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I hereby declare that this thesis was formulated by myself and that no sources or tools other than those cited were used.

Bonn, ...

Date Signature

1. Gutachter: Prof. Dr. John Smith
2. Gutachterin: Prof. Dr. Anne Jones

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I would like to thank ...

You should probably use `\chapter*` for acknowledgements at the beginning of a thesis and `\chapter` for the end.

Contents

1 Introduction **1**

2 Theoretical Overview **3**
- 2.1 Excitons . . . 3
- 2.2 Hubbard-Model . . . 3
- 2.3 HMC . . . 3
- 2.4 Correlation Functions . . . 3

3 Symmetries **5**
- 3.1 Lattice . . . 5
- 3.2 Symmetry Operators . . . 7
  - 3.2.1 Spin-flip transformation . . . 7
  - 3.2.2 Charge transformation . . . 9
  - 3.2.3 Particle-hole transformation . . . 10
- 3.3 Correlation functions . . . 13
  - 3.3.1 One-particle correlation function . . . 13
  - 3.3.2 Two-particle correlation function . . . 14

4 Spectrum of Excitons **31**
- 4.1 Data Generating . . . 31
- 4.2 Fitting Data . . . 31

A Useful information **33**

Bibliography **35**

List of Figures **37**

List of Tables **39**

CHAPTER 1

Introduction

The introduction usually gives a few pages of introduction to the whole subject, maybe even starting with the Greeks.

For more information on LaTeX and the packages that are available see for example the books of Kopka [1] and Goossens et al [2].

A lot of useful information on particle physics can be found in the "Particle Data Book" [3].

I have resisted the temptation to put a lot of definitions into the file `thesis_defs.sty`, as everyone has their own taste as to what scheme they want to use for names. However, a few examples are included to help you get started:

- cross-sections are measured in pb and integrated luminosity in pb^{-1};
- the K_S^0 is an interesting particle;
- the missing transverse momentum, $p_{\mathrm{T}}^{\mathrm{miss}}$, is often called missing transverse energy, even though it is calculated using a vector sum.

Note that the examples of units assume that you are using the siunitx package.

It also is probably a good idea to include a few well formatted references in the thesis skeleton. More detailed suggestions on what citation types to use can be found in the "Thesis Guide" [4]:

- articles in refereed journals [3, 5];
- a book [6];
- a PhD thesis [7] and a Diplom thesis [8];
- a collection of articles [9];
- a conference note [10];
- a preprint [11] (you can also use `@online` or `@booklet` for such things);
- something that is only available online [4].

At the end of the introduction it is normal to say briefly what comes in the following chapters.

The line at the beginning of this file is used by TeXstudio etc. to specify which is the master LaTeX file, so that you can compile your thesis directly from this file. If your thesis is called something other than `mythesis`, adjust it as appropriate.

CHAPTER 2

Theoretical Overview

2.1 Excitons

2.2 Hubbard-Model

2.3 HMC

2.4 Correlation Functions

CHAPTER 3

Symmetries

A very important aspect of the analysis are the symmetries. They let us extract useful information from the available data. We can leverage the symmetries we have found, so that we can reduce the numerical uncertainties of our data by averaging, or we can reduce the computational cost by declaring two results are the same by symmetry. The term means any operation one can perform on an operator that leaves it invariant.

In this chapter we will see what is the symmetry of the lattice, which transformations are symmetry operators of the Hubbard model Hamiltonian, and how they transform the correlation functions that we are interested in. This will help us better our data and the results we extract from it.

3.1 Lattice

The honeycomb lattice is a 2D lattice where the points of the primitive cell lie on the vertices of a hexagon. The reciprocal lattice is also hexagonal. The honeycomb lattice can be seen as made of two triangular lattices. Lattices with this property are called a bipartite. In general, a bipartite lattice is any one which we can make from two overlapping lattices, where every point of one of the sublattices is only neighboring points from the other sublattice. One of the triangular sublattices of the honeycomb plane is generated by two non-orthogonal vectors

$$\vec{a}_1 = \frac{\sqrt{3}}{2}\begin{pmatrix}\sqrt{3}\\+1\end{pmatrix} \qquad \vec{a}_2 = \frac{\sqrt{3}}{2}\begin{pmatrix}\sqrt{3}\\-1\end{pmatrix} \tag{3.1}$$

By repeating them L_1 and L_2 times respectively, we can construct the whole finite sublattice. The other one can be constructed simply by offsetting the points with a translation vector $\vec{r} = \pm(\vec{a}_1 + \vec{a}_2)$. On Fig. 3.1 is shown the bipartite of the hexagonal lattice.

In order to investigate the symmetry operators and how they transform the correlation functions, we work with the reciprocal lattice. The primitive cell of the reciprocal is called Brillouin zone (BZ). Some of the points on the lattice which are of special interest have labels that are used in the literature and in this thesis. These are the center of the first Brillouin zone called the Gamma point (Γ), the two points on the vertices of the BZ called the Dirac points (K, K'), and the three middle of the edges of the BZ points M, M', M''. They are all shown on Fig. 3.2 as well as the first neighbors. The graph shows that the first BZ exhibits all the symmetries of the lattice, since the plane wave with a

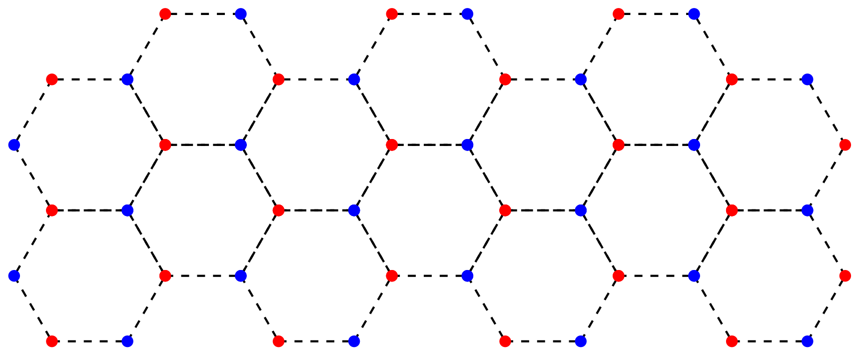

Figure 3.1: Honeycomb lattice is a bipartite lattice because we can separate and label the lattice points with different colors (in this example blue and red) in such a way that every point in blue color neighbors only points with red color.

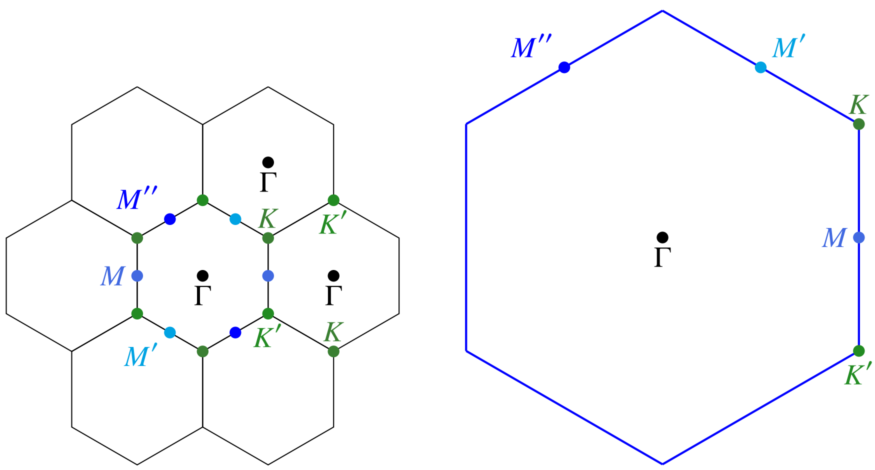

Figure 3.2: LEFT: All points of special interest shown on the lattice and how they are copied over to other lattice cells. RIGHT: Special points only in the first BZ since it has all the symmetries of the lattice

specific mode outside the 1st BZ is exactly equal to a plane wave with a momentum inside the BZ. Therefore, we do not need to investigate beyond it. So, we consider only the first Brillouin zone in our calculations. The primitive cell of the reciprocal lattice of the honeycomb is a hexagon, meaning it must have all the symmetries of the dihedral group. The result is that each set of momenta that is related by symmetry of the lattice can be averaged (e.g. K with K', M with M' and M'', etc.). The key moment here is that we have not used any model here. This is only due to the symmetry of the lattice itself. Therefore, this averaging of momenta could be performed anytime but the group symmetry operations are lattice dependent (Square, Honeycomb, Kagome, and others).

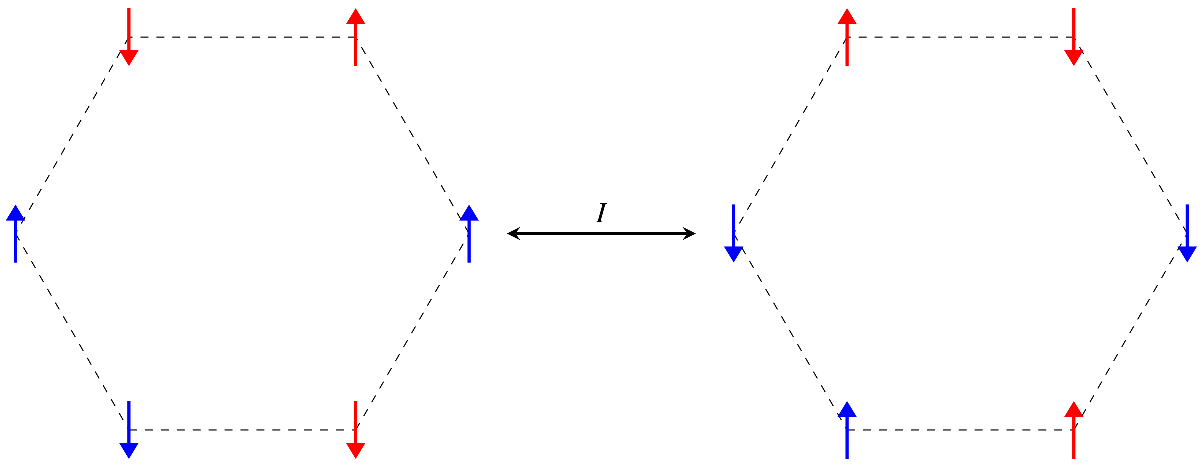

Figure 3.3: An example of the spin-flip transformation with all possibilities of creation/annihilation operators. The graphs show only the transformation of the charge and the spin of the operators. The color of the arrows shows the charge (blue = +1, red = −1) and the orientation shows the spin. We can can go back and forth between the left and the right graphs by using the spin-flip conjugation.

3.2 Symmetry Operators

In this section, we are going to focus only on transformation that do not change the Hamiltonian in general or at half filling ($\mu = 0$) i.e. symmetry operations which means they commute with it

$$[H, R] = 0. \tag{3.2}$$

We can show that the expectation value of the operator does not change when we apply the symmetry to it.

$$\langle O \rangle = \frac{1}{Z} tr[OR^{-1}Re^{-\beta H}] = \frac{1}{Z} tr[OR^{-1}e^{-\beta H}R] = \frac{1}{Z} tr[ROR^{-1}e^{-\beta H}] = \langle ROR^{-1} \rangle \tag{3.3}$$

For our chosen model, there are only a few symmetry operators that satisfy this condition. We are going to see how they can be used to our advantage to increase our statistics.

3.2.1 Spin-flip transformation

On a bipartite lattice, we have a spin-flip operation that exchanges the particles and holes but leaves the charge untouched. On Figure 3.3 there is shown an example of how the spin-flip operation transforms each operator when applied. Unfortunately, the example can show only how the spin and the charge transform, but not the Hamiltonian terms and amplitudes.

When applied onto the creation/annihilation operators of the particles and the holes, this spin transformation gives us

$$
\begin{aligned}
IpI^{-1} &= \Sigma h^\dagger = h^\dagger \Sigma & \qquad IhI^{-1} &= \Sigma p^\dagger = p^\dagger \Sigma \\
Ip^\dagger I^{-1} &= \Sigma h = h\Sigma & \qquad Ih^\dagger I^{-1} &= \Sigma p = p\Sigma
\end{aligned}
$$

where Σ is a matrix which shows us the behavior of the bipartite lattice

$$\Sigma_{xy} = (-1)^x \delta_{xy} \tag{3.4}$$

We can check if this symmetry commutes with the Hamiltonian by first showing that it leaves the charge invariant,

$$IqI^{-1} = Ip^\dagger I^{-1} IpI^{-1} - Ih^\dagger I^{-1} IhI^{-1} = h\Sigma\Sigma h^\dagger - p\Sigma\Sigma p^\dagger = (1 - h^\dagger h) - (1 - p^\dagger p) = p^\dagger p - h^\dagger h = q \tag{3.5}$$

It can also be shown that the third component of the spin gets flipped after we apply this operator

$$Is^3 I^{-1} = -s^3, \tag{3.6}$$

where

$$s^3_x = \frac{1}{2}\left(\delta_{xx} - p^\dagger_x p_x - h^\dagger_x h_x\right) \tag{3.7}$$

Finally, it is important to see how the hopping term transforms under this symmetry operation

$$
\begin{aligned}
I\left(-\sum_{xy} p^\dagger_x K_{xy} p_y + \sigma_k h^\dagger_x K_{xy} h_y\right) I^{-1} &= -\sum_{xy} Ip^\dagger_x I^{-1} IK_{xy} I^{-1} Ip_y I^{-1} + \sigma_k Ih^\dagger_x I^{-1} IK_{xy} I^{-1} Ih_y I^{-1} \\
&= -\sum_{xy} h_x (\Sigma IKI^{-1}\Sigma)_{xy} h^\dagger_y + \sigma_k p_x (\Sigma IKI^{-1}\Sigma)_{xy} p^\dagger_y \\
&= -\sum_{xy} -(\delta_{xy}(IKI^{-1})_{xy} - h^\dagger_y (IKI^{-1})_{xy} h_x) - \\
&- \sigma_k(\delta_{xy}(IKI^{-1})_{xy} - p^\dagger_y (IKI^{-1})_{xy} p_x) \\
&= \sigma_k \left[-\sum_{xy} p^\dagger_x (IKI^{-1})^\top_{xy} p_y + \sigma_k h^\dagger_x (IKI^{-1})^\top_{xy} h_y\right]
\end{aligned}
$$

where $\Sigma K \Sigma = -K$, $tr[K] = 0$, and $\sigma_k^2 = 1$. In general, to return to our initial hopping term, we need I to be anti-unitary where $IKI^{-1} = K^*$ and then we could replace $K^\dagger \to K$. Since we are working on a bipartite lattice, we can choose $\sigma_k = 1$ which even for $\mu \neq 0$ gives us the commutator $[H - \vec{\mu}\cdot\vec{q}, I] = 0$. This is true because as shown in (3.5) the charge is even in relation to the spin-flip symmetry operation.

