journal of Humanoid Robotics 1, 157-173, 2004.

<http://www.cs.cmu.edu/~cga/legs/vukobratovic.pdf>

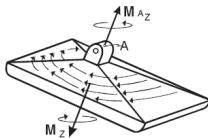
# Zero Moment Point



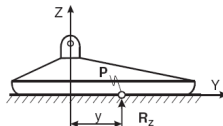
(a)



(b)



(a)

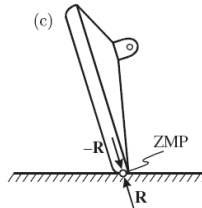
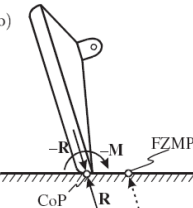
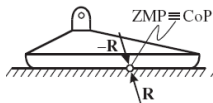


(b)

(a)

(b)

(c)



# Zero Moment Point

- “ZMP is defined as that point on the ground at which the net moment of the inertial forces and the gravity forces has no component along the horizontal axes.” (Vukobratovic & Borovac, 2004)

- $f_i$  = force vector acting on body  $i$  (gravity plus external)

$w_i$  = angular vel vector of body  $i$

$I_i$  = inertia tensor ( $\in \mathbb{R}^{3 \times 3}$ ) of body  $i$

$r_i = p_i - p_{\text{ZMP}}$  = relative position of body  $i$  to ZMP

- Definition:  $p_{\text{ZMP}}$  is the point on the ground for which

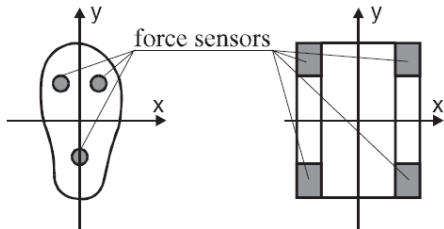
$$\sum_i r_i \times f_i + I_i \dot{w}_i + w_i \times I_i w_i \stackrel{!}{=} (0, 0, *)^T$$

See also: Popovic, Goswami & Herr: *Ground Reference Points in Legged Locomotion: Definitions, Biological Trajectories and Control Implications*. International Journal of Robotics Research 24(12), 2005.

[http://www.cs.cmu.edu/~cga/legs/Popovic\\_Goswami\\_Herr\\_IJRR\\_Dec\\_2005.pdf](http://www.cs.cmu.edu/~cga/legs/Popovic_Goswami_Herr_IJRR_Dec_2005.pdf)

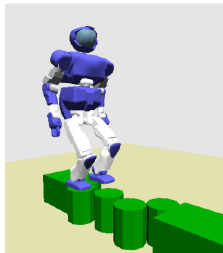
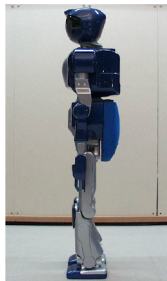
# Zero Moment Point

- If the ZMP is outside the support polygon  $\rightarrow$  foot rolls over the edge (presumes foot is a rigid body, and non-compliant control)
- Locomotion is *dynamically stable* if the ZMP remains within the foot-print polygons.  
( $\leftrightarrow$  the support can always apply some momentum, if necessary)
- Computing the ZMP in practise:
  - either exact robot model
  - or foot pressure sensors



# Zero Moment Point – example

- combine ZMP with 3D linear inverted pendulum model and model-predictive control



HRP-2 stair climbing

Kajita et al.: *Biped Walking Pattern Generation by using Preview Control of Zero-Moment Point*. ICRA 2003. [http://eref.uqu.edu.sa/files/eref2/folder1/biped\\_walking\\_pattern\\_generation\\_by\\_usin\\_53925.pdf](http://eref.uqu.edu.sa/files/eref2/folder1/biped_walking_pattern_generation_by_usin_53925.pdf)

# ZMP Summary

- ZMP is the “rescue” for conventional methods:
  - ZMP-stability  $\rightarrow$  the robot can be controlled as if foot is “glued” (virtually) to the ground!
  - The whole body behaves like a “conventional arm”
  - Can accelerate  $\ddot{q}$  any DoF  $\rightarrow$  conventional dynamic control  
*fully actuated system*
- Limitations:
  - Humans don’t use ZMP stability, we allow our feet to roll (toe-off, heel-strike: ZMP at edge of support polygon)
  - Can’t describe robots with point feet (walking on stilts)

