Title of the Thesis

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I hereby declare that this thesis was formulated by myself and that no sources or tools other than those cited were used.

Bonn, ... Date . . . . Signature

1. Gutachter: Prof. Dr. John Smith
2. Gutachterin: Prof. Dr. Anne Jones

Acknowledgements

I would like to thank ...

You should probably use `\chapter*` for acknowledgements at the beginning of a thesis and `\chapter` for the end.

Contents

CHAPTER 1

Introduction

The introduction usually gives a few pages of introduction to the whole subject, maybe even starting with the Greeks.

For more information on LaTeX and the packages that are available see for example the books of Kopka [1] and Goossens et al [2].

A lot of useful information on particle physics can be found in the “Particle Data Book” [3].

I have resisted the temptation to put a lot of definitions into the file `thesis_defs.sty`, as everyone has their own taste as to what scheme they want to use for names. However, a few examples are included to help you get started:

- cross-sections are measured in pb and integrated luminosity in pb^{-1};
- the K_S^0 is an interesting particle;
- the missing transverse momentum, $p_{\mathrm{T}}^{\mathrm{miss}}$, is often called missing transverse energy, even though it is calculated using a vector sum.

Note that the examples of units assume that you are using the siunitx package.

It also is probably a good idea to include a few well formatted references in the thesis skeleton. More detailed suggestions on what citation types to use can be found in the “Thesis Guide” [4]:

- articles in refereed journals [3, 5];
- a book [6];
- a PhD thesis [7] and a Diplom thesis [8];
- a collection of articles [9];
- a conference note [10];
- a preprint [11] (you can also use `@online` or `@booklet` for such things);
- something that is only available online [4].

At the end of the introduction it is normal to say briefly what comes in the following chapters.

The line at the beginning of this file is used by TeXstudio etc. to specify which is the master LaTeX file, so that you can compile your thesis directly from this file. If your thesis is called something other than `mythesis`, adjust it as appropriate.

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Theorical Overview

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2.2 Hubbard-Model

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CHAPTER 3

Symmetries

3.1 Lattice

3.1.1 Symmetry Operators

3.2 Correlation functions

After we have found out what the symmetries of the lattice are and how they work, we can take them into consideration and apply them onto the one- and two-body correlation functions. We use them in order to average data with the same total momentum. This will increase our statistics and better our precision when performing the fitting of the final data.

3.2.1 One-particle correlation function

First, we start with the one-body correlators which will give us a valuable insight on how to work with the two-body correlation functions. The full derivation is provided in EVAN'S PAPER, so only the relevant information for this thesis will be shown here.

		Q	S	S^3	trafo	$kqrs$	τ
Start with	$QSkt$	+1	1	-1	1	1234	τ
trace cycl	$\overline{QSkt}$	0	1	+1	$\sigma_5 \top \sigma_5$	4321	$\beta - \tau$
I	$Q\overline{Sk}t$	0	1	-1	$\sigma_1 \top \sigma_1$	3412	$\beta - \tau$
		0	1	+1	$\sigma_5\sigma_1 \cdot \sigma_1\sigma_5$	2143	τ
XF	$\overline{QSk}t$	0	1	+1	$\top$	3412	$\beta - \tau$
		0	1	-1	$\sigma_5 \cdot \sigma_5$	2143	τ
C	$\overline{Q}Skt$	0	1	+1	$\sigma_1 \cdot \sigma_1$	1234	τ
		0	1	-1	$\sigma_5\sigma_1 \top \sigma_1\sigma_5$	4321	$\beta - \tau$

3.2.2 Two-particle correlation function

In this thesis we are working with only two types of two-body correlators. There are correlators with particles and holes, and only particles. We start with the former correlator $\langle p h h^\dagger p^\dagger \rangle$. In matrix form with all labels it looks like this

$$C_{Q=0,S=1,S^3=-1;k,q,r,s} = \begin{bmatrix} p_+h_+h_+^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_-^\dagger \\ p_+h_-h_+^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_-^\dagger \\ p_-h_+h_+^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_-^\dagger \\ p_-h_-h_+^\dagger p_+^\dagger & p_-h_-h_+^\dagger p_-^\dagger & p_-h_-h_-^\dagger p_+^\dagger & p_-h_-h_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau)$$

Now, we apply different symmetry operator to the two particle correlation function. When we apply them, we first start with the thermal trace and then switch to matrix form, so that it is easier to see the transformations needed in order to get another two particle correlator.

Trace cyclicity

We begin with the trace cyclicity because it will give us how the correlation function transformations when timereversed.

$$C_{Q=0,S=1,S^3=-1;k,q,r,s} = \frac{1}{Z}tr\left[p_k h_q e^{-H\tau} h_r^\dagger p_s^\dagger e^{-H(\beta-\tau)}\right] = \frac{1}{Z}tr\left[h_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} p_k h_q e^{-H\tau}\right]$$

Now we apply time reverse

$$\beta \to \beta - \tau$$

and use the anti-commutation relations of the operators to get

$$\frac{1}{Z}tr\left[h_r^\dagger p_s^\dagger e^{-H(\beta-(\beta-\tau))} p_k h_q e^{-H(\beta-\tau)}\right] = \frac{1}{Z}tr\left[p_s^\dagger h_r^\dagger e^{-H\tau} h_q p_k e^{-H(\beta-\tau)}\right] \tag{3.1}$$