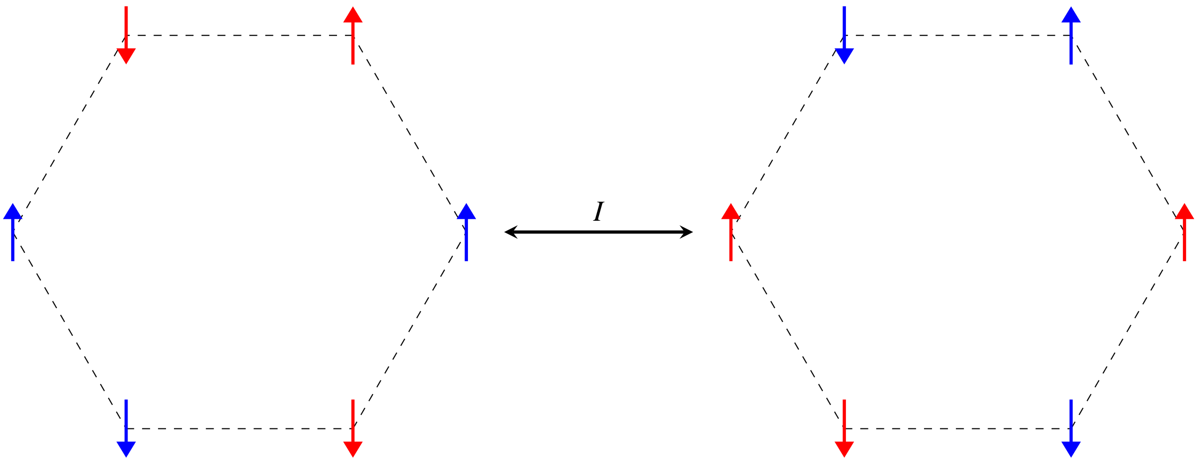

Figure 3.4: An example of the charge transformation with all possibilities of creation/annihilation operators. The graphs show only the transformation of the charge and the spin of the operators. The color of the arrows shows the charge (blue = +1, red = −1) and the orientation shows the spin. We can can go back and forth between the left and the right graphs by using the charge-flip conjugation.

3.2.2 Charge transformation

Another symmetry we can consider is the charge symmetry operation. It exchanges the charges but leaves the spin fixed. When applied onto the creation and annihilation operators, it transforms them as

$$\begin{aligned} CpC^{-1} &= h & \qquad ChC^{-1} &= p \\ Cp^\dagger C^{-1} &= h^\dagger & \qquad Ch^\dagger C^{-1} &= p^\dagger \end{aligned}$$

On Figure 3.4 is shown how the charge and spin of all creation/annihilation operators transform under this symmetry operation. If we conjugate with the charge operator, we get

$$CqC^{-1} = Cp^\dagger C^{-1}CpC^{-1} - Ch^\dagger C^{-1}ChC^{-1} = h^\dagger h - p^\dagger p = -q \tag{3.8}$$

and when conjugated with the spin operator

$$Cs^3C^{-1} = s^3, \tag{3.9}$$

so the spin is fixed for this operation. Again, we want to calculate the conjugation of the tight-binding term in the Hamiltonian with the charge-flip operator

$$
\begin{aligned}
C\left(-\sum_{xy} p^\dagger_x K_{xy} p_y + \sigma_k h^\dagger_x K_{xy} h_y\right) C^{-1} &= -\sum_{xy} C p^\dagger_x C^{-1} C K_{xy} C^{-1} C p_y C^{-1} + \sigma_k C h^\dagger_x C^{-1} C K_{xy} C^{-1} C h_y C^{-1} \\
&= -\sum_{xy} h^\dagger_x (CKC^{-1})_{xy} h_y + \sigma_k p^\dagger_x (CKC^{-1})_{xy} p_y \\
&= -\sum_{xy} \sigma_k p^\dagger_x (CKC^{-1})_{xy} p_y + h^\dagger_x (CKC^{-1})_{xy} h_y
\end{aligned}
$$

where if we choose $CKC^{-1} = K$, we get to the initial term. Therefore, we are shown that on a bipartite lattice where we can choose $\sigma_k = +1$, we have a charge conjugation symmetry i.e. it commutes with the tight-binding Hamiltonian $[H, C] = 0$. This is because as shown in (3.8) the charge operator is flipped, but the interaction is even under this transformation because it is of the form qVq.

3.2.3 Particle-hole transformation

The particle-hole symmetry operator is a combination of two symmetry operators that do not transform the Hamiltonian nicely, but their combination does. It preserves the energy of the system, but flips the charge and the spin. This means that we should only consider the tight-binding Hamiltonian, i.e. the Hamiltonian without the chemical potential term.

In order to convince ourselves of the neat transformation and also the commutation of the Hamiltonian with the symmetry operator, we should first see how the constituent operators transform the Hubbard model Hamiltonian.

First, we look at the staggering operator X. It transforms the creation/annihilation operators like

$$
\begin{aligned}
Xp^\dagger X^{-1} &= \Sigma p^\dagger = p^\dagger \Sigma & \qquad Xh^\dagger X^{-1} &= \Sigma h^\dagger = h^\dagger \Sigma \\
XpX^{-1} &= \Sigma p = p\Sigma & XhX^{-1} &= \Sigma h = h\Sigma
\end{aligned}
$$

It leaves the charge, spin, and number operators invariant, but it flips the sign of the hopping term

$$
\begin{aligned}
X\left(-\sum_{xy} p^\dagger_x K_{xy} p_y + \sigma_k h^\dagger_x K_{xy} h_y\right) X^{-1} &= -\sum_{xy} X p^\dagger_x X^{-1} X K_{xy} X^{-1} X p_y X^{-1} + \sigma_k X h^\dagger_x X^{-1} X K_{xy} X^{-1} X h_y X^{-1} \\
&= -\sum_{xy} p^\dagger_x (\Sigma XKX^{-1}\Sigma)_{xy} p_y + \sigma_k h^\dagger_x (\Sigma XKX^{-1}\Sigma)_{xy} h_y \\
&= -\sum_{xy} p^\dagger_x (\Sigma K \Sigma)_{xy} p_y + \sigma_k h^\dagger_x (\Sigma K \Sigma)_{xy} h_y \\
&= -\left[-\sum_{xy} p^\dagger_x K_{xy} p_y + \sigma_k h^\dagger_x K_{xy} h_y\right]
\end{aligned}
$$

The other operator that we consider is the one that "flips" the dagger of the operators. It transforms the particle/hole operators as

$$FpF^{-1} = p^\dagger \qquad FhF^{-1} = h^\dagger$$
$$Fp^\dagger F^{-1} = p \qquad Fh^\dagger F^{-1} = h$$

When applied onto the charge operator, it flips the sign $q \to -q$ which is shown below

$$FqF^{-1} = Fp^\dagger F^{-1}FpF^{-1} - Fh^\dagger F^{-1}FhF^{-1} = pp^\dagger - hh^\dagger = (1 - p^\dagger p) - (1 - h^\dagger h) = -q \tag{3.10}$$

This in turn means that this symmetry operation flips the chemical potential and more importantly the hopping term

$$\begin{aligned}
F\left(-\sum_{xy} p_x^\dagger K_{xy} p_y + \sigma_k h_x^\dagger K_{xy} h_y\right) F^{-1} &= -\sum_{xy} Fp_x^\dagger F^{-1} FK_{xy}F^{-1} Fp_y F^{-1} + \sigma_k Fh_x^\dagger F^{-1} FK_{xy} F^{-1} Fh_y F^{-1} \\
&= -\sum_{xy} p_x (FKF^{-1})_{xy} p_y^\dagger + \sigma_k h_x (FKF^{-1})_{xy} h_y^\dagger \\
&= -\sum_{xy} (\delta_{xy}(FKF^{-1})_{xy} - p_y^\dagger (FKF^{-1})_{xy} p_x) + \sigma_k(\delta_{xy}(FKF^{-1})_{xy} - h_y^\dagger (FKF^{-1}) \\
&= -\left[-\sum_{xy} p_y^\dagger (FKF^{-1})_{xy} p_x + \sigma_k h_y^\dagger (FKF^{-1})_{xy} h_x\right] \\
&= -\left[-\sum_{xy} p_x^\dagger (FKF^{-1})^\top_{xy} p_y + \sigma_k h_x^\dagger (FKF^{-1})^\top_{xy} h_y\right]
\end{aligned}$$

where $tr[K] = 0$, if we want to return to the original hopping term then we have to choose $FKF^{-1} = K^*$ and then we can substitute $K^\dagger \to K$. We see on Figure 3.5 an example of the dagger-flip operator switching the signs of the charge and the spin.

After we have shown how the different term of the Hamiltonian transform under both symmetries separately, we can answer how the composition of the two transforms each term of the Hubbard model's Hamiltonian. Starting from the hopping term, we know that each operation flips the sign of the kinetic term, meaning it is even under the composed operator. Next, we know that the charge operator is invariant under the staggering operator, and it flips sign when conjugated with the dagger-flip operator. This means that the interaction term qVq must preserve its sign with the particle-hole transformation because the interaction is bilinear. Finally, the chemical potential unfortunately is linear $\vec{\mu} \cdot \vec{q}$, so the sign of this term is flipped. After all these considerations, we could conclude that at half-filling ($\mu = 0$), the tight-binding Hamiltonian commutates with the particle-hole transformation

$$[H, XF] = 0 \tag{3.11}$$

and we can use this symmetry operation to our advantage. On Figure 3.6 it is shown again an example of the transformation of spin and charge. On this graph, we can only visualize the spin and the charge of the operators but not the transformed terms of the Hamiltonian, resulting in the same looking example as if only we consider F in spite of the kinetic terms having opposite signs.

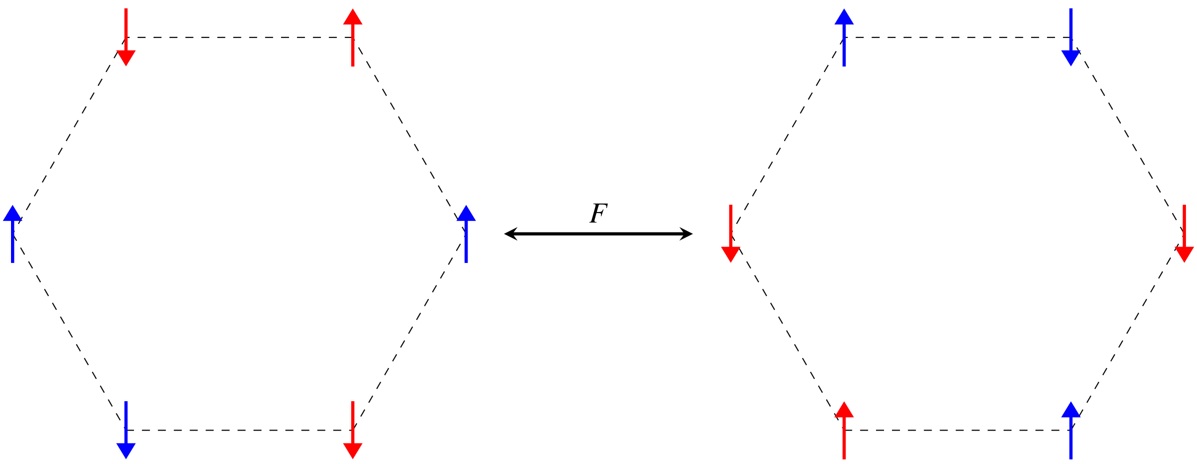

Figure 3.5: An example of the dagger-flip transformation with all possibilities of creation/annihilation operators. The graphs show only the transformation of the charge and the spin of the operators. The color of the arrows shows the charge (blue = +1, red = −1) and the orientation shows the spin. We can can go back and forth between the left and the right graphs by using the dagger-flip conjugation.