# Models of human bipedal locomotion

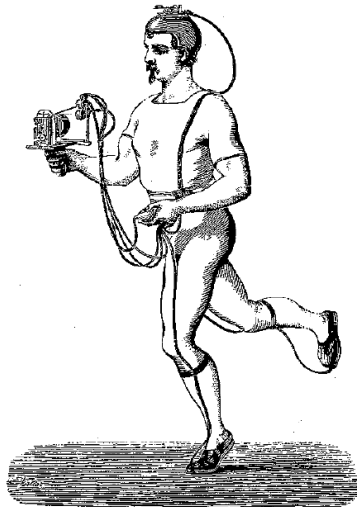
The following illustrations are from:

McMahon: *Mechanics of Locomotion*. IJRR 3:4-28, 1984

<http://www.cs.cmu.edu/~cga/legs/mcmahon1.pdf>

<http://www.cs.cmu.edu/~cga/legs/mcmahon2.pdf>

Walking research from Marey 1874:

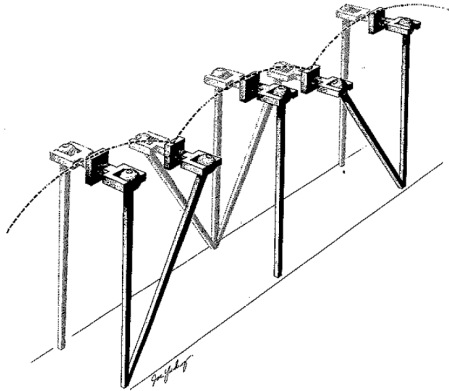




# Six determinants of gait

following Saunders, Inman & Eberhart (1953)

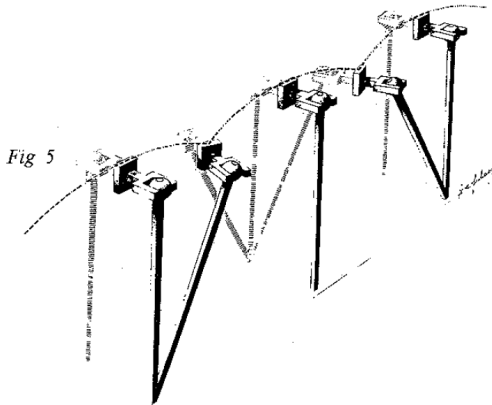
## 1. Compass Gait:



# Six determinants of gait

following Saunders, Inman & Eberhart (1953)

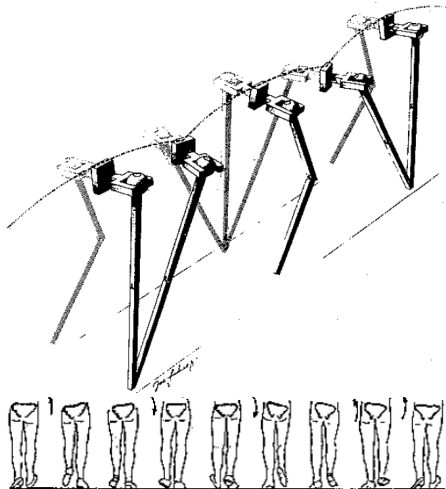
## 2. Pelvic Rotation:



# Six determinants of gait

following Saunders, Inman & Eberhart (1953)

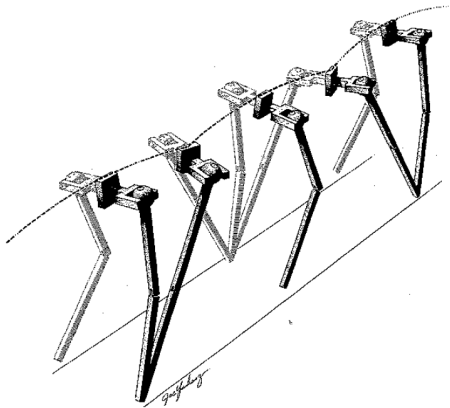
## 3. Pelvic Tilt:



# Six determinants of gait

following Saunders, Inman & Eberhart (1953)

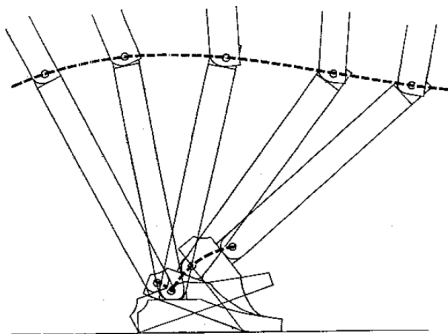
## 4. Stance Knee Flexion:



# Six determinants of gait

following Saunders, Inman & Eberhart (1953)

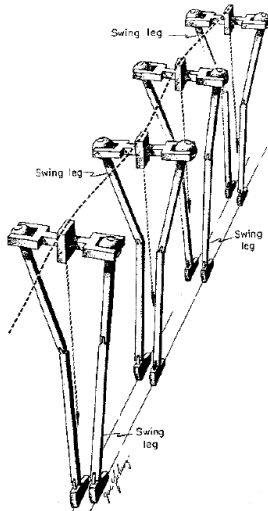
## 5. Stance Ankle Flexion:



# Six determinants of gait

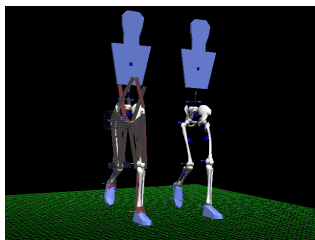
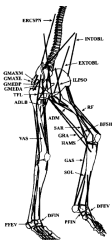
following Saunders, Inman & Eberhart (1953)

## 6. Pelvis Lateral Displacement:



# Models of human bipedal locomotion

- Human model with 23 DoFs, 54 muscles
  - Compare human walking data with model
  - Model: optimize energy-per-distance
  - Energy estimated based on metabolism and muscle heat rate models



Anderson & Pandy: *Dynamic Optimization of Human Walking*. Journal of Biomechanical Engineering 123:381-390, 2001.

<http://e.guigon.free.fr/rsc/article/AndersonPandy01.pdf>

Anderson & Pandy: *Static and dynamic optimization solutions for gait are practically equivalent*. Journal of Biomechanics 34 (2001) 153-161.

<http://www.bme.utexas.edu/faculty/pandy/StaticOptWalking2001.pdf>

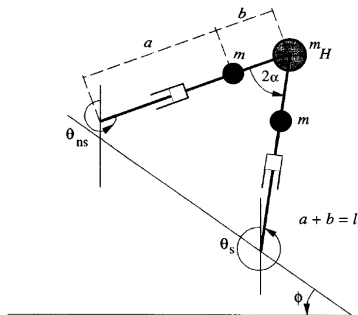
# Models of human bipedal locomotion

- Suggest different principles of human motion:
  - passive dynamics (Compass Gait)  $\leftrightarrow$  underactuated system
  - modulation of basic passive dynamics
  - Energy minimization



# Passive dynamic walking: Compass Gait

- Basic 2D planar model of the Compass Gait:



The pose is described by  $q = (\theta_s, \theta_{ns})$ , the state by  $(q, \dot{q})$

Goswami, Thuilot & Espiau: *A study of the passive gait of a compass-like biped robot: symmetry and chaos*. International Journal of Robotics Research 17, 1998.

[http://www.ambarish.com/paper/COMPASS\\_IJRR\\_Goswami.pdf](http://www.ambarish.com/paper/COMPASS_IJRR_Goswami.pdf)

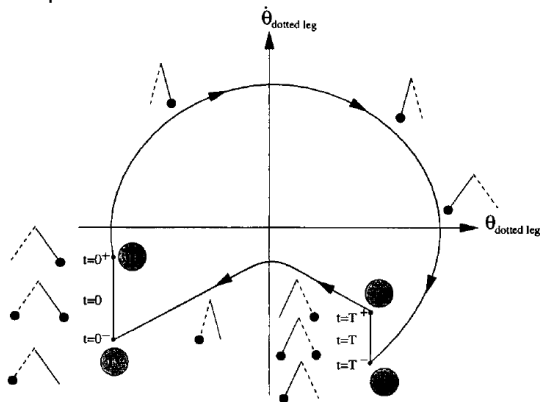
# Passive dynamic walking: Compass Gait

- Swing phase has analytic equations of motions

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = 0$$

but can't be solved analytically...