We switch to matrix form to see observe better how all labels transform

$$\begin{aligned} &C_{Q=0,S=1,S^3=-1;k,q,r,s} \\ &= \begin{bmatrix} p_+h_+h_+^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_-^\dagger \\ p_+h_-h_+^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_-^\dagger \\ p_-h_+h_+^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_-^\dagger \\ p_-h_-h_+^\dagger p_+^\dagger & p_-h_-h_+^\dagger p_-^\dagger & p_-h_-h_-^\dagger p_+^\dagger & p_-h_-h_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau) \\ &= \begin{bmatrix} h_+^\dagger p_+^\dagger p_+h_+ & h_+^\dagger p_-^\dagger p_+h_+ & h_-^\dagger p_+^\dagger p_+h_+ & h_-^\dagger p_-^\dagger p_+h_+ \\ h_+^\dagger p_+^\dagger p_+h_- & h_+^\dagger p_-^\dagger p_+h_- & h_-^\dagger p_+^\dagger p_+h_- & h_-^\dagger p_-^\dagger p_+h_- \\ h_+^\dagger p_+^\dagger p_-h_+ & h_+^\dagger p_-^\dagger p_-h_+ & h_-^\dagger p_+^\dagger p_-h_+ & h_-^\dagger p_-^\dagger p_-h_+ \\ h_+^\dagger p_+^\dagger p_-h_- & h_+^\dagger p_-^\dagger p_-h_- & h_-^\dagger p_+^\dagger p_-h_- & h_-^\dagger p_-^\dagger p_-h_- \end{bmatrix}_{r,s,k,q} (\beta-\tau) \end{aligned}$$

$$= \begin{bmatrix} p_+^\dagger h_+^\dagger h_+ p_+ & p_-^\dagger h_+^\dagger h_+ p_+ & p_+^\dagger h_-^\dagger h_+ p_+ & p_-^\dagger h_-^\dagger h_+ p_+ \\ p_+^\dagger h_+^\dagger h_- p_+ & p_-^\dagger h_+^\dagger h_- p_+ & p_+^\dagger h_-^\dagger h_- p_+ & p_-^\dagger h_-^\dagger h_- p_+ \\ p_+^\dagger h_+^\dagger h_+ p_- & p_-^\dagger h_+^\dagger h_+ p_- & p_+^\dagger h_-^\dagger h_+ p_- & p_-^\dagger h_-^\dagger h_+ p_- \\ p_+^\dagger h_+^\dagger h_- p_- & p_-^\dagger h_+^\dagger h_- p_- & p_+^\dagger h_-^\dagger h_- p_- & p_-^\dagger h_-^\dagger h_- p_- \end{bmatrix}_{s,r,q,k} (\beta - \tau)$$

$$= C_{Q=0,S=1,S^3=+1;s,r,q,k}$$

We notice that order reverse is actually a composed of two transformations – particle-hole momentum switch and time reverse. After we stare at the matrix for an hour, we can figure out what the transformation between the correlation functions is

$$C_{Q=0,S=1,S^3=-1;k,q,r,s}(\tau) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} C_{Q=0,S=1,S^3=+1;s,r,q,k}{}^{\top}(\beta - \tau) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We repeat the steps again for each symmetry operator. First we start with the thermal trace and then switch to matrix for easy transformations. It is shown on the right side of the column if the operator transforms well the full hamiltonian or only at half-filling.

$\underline{\boldsymbol{I : H - \vec{\mu} \cdot \vec{q}}}$

$$\begin{aligned} C_{Q=0,S=1,S^3=-1;k,q,r,s} \\ &= \frac{1}{Z} tr \left[p_k h_q e^{-H\tau} h_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} \right] \\ &= \frac{1}{Z} tr \left[I^{-1} I p_k I^{-1} I h_q I^{-1} I e^{-H\tau} I^{-1} I h_r^\dagger I^{-1} I p_s^\dagger I^{-1} I e^{-H(\beta-\tau)} \right] \\ &= \frac{1}{Z} tr \left[I p_k I^{-1} I h_q I^{-1} I e^{-H\tau} I^{-1} I h_r^\dagger I^{-1} I p_s^\dagger I^{-1} I e^{-H(\beta-\tau) I^{-1}} \right] \\ &= \frac{1}{Z} tr \left[(\Sigma h)_k^\dagger (\Sigma p)_q^\dagger e^{-H\tau} (\Sigma p)_r (\Sigma h)_s e^{-H(\beta-\tau)} \right] \\ &= \frac{1}{Z} tr \left[(\Sigma p)_q^\dagger (\Sigma h)_k^\dagger e^{-H\tau} (\Sigma h)_s (\Sigma p)_r e^{-H(\beta-\tau)} \right] \\ &= \frac{1}{Z} tr \left[(\Sigma p)_r (\Sigma h)_s e^{-H(\beta-\tau)} (\Sigma h)_k^\dagger (\Sigma p)_q^\dagger e^{-H\tau} \right] \end{aligned}$$

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=-1;k,q,r,s}\\
&= I^{-1}\begin{bmatrix}
p_+h_+h_+^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_-^\dagger \\
p_+h_-h_+^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_-^\dagger \\
p_-h_+h_+^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_-^\dagger \\
p_-h_-h_+^\dagger p_+^\dagger & p_-h_-h_+^\dagger p_-^\dagger & p_-h_-h_-^\dagger p_+^\dagger & p_-h_-h_-^\dagger p_-^\dagger
\end{bmatrix}_{k,q,r,s} (\tau)\, I\\
&= \left[\begin{matrix}
(\Sigma h)_+^\dagger (\Sigma p)_+^\dagger (\Sigma p)_+ (\Sigma h)_+ & (\Sigma h)_+^\dagger (\Sigma p)_+^\dagger (\Sigma p)_+ (\Sigma h)_- & (\Sigma h)_+^\dagger (\Sigma p)_+^\dagger (\Sigma p)_- (\Sigma h)_+ & (\Sigma h)_+^\dagger (\Sigma p)_+^\dagger (\Sigma p)_- \\
(\Sigma h)_+^\dagger (\Sigma p)_-^\dagger (\Sigma p)_+ (\Sigma h)_+ & (\Sigma h)_+^\dagger (\Sigma p)_-^\dagger (\Sigma p)_+ (\Sigma h)_- & (\Sigma h)_+^\dagger (\Sigma p)_-^\dagger (\Sigma p)_- (\Sigma h)_+ & (\Sigma h)_+^\dagger (\Sigma p)_-^\dagger (\Sigma p)_- \\
(\Sigma h)_-^\dagger (\Sigma p)_+^\dagger (\Sigma p)_+ (\Sigma h)_+ & (\Sigma h)_-^\dagger (\Sigma p)_+^\dagger (\Sigma p)_+ (\Sigma h)_- & (\Sigma h)_-^\dagger (\Sigma p)_+^\dagger (\Sigma p)_- (\Sigma h)_+ & (\Sigma h)_-^\dagger (\Sigma p)_+^\dagger (\Sigma p)_- \\
(\Sigma h)_-^\dagger (\Sigma p)_-^\dagger (\Sigma p)_+ (\Sigma h)_+ & (\Sigma h)_-^\dagger (\Sigma p)_-^\dagger (\Sigma p)_+ (\Sigma h)_- & (\Sigma h)_-^\dagger (\Sigma p)_-^\dagger (\Sigma p)_- (\Sigma h)_+ & (\Sigma h)_-^\dagger (\Sigma p)_-^\dagger (\Sigma p)_-
\end{matrix}\right.\\
&= \begin{bmatrix}
h_-^\dagger p_-^\dagger p_-h_- & h_-^\dagger p_-^\dagger p_-h_+ & h_-^\dagger p_-^\dagger p_+h_- & h_-^\dagger p_-^\dagger p_+h_+ \\
h_-^\dagger p_+^\dagger p_-h_- & h_-^\dagger p_+^\dagger p_-h_+ & h_-^\dagger p_+^\dagger p_+h_- & h_-^\dagger p_+^\dagger p_+h_+ \\
h_+^\dagger p_-^\dagger p_-h_- & h_+^\dagger p_-^\dagger p_-h_+ & h_+^\dagger p_-^\dagger p_+h_- & h_+^\dagger p_-^\dagger p_+h_+ \\
h_+^\dagger p_+^\dagger p_-h_- & h_+^\dagger p_+^\dagger p_-h_+ & h_+^\dagger p_+^\dagger p_+h_- & h_+^\dagger p_+^\dagger p_+h_+
\end{bmatrix}_{k,q,r,s} (\tau)
\end{aligned}
$$