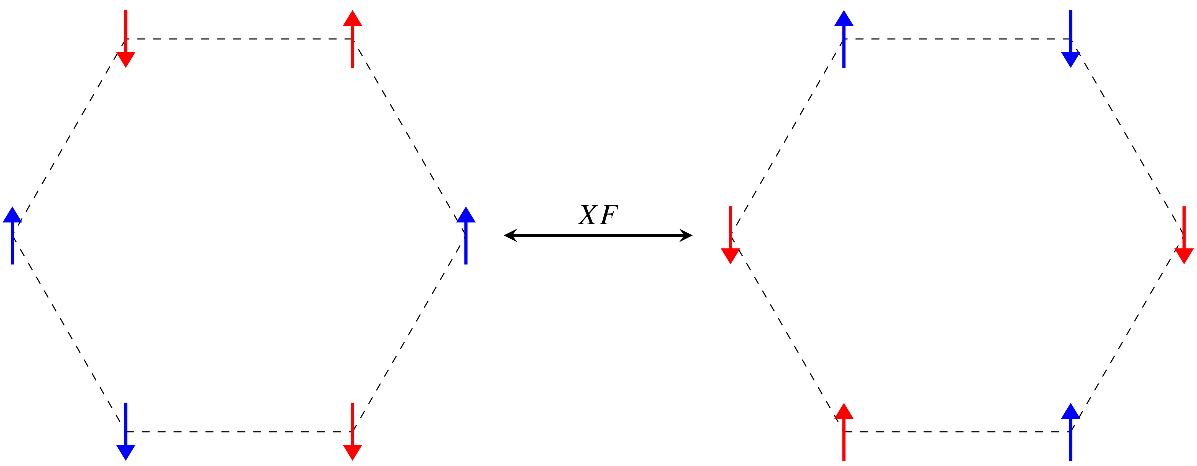

Figure 3.6: An example of the particle-hole transformation with all possibilities of creation/annihilation operators. The graphs show only the transformation of the charge and the spin of the operators. The color of the arrows shows the charge (blue = +1, red = −1) and the orientation shows the spin. We can can go back and forth between the left and the right graphs by using the particle-hole conjugation.

It is worth noting that after working out how these symmetry operations transform the Hamiltonian, we could write out $I = XFC$ but we should still examine them all separately because I commutes with the whole Hamiltonian and the other two only with the tight-binding one.

3.3 Correlation functions

After we have found out what the symmetries of the lattice are and how they transform the relevant operators, we can take them into consideration and apply them onto the one- and two-body correlation functions. We use them in order to average data with the same total momentum. This will increase our statistics and better our precision when performing the fitting of the final data.

3.3.1 One-particle correlation function

First, we start with the one-body correlators which will give us a valuable insight on how to work with the two-body correlation functions. The full derivation is provided in EVAN'S PAPER, so only the relevant information for this thesis will be shown here.

We start with the $\langle pp^\dagger \rangle$ correlation function which we can write as $C_Q = +1, S = 1, S^3 = -1, \vec{k}$. It can be represented as a 2×2 matrix with two one-body correlators due to the internal band labels

$$C_{Q=+1,S=1,S^3=-1,\vec{k}} = \begin{bmatrix} p_+p_+^\dagger & p_+p_-^\dagger \\ p_-p_+^\dagger & p_-p_-^\dagger \end{bmatrix}_{\vec{k}} (\tau) \tag{3.12}$$

This matrix of correlators can be calculated and then diagonalized to

$$C_{Q=+1,S=1,S^3=-1,\vec{k}} = \frac{1}{2}\left(C_{p_+p_+^\dagger} + C_{p_-p_-^\dagger}\right)\mathbb{1} + \frac{1}{2}\left(\sqrt{\left(C_{p_+p_+^\dagger} - C_{p_-p_-^\dagger}\right)^2 + 4C_{p_+p_-^\dagger}C_{p_-p_+^\dagger}}\right)\sigma_3 \tag{3.13}$$

In the tight-binding case of the Hamiltonian, the off-diagonal components vanish and the diagonal are equal. However, in general one must be careful because this is discretization dependent. In order to avoid this problem, we must always compute the whole matrix and then diagonalize. The whole argument can also be done for $\langle hh^\dagger \rangle$.

To increase the precision of our data, we apply the symmetry operators that we calculated in Section 3.2. Also, we use the cyclic property of the trace, and the final results can be seen in Table 3.1. A detailed derivation of these results as mentioned previously can be found in EVAN'S PAPER. It is important to note that we can increase the precision up to eightfold not sixteenfold which is the number of cases in the table. This is because some of the entries are duplicates and they can be found by examining the quantum numbers in the second column. The averaging put into equations looks like this

$$\tilde{C}_{QS^3\vec{k}}(\tau) = \frac{1}{4}\left(C_{QS^3\vec{k}}(\tau) + C_{\overline{QS^3}\vec{k}}(\beta-\tau)^\top + \sigma_1 \cdot C_{Q\overline{S^3}\vec{k}}(\tau)\cdot\sigma_1 + \sigma_1\cdot C_{\overline{Q}S^3\vec{k}}(\beta-\tau)^\top\cdot\sigma_1\right) \tag{3.14}$$

and

$$\tilde{C}_{\vec{k}}(\tau) = \frac{1}{2}\left(\tilde{C}_{QS^3\vec{k}}(\tau) + \tilde{C}_{\overline{Q}S^3\vec{k}}(\tau)\right) \tag{3.15}$$

We could always use (3.14) to average and be four times more precise and only when $\mu = 0$, we can also use (3.15) for a total of eight times more statistics. After that, one must always diagonalize.

		Q	S	S^3	$\vec{k}$	τ
Start with	$QSkt$	+1	$\frac{1}{2}$	$-\frac{1}{2}$	1	τ
trace cycl	$\overline{QSkt}$	-1	$\frac{1}{2}$	$+\frac{1}{2}$	$\top$	$\beta - \tau$
I	$Q\overline{Sk}t$	+1	$\frac{1}{2}$	$+\frac{1}{2}$	$\sigma_1 \cdot \sigma_1$	τ
		-1	$\frac{1}{2}$	$-\frac{1}{2}$	$\sigma_1 \top \sigma_1$	$\beta - \tau$
XF	$\overline{QSk}t$	-1	$\frac{1}{2}$	$+\frac{1}{2}$	$\sigma_1 \cdot \sigma_1$	τ
		+1	$\frac{1}{2}$	$-\frac{1}{2}$	$\sigma_1 \cdot \sigma_1$	$\beta - \tau$
		-1	$\frac{1}{2}$	$-\frac{1}{2}$	1	τ
		+1	$\frac{1}{2}$	$+\frac{1}{2}$	$\top$	$\beta - \tau$
C	$\overline{Q}Skt$	-1	$\frac{1}{2}$	$-\frac{1}{2}$	1	τ
		+1	$\frac{1}{2}$	$+\frac{1}{2}$	$\top$	$\beta - \tau$
		-1	$\frac{1}{2}$	$+\frac{1}{2}$	$\sigma_1 \cdot \sigma_1$	τ
		+1	$\frac{1}{2}$	$-\frac{1}{2}$	$\sigma_1 \top \sigma_1$	$\beta - \tau$
		+1	$\frac{1}{2}$	$+\frac{1}{2}$	$\sigma_1 \cdot \sigma_1$	τ
		-1	$\frac{1}{2}$	$-\frac{1}{2}$	$\sigma_1 \top \sigma_1$	$\beta - \tau$
		+1	$\frac{1}{2}$	$-\frac{1}{2}$	1	τ
		-1	$\frac{1}{2}$	$+\frac{1}{2}$	$\top$	$\beta - \tau$

Table 3.1: All the correlators that can be directly averaged, assuming they all take the 2×2 matrix form. The correlators above the double line may be averaged even at finite μ and everything below may be averaged when $\mu = 0$. The combination of charge and spin indicate whether $p^\dagger$, p, $h^\dagger$, or h is right-most. The $\vec{k}$ column indicates what must be done in the interal 2×2 momentum space before averaging; $\top$ indicates transpose, $\sigma_1 \cdot \sigma_1$ indicates conjugation by σ_1, 1 indicates that no operation is needed. The τ column indicates whether the given correlator needs to be time-reversed.

3.3.2 Two-particle correlation function

In this thesis, we are working with only two types of two-body correlators. There are correlators with particles and holes, and only particles. We start with the former correlator $\langle p h h^\dagger p^\dagger \rangle$. In matrix form with all labels it looks like this

$$C_{Q=0,S=1,S^3=-1;k,q,r,s} = \begin{bmatrix} p_+h_+h_+^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_-^\dagger \\ p_+h_-h_+^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_-^\dagger \\ p_-h_+h_+^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_-^\dagger \\ p_-h_-h_+^\dagger p_+^\dagger & p_-h_-h_+^\dagger p_-^\dagger & p_-h_-h_-^\dagger p_+^\dagger & p_-h_-h_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau) \quad (3.16)$$

Now, we apply a different symmetry operator to the two particle correlation function. The derivations of the conjugation with the symmetry operators are always done in two steps. First, we start with the thermal trace and then switch to matrix form, so that it is easier to see the transformations needed in order to get another two particle correlator.

Trace cyclicity

We begin with the trace cyclicity because it will give us how the correlation function transformations when time reversed.

$$C_{Q=0,S=1,S^3=-1;k,q,r,s} = \frac{1}{Z}tr\left[p_k h_q e^{-H\tau} h_r^\dagger p_s^\dagger e^{-H(\beta-\tau)}\right] = \frac{1}{Z}tr\left[h_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} p_k h_q e^{-H\tau}\right] \tag{3.17}$$

Now we apply time reverse

$$\beta \to \beta - \tau$$

and use the anti-commutation relations of the operators to get

$$\frac{1}{Z}tr\left[h_r^\dagger p_s^\dagger e^{-H(\beta-(\beta-\tau))} p_k h_q e^{-H(\beta-\tau)}\right] = \frac{1}{Z}tr\left[p_s^\dagger h_r^\dagger e^{-H\tau} h_q p_k e^{-H(\beta-\tau)}\right] \tag{3.18}$$

We switch to matrix form to see observe better how all labels transform

$$\begin{aligned}
&C_{Q=0,S=1,S^3=-1;k,q,r,s} \\
&= \begin{bmatrix} p_+h_+h_+^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_-^\dagger \\ p_+h_-h_+^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_-^\dagger \\ p_-h_+h_+^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_-^\dagger \\ p_-h_-h_+^\dagger p_+^\dagger & p_-h_-h_+^\dagger p_-^\dagger & p_-h_-h_-^\dagger p_+^\dagger & p_-h_-h_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau) \qquad (3.19) \\
&= \begin{bmatrix} h_+^\dagger p_+^\dagger p_+h_+ & h_+^\dagger p_-^\dagger p_+h_+ & h_-^\dagger p_+^\dagger p_+h_+ & h_-^\dagger p_-^\dagger p_+h_+ \\ h_+^\dagger p_+^\dagger p_+h_- & h_+^\dagger p_-^\dagger p_+h_- & h_-^\dagger p_+^\dagger p_+h_- & h_-^\dagger p_-^\dagger p_+h_- \\ h_+^\dagger p_+^\dagger p_-h_+ & h_+^\dagger p_-^\dagger p_-h_+ & h_-^\dagger p_+^\dagger p_-h_+ & h_-^\dagger p_-^\dagger p_-h_+ \\ h_+^\dagger p_+^\dagger p_-h_- & h_+^\dagger p_-^\dagger p_-h_- & h_-^\dagger p_+^\dagger p_-h_- & h_-^\dagger p_-^\dagger p_-h_- \end{bmatrix}_{r,s,k,q} (\beta-\tau) \\
&= \begin{bmatrix} p_+^\dagger h_+^\dagger h_+p_+ & p_-^\dagger h_+^\dagger h_+p_+ & p_+^\dagger h_-^\dagger h_+p_+ & p_-^\dagger h_-^\dagger h_+p_+ \\ p_+^\dagger h_+^\dagger h_-p_+ & p_-^\dagger h_+^\dagger h_-p_+ & p_+^\dagger h_-^\dagger h_-p_+ & p_-^\dagger h_-^\dagger h_-p_+ \\ p_+^\dagger h_+^\dagger h_+p_- & p_-^\dagger h_+^\dagger h_+p_- & p_+^\dagger h_-^\dagger h_+p_- & p_-^\dagger h_-^\dagger h_+p_- \\ p_+^\dagger h_+^\dagger h_-p_- & p_-^\dagger h_+^\dagger h_-p_- & p_+^\dagger h_-^\dagger h_-p_- & p_-^\dagger h_-^\dagger h_-p_- \end{bmatrix}_{s,r,q,k} (\beta-\tau) \qquad (3.20) \\
&= C_{Q=0,S=1,S^3=+1;s,r,q,k}
\end{aligned}$$

We notice that order reverse is actually composed of two transformations – particle-hole momentum switch and time reverse. After we stare at the matrix for an hour, we can figure out what the transformation between the correlation functions is

$$C_{Q=0,S=1,S^3=-1;k,q,r,s}(\tau) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} C_{Q=0,S=1,S^3=+1;s,r,q,k}{}^{\top}(\beta-\tau) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.21}$$

We repeat the steps again for each symmetry operator. First, we start with the thermal trace and then switch to matrix for easy transformations. It is shown on the right side of the column sign if the operator transforms well the full Hamiltonian or only at half-filling.

