Advanced Numerical Methods

Assignment 1

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Just uploading something.

1 Task 1

1.1 Energy method

The energy method on the continuous problem is used to derive the boundary terms.

$$\begin{gathered} Cu_t = Au_x \\ u = \begin{bmatrix} E \\ H \end{bmatrix} A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} C = \begin{bmatrix} \epsilon & 0 \\ 0 & \mu \end{bmatrix} \end{gathered} \tag{1}$$

Multiply by u^T from the left an integrate by parts:

$$(u^T, Cu_t) = (u^T, Au_x) = -(u_x^T, Au) + [u^T Au]_{x_l}^{x_r} \tag{2}$$

Calculate the transpose:

$$((Cu_t)^T, u) = ((Au_x)^T, u) = (u_x^T A^T, u) = (u_x^T A, u) = (u_x^T, Au) \tag{3}$$

Adding (2) and (3):

$$(u^T, Cu_t) + ((Cu_t)^T, u) = \frac{d}{dt} \|u\|^2 = (u_x^T, Au) - (u_x^T, Au) + [u^T Au]_{x_l}^{x_r} = [u^T Au]_{x_l}^{x_r} \tag{4}$$

We get the boundary terms:

$$[u^T Au]_{x_l}^{x_r} = u^{(1)} u^{(2)}|_{x_r} - u^{(1)} u^{(2)}|_{x_l} \tag{5}$$

2 Task 2

3 Task 3