$$
= \begin{bmatrix}
p_-h_-h_-^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_-^\dagger \\
p_-h_-h_-^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_+^\dagger \\
p_-h_-h_+^\dagger p_-^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_+h_+h_+^\dagger p_-^\dagger \\
p_-h_-h_+^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_+^\dagger
\end{bmatrix}_{r,s,k,q} (\beta-\tau)
$$

$$
= \begin{bmatrix}
p_-^\dagger h_-^\dagger h_-p_- & p_-^\dagger h_-^\dagger h_+p_- & p_-^\dagger h_-^\dagger h_-p_+ & p_-^\dagger h_-^\dagger h_+p_+ \\
p_+^\dagger h_-^\dagger h_-p_- & p_+^\dagger h_-^\dagger h_+p_- & p_+^\dagger h_-^\dagger h_-p_+ & p_+^\dagger h_-^\dagger h_+p_+ \\
p_-^\dagger h_+^\dagger h_-p_- & p_-^\dagger h_+^\dagger h_+p_- & p_-^\dagger h_+^\dagger h_-p_+ & p_-^\dagger h_+^\dagger h_+p_+ \\
p_+^\dagger h_+^\dagger h_-p_- & p_+^\dagger h_+^\dagger h_+p_- & p_+^\dagger h_+^\dagger h_-p_+ & p_+^\dagger h_+^\dagger h_+p_+
\end{bmatrix}_{q,k,s,r} (\tau)
$$

Depending on what transformation we use to get back to $\langle phh^\dagger p^\dagger\rangle$, we get two transformations which are time reverse to one another. We notice the same behaviour for all operators.

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=-1;k,q,r,s}(\tau)\\
&\qquad = \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} C_{Q=0,S=1,S^3=-1;r,s,k,q}{}^\top(\beta-\tau) \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=-1;k,q,r,s}(\tau)\\
&\qquad = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} C_{Q=0,S=1,S^3=+1;q,k,s,r}(\tau) \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

C : H

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=-1;k,q,r,s}\\
&\quad=\frac{1}{Z}tr\left[p_k h_q e^{-H\tau} h_r^\dagger p_s^\dagger e^{-H(\beta-\tau)}\right]\\
&\quad=\frac{1}{Z}tr\left[C^{-1}Cp_kC^{-1}Ch_qC^{-1}Ce^{-H\tau}C^{-1}Ch_r^\dagger C^{-1}Cp_s^\dagger C^{-1}Ce^{-H(\beta-\tau)}\right]\\
&\quad=\frac{1}{Z}tr\left[h_k p_q e^{-H\tau} p_r^\dagger h_s^\dagger e^{-H(\beta-\tau)}\right]\\
&\quad=\frac{1}{Z}tr\left[p_q h_k e^{-H\tau} h_s^\dagger p_r^\dagger e^{-H(\beta-\tau)}\right]
\end{aligned}
$$

$$
C_{Q=0,S=1,S^3=-1;k,q,r,s}
= C^{-1}\begin{bmatrix}
p_+h_+h_+^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_-^\dagger\\
p_+h_-h_+^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_-^\dagger\\
p_-h_+h_+^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_-^\dagger\\
p_-h_-h_+^\dagger p_+^\dagger & p_-h_-h_+^\dagger p_-^\dagger & p_-h_-h_-^\dagger p_+^\dagger & p_-h_-h_-^\dagger p_-^\dagger
\end{bmatrix}_{k,q,r,s}(\tau)\,C
$$

$$
=\begin{bmatrix}
h_+p_+p_+^\dagger h_+^\dagger & h_+p_+p_+^\dagger h_-^\dagger & h_+p_+p_-^\dagger h_+^\dagger & h_+p_+p_-^\dagger h_-^\dagger\\
h_+p_-p_+^\dagger h_+^\dagger & h_+p_-p_+^\dagger h_-^\dagger & h_+p_-p_-^\dagger h_+^\dagger & h_+p_-p_-^\dagger h_-^\dagger\\
h_-p_+p_+^\dagger h_+^\dagger & h_-p_+p_+^\dagger h_-^\dagger & h_-p_+p_-^\dagger h_+^\dagger & h_-p_+p_-^\dagger h_-^\dagger\\
h_-p_-p_+^\dagger h_+^\dagger & h_-p_-p_+^\dagger h_-^\dagger & h_-p_-p_-^\dagger h_+^\dagger & h_-p_-p_-^\dagger h_-^\dagger
\end{bmatrix}_{k,q,r,s}(\tau)
$$