$\underline{\boldsymbol{I : H - \vec{\mu} \cdot \vec{q}}}$

$$
\begin{aligned}
C&_{Q=0,S=1,S^3=-1;k,q,r,s} \\
&= \frac{1}{Z} tr \left[p_k h_q e^{-H\tau} h^\dagger_r p^\dagger_s e^{-H(\beta-\tau)} \right] \\
&= \frac{1}{Z} tr \left[I^{-1} I p_k I^{-1} I h_q I^{-1} I e^{-H\tau} I^{-1} I h^\dagger_r I^{-1} I p^\dagger_s I^{-1} I e^{-H(\beta-\tau)} \right] \\
&= \frac{1}{Z} tr \left[I p_k I^{-1} I h_q I^{-1} I e^{-H\tau} I^{-1} I h^\dagger_r I^{-1} I p^\dagger_s I^{-1} I e^{-H(\beta-\tau) I^{-1}} \right] \\
&= \frac{1}{Z} tr \left[(\Sigma h)^\dagger_k (\Sigma p)^\dagger_q e^{-H\tau} (\Sigma p)_r (\Sigma h)_s e^{-H(\beta-\tau)} \right] \\
&= \frac{1}{Z} tr \left[(\Sigma p)^\dagger_q (\Sigma h)^\dagger_k e^{-H\tau} (\Sigma h)_s (\Sigma p)_r e^{-H(\beta-\tau)} \right] \\
&= \frac{1}{Z} tr \left[(\Sigma p)_r (\Sigma h)_s e^{-H(\beta-\tau)} (\Sigma h)^\dagger_k (\Sigma p)^\dagger_q e^{-H\tau} \right]
\end{aligned}
\tag{3.22}
$$

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=-1;k,q,r,s} \\
&= I^{-1} \begin{bmatrix}
p_+h_+h^\dagger_+p^\dagger_+ & p_+h_+h^\dagger_+p^\dagger_- & p_+h_+h^\dagger_-p^\dagger_+ & p_+h_+h^\dagger_-p^\dagger_- \\
p_+h_-h^\dagger_+p^\dagger_+ & p_+h_-h^\dagger_+p^\dagger_- & p_+h_-h^\dagger_-p^\dagger_+ & p_+h_-h^\dagger_-p^\dagger_- \\
p_-h_+h^\dagger_+p^\dagger_+ & p_-h_+h^\dagger_+p^\dagger_- & p_-h_+h^\dagger_-p^\dagger_+ & p_-h_+h^\dagger_-p^\dagger_- \\
p_-h_-h^\dagger_+p^\dagger_+ & p_-h_-h^\dagger_+p^\dagger_- & p_-h_-h^\dagger_-p^\dagger_+ & p_-h_-h^\dagger_-p^\dagger_-
\end{bmatrix}_{k,q,r,s} (\tau)\, I \\
&= \begin{bmatrix}
(\Sigma h)^\dagger_+ (\Sigma p)^\dagger_+ (\Sigma p)_+ (\Sigma h)_+ & (\Sigma h)^\dagger_+ (\Sigma p)^\dagger_+ (\Sigma p)_+ (\Sigma h)_- & (\Sigma h)^\dagger_+ (\Sigma p)^\dagger_+ (\Sigma p)_- (\Sigma h)_+ & (\Sigma h)^\dagger_+ (\Sigma p)^\dagger_+ (\Sigma p)_- \\
(\Sigma h)^\dagger_+ (\Sigma p)^\dagger_- (\Sigma p)_+ (\Sigma h)_+ & (\Sigma h)^\dagger_+ (\Sigma p)^\dagger_- (\Sigma p)_+ (\Sigma h)_- & (\Sigma h)^\dagger_+ (\Sigma p)^\dagger_- (\Sigma p)_- (\Sigma h)_+ & (\Sigma h)^\dagger_+ (\Sigma p)^\dagger_- (\Sigma p)_- \\
(\Sigma h)^\dagger_- (\Sigma p)^\dagger_+ (\Sigma p)_+ (\Sigma h)_+ & (\Sigma h)^\dagger_- (\Sigma p)^\dagger_+ (\Sigma p)_+ (\Sigma h)_- & (\Sigma h)^\dagger_- (\Sigma p)^\dagger_+ (\Sigma p)_- (\Sigma h)_+ & (\Sigma h)^\dagger_- (\Sigma p)^\dagger_+ (\Sigma p)_- \\
(\Sigma h)^\dagger_- (\Sigma p)^\dagger_- (\Sigma p)_+ (\Sigma h)_+ & (\Sigma h)^\dagger_- (\Sigma p)^\dagger_- (\Sigma p)_+ (\Sigma h)_- & (\Sigma h)^\dagger_- (\Sigma p)^\dagger_- (\Sigma p)_- (\Sigma h)_+ & (\Sigma h)^\dagger_- (\Sigma p)^\dagger_- (\Sigma p)_-
\end{bmatrix} \\
&= \begin{bmatrix}
h^\dagger_-p^\dagger_-p_-h_- & h^\dagger_-p^\dagger_-p_-h_+ & h^\dagger_-p^\dagger_-p_+h_- & h^\dagger_-p^\dagger_-p_+h_+ \\
h^\dagger_-p^\dagger_+p_-h_- & h^\dagger_-p^\dagger_+p_-h_+ & h^\dagger_-p^\dagger_+p_+h_- & h^\dagger_-p^\dagger_+p_+h_+ \\
h^\dagger_+p^\dagger_-p_-h_- & h^\dagger_+p^\dagger_-p_-h_+ & h^\dagger_+p^\dagger_-p_+h_- & h^\dagger_+p^\dagger_-p_+h_+ \\
h^\dagger_+p^\dagger_+p_-h_- & h^\dagger_+p^\dagger_+p_-h_+ & h^\dagger_+p^\dagger_+p_+h_- & h^\dagger_+p^\dagger_+p_+h_+
\end{bmatrix}_{k,q,r,s} (\tau)
\end{aligned}
\tag{3.23}
$$

$$
= \begin{bmatrix}
p_-h_-h^\dagger_-p^\dagger_- & p_-h_+h^\dagger_-p^\dagger_- & p_+h_-h^\dagger_-p^\dagger_- & p_+h_+h^\dagger_-p^\dagger_- \\
p_-h_-h^\dagger_-p^\dagger_+ & p_-h_+h^\dagger_-p^\dagger_+ & p_+h_-h^\dagger_-p^\dagger_+ & p_+h_+h^\dagger_-p^\dagger_+ \\
p_-h_-h^\dagger_+p^\dagger_- & p_-h_+h^\dagger_+p^\dagger_- & p_+h_-h^\dagger_+p^\dagger_- & p_+h_+h^\dagger_+p^\dagger_- \\
p_-h_-h^\dagger_+p^\dagger_+ & p_-h_+h^\dagger_+p^\dagger_+ & p_+h_-h^\dagger_+p^\dagger_+ & p_+h_+h^\dagger_+p^\dagger_+
\end{bmatrix}_{r,s,k,q} (\beta - \tau)
\tag{3.24}
$$

$$= \begin{bmatrix} p_-^\dagger h_-^\dagger h_- p_- & p_-^\dagger h_-^\dagger h_+ p_- & p_-^\dagger h_-^\dagger h_- p_+ & p_-^\dagger h_-^\dagger h_+ p_+ \\ p_+^\dagger h_-^\dagger h_- p_- & p_+^\dagger h_-^\dagger h_+ p_- & p_+^\dagger h_-^\dagger h_- p_+ & p_+^\dagger h_-^\dagger h_+ p_+ \\ p_-^\dagger h_+^\dagger h_- p_- & p_-^\dagger h_+^\dagger h_+ p_- & p_-^\dagger h_+^\dagger h_- p_+ & p_-^\dagger h_+^\dagger h_+ p_+ \\ p_+^\dagger h_+^\dagger h_- p_- & p_+^\dagger h_+^\dagger h_+ p_- & p_+^\dagger h_+^\dagger h_- p_+ & p_+^\dagger h_+^\dagger h_+ p_+ \end{bmatrix}_{q,k,s,r} (\tau) \tag{3.25}$$

Depending on what transformation we use to get back to $\langle p h h^\dagger p^\dagger \rangle$, we get two transformations which are time reverse to one another. We notice the same behavior for all operators.

$$\begin{aligned} &C_{Q=0,S=1,S^3=-1;k,q,r,s}(\tau) \\ &\qquad = \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} C_{Q=0,S=1,S^3=-1;r,s,k,q}{}^\top(\beta-\tau) \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} \end{aligned} \tag{3.26}$$

$$\begin{aligned} &C_{Q=0,S=1,S^3=-1;k,q,r,s}(\tau) \\ &\qquad = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} C_{Q=0,S=1,S^3=+1;q,k,s,r}(\tau) \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{3.27}$$

C : H

$$\begin{aligned} &C_{Q=0,S=1,S^3=-1;k,q,r,s} \\ &\qquad = \frac{1}{Z} tr\left[p_k h_q e^{-H\tau} h_r^\dagger p_s^\dagger e^{-H(\beta-\tau)}\right] \\ &\qquad = \frac{1}{Z} tr\left[C^{-1} C p_k C^{-1} C h_q C^{-1} C e^{-H\tau} C^{-1} C h_r^\dagger C^{-1} C p_s^\dagger C^{-1} C e^{-H(\beta-\tau)}\right] \\ &\qquad = \frac{1}{Z} tr\left[h_k p_q e^{-H\tau} p_r^\dagger h_s^\dagger e^{-H(\beta-\tau)}\right] \\ &\qquad = \frac{1}{Z} tr\left[p_q h_k e^{-H\tau} h_s^\dagger p_r^\dagger e^{-H(\beta-\tau)}\right] \end{aligned} \tag{3.28}$$

$$\begin{aligned} &C_{Q=0,S=1,S^3=-1;k,q,r,s} \\ &\qquad = C^{-1} \begin{bmatrix} p_+ h_+ h_+^\dagger p_+^\dagger & p_+ h_+ h_+^\dagger p_-^\dagger & p_+ h_+ h_-^\dagger p_+^\dagger & p_+ h_+ h_-^\dagger p_-^\dagger \\ p_+ h_- h_+^\dagger p_+^\dagger & p_+ h_- h_+^\dagger p_-^\dagger & p_+ h_- h_-^\dagger p_+^\dagger & p_+ h_- h_-^\dagger p_-^\dagger \\ p_- h_+ h_+^\dagger p_+^\dagger & p_- h_+ h_+^\dagger p_-^\dagger & p_- h_+ h_-^\dagger p_+^\dagger & p_- h_+ h_-^\dagger p_-^\dagger \\ p_- h_- h_+^\dagger p_+^\dagger & p_- h_- h_+^\dagger p_-^\dagger & p_- h_- h_-^\dagger p_+^\dagger & p_- h_- h_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau)\, C \end{aligned} \tag{3.29}$$

$$
\begin{aligned}
&= \begin{bmatrix}
h_+p_+p_+^\dagger h_+^\dagger & h_+p_+p_+^\dagger h_-^\dagger & h_+p_+p_-^\dagger h_+^\dagger & h_+p_+p_-^\dagger h_-^\dagger \\
h_+p_-p_+^\dagger h_+^\dagger & h_+p_-p_+^\dagger h_-^\dagger & h_+p_-p_-^\dagger h_+^\dagger & h_+p_-p_-^\dagger h_-^\dagger \\
h_-p_+p_+^\dagger h_+^\dagger & h_-p_+p_+^\dagger h_-^\dagger & h_-p_+p_-^\dagger h_+^\dagger & h_-p_+p_-^\dagger h_-^\dagger \\
h_-p_-p_+^\dagger h_+^\dagger & h_-p_-p_+^\dagger h_-^\dagger & h_-p_-p_-^\dagger h_+^\dagger & h_-p_-p_-^\dagger h_-^\dagger
\end{bmatrix}_{k,q,r,s} (\tau) \\
&= \begin{bmatrix}
p_+^\dagger h_+^\dagger h_+p_+ & p_+^\dagger h_-^\dagger h_+p_+ & p_-^\dagger h_+^\dagger h_+p_+ & p_-^\dagger h_-^\dagger h_+p_+ \\
p_+^\dagger h_+^\dagger h_+p_- & p_+^\dagger h_-^\dagger h_+p_- & p_-^\dagger h_+^\dagger h_+p_- & p_-^\dagger h_-^\dagger h_+p_- \\
p_+^\dagger h_+^\dagger h_-p_+ & p_+^\dagger h_-^\dagger h_-p_+ & p_-^\dagger h_+^\dagger h_-p_+ & p_-^\dagger h_-^\dagger h_-p_+ \\
p_+^\dagger h_+^\dagger h_-p_- & p_+^\dagger h_-^\dagger h_-p_- & p_-^\dagger h_+^\dagger h_-p_- & p_-^\dagger h_-^\dagger h_-p_-
\end{bmatrix}_{r,s,k,q} (\beta-\tau)
\end{aligned} \tag{3.30}
$$

$$
= \begin{bmatrix}
p_+h_+h_+^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_-^\dagger \\
p_-h_+h_+^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_-^\dagger \\
p_+h_-h_+^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_-^\dagger \\
p_-h_-h_+^\dagger p_+^\dagger & p_-h_-h_-^\dagger p_+^\dagger & p_-h_-h_+^\dagger p_-^\dagger & p_-h_-h_-^\dagger p_-^\dagger
\end{bmatrix}_{q,k,s,r} (\tau) \tag{3.31}
$$

$$
C_{Q=0,S=1,S^3=-1;k,q,r,s}(\tau) = C_{Q=0,S=1,S^3=+1;r,s,k,q}{}^{\top}(\beta-\tau) \tag{3.32}
$$

$$
C_{Q=0,S=1,S^3=-1;k,q,r,s}(\tau) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} C_{Q=0,S=1,S^3=-1;q,k,s,r}(\tau) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.33}
$$