- Phase space plot of numeric solution:



# Passive walker examples

compass gait simulation

controlled on a circle

passive walker

- Minimally actuated: Minimal Control on rough terrain

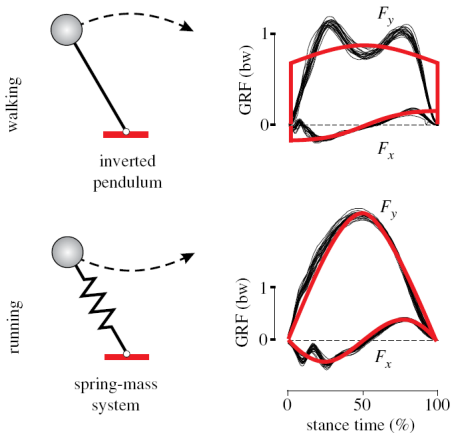
# Impact Models in the Compass Gait

- Switch between two consecutive swing phases: depends on slope!
- Typical assumptions made in simulation models:
  - The contact of the swing leg with the ground results in no rebound and no slipping of the swing leg.
  - At the moment of impact, the stance leg lifts from the ground without interaction.
  - The impact is instantaneous.
  - The external forces during the impact can be represented by impulses.
  - The impulsive forces may result in an instantaneous change in the velocities, but there is no instantaneous change in the configuration.
  - The actuators cannot generate impulses and, hence, can be ignored during impact.

Westervelt, Grizzle & Koditschek: *Hybrid Zero Dynamics of Planar Biped Walkers*. IEEE Trans. on Automatic Control 48(1), 2003.

[http://repository.upenn.edu/cgi/viewcontent.cgi?article=1124&context=ese\\_papers](http://repository.upenn.edu/cgi/viewcontent.cgi?article=1124&context=ese_papers)

# Implausibility of the stiff Compass Gait leg



Geyer, Seyfarth & Blickhan: *Compliant leg behaviour explains basic dynamics of walking and running*. Proc. Roy. Soc. Lond. B, 273(1603): 2861-2867, 2006.

<http://www.cs.cmu.edu/~cga/legs/GeyerEA06RoySocBiolSci.pdf>

# Learning to walk in 20 Minutes

- Policy Gradient method (Reinforcement Learning)  
Stationary policy parameterized as linear in features  $u = \sum_i w_i \phi_i(q, \dot{q})$
- Problem: find parameters  $w_i$  to minimize expected costs  
cost = mimick  $(q, \dot{q})$  of the passive down-hill walker at “certain point in cycle”



Learning To Walk

Tedrake, Zhang & Seung: *Stochastic policy gradient reinforcement learning on a simple 3D biped*. IROS, 2849-2854, 2004.

[http://groups.csail.mit.edu/robotics-center/public\\_papers/Tedrake04a.pdf](http://groups.csail.mit.edu/robotics-center/public_papers/Tedrake04a.pdf)

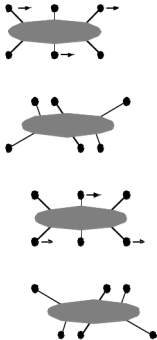
# Summary

- ZMP type walking was successful (ASIMO, HRP-2, etc), but limited
- Future types of walking:
  - Exploit passive dynamics, cope with *underactuation*
  - Follow some general optimality principle (but real-time!)
  - Learn (esp. Reinforcement Learning)
  - Compliant hardware! (controllable elasticity & damping)
- Recommended reading: Tedrake: *Underactuated Robotics: Learning, Planning, and Control for Efficient and Agile Machines*. Course Notes for MIT 6.832

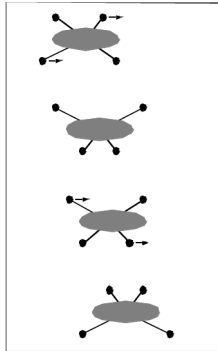
[www.cs.berkeley.edu/~pabbeel/cs287-fa09/readings/Tedrake-Aug09.pdf](http://www.cs.berkeley.edu/~pabbeel/cs287-fa09/readings/Tedrake-Aug09.pdf)

# Finally, multi-legged locomotion

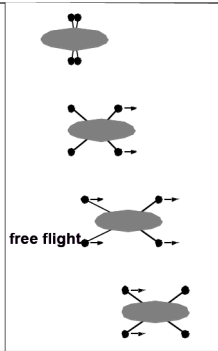
Static walking (6 legs)



Changeover walking

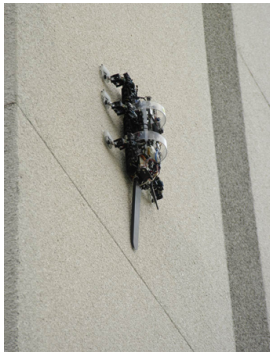


Gallop (4 legs)





# Finally, multi-legged locomotion



[http://www.bostondynamics.com/robot\\_rise.html](http://www.bostondynamics.com/robot_rise.html)