$$
=\begin{bmatrix}
p_+^\dagger h_+^\dagger h_+p_+ & p_+^\dagger h_-^\dagger h_+p_+ & p_-^\dagger h_+^\dagger h_+p_+ & p_-^\dagger h_-^\dagger h_+p_+\\
p_+^\dagger h_+^\dagger h_+p_- & p_+^\dagger h_-^\dagger h_+p_- & p_-^\dagger h_+^\dagger h_+p_- & p_-^\dagger h_-^\dagger h_+p_-\\
p_+^\dagger h_+^\dagger h_-p_+ & p_+^\dagger h_-^\dagger h_-p_+ & p_-^\dagger h_+^\dagger h_-p_+ & p_-^\dagger h_-^\dagger h_-p_+\\
p_+^\dagger h_+^\dagger h_-p_- & p_+^\dagger h_-^\dagger h_-p_- & p_-^\dagger h_+^\dagger h_-p_- & p_-^\dagger h_-^\dagger h_-p_-
\end{bmatrix}_{r,s,k,q}(\beta-\tau)
$$

$$
=\begin{bmatrix}
p_+h_+h_+^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_-^\dagger\\
p_-h_+h_+^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_-^\dagger\\
p_+h_-h_+^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_-^\dagger\\
p_-h_-h_+^\dagger p_+^\dagger & p_-h_-h_-^\dagger p_+^\dagger & p_-h_-h_+^\dagger p_-^\dagger & p_-h_-h_-^\dagger p_-^\dagger
\end{bmatrix}_{q,k,s,r}(\tau)
$$

$$
C_{Q=0,S=1,S^3=-1;k,q,r,s}(\tau) = C_{Q=0,S=1,S^3=+1;r,s,k,q}{}^\top(\beta-\tau)
$$

$$
C_{Q=0,S=1,S^3=-1;k,q,r,s}(\tau) = \begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix} C_{Q=0,S=1,S^3=-1;q,k,s,r}(\tau)\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}
$$

<u>XF : H</u>

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=-1;k,q,r,s} \\
&= \frac{1}{Z} tr \left[p_k h_q e^{-H\tau} h_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} \right] \\
&= \frac{1}{Z} tr \left[(XF)^{-1}(XF) p_k (XF)^{-1}(XF) h_q (XF)^{-1}(XF) e^{-H\tau} (XF)^{-1}(XF) h_r^\dagger (XF)^{-1}(XF) p_s^\dagger (XF)^{-1}(XF) e^{-H(\beta-\right. \\
&= \frac{1}{Z} tr \left[(\Sigma p)_k^\dagger (\Sigma h)_q^\dagger e^{-H\tau} (\Sigma h)_r (\Sigma p)_s e^{-H(\beta-\tau)} \right]
\end{aligned}
$$

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=-1;k,q,r,s} \\
&= (XF)^{-1} \begin{bmatrix} p_+h_+h_+^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_-^\dagger \\ p_+h_-h_+^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_-^\dagger \\ p_-h_+h_+^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_-^\dagger \\ p_-h_-h_+^\dagger p_+^\dagger & p_-h_-h_+^\dagger p_-^\dagger & p_-h_-h_-^\dagger p_+^\dagger & p_-h_-h_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau)\ (XF) \\
&= \left[\begin{matrix} (\Sigma p)_+^\dagger (\Sigma h)_+^\dagger (\Sigma h)_+ (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma h)_+^\dagger (\Sigma h)_+ (\Sigma p)_- & (\Sigma p)_+^\dagger (\Sigma h)_+^\dagger (\Sigma h)_- (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma h)_+^\dagger (\Sigma h)_- (\\ (\Sigma p)_+^\dagger (\Sigma h)_-^\dagger (\Sigma h)_+ (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma h)_-^\dagger (\Sigma h)_+ (\Sigma p)_- & (\Sigma p)_+^\dagger (\Sigma h)_-^\dagger (\Sigma h)_- (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma h)_-^\dagger (\Sigma h)_- (\\ (\Sigma p)_-^\dagger (\Sigma h)_+^\dagger (\Sigma h)_+ (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma h)_+^\dagger (\Sigma h)_+ (\Sigma p)_- & (\Sigma p)_-^\dagger (\Sigma h)_+^\dagger (\Sigma h)_- (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma h)_+^\dagger (\Sigma h)_- (\\ (\Sigma p)_-^\dagger (\Sigma h)_-^\dagger (\Sigma h)_+ (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma h)_-^\dagger (\Sigma h)_+ (\Sigma p)_- & (\Sigma p)_-^\dagger (\Sigma h)_-^\dagger (\Sigma h)_- (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma h)_-^\dagger (\Sigma h)_- \end{matrix} \right. \\
&= \begin{bmatrix} p_-^\dagger h_-^\dagger h_- p_- & p_-^\dagger h_-^\dagger h_- p_+ & p_-^\dagger h_-^\dagger h_+ p_- & p_-^\dagger h_-^\dagger h_+ p_+ \\ p_-^\dagger h_+^\dagger h_- p_- & p_-^\dagger h_+^\dagger h_- p_+ & p_-^\dagger h_+^\dagger h_+ p_- & p_-^\dagger h_+^\dagger h_+ p_+ \\ p_+^\dagger h_-^\dagger h_- p_- & p_+^\dagger h_-^\dagger h_- p_+ & p_+^\dagger h_-^\dagger h_+ p_- & p_+^\dagger h_-^\dagger h_+ p_+ \\ p_+^\dagger h_+^\dagger h_- p_- & p_+^\dagger h_+^\dagger h_- p_+ & p_+^\dagger h_+^\dagger h_+ p_- & p_+^\dagger h_+^\dagger h_+ p_+ \end{bmatrix}_{k,q,r,s} (\tau)
\end{aligned}
$$

$$
= \begin{bmatrix} p_-h_-h_-^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_-^\dagger \\ p_-h_-h_+^\dagger p_-^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_+h_+h_+^\dagger p_-^\dagger \\ p_-h_-h_-^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_+^\dagger \\ p_-h_-h_+^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_+^\dagger \end{bmatrix}_{s,r,k,q} (\beta-\tau)
$$