XF : H

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=-1;k,q,r,s} \\
&= \frac{1}{Z} tr \left[p_k h_q e^{-H\tau} h_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} \right] \\
&= \frac{1}{Z} tr \left[(XF)^{-1}(XF) p_k (XF)^{-1}(XF) h_q (XF)^{-1}(XF) e^{-H\tau} (XF)^{-1}(XF) h_r^\dagger (XF)^{-1}(XF) p_s^\dagger (XF)^{-1}(XF) e^{-H(\beta-\tau)} \right. \\
&= \frac{1}{Z} tr \left[(\Sigma p)_k^\dagger (\Sigma h)_q^\dagger e^{-H\tau} (\Sigma h)_r (\Sigma p)_s e^{-H(\beta-\tau)} \right]
\end{aligned} \tag{3.34}
$$

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=-1;k,q,r,s}\\
&= (XF)^{-1}\begin{bmatrix} p_+h_+h_+^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_-^\dagger \\ p_+h_-h_+^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_-^\dagger \\ p_-h_+h_+^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_-^\dagger \\ p_-h_-h_+^\dagger p_+^\dagger & p_-h_-h_+^\dagger p_-^\dagger & p_-h_-h_-^\dagger p_+^\dagger & p_-h_-h_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau)\ (XF)\\
&= \begin{bmatrix} (\Sigma p)_+^\dagger (\Sigma h)_+^\dagger (\Sigma h)_+ (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma h)_+^\dagger (\Sigma h)_+ (\Sigma p)_- & (\Sigma p)_+^\dagger (\Sigma h)_+^\dagger (\Sigma h)_- (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma h)_+^\dagger (\Sigma h)_- (\Sigma p)_- \\ (\Sigma p)_+^\dagger (\Sigma h)_-^\dagger (\Sigma h)_+ (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma h)_-^\dagger (\Sigma h)_+ (\Sigma p)_- & (\Sigma p)_+^\dagger (\Sigma h)_-^\dagger (\Sigma h)_- (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma h)_-^\dagger (\Sigma h)_- (\Sigma p)_- \\ (\Sigma p)_-^\dagger (\Sigma h)_+^\dagger (\Sigma h)_+ (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma h)_+^\dagger (\Sigma h)_+ (\Sigma p)_- & (\Sigma p)_-^\dagger (\Sigma h)_+^\dagger (\Sigma h)_- (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma h)_+^\dagger (\Sigma h)_- (\Sigma p)_- \\ (\Sigma p)_-^\dagger (\Sigma h)_-^\dagger (\Sigma h)_+ (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma h)_-^\dagger (\Sigma h)_+ (\Sigma p)_- & (\Sigma p)_-^\dagger (\Sigma h)_-^\dagger (\Sigma h)_- (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma h)_-^\dagger (\Sigma h)_- (\Sigma p)_- \end{bmatrix}\\
&= \begin{bmatrix} p_-^\dagger h_-^\dagger h_- p_- & p_-^\dagger h_-^\dagger h_- p_+ & p_-^\dagger h_-^\dagger h_+ p_- & p_-^\dagger h_-^\dagger h_+ p_+ \\ p_-^\dagger h_+^\dagger h_- p_- & p_-^\dagger h_+^\dagger h_- p_+ & p_-^\dagger h_+^\dagger h_+ p_- & p_-^\dagger h_+^\dagger h_+ p_+ \\ p_+^\dagger h_-^\dagger h_- p_- & p_+^\dagger h_-^\dagger h_- p_+ & p_+^\dagger h_-^\dagger h_+ p_- & p_+^\dagger h_-^\dagger h_+ p_+ \\ p_+^\dagger h_+^\dagger h_- p_- & p_+^\dagger h_+^\dagger h_- p_+ & p_+^\dagger h_+^\dagger h_+ p_- & p_+^\dagger h_+^\dagger h_+ p_+ \end{bmatrix}_{k,q,r,s} (\tau)
\end{aligned}
\tag{3.35}
$$

$$
= \begin{bmatrix} p_-h_-h_-^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_-^\dagger \\ p_-h_-h_+^\dagger p_-^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_+h_+h_+^\dagger p_-^\dagger \\ p_-h_-h_-^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_+^\dagger \\ p_-h_-h_+^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_+^\dagger \end{bmatrix}_{s,r,k,q} (\beta-\tau) \tag{3.36}
$$

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=-1;k,q,r,s}(\tau)\\
&\quad = \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} C_{Q=0,S=1,S^3=+1;k,q,r,s}(\tau) \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix}
\end{aligned}
\tag{3.37}
$$

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=-1;k,q,r,s}(\tau)\\
&= \begin{bmatrix} 1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1 \end{bmatrix} \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} C_{Q=0,S=1,S^3=-1;s,r,q,k}{}^\top(\beta-\tau) \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} \begin{bmatrix} 1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1 \end{bmatrix}
\end{aligned}
\tag{3.38}
$$

The table below shows the summary of all symmetries that we calculated.

For more compact form, we write

$$
\sigma_5 = \begin{bmatrix} 1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1 \end{bmatrix} \tag{3.39}
$$

The other correlation function that we are interested in is with two particles at the source and two at the sink. Again, we start with correlator $\langle ppp^\dagger p^\dagger\rangle$. In trace and matrix forms it look like this.

	Q	S	S^3	trafo	$kqrs$	τ
Start with	0	1	-1	1	1234	τ
trace cycl	0	1	+1	$\sigma_5 \top \sigma_5$	4321	$\beta - \tau$
I	0	1	-1	$\sigma_1 \top \sigma_1$	3412	$\beta - \tau$
	0	1	+1	$\sigma_5\sigma_1 \cdot \sigma_1\sigma_5$	2143	τ
C	0	1	+1	$\top$	3412	$\beta - \tau$
	0	1	-1	$\sigma_5 \cdot \sigma_5$	2143	τ
XF	0	1	+1	$\sigma_1 \cdot \sigma_1$	1234	τ
	0	1	-1	$\sigma_5\sigma_1 \top \sigma_1\sigma_5$	4321	$\beta - \tau$

Table 3.2: All the correlators that can be directly averaged, if they all take a 4×4 matrix form. The correlators above the double line may be averaged even at finite μ and everything below may be averaged when $\mu = 0$. The combination of charge and spin indicate whether $h^\dagger p^\dagger$ or hp is at the source (right-most). The *trafo* column indicates what must be done in the interal 4×4 momentum space before averaging; $\top$ indicates transpose, $\sigma_1 \cdot \sigma_1$ indicates conjugation by σ_1, $\sigma_5 \cdot \sigma_5$ indicates conjugation by σ_5, and 1 indicates that no operation is needed. The next column $kqrs$ indicates the position of the momenta at the sink and source relative to the initial correlator is. The τ column indicates whether the given correlator needs to be time-reversed.

$$C_{Q=2,S=1,S^3=-1;k,q,r,s} = \frac{1}{Z} tr \left[p_k p_q e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} \right] \tag{3.40}$$

$$C_{Q=2,S=1,S^3=-1;k,q,r,s} = \begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} \quad (\tau) \tag{3.41}$$

First, we implement the anti-commutation relations which will give us the identities of this correlation function. Again, we use derive the the results through traces and then continue with the matrix form which will give us a better insight on the transformation of all labels that are included.

Anti-commutation

$$\begin{aligned} C_{Q=2,S=1,S^3=-1;k,q,r,s} &= \frac{1}{Z} tr \left[p_k p_q e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} \right] = \\ &= \frac{1}{Z} tr \left[p_q p_k e^{-H\tau} p_s^\dagger p_r^\dagger e^{-H(\beta-\tau)} \right] \\ &= -\frac{1}{Z} tr \left[p_q p_k e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} \right] \\ &= -\frac{1}{Z} tr \left[p_k p_q e^{-H\tau} p_s^\dagger p_r^\dagger e^{-H(\beta-\tau)} \right] \end{aligned} \tag{3.42}$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s} &= \begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau) \\
&= \begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{q,k,s,r} (\tau) \\
&= -\begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{q,k,r,s} (\tau) \\
&= -\begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,s,r} (\tau)
\end{aligned} \tag{3.43}
$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_5 \cdot C_{Q=2,S=1,S^3=-1;q,k,s,r}(\tau) \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_5 \cdot C_{Q=2,S=1,S^3=-1;q,k,s,r}(\tau) \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -C_{Q=2,S=1,S^3=-1;q,k,s,r}(\tau) \cdot \sigma_5
\end{aligned} \tag{3.44}
$$

These identities show us that we can, average our data four fold only because we are using this kind of correlator. Using the cyclicity of the trace, we find how the correlation function transforms when time is reversed. This will further increase our statistics two times.

Trace ciclicity

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s} &= \frac{1}{Z} tr \left[p_k p_q e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} \right] = \\
&= \frac{1}{Z} tr \left[p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} p_k p_q e^{-H\tau} \right] \\
&= \frac{1}{Z} tr \left[p_s^\dagger p_r^\dagger e^{-H(\beta-\tau)} p_q p_k e^{-H\tau} \right] \\
&= -\frac{1}{Z} tr \left[p_s^\dagger p_r^\dagger e^{-H(\beta-\tau)} p_k p_q e^{-H\tau} \right] \\
&= -\frac{1}{Z} tr \left[p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} p_q p_k e^{-H\tau} \right]
\end{aligned} \tag{3.45}
$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s} &= \begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau) = \\
&= \begin{bmatrix} p_+^\dagger p_+^\dagger p_+p_+ & p_+^\dagger p_-^\dagger p_+p_+ & p_-^\dagger p_+^\dagger p_+p_+ & p_-^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_+p_- \\ p_+^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_-p_+ \\ p_+^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_-p_- \end{bmatrix}_{r,s,k,q} (\beta-\tau) \\
&= \begin{bmatrix} p_+^\dagger p_+^\dagger p_+p_+ & p_-^\dagger p_+^\dagger p_+p_+ & p_+^\dagger p_-^\dagger p_+p_+ & p_-^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_-p_+ \\ p_+^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_+p_- \\ p_+^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_-p_- \end{bmatrix}_{s,r,q,k} (\beta-\tau) \\
&= -\begin{bmatrix} p_+^\dagger p_+^\dagger p_+p_+ & p_-^\dagger p_+^\dagger p_+p_+ & p_+^\dagger p_-^\dagger p_+p_+ & p_-^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_+p_- \\ p_+^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_-p_+ \\ p_+^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_-p_- \end{bmatrix}_{s,r,k,q} (\beta-\tau) \\
&= -\begin{bmatrix} p_+^\dagger p_+^\dagger p_+p_+ & p_+^\dagger p_-^\dagger p_+p_+ & p_-^\dagger p_+^\dagger p_+p_+ & p_-^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_-p_+ \\ p_+^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_+p_- \\ p_+^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_-p_- \end{bmatrix}_{r,s,q,k} (\beta-\tau)
\end{aligned}
\tag{3.46}
$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= C_{Q=2,S=1,S^3=+1;r,s,k,q}{}^\top(\beta-\tau) \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_5 \cdot C_{Q=2,S=1,S^3=+1;s,r,q,k}{}^\top(\beta-\tau) \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -C_{Q=2,S=1,S^3=+1;s,r,k,q}{}^\top(\beta-\tau) \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_5 \cdot C_{Q=2,S=1,S^3=+1;r,s,q,k}{}^\top(\beta-\tau)
\end{aligned}
\tag{3.47}
$$