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=-1;k,q,r,s}(\tau) \\
&\qquad = \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} C_{Q=0,S=1,S^3=+1;k,q,r,s}(\tau) \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=-1;k,q,r,s}(\tau) \\
&\qquad = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} C_{Q=0,S=1,S^3=-1;s,r,q,k}{}^\top(\beta-\tau) \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

The table below shows the summary of all symmetries that we calculated.
For more compact form, we write

	Q	S	S^3	trafo	$kqrs$	τ
Start with	0	1	-1	1	1234	τ
trace cycl	0	1	+1	$\sigma_5 \top \sigma_5$	4321	$\beta - \tau$
I	0	1	-1	$\sigma_1 \top \sigma_1$	3412	$\beta - \tau$
	0	1	+1	$\sigma_5\sigma_1 \cdot \sigma_1\sigma_5$	2143	τ
C	0	1	+1	$\top$	3412	$\beta - \tau$
	0	1	-1	$\sigma_5 \cdot \sigma_5$	2143	τ
XF	0	1	+1	$\sigma_1 \cdot \sigma_1$	1234	τ
	0	1	-1	$\sigma_5\sigma_1 \top \sigma_1\sigma_5$	4321	$\beta - \tau$

$$\sigma_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The other correlation function that we are interested in is with two particles at the source and two at the sink. Again, we start with correlator $\langle ppp^\dagger p^\dagger \rangle$. In trace and matrix forms it look like this.

$$C_{Q=2,S=1,S^3=-1;k,q,r,s} = \frac{1}{Z} tr \left[p_k p_q e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} \right]$$

$$C_{Q=2,S=1,S^3=-1;k,q,r,s} = \begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau)$$

First, we implement the anti-commutation relations which will give us the identities of this correlation function. Again, we use derive the the results through traces and then continue with the matrix form which will give us a better insight on the transformation of all labels that are included.

Anti-commutation

$$\begin{aligned} C_{Q=2,S=1,S^3=-1;k,q,r,s} &= \frac{1}{Z} tr \left[p_k p_q e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} \right] = \\ &= \frac{1}{Z} tr \left[p_q p_k e^{-H\tau} p_s^\dagger p_r^\dagger e^{-H(\beta-\tau)} \right] \\ &= -\frac{1}{Z} tr \left[p_q p_k e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} \right] \\ &= -\frac{1}{Z} tr \left[p_k p_q e^{-H\tau} p_s^\dagger p_r^\dagger e^{-H(\beta-\tau)} \right] \end{aligned}$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s} &= \begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau) \\
&= \begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{q,k,s,r} (\tau) \\
&= - \begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{q,k,r,s} (\tau) \\
&= - \begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,s,r} (\tau)
\end{aligned}
$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_5 \cdot C_{Q=2,S=1,S^3=-1;q,k,s,r}(\tau) \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_5 \cdot C_{Q=2,S=1,S^3=-1;q,k,s,r}(\tau) \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -C_{Q=2,S=1,S^3=-1;q,k,s,r}(\tau) \cdot \sigma_5
\end{aligned}
$$

These identities show us that we can, average our data four fold only because we are using this kind of correlator. Using the cyclicity of the trace, we find how the correlation function transforms when time is reversed. This will further increase our statistics two times.

Trace ciclicity

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s} &= \frac{1}{Z} tr \left[p_k p_q e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} \right] = \\
&= \frac{1}{Z} tr \left[p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} p_k p_q e^{-H\tau} \right] \\
&= \frac{1}{Z} tr \left[p_s^\dagger p_r^\dagger e^{-H(\beta-\tau)} p_q p_k e^{-H\tau} \right] \\
&= -\frac{1}{Z} tr \left[p_s^\dagger p_r^\dagger e^{-H(\beta-\tau)} p_k p_q e^{-H\tau} \right] \\
&= -\frac{1}{Z} tr \left[p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} p_q p_k e^{-H\tau} \right]
\end{aligned}
$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s} &= \begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau) = \\
&= \begin{bmatrix} p_+^\dagger p_+^\dagger p_+p_+ & p_+^\dagger p_-^\dagger p_+p_+ & p_-^\dagger p_+^\dagger p_+p_+ & p_-^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_+p_- \\ p_+^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_-p_+ \\ p_+^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_-p_- \end{bmatrix}_{r,s,k,q} (\beta-\tau) \\
&= \begin{bmatrix} p_+^\dagger p_+^\dagger p_+p_+ & p_-^\dagger p_+^\dagger p_+p_+ & p_+^\dagger p_-^\dagger p_+p_+ & p_-^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_-p_+ \\ p_+^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_+p_- \\ p_+^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_-p_- \end{bmatrix}_{s,r,q,k} (\beta-\tau) \\
&= -\begin{bmatrix} p_+^\dagger p_+^\dagger p_+p_+ & p_-^\dagger p_+^\dagger p_+p_+ & p_+^\dagger p_-^\dagger p_+p_+ & p_-^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_+p_- \\ p_+^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_-p_+ \\ p_+^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_-p_- \end{bmatrix}_{s,r,k,q} (\beta-\tau) \\
&= -\begin{bmatrix} p_+^\dagger p_+^\dagger p_+p_+ & p_+^\dagger p_-^\dagger p_+p_+ & p_-^\dagger p_+^\dagger p_+p_+ & p_-^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_-p_+ \\ p_+^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_+p_- \\ p_+^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_-p_- \end{bmatrix}_{r,s,q,k} (\beta-\tau)
\end{aligned}
$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= C_{Q=2,S=1,S^3=+1;r,s,k,q}{}^\top(\beta-\tau) \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_5 \cdot C_{Q=2,S=1,S^3=+1;s,r,q,k}{}^\top(\beta-\tau) \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -C_{Q=2,S=1,S^3=+1;s,r,k,q}{}^\top(\beta-\tau) \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_5 \cdot C_{Q=2,S=1,S^3=+1;r,s,q,k}{}^\top(\beta-\tau)
\end{aligned}
$$

***I* : $H - \vec{\mu} \cdot \vec{q}$**

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s} &= \frac{1}{Z} tr \left[p_k p_q e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} \right] = \\
&= \frac{1}{Z} tr \left[I^{-1} I p_k I^{-1} I p_q I^{-1} I e^{-H\tau} I^{-1} I p_r^\dagger I^{-1} I p_s^\dagger I^{-1} I e^{-H(\beta-\tau)} \right] = \\
&= \frac{1}{Z} tr \left[I p_k I^{-1} I p_q I^{-1} I e^{-H\tau} I^{-1} I p_r^\dagger I^{-1} I p_s^\dagger I^{-1} I e^{-H(\beta-\tau)} I^{-1} \right] = \\
&= \frac{1}{Z} tr \left[(\Sigma h)_k^\dagger (\Sigma h)_q^\dagger e^{-H\tau} (\Sigma h)_r (\Sigma h)_s e^{-H(\beta-\tau)} \right] \\
&= \frac{1}{Z} tr \left[(\Sigma h)_q^\dagger (\Sigma h)_k^\dagger e^{-H\tau} (\Sigma h)_s (\Sigma h)_r e^{-H(\beta-\tau)} \right] \\
&= -\frac{1}{Z} tr \left[(\Sigma h)_q^\dagger (\Sigma h)_k^\dagger e^{-H\tau} (\Sigma h)_r (\Sigma h)_s e^{-H(\beta-\tau)} \right] \\
&= -\frac{1}{Z} tr \left[(\Sigma h)_k^\dagger (\Sigma h)_q^\dagger e^{-H\tau} (\Sigma h)_s (\Sigma h)_r e^{-H(\beta-\tau)} \right] \\
&= \frac{1}{Z} tr \left[(\Sigma h)_r (\Sigma h)_s e^{-H(\beta-\tau)} (\Sigma h)_k^\dagger (\Sigma h)_q^\dagger e^{-H\tau} \right] \\
&= \frac{1}{Z} tr \left[(\Sigma h)_s (\Sigma h)_r e^{-H(\beta-\tau)} (\Sigma h)_q^\dagger (\Sigma h)_k^\dagger e^{-H\tau} \right] \\
&= -\frac{1}{Z} tr \left[(\Sigma h)_s (\Sigma h)_r e^{-H(\beta-\tau)} (\Sigma h)_k^\dagger (\Sigma h)_q^\dagger e^{-H\tau} \right] \\
&= -\frac{1}{Z} tr \left[(\Sigma h)_r (\Sigma h)_s e^{-H(\beta-\tau)} (\Sigma h)_q^\dagger (\Sigma h)_k^\dagger e^{-H\tau} \right]
\end{aligned}
$$

$$C_{Q=2,S=1,S^3=-1;k,q,r,s} = I^{-1} \begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau) I =$$

$$= \begin{bmatrix} (\Sigma h)_+^\dagger (\Sigma h)_+^\dagger (\Sigma h)_+ (\Sigma h)_+ & (\Sigma h)_+^\dagger (\Sigma h)_+^\dagger (\Sigma h)_+ (\Sigma h)_- & (\Sigma h)_+^\dagger (\Sigma h)_+^\dagger (\Sigma h)_- (\Sigma h)_+ & (\Sigma h)_+^\dagger (\Sigma h)_+^\dagger (\Sigma h)_- (\Sigma h)_- \\ (\Sigma h)_+^\dagger (\Sigma h)_-^\dagger (\Sigma h)_+ (\Sigma h)_+ & (\Sigma h)_+^\dagger (\Sigma h)_-^\dagger (\Sigma h)_+ (\Sigma h)_- & (\Sigma h)_+^\dagger (\Sigma h)_-^\dagger (\Sigma h)_- (\Sigma h)_+ & (\Sigma h)_+^\dagger (\Sigma h)_-^\dagger (\Sigma h)_- (\Sigma h)_- \\ (\Sigma h)_-^\dagger (\Sigma h)_+^\dagger (\Sigma h)_+ (\Sigma h)_+ & (\Sigma h)_-^\dagger (\Sigma h)_+^\dagger (\Sigma h)_+ (\Sigma h)_- & (\Sigma h)_-^\dagger (\Sigma h)_+^\dagger (\Sigma h)_- (\Sigma h)_+ & (\Sigma h)_-^\dagger (\Sigma h)_+^\dagger (\Sigma h)_- (\Sigma h)_- \\ (\Sigma h)_-^\dagger (\Sigma h)_-^\dagger (\Sigma h)_+ (\Sigma h)_+ & (\Sigma h)_-^\dagger (\Sigma h)_-^\dagger (\Sigma h)_+ (\Sigma h)_- & (\Sigma h)_-^\dagger (\Sigma h)_-^\dagger (\Sigma h)_- (\Sigma h)_+ & (\Sigma h)_-^\dagger (\Sigma h)_-^\dagger (\Sigma h)_- (\Sigma h)_- \end{bmatrix}$$

$$= \begin{bmatrix} h_-^\dagger h_-^\dagger h_-h_- & h_-^\dagger h_-^\dagger h_-h_+ & h_-^\dagger h_-^\dagger h_+h_- & h_-^\dagger h_-^\dagger h_+h_+ \\ h_-^\dagger h_+^\dagger h_-h_- & h_-^\dagger h_+^\dagger h_-h_+ & h_-^\dagger h_+^\dagger h_+h_- & h_-^\dagger h_+^\dagger h_+h_+ \\ h_+^\dagger h_-^\dagger h_-h_- & h_+^\dagger h_-^\dagger h_-h_+ & h_+^\dagger h_-^\dagger h_+h_- & h_+^\dagger h_-^\dagger h_+h_+ \\ h_+^\dagger h_+^\dagger h_-h_- & h_+^\dagger h_+^\dagger h_-h_+ & h_+^\dagger h_+^\dagger h_+h_- & h_+^\dagger h_+^\dagger h_+h_+ \end{bmatrix}_{k,q,r,s} (\tau)$$