I **:** $\boldsymbol{H - \vec{\mu} \cdot \vec{q}}$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s} &= \frac{1}{Z} tr \left[p_k p_q e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} \right] = \\
&= \frac{1}{Z} tr \left[I^{-1} I p_k I^{-1} I p_q I^{-1} I e^{-H\tau} I^{-1} I p_r^\dagger I^{-1} I p_s^\dagger I^{-1} I e^{-H(\beta-\tau)} \right] = \\
&= \frac{1}{Z} tr \left[I p_k I^{-1} I p_q I^{-1} I e^{-H\tau} I^{-1} I p_r^\dagger I^{-1} I p_s^\dagger I^{-1} I e^{-H(\beta-\tau)} I^{-1} \right] = \\
&= \frac{1}{Z} tr \left[(\Sigma h)_k^\dagger (\Sigma h)_q^\dagger e^{-H\tau} (\Sigma h)_r (\Sigma h)_s e^{-H(\beta-\tau)} \right] \\
&= \frac{1}{Z} tr \left[(\Sigma h)_q^\dagger (\Sigma h)_k^\dagger e^{-H\tau} (\Sigma h)_s (\Sigma h)_r e^{-H(\beta-\tau)} \right] \\
&= -\frac{1}{Z} tr \left[(\Sigma h)_q^\dagger (\Sigma h)_k^\dagger e^{-H\tau} (\Sigma h)_r (\Sigma h)_s e^{-H(\beta-\tau)} \right] \\
&= -\frac{1}{Z} tr \left[(\Sigma h)_k^\dagger (\Sigma h)_q^\dagger e^{-H\tau} (\Sigma h)_s (\Sigma h)_r e^{-H(\beta-\tau)} \right] \\
&= \frac{1}{Z} tr \left[(\Sigma h)_r (\Sigma h)_s e^{-H(\beta-\tau)} (\Sigma h)_k^\dagger (\Sigma h)_q^\dagger e^{-H\tau} \right] \\
&= \frac{1}{Z} tr \left[(\Sigma h)_s (\Sigma h)_r e^{-H(\beta-\tau)} (\Sigma h)_q^\dagger (\Sigma h)_k^\dagger e^{-H\tau} \right] \\
&= -\frac{1}{Z} tr \left[(\Sigma h)_s (\Sigma h)_r e^{-H(\beta-\tau)} (\Sigma h)_k^\dagger (\Sigma h)_q^\dagger e^{-H\tau} \right] \\
&= -\frac{1}{Z} tr \left[(\Sigma h)_r (\Sigma h)_s e^{-H(\beta-\tau)} (\Sigma h)_q^\dagger (\Sigma h)_k^\dagger e^{-H\tau} \right]
\end{aligned}
\tag{3.48}
$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s} &= I^{-1}\begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau)I = \\
&= \left[\begin{matrix} (\Sigma h)_+^\dagger\,(\Sigma h)_+^\dagger\,(\Sigma h)_+\,(\Sigma h)_+ & (\Sigma h)_+^\dagger\,(\Sigma h)_+^\dagger\,(\Sigma h)_+\,(\Sigma h)_- & (\Sigma h)_+^\dagger\,(\Sigma h)_+^\dagger\,(\Sigma h)_-\,(\Sigma h)_+ & (\Sigma h)_+^\dagger\,(\Sigma h)_+^\dagger\,(\Sigma h)_- \\ (\Sigma h)_+^\dagger\,(\Sigma h)_-^\dagger\,(\Sigma h)_+\,(\Sigma h)_+ & (\Sigma h)_+^\dagger\,(\Sigma h)_-^\dagger\,(\Sigma h)_+\,(\Sigma h)_- & (\Sigma h)_+^\dagger\,(\Sigma h)_-^\dagger\,(\Sigma h)_-\,(\Sigma h)_+ & (\Sigma h)_+^\dagger\,(\Sigma h)_-^\dagger\,(\Sigma h)_- \\ (\Sigma h)_-^\dagger\,(\Sigma h)_+^\dagger\,(\Sigma h)_+\,(\Sigma h)_+ & (\Sigma h)_-^\dagger\,(\Sigma h)_+^\dagger\,(\Sigma h)_+\,(\Sigma h)_- & (\Sigma h)_-^\dagger\,(\Sigma h)_+^\dagger\,(\Sigma h)_-\,(\Sigma h)_+ & (\Sigma h)_-^\dagger\,(\Sigma h)_+^\dagger\,(\Sigma h)_- \\ (\Sigma h)_-^\dagger\,(\Sigma h)_-^\dagger\,(\Sigma h)_+\,(\Sigma h)_+ & (\Sigma h)_-^\dagger\,(\Sigma h)_-^\dagger\,(\Sigma h)_+\,(\Sigma h)_- & (\Sigma h)_-^\dagger\,(\Sigma h)_-^\dagger\,(\Sigma h)_-\,(\Sigma h)_+ & (\Sigma h)_-^\dagger\,(\Sigma h)_-^\dagger\,(\Sigma h)_- \end{matrix}\right. \\
&= \begin{bmatrix} h_-^\dagger h_-^\dagger h_-h_- & h_-^\dagger h_-^\dagger h_-h_+ & h_-^\dagger h_-^\dagger h_+h_- & h_-^\dagger h_-^\dagger h_+h_+ \\ h_-^\dagger h_+^\dagger h_-h_- & h_-^\dagger h_+^\dagger h_-h_+ & h_-^\dagger h_+^\dagger h_+h_- & h_-^\dagger h_+^\dagger h_+h_+ \\ h_+^\dagger h_-^\dagger h_-h_- & h_+^\dagger h_-^\dagger h_-h_+ & h_+^\dagger h_-^\dagger h_+h_- & h_+^\dagger h_-^\dagger h_+h_+ \\ h_+^\dagger h_+^\dagger h_-h_- & h_+^\dagger h_+^\dagger h_-h_+ & h_+^\dagger h_+^\dagger h_+h_- & h_+^\dagger h_+^\dagger h_+h_+ \end{bmatrix}_{k,q,r,s} (\tau) \\
&= \begin{bmatrix} h_-^\dagger h_-^\dagger h_-h_- & h_-^\dagger h_-^\dagger h_+h_- & h_-^\dagger h_-^\dagger h_-h_+ & h_-^\dagger h_-^\dagger h_+h_+ \\ h_+^\dagger h_-^\dagger h_-h_- & h_+^\dagger h_-^\dagger h_+h_- & h_+^\dagger h_-^\dagger h_-h_+ & h_+^\dagger h_-^\dagger h_+h_+ \\ h_-^\dagger h_+^\dagger h_-h_- & h_-^\dagger h_+^\dagger h_+h_- & h_-^\dagger h_+^\dagger h_-h_+ & h_-^\dagger h_+^\dagger h_+h_+ \\ h_+^\dagger h_+^\dagger h_-h_- & h_+^\dagger h_+^\dagger h_+h_- & h_+^\dagger h_+^\dagger h_-h_+ & h_+^\dagger h_+^\dagger h_+h_+ \end{bmatrix}_{q,k,s,r} (\tau) \\
&= -\begin{bmatrix} h_-^\dagger h_-^\dagger h_-h_- & h_-^\dagger h_-^\dagger h_-h_+ & h_-^\dagger h_-^\dagger h_+h_- & h_-^\dagger h_-^\dagger h_+h_+ \\ h_+^\dagger h_-^\dagger h_-h_- & h_+^\dagger h_-^\dagger h_-h_+ & h_+^\dagger h_-^\dagger h_+h_- & h_+^\dagger h_-^\dagger h_+h_+ \\ h_-^\dagger h_+^\dagger h_-h_- & h_-^\dagger h_+^\dagger h_-h_+ & h_-^\dagger h_+^\dagger h_+h_- & h_-^\dagger h_+^\dagger h_+h_+ \\ h_+^\dagger h_+^\dagger h_-h_- & h_+^\dagger h_+^\dagger h_-h_+ & h_+^\dagger h_+^\dagger h_+h_- & h_+^\dagger h_+^\dagger h_+h_+ \end{bmatrix}_{q,k,r,s} (\tau) \\
&= -\begin{bmatrix} h_-^\dagger h_-^\dagger h_-h_- & h_-^\dagger h_-^\dagger h_+h_- & h_-^\dagger h_-^\dagger h_-h_+ & h_-^\dagger h_-^\dagger h_+h_+ \\ h_-^\dagger h_+^\dagger h_-h_- & h_-^\dagger h_+^\dagger h_+h_- & h_-^\dagger h_+^\dagger h_-h_+ & h_-^\dagger h_+^\dagger h_+h_+ \\ h_+^\dagger h_-^\dagger h_-h_- & h_+^\dagger h_-^\dagger h_+h_- & h_+^\dagger h_-^\dagger h_-h_+ & h_+^\dagger h_-^\dagger h_+h_+ \\ h_+^\dagger h_+^\dagger h_-h_- & h_+^\dagger h_+^\dagger h_+h_- & h_+^\dagger h_+^\dagger h_-h_+ & h_+^\dagger h_+^\dagger h_+h_+ \end{bmatrix}_{k,q,s,r} (\tau) \\
&= \begin{bmatrix} h_-h_-h_-^\dagger h_-^\dagger & h_-h_+h_-^\dagger h_-^\dagger & h_+h_-h_-^\dagger h_-^\dagger & h_+h_+h_-^\dagger h_-^\dagger \\ h_-h_-h_-^\dagger h_+^\dagger & h_-h_+h_-^\dagger h_+^\dagger & h_+h_-h_-^\dagger h_+^\dagger & h_+h_+h_-^\dagger h_+^\dagger \\ h_-h_-h_+^\dagger h_-^\dagger & h_-h_+h_+^\dagger h_-^\dagger & h_+h_-h_+^\dagger h_-^\dagger & h_+h_+h_+^\dagger h_-^\dagger \\ h_-h_-h_+^\dagger h_+^\dagger & h_-h_+h_+^\dagger h_+^\dagger & h_+h_-h_+^\dagger h_+^\dagger & h_+h_+h_+^\dagger h_+^\dagger \end{bmatrix}_{r,s,k,q} (\beta-\tau) \\
&= \begin{bmatrix} h_-h_-h_-^\dagger h_-^\dagger & h_+h_-h_-^\dagger h_-^\dagger & h_-h_+h_-^\dagger h_-^\dagger & h_+h_+h_-^\dagger h_-^\dagger \\ h_-h_-h_+^\dagger h_-^\dagger & h_+h_-h_+^\dagger h_-^\dagger & h_-h_+h_+^\dagger h_-^\dagger & h_+h_+h_+^\dagger h_-^\dagger \\ h_-h_-h_-^\dagger h_+^\dagger & h_+h_-h_-^\dagger h_+^\dagger & h_-h_+h_-^\dagger h_+^\dagger & h_+h_+h_-^\dagger h_+^\dagger \\ h_-h_-h_+^\dagger h_+^\dagger & h_+h_-h_+^\dagger h_+^\dagger & h_-h_+h_+^\dagger h_+^\dagger & h_+h_+h_+^\dagger h_+^\dagger \end{bmatrix}_{s,r,q,k} (\beta-\tau) \\
&= -\begin{bmatrix} h_-h_-h_-^\dagger h_-^\dagger & h_+h_-h_-^\dagger h_-^\dagger & h_-h_+h_-^\dagger h_-^\dagger & h_+h_+h_-^\dagger h_-^\dagger \\ h_-h_-h_-^\dagger h_+^\dagger & h_+h_-h_-^\dagger h_+^\dagger & h_-h_+h_-^\dagger h_+^\dagger & h_+h_+h_-^\dagger h_+^\dagger \\ h_-h_-h_+^\dagger h_-^\dagger & h_+h_-h_+^\dagger h_-^\dagger & h_-h_+h_+^\dagger h_-^\dagger & h_+h_+h_+^\dagger h_-^\dagger \\ h_-h_-h_+^\dagger h_+^\dagger & h_+h_-h_+^\dagger h_+^\dagger & h_-h_+h_+^\dagger h_+^\dagger & h_+h_+h_+^\dagger h_+^\dagger \end{bmatrix}_{s,r,k,q} (\beta-\tau) \\
&= -\begin{bmatrix} h_-h_-h_-^\dagger h_-^\dagger & h_-h_+h_-^\dagger h_-^\dagger & h_+h_-h_-^\dagger h_-^\dagger & h_+h_+h_-^\dagger h_-^\dagger \\ h_-h_-h_+^\dagger h_-^\dagger & h_-h_+h_+^\dagger h_-^\dagger & h_+h_-h_+^\dagger h_-^\dagger & h_+h_+h_+^\dagger h_-^\dagger \\ h_-h_-h_-^\dagger h_+^\dagger & h_-h_+h_-^\dagger h_+^\dagger & h_+h_-h_-^\dagger h_+^\dagger & h_+h_+h_-^\dagger h_+^\dagger \\ h_-h_-h_+^\dagger h_+^\dagger & h_-h_+h_+^\dagger h_+^\dagger & h_+h_-h_+^\dagger h_+^\dagger & h_+h_+h_+^\dagger h_+^\dagger \end{bmatrix}_{r,s,q,k} (\beta-\tau)
\end{aligned}
\tag{3.49}
$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_1 \cdot C_{Q=-2,S=1,S^3=+1;k,q,r,s}(\tau) \cdot \sigma_1 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_5 \cdot \sigma_1 \cdot C_{Q=-2,S=1,S^3=+1;q,k,s,r}(\tau) \cdot \sigma_1 \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_5 \cdot \sigma_1 \cdot C_{Q=-2,S=1,S^3=+1;q,k,r,s}(\tau) \cdot \sigma_1 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_1 \cdot C_{Q=-2,S=1,S^3=+1;k,q,s,r}(\tau) \cdot \sigma_1 \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_1 \cdot C_{Q=-2,S=1,S^3=-1;r,s,k,q}{}^{\top}(\beta-\tau) \cdot \sigma_1 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_5 \cdot \sigma_1 \cdot C_{Q=-2,S=1,S^3=-1;s,r,q,k}{}^{\top}(\beta-\tau) \cdot \sigma_1 \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_1 \cdot C_{Q=-2,S=1,S^3=-1;s,r,k,q}{}^{\top}(\beta-\tau) \cdot \sigma_1 \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_5 \cdot \sigma_1 \cdot C_{Q=-2,S=1,S^3=-1;r,s,q,k}{}^{\top}(\beta-\tau) \cdot \sigma_1
\end{aligned}
\tag{3.50}
$$

C* : *H

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s} &= \frac{1}{Z} tr\left[p_k p_q e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)}\right] = \\
&= \frac{1}{Z} tr\left[C^{-1} C p_k C^{-1} C p_q C^{-1} C e^{-H\tau} C^{-1} C p_r^\dagger C^{-1} C p_s^\dagger C^{-1} C e^{-H(\beta-\tau)}\right] = \\
&= \frac{1}{Z} tr\left[C p_k C^{-1} C p_q C^{-1} C e^{-H\tau} C^{-1} C p_r^\dagger C^{-1} C p_s^\dagger C^{-1} C e^{-H(\beta-\tau)} C^{-1}\right] = \\
&= \frac{1}{Z} tr\left[h_k h_q e^{-H\tau} h_r^\dagger h_s^\dagger e^{-H(\beta-\tau)}\right] \\
&= \frac{1}{Z} tr\left[h_q h_k - H\tau h_s^\dagger h_r^\dagger e^{-H(\beta-\tau)}\right] \\
&= -\frac{1}{Z} tr\left[h_q h_k e^{-H\tau} h_r^\dagger h_s^\dagger e^{-H(\beta-\tau)}\right] \\
&= -\frac{1}{Z} tr\left[h_k h_q e^{-H\tau} h_s^\dagger h_r^\dagger e^{-H(\beta-\tau)}\right] \\
&= \frac{1}{Z} tr\left[h_r^\dagger h_s^\dagger e^{-H(\beta-\tau)} h_k h_q e^{-H\tau}\right] \\
&= \frac{1}{Z} tr\left[h_s^\dagger h_r^\dagger e^{-H(\beta-\tau)} h_q h_k e^{-H\tau}\right] \\
&= -\frac{1}{Z} tr\left[h_s^\dagger h_r^\dagger e^{-H(\beta-\tau)} h_k h_q e^{-H\tau}\right] \\
&= -\frac{1}{Z} tr\left[h_r^\dagger h_s^\dagger e^{-H(\beta-\tau)} h_q h_k e^{-H\tau}\right]
\end{aligned}
\tag{3.51}
$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s} = C^{-1} & \begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau)C = \\
= & \begin{bmatrix} h_+h_+h_+^\dagger h_+^\dagger & h_+h_+h_+^\dagger h_-^\dagger & h_+h_+h_-^\dagger h_+^\dagger & h_+h_+h_-^\dagger h_-^\dagger \\ h_+h_-h_+^\dagger h_+^\dagger & h_+h_-h_+^\dagger h_-^\dagger & h_+h_-h_-^\dagger h_+^\dagger & h_+h_-h_-^\dagger h_-^\dagger \\ h_-h_+h_+^\dagger h_+^\dagger & h_-h_+h_+^\dagger h_-^\dagger & h_-h_+h_-^\dagger h_+^\dagger & h_-h_+h_-^\dagger h_-^\dagger \\ h_-h_-h_+^\dagger h_+^\dagger & h_-h_-h_+^\dagger h_-^\dagger & h_-h_-h_-^\dagger h_+^\dagger & h_-h_-h_-^\dagger h_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau) \\
= & \begin{bmatrix} h_+h_+h_+^\dagger h_+^\dagger & h_+h_+h_-^\dagger h_+^\dagger & h_+h_+h_+^\dagger h_-^\dagger & h_+h_+h_-^\dagger h_-^\dagger \\ h_-h_+h_+^\dagger h_+^\dagger & h_-h_+h_-^\dagger h_+^\dagger & h_-h_+h_+^\dagger h_-^\dagger & h_-h_+h_-^\dagger h_-^\dagger \\ h_+h_-h_+^\dagger h_+^\dagger & h_+h_-h_-^\dagger h_+^\dagger & h_+h_-h_+^\dagger h_-^\dagger & h_+h_-h_-^\dagger h_-^\dagger \\ h_-h_-h_+^\dagger h_+^\dagger & h_-h_-h_-^\dagger h_+^\dagger & h_-h_-h_+^\dagger h_-^\dagger & h_-h_-h_-^\dagger h_-^\dagger \end{bmatrix}_{q,k,s,r} (\tau) \\
= - & \begin{bmatrix} h_+h_+h_+^\dagger h_+^\dagger & h_+h_+h_+^\dagger h_-^\dagger & h_+h_+h_-^\dagger h_+^\dagger & h_+h_+h_-^\dagger h_-^\dagger \\ h_-h_+h_+^\dagger h_+^\dagger & h_-h_+h_+^\dagger h_-^\dagger & h_-h_+h_-^\dagger h_+^\dagger & h_-h_+h_-^\dagger h_-^\dagger \\ h_+h_-h_+^\dagger h_+^\dagger & h_+h_-h_+^\dagger h_-^\dagger & h_+h_-h_-^\dagger h_+^\dagger & h_+h_-h_-^\dagger h_-^\dagger \\ h_-h_-h_+^\dagger h_+^\dagger & h_-h_-h_+^\dagger h_-^\dagger & h_-h_-h_-^\dagger h_+^\dagger & h_-h_-h_-^\dagger h_-^\dagger \end{bmatrix}_{q,k,r,s} (\tau) \\
= - & \begin{bmatrix} h_+^\dagger h_+^\dagger h_+h_+ & h_+^\dagger h_-^\dagger h_+h_+ & h_-^\dagger h_+^\dagger h_+h_+ & h_-^\dagger h_-^\dagger h_+h_+ \\ h_+^\dagger h_+^\dagger h_-h_+ & h_+^\dagger h_-^\dagger h_-h_+ & h_-^\dagger h_+^\dagger h_-h_+ & h_-^\dagger h_-^\dagger h_-h_+ \\ h_+^\dagger h_+^\dagger h_+h_- & h_+^\dagger h_-^\dagger h_+h_- & h_-^\dagger h_+^\dagger h_+h_- & h_-^\dagger h_-^\dagger h_+h_- \\ h_+^\dagger h_+^\dagger h_-h_- & h_+^\dagger h_-^\dagger h_-h_- & h_-^\dagger h_+^\dagger h_-h_- & h_-^\dagger h_-^\dagger h_-h_- \end{bmatrix}_{k,q,s,r} (\tau) \\
= & \begin{bmatrix} h_+^\dagger h_+^\dagger h_+h_+ & h_+^\dagger h_-^\dagger h_+h_+ & h_-^\dagger h_+^\dagger h_+h_+ & h_-^\dagger h_-^\dagger h_+h_+ \\ h_+^\dagger h_+^\dagger h_+h_- & h_+^\dagger h_-^\dagger h_+h_- & h_-^\dagger h_+^\dagger h_+h_- & h_-^\dagger h_-^\dagger h_+h_- \\ h_+^\dagger h_+^\dagger h_-h_+ & h_+^\dagger h_-^\dagger h_-h_+ & h_-^\dagger h_+^\dagger h_-h_+ & h_-^\dagger h_-^\dagger h_-h_+ \\ h_+^\dagger h_+^\dagger h_-h_- & h_+^\dagger h_-^\dagger h_-h_- & h_-^\dagger h_+^\dagger h_-h_- & h_-^\dagger h_-^\dagger h_-h_- \end{bmatrix}_{r,s,k,q} (\beta-\tau) \\
= & \begin{bmatrix} h_+^\dagger h_+^\dagger h_+h_+ & h_-^\dagger h_+^\dagger h_+h_+ & h_+^\dagger h_-^\dagger h_+h_+ & h_-^\dagger h_-^\dagger h_+h_+ \\ h_+^\dagger h_+^\dagger h_-h_+ & h_-^\dagger h_+^\dagger h_-h_+ & h_+^\dagger h_-^\dagger h_-h_+ & h_-^\dagger h_-^\dagger h_-h_+ \\ h_+^\dagger h_+^\dagger h_+h_- & h_-^\dagger h_+^\dagger h_+h_- & h_+^\dagger h_-^\dagger h_+h_- & h_-^\dagger h_-^\dagger h_+h_- \\ h_+^\dagger h_+^\dagger h_-h_- & h_-^\dagger h_+^\dagger h_-h_- & h_+^\dagger h_-^\dagger h_-h_- & h_-^\dagger h_-^\dagger h_-h_- \end{bmatrix}_{s,r,q,k} (\beta-\tau) \\
= - & \begin{bmatrix} h_+^\dagger h_+^\dagger h_+h_+ & h_-^\dagger h_+^\dagger h_+h_+ & h_+^\dagger h_-^\dagger h_+h_+ & h_-^\dagger h_-^\dagger h_+h_+ \\ h_+^\dagger h_+^\dagger h_+h_- & h_-^\dagger h_+^\dagger h_+h_- & h_+^\dagger h_-^\dagger h_+h_- & h_-^\dagger h_-^\dagger h_+h_- \\ h_+^\dagger h_+^\dagger h_-h_+ & h_-^\dagger h_+^\dagger h_-h_+ & h_+^\dagger h_-^\dagger h_-h_+ & h_-^\dagger h_-^\dagger h_-h_+ \\ h_+^\dagger h_+^\dagger h_-h_- & h_-^\dagger h_+^\dagger h_-h_- & h_+^\dagger h_-^\dagger h_-h_- & h_-^\dagger h_-^\dagger h_-h_- \end{bmatrix}_{s,r,k,q} (\beta-\tau) \\
= - & \begin{bmatrix} h_+^\dagger h_+^\dagger h_+h_+ & h_+^\dagger h_-^\dagger h_+h_+ & h_-^\dagger h_+^\dagger h_+h_+ & h_-^\dagger h_-^\dagger h_+h_+ \\ h_+^\dagger h_+^\dagger h_-h_+ & h_+^\dagger h_-^\dagger h_-h_+ & h_-^\dagger h_+^\dagger h_-h_+ & h_-^\dagger h_-^\dagger h_-h_+ \\ h_+^\dagger h_+^\dagger h_+h_- & h_+^\dagger h_-^\dagger h_+h_- & h_-^\dagger h_+^\dagger h_+h_- & h_-^\dagger h_-^\dagger h_+h_- \\ h_+^\dagger h_+^\dagger h_-h_- & h_+^\dagger h_-^\dagger h_-h_- & h_-^\dagger h_+^\dagger h_-h_- & h_-^\dagger h_-^\dagger h_-h_- \end{bmatrix}_{r,s,q,k} (\beta-\tau)
\end{aligned}
\tag{3.52}
$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= C_{Q=-2,S=1,S^3=-1;k,q,r,s}(\tau) \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_5 \cdot C_{Q=-2,S=1,S^3=-1;q,k,s,r}(\tau) \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_5 \cdot C_{Q=-2,S=1,S^3=-1;q,k,r,s}(\tau) \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -C_{Q=-2,S=1,S^3=-1;k,q,s,r}(\tau) \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= C_{Q=-2,S=1,S^3=+1;r,s,k,q}{}^{\top}(\beta-\tau) \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_5 \cdot C_{Q=-2,S=1,S^3=+1;s,r,q,k}{}^{\top}(\beta-\tau) \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -C_{Q=-2,S=1,S^3=+1;s,r,k,q}{}^{\top}(\beta-\tau) \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_5 \cdot C_{Q=-2,S=1,S^3=+1;r,s,q,k}{}^{\top}(\beta-\tau)
\end{aligned}
\tag{3.53}
$$