$$= \begin{bmatrix} h_-^\dagger h_-^\dagger h_-h_- & h_-^\dagger h_-^\dagger h_+h_- & h_-^\dagger h_-^\dagger h_-h_+ & h_-^\dagger h_-^\dagger h_+h_+ \\ h_+^\dagger h_-^\dagger h_-h_- & h_+^\dagger h_-^\dagger h_+h_- & h_+^\dagger h_-^\dagger h_-h_+ & h_+^\dagger h_-^\dagger h_+h_+ \\ h_-^\dagger h_+^\dagger h_-h_- & h_-^\dagger h_+^\dagger h_+h_- & h_-^\dagger h_+^\dagger h_-h_+ & h_-^\dagger h_+^\dagger h_+h_+ \\ h_+^\dagger h_+^\dagger h_-h_- & h_+^\dagger h_+^\dagger h_+h_- & h_+^\dagger h_+^\dagger h_-h_+ & h_+^\dagger h_+^\dagger h_+h_+ \end{bmatrix}_{q,k,s,r} (\tau)$$

$$= - \begin{bmatrix} h_-^\dagger h_-^\dagger h_-h_- & h_-^\dagger h_-^\dagger h_-h_+ & h_-^\dagger h_-^\dagger h_+h_- & h_-^\dagger h_-^\dagger h_+h_+ \\ h_+^\dagger h_-^\dagger h_-h_- & h_+^\dagger h_-^\dagger h_-h_+ & h_+^\dagger h_-^\dagger h_+h_- & h_+^\dagger h_-^\dagger h_+h_+ \\ h_-^\dagger h_+^\dagger h_-h_- & h_-^\dagger h_+^\dagger h_-h_+ & h_-^\dagger h_+^\dagger h_+h_- & h_-^\dagger h_+^\dagger h_+h_+ \\ h_+^\dagger h_+^\dagger h_-h_- & h_+^\dagger h_+^\dagger h_-h_+ & h_+^\dagger h_+^\dagger h_+h_- & h_+^\dagger h_+^\dagger h_+h_+ \end{bmatrix}_{q,k,r,s} (\tau)$$

$$= - \begin{bmatrix} h_-^\dagger h_-^\dagger h_-h_- & h_-^\dagger h_-^\dagger h_+h_- & h_-^\dagger h_-^\dagger h_-h_+ & h_-^\dagger h_-^\dagger h_+h_+ \\ h_-^\dagger h_+^\dagger h_-h_- & h_-^\dagger h_+^\dagger h_+h_- & h_-^\dagger h_+^\dagger h_-h_+ & h_-^\dagger h_+^\dagger h_+h_+ \\ h_+^\dagger h_-^\dagger h_-h_- & h_+^\dagger h_-^\dagger h_+h_- & h_+^\dagger h_-^\dagger h_-h_+ & h_+^\dagger h_-^\dagger h_+h_+ \\ h_+^\dagger h_+^\dagger h_-h_- & h_+^\dagger h_+^\dagger h_+h_- & h_+^\dagger h_+^\dagger h_-h_+ & h_+^\dagger h_+^\dagger h_+h_+ \end{bmatrix}_{k,q,s,r} (\tau)$$

$$= \begin{bmatrix} h_-h_-h_-^\dagger h_-^\dagger & h_-h_+h_-^\dagger h_-^\dagger & h_+h_-h_-^\dagger h_-^\dagger & h_+h_+h_-^\dagger h_-^\dagger \\ h_-h_-h_-^\dagger h_+^\dagger & h_-h_+h_-^\dagger h_+^\dagger & h_+h_-h_-^\dagger h_+^\dagger & h_+h_+h_-^\dagger h_+^\dagger \\ h_-h_-h_+^\dagger h_-^\dagger & h_-h_+h_+^\dagger h_-^\dagger & h_+h_-h_+^\dagger h_-^\dagger & h_+h_+h_+^\dagger h_-^\dagger \\ h_-h_-h_+^\dagger h_+^\dagger & h_-h_+h_+^\dagger h_+^\dagger & h_+h_-h_+^\dagger h_+^\dagger & h_+h_+h_+^\dagger h_+^\dagger \end{bmatrix}_{r,s,k,q} (\beta - \tau)$$

$$= \begin{bmatrix} h_-h_-h_-^\dagger h_-^\dagger & h_+h_-h_-^\dagger h_-^\dagger & h_-h_+h_-^\dagger h_-^\dagger & h_+h_+h_-^\dagger h_-^\dagger \\ h_-h_-h_+^\dagger h_-^\dagger & h_+h_-h_+^\dagger h_-^\dagger & h_-h_+h_+^\dagger h_-^\dagger & h_+h_+h_+^\dagger h_-^\dagger \\ h_-h_-h_-^\dagger h_+^\dagger & h_+h_-h_-^\dagger h_+^\dagger & h_-h_+h_-^\dagger h_+^\dagger & h_+h_+h_-^\dagger h_+^\dagger \\ h_-h_-h_+^\dagger h_+^\dagger & h_+h_-h_+^\dagger h_+^\dagger & h_-h_+h_+^\dagger h_+^\dagger & h_+h_+h_+^\dagger h_+^\dagger \end{bmatrix}_{s,r,q,k} (\beta - \tau)$$

$$= - \begin{bmatrix} h_-h_-h_-^\dagger h_-^\dagger & h_+h_-h_-^\dagger h_-^\dagger & h_-h_+h_-^\dagger h_-^\dagger & h_+h_+h_-^\dagger h_-^\dagger \\ h_-h_-h_-^\dagger h_+^\dagger & h_+h_-h_-^\dagger h_+^\dagger & h_-h_+h_-^\dagger h_+^\dagger & h_+h_+h_-^\dagger h_+^\dagger \\ h_-h_-h_+^\dagger h_-^\dagger & h_+h_-h_+^\dagger h_-^\dagger & h_-h_+h_+^\dagger h_-^\dagger & h_+h_+h_+^\dagger h_-^\dagger \\ h_-h_-h_+^\dagger h_+^\dagger & h_+h_-h_+^\dagger h_+^\dagger & h_-h_+h_+^\dagger h_+^\dagger & h_+h_+h_+^\dagger h_+^\dagger \end{bmatrix}_{s,r,k,q} (\beta - \tau)$$