XF* : *H

$$
\begin{aligned}
&C_{Q=2,S=1,S^3=-1;k,q,r,s} = \frac{1}{Z} tr\left[p_k p_q e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)}\right] = \\
&= \frac{1}{Z} tr\left[(XF)^{-1}(XF)p_k(XF)^{-1}(XF)p_q(XF)^{-1}(XF)e^{-H\tau}(XF)^{-1}(XF)p_r^\dagger(XF)^{-1}(XF)p_s^\dagger(XF)^{-1}(XF)e^{-H(\beta-\tau)}\right] = \\
&= \frac{1}{Z} tr\left[(XF)p_k(XF)^{-1}(XF)p_q(XF)^{-1}(XF)e^{-H\tau}(XF)^{-1}(XF)p_r^\dagger(XF)^{-1}(XF)p_s^\dagger(XF)^{-1}(XF)e^{-H(\beta-\tau)}(XF)^{-1}\right] = \\
&= \frac{1}{Z} tr\left[(\Sigma p)_k^\dagger(\Sigma p)_q^\dagger e^{-H\tau}(\Sigma p)_r(\Sigma p)_s e^{-H(\beta-\tau)}\right] \\
&= \frac{1}{Z} tr\left[(\Sigma p)_q^\dagger(\Sigma p)_k^\dagger e^{-H\tau}(\Sigma p)_s(\Sigma p)_r e^{-H(\beta-\tau)}\right] \\
&= -\frac{1}{Z} tr\left[(\Sigma p)_q^\dagger(\Sigma p)_k^\dagger e^{-H\tau}(\Sigma p)_r(\Sigma p)_s e^{-H(\beta-\tau)}\right] \\
&= -\frac{1}{Z} tr\left[(\Sigma p)_k^\dagger(\Sigma p)_q^\dagger e^{-H\tau}(\Sigma p)_s(\Sigma p)_r e^{-H(\beta-\tau)}\right] \\
&= \frac{1}{Z} tr\left[(\Sigma p)_r(\Sigma p)_s e^{-H(\beta-\tau)}(\Sigma p)_k^\dagger(\Sigma p)_q^\dagger e^{-H\tau}\right] \\
&= \frac{1}{Z} tr\left[(\Sigma p)_s(\Sigma p)_r e^{-H(\beta-\tau)}(\Sigma p)_q^\dagger(\Sigma p)_k^\dagger e^{-H\tau}\right] \\
&= -\frac{1}{Z} tr\left[(\Sigma p)_s(\Sigma p)_r e^{-H(\beta-\tau)}(\Sigma p)_k^\dagger(\Sigma p)_q^\dagger e^{-H\tau}\right] \\
&= -\frac{1}{Z} tr\left[(\Sigma p)_r(\Sigma p)_s e^{-H(\beta-\tau)}(\Sigma p)_q^\dagger(\Sigma p)_k^\dagger e^{-H\tau}\right]
\end{aligned}
\tag{3.54}
$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s} &= (XF)^{-1}\begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} \qquad (\tau)(XF) = \\
&= \begin{bmatrix} (\Sigma p)_+^\dagger (\Sigma p)_+^\dagger (\Sigma p)_+ (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma p)_+^\dagger (\Sigma p)_+ (\Sigma p)_- & (\Sigma p)_+^\dagger (\Sigma p)_+^\dagger (\Sigma p)_- (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma p)_+^\dagger (\Sigma p)_- \\ (\Sigma p)_+^\dagger (\Sigma p)_-^\dagger (\Sigma p)_+ (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma p)_-^\dagger (\Sigma p)_+ (\Sigma p)_- & (\Sigma p)_+^\dagger (\Sigma p)_-^\dagger (\Sigma p)_- (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma p)_-^\dagger (\Sigma p)_- \\ (\Sigma p)_-^\dagger (\Sigma p)_+^\dagger (\Sigma p)_+ (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma p)_+^\dagger (\Sigma p)_+ (\Sigma p)_- & (\Sigma p)_-^\dagger (\Sigma p)_+^\dagger (\Sigma p)_- (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma p)_+^\dagger (\Sigma p)_- \\ (\Sigma p)_-^\dagger (\Sigma p)_-^\dagger (\Sigma p)_+ (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma p)_-^\dagger (\Sigma p)_+ (\Sigma p)_- & (\Sigma p)_-^\dagger (\Sigma p)_-^\dagger (\Sigma p)_- (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma p)_-^\dagger (\Sigma p)_- \end{bmatrix} \\
&= \begin{bmatrix} p_-^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_+p_+ \\ p_-^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_+p_+ \\ p_+^\dagger p_-^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_+^\dagger p_+p_+ \end{bmatrix}_{k,q,r,s} \qquad (\tau) \\
&= \begin{bmatrix} p_-^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_-^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_+p_+ \\ p_-^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_+^\dagger p_+p_+ \end{bmatrix}_{q,k,s,r} \qquad (\tau) \\
&= -\begin{bmatrix} p_-^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_-^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_+p_+ \\ p_-^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_+^\dagger p_+p_+ \end{bmatrix}_{q,k,r,s} \qquad (\tau) \\
&= -\begin{bmatrix} p_-^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_+p_+ \\ p_-^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_+p_+ \\ p_+^\dagger p_-^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_+^\dagger p_+p_+ \end{bmatrix}_{k,q,s,r} \qquad (\tau) \\
&= \begin{bmatrix} p_-p_-p_-^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_+^\dagger \\ p_-p_-p_+^\dagger p_-^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_+p_+^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_+^\dagger \end{bmatrix}_{r,s,k,q} \qquad (\beta-\tau) \\
&= \begin{bmatrix} p_-p_-p_-^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_-^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_+p_+p_+^\dagger p_-^\dagger \\ p_-p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_+^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_+^\dagger \end{bmatrix}_{s,r,q,k} \qquad (\beta-\tau) \\
&= -\begin{bmatrix} p_-p_-p_-^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_+^\dagger \\ p_-p_-p_+^\dagger p_-^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_+p_+p_+^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_+^\dagger \end{bmatrix}_{s,r,k,q} \qquad (\beta-\tau) \\
&= -\begin{bmatrix} p_-p_-p_-^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_-^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_+p_+^\dagger p_-^\dagger \\ p_-p_-p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_+^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_+^\dagger \end{bmatrix}_{r,s,q,k} \qquad (\beta-\tau)
\end{aligned}
\tag{3.55}
$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_1 \cdot C_{Q=2,S=1,S^3=+1;k,q,r,s}(\tau) \cdot \sigma_1 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_5 \cdot \sigma_1 \cdot C_{Q=2,S=1,S^3=+1;q,k,s,r}(\tau) \cdot \sigma_1 \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_5 \cdot \sigma_1 \cdot C_{Q=2,S=1,S^3=+1;q,k,r,s}(\tau) \cdot \sigma_1 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_1 \cdot C_{Q=2,S=1,S^3=+1;k,q,s,r}(\tau) \cdot \sigma_1 \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_1 \cdot {C_{Q=2,S=1,S^3=-1;r,s,k,q}}^{\top}(\beta-\tau) \cdot \sigma_1 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_5 \cdot \sigma_1 \cdot {C_{Q=2,S=1,S^3=-1;s,r,q,k}}^{\top}(\beta-\tau) \cdot \sigma_1 \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_1 \cdot {C_{Q=2,S=1,S^3=-1;s,r,k,q}}^{\top}(\beta-\tau) \cdot \sigma_1 \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_5 \cdot \sigma_1 \cdot {C_{Q=2,S=1,S^3=-1;r,s,q,k}}^{\top}(\beta-\tau) \cdot \sigma_1
\end{aligned}
\tag{3.56}
$$

Table 3.3 shows the summary of all symmetries that we derived. From all these symmetries, with our data we can use only the anti-commutation and trace cyclicity, and if we calculate at half-filling, we could use also the XF symmetry. In the worst case, we can make our statistics 8 times better and 16 times better if we work at $\mu = 0$.

	Q	S	S^3	trafo	$kqrs$	τ
Start with	2	1	-1	1	1234	τ
Anti-commutation	2	1	-1	$\sigma_5 \cdot \sigma_5$	2143	τ
	2	1	-1	$-\sigma_5 \cdot$	2134	τ
	2	1	-1	$- \cdot \sigma_5$	1243	τ
trace cycl	2	1	+1	$\top$	3412	$\beta - \tau$
	2	1	+1	$\sigma_5 \top \sigma_5$	4321	$\beta - \tau$
	2	1	+1	$-\top \sigma_5$	4312	$\beta - \tau$
	2	1	+1	$-\sigma_5 \top$	3421	$\beta - \tau$
I	-2	1	+1	$\sigma_1 \cdot \sigma_1$	1234	τ
	-2	1	+1	$\sigma_5 \sigma_1 \cdot \sigma_1 \sigma_5$	2143	τ
	-2	1	+1	$-\sigma_5 \sigma_1 \cdot \sigma_1$	2134	τ
	-2	1	+1	$-\sigma_1 \cdot \sigma_1 \sigma_5$	1243	τ
	-2	1	-1	$\sigma_1 \top \sigma_1$	3412	$\beta - \tau$
	-2	1	-1	$\sigma_5 \sigma_1 \top \sigma_1 \sigma_5$	4321	$\beta - \tau$
	-2	1	-1	$-\sigma_1 \top \sigma_1 \sigma_5$	4312	$\beta - \tau$
	-2	1	-1	$-\sigma_5 \sigma_1 \top \sigma_1$	3421	$\beta - \tau$
C	-2	1	-1	1	1234	τ
	-2	1	-1	$\sigma_5 \cdot \sigma_5$	2143	τ
	-2	1	-1	$-\sigma_5 \cdot$	2134	τ
	-2	1	-1	$- \cdot \sigma_5$	1243	τ
	-2	1	+1	$\top$	3412	$\beta - \tau$
	-2	1	+1	$\sigma_5 \top \sigma_5$	4321	$\beta - \tau$
	-2	1	+1	$-\top \sigma_5$	4312	$\beta - \tau$
	-2	1	+1	$-\sigma_5 \top$	3421	$\beta - \tau$
XF	2	1	+1	$\sigma_1 \cdot \sigma_1$	1234	τ
	2	1	+1	$\sigma_5 \sigma_1 \cdot \sigma_1 \sigma_5$	2143	τ
	2	1	+1	$-\sigma_5 \sigma_1 \cdot \sigma_1$	2134	τ
	2	1	+1	$-\sigma_1 \cdot \sigma_1 \sigma_5$	1243	τ
	2	1	-1	$\sigma_1 \top \sigma_1$	3412	$\beta - \tau$
	2	1	-1	$\sigma_5 \sigma_1 \top \sigma_1 \sigma_5$	4321	$\beta - \tau$
	2	1	-1	$-\sigma_1 \top \sigma_1 \sigma_5$	4312	$\beta - \tau$
	2	1	-1	$-\sigma_5 \sigma_1 \top \sigma_1$	3421	$\beta - \tau$

Table 3.3: All the correlators that can be directly averaged, if they all take a 4×4 matrix form. The correlators above the double line may be averaged even at finite μ and everything below may be averaged when $\mu = 0$. The combination of charge and spin indicate whether $p^\dagger p^\dagger$ or pp is at the source (right-most). The *trafo* column indicates what must be done in the interal 4×4 momentum space before averaging; $\top$ indicates transpose, $\sigma_1 \cdot \sigma_1$ indicates conjugation by σ_1, $\sigma_5 \cdot \sigma_5$ indicates conjugation by σ_5, minus sign indicates that there must be added one minus in front, and 1 indicates that no operation is needed. The next column $kqrs$ indicates the position of the momenta at the sink and source relative to the initial correlator is. The τ column indicates whether the given correlator needs to be time-reversed.

CHAPTER 4

Spectrum of Excitons

4.1 Data Generating

4.2 Fitting Data

APPENDIX A

Useful information

In the appendix you usually include extra information that should be documented in your thesis, but not interrupt the flow.

The LaTeX WikiBook [12] is a useful source of information on LaTeX.

Bibliography

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List of Figures

3.1 Honeycomb lattice is a bipartite lattice because we can separate and label the lattice points with different colors (in this example blue and red) in such a way that every point in blue color neighbors only points with red color. 6
3.2 LEFT: All points of special interest shown on the lattice and how they are copied over to other lattice cells. RIGHT: Special points only in the first BZ since it has all the symmetries of the lattice 6
3.3 An example of the spin-flip transformation with all possibilities of creation/annihilation operators. The graphs show only the transformation of the charge and the spin of the operators. The color of the arrows shows the charge (blue = $+1$, red = -1) and the orientation shows the spin. We can can go back and forth between the left and the right graphs by using the spin-flip conjugation. 7
3.4 An example of the charge transformation with all possibilities of creation/annihilation operators. The graphs show only the transformation of the charge and the spin of the operators. The color of the arrows shows the charge (blue = $+1$, red = -1) and the orientation shows the spin. We can can go back and forth between the left and the right graphs by using the charge-flip conjugation. 9
3.5 An example of the dagger-flip transformation with all possibilities of creation/annihilation operators. The graphs show only the transformation of the charge and the spin of the operators. The color of the arrows shows the charge (blue = $+1$, red = -1) and the orientation shows the spin. We can can go back and forth between the left and the right graphs by using the dagger-flip conjugation. 12
3.6 An example of the particle-hole transformation with all possibilities of creation/annihilation operators. The graphs show only the transformation of the charge and the spin of the operators. The color of the arrows shows the charge (blue = $+1$, red = -1) and the orientation shows the spin. We can can go back and forth between the left and the right graphs by using the particle-hole conjugation. 12

List of Tables

3.1 All the correlators that can be directly averaged, assuming they all take the 2×2 matrix form. The correlators above the double line may be averaged even at finite μ and everything below may be averaged when $\mu = 0$. The combination of charge and spin indicate whether $p^\dagger$, p, $h^\dagger$, or h is right-most. The $\vec{k}$ column indicates what must be done in the interal 2×2 momentum space before averaging; $\top$ indicates transpose, $\sigma_1 \cdot \sigma_1$ indicates conjugation by σ_1, 1 indicates that no operation is needed. The τ column indicates whether the given correlator needs to be time-reversed. 14

3.2 All the correlators that can be directly averaged, if they all take a 4×4 matrix form. The correlators above the double line may be averaged even at finite μ and everything below may be averaged when $\mu = 0$. The combination of charge and spin indicate whether $h^\dagger p^\dagger$ or hp is at the source (right-most). The *trafo* column indicates what must be done in the interal 4×4 momentum space before averaging; $\top$ indicates transpose, $\sigma_1 \cdot \sigma_1$ indicates conjugation by σ_1, $\sigma_5 \cdot \sigma_5$ indicates conjugation by σ_5, and 1 indicates that no operation is needed. The next column $kqrs$ indicates the position of the momenta at the sink and source relative to the initial correlator is. The τ column indicates whether the given correlator needs to be time-reversed. 20

3.3 All the correlators that can be directly averaged, if they all take a 4×4 matrix form. The correlators above the double line may be averaged even at finite μ and everything below may be averaged when $\mu = 0$. The combination of charge and spin indicate whether $p^\dagger p^\dagger$ or pp is at the source (right-most). The *trafo* column indicates what must be done in the interal 4×4 momentum space before averaging; $\top$ indicates transpose, $\sigma_1 \cdot \sigma_1$ indicates conjugation by σ_1, $\sigma_5 \cdot \sigma_5$ indicates conjugation by σ_5, minus sign indicates that there must be added one minus in front, and 1 indicates that no operation is needed. The next column $kqrs$ indicates the position of the momenta at the sink and source relative to the initial correlator is. The τ column indicates whether the given correlator needs to be time-reversed. 30