$$= - \begin{bmatrix} h_-h_-h_-^\dagger h_-^\dagger & h_-h_+h_-^\dagger h_-^\dagger & h_+h_-h_-^\dagger h_-^\dagger & h_+h_+h_-^\dagger h_-^\dagger \\ h_-h_-h_+^\dagger h_-^\dagger & h_-h_+h_+^\dagger h_-^\dagger & h_+h_-h_+^\dagger h_-^\dagger & h_+h_+h_+^\dagger h_-^\dagger \\ h_-h_-h_-^\dagger h_+^\dagger & h_-h_+h_-^\dagger h_+^\dagger & h_+h_-h_-^\dagger h_+^\dagger & h_+h_+h_-^\dagger h_+^\dagger \\ h_-h_-h_+^\dagger h_+^\dagger & h_-h_+h_+^\dagger h_+^\dagger & h_+h_-h_+^\dagger h_+^\dagger & h_+h_+h_+^\dagger h_+^\dagger \end{bmatrix}_{r,s,q,k} (\beta - \tau)$$

$$C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) = \sigma_1 \cdot C_{Q=-2,S=1,S^3=+1;k,q,r,s}(\tau) \cdot \sigma_1$$
$$C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) = \sigma_5 \cdot \sigma_1 \cdot C_{Q=-2,S=1,S^3=+1;q,k,s,r}(\tau) \cdot \sigma_1 \cdot \sigma_5$$
$$C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) = -\sigma_5 \cdot \sigma_1 \cdot C_{Q=-2,S=1,S^3=+1;q,k,r,s}(\tau) \cdot \sigma_1$$
$$C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) = -\sigma_1 \cdot C_{Q=-2,S=1,S^3=+1;k,q,s,r}(\tau) \cdot \sigma_1 \cdot \sigma_5$$
$$C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) = \sigma_1 \cdot C_{Q=-2,S=1,S^3=-1;r,s,k,q}{}^\top(\beta-\tau) \cdot \sigma_1$$
$$C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) = \sigma_5 \cdot \sigma_1 \cdot C_{Q=-2,S=1,S^3=-1;s,r,q,k}{}^\top(\beta-\tau) \cdot \sigma_1 \cdot \sigma_5$$
$$C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) = -\sigma_1 \cdot C_{Q=-2,S=1,S^3=-1;s,r,k,q}{}^\top(\beta-\tau) \cdot \sigma_1 \cdot \sigma_5$$
$$C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) = -\sigma_5 \cdot \sigma_1 \cdot C_{Q=-2,S=1,S^3=-1;r,s,q,k}{}^\top(\beta-\tau) \cdot \sigma_1$$

C* : *H

$$\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s} &= \frac{1}{Z} tr\left[p_k p_q e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)}\right] = \\
&= \frac{1}{Z} tr\left[C^{-1}Cp_k C^{-1}Cp_q C^{-1}Ce^{-H\tau}C^{-1}Cp_r^\dagger C^{-1}Cp_s^\dagger C^{-1}Ce^{-H(\beta-\tau)}\right] = \\
&= \frac{1}{Z} tr\left[Cp_k C^{-1}Cp_q C^{-1}Ce^{-H\tau}C^{-1}Cp_r^\dagger C^{-1}Cp_s^\dagger C^{-1}Ce^{-H(\beta-\tau)}C^{-1}\right] = \\
&= \frac{1}{Z} tr\left[h_k h_q e^{-H\tau} h_r^\dagger h_s^\dagger e^{-H(\beta-\tau)}\right] \\
&= \frac{1}{Z} tr\left[h_q h_k - H\tau h_s^\dagger h_r^\dagger e^{-H(\beta-\tau)}\right] \\
&= -\frac{1}{Z} tr\left[h_q h_k e^{-H\tau} h_r^\dagger h_s^\dagger e^{-H(\beta-\tau)}\right] \\
&= -\frac{1}{Z} tr\left[h_k h_q e^{-H\tau} h_s^\dagger h_r^\dagger e^{-H(\beta-\tau)}\right] \\
&= \frac{1}{Z} tr\left[h_r^\dagger h_s^\dagger e^{-H(\beta-\tau)} h_k h_q e^{-H\tau}\right] \\
&= \frac{1}{Z} tr\left[h_s^\dagger h_r^\dagger e^{-H(\beta-\tau)} h_q h_k e^{-H\tau}\right] \\
&= -\frac{1}{Z} tr\left[h_s^\dagger h_r^\dagger e^{-H(\beta-\tau)} h_k h_q e^{-H\tau}\right] \\
&= -\frac{1}{Z} tr\left[h_r^\dagger h_s^\dagger e^{-H(\beta-\tau)} h_q h_k e^{-H\tau}\right]
\end{aligned}$$

$$
C_{Q=2,S=1,S^3=-1;k,q,r,s} = C^{-1} \begin{bmatrix} p_+p_+p^\dagger_+p^\dagger_+ & p_+p_+p^\dagger_+p^\dagger_- & p_+p_+p^\dagger_-p^\dagger_+ & p_+p_+p^\dagger_-p^\dagger_- \\ p_+p_-p^\dagger_+p^\dagger_+ & p_+p_-p^\dagger_+p^\dagger_- & p_+p_-p^\dagger_-p^\dagger_+ & p_+p_-p^\dagger_-p^\dagger_- \\ p_-p_+p^\dagger_+p^\dagger_+ & p_-p_+p^\dagger_+p^\dagger_- & p_-p_+p^\dagger_-p^\dagger_+ & p_-p_+p^\dagger_-p^\dagger_- \\ p_-p_-p^\dagger_+p^\dagger_+ & p_-p_-p^\dagger_+p^\dagger_- & p_-p_-p^\dagger_-p^\dagger_+ & p_-p_-p^\dagger_-p^\dagger_- \end{bmatrix}_{k,q,r,s} (\tau)C =
$$

$$
= \begin{bmatrix} h_+h_+h^\dagger_+h^\dagger_+ & h_+h_+h^\dagger_+h^\dagger_- & h_+h_+h^\dagger_-h^\dagger_+ & h_+h_+h^\dagger_-h^\dagger_- \\ h_+h_-h^\dagger_+h^\dagger_+ & h_+h_-h^\dagger_+h^\dagger_- & h_+h_-h^\dagger_-h^\dagger_+ & h_+h_-h^\dagger_-h^\dagger_- \\ h_-h_+h^\dagger_+h^\dagger_+ & h_-h_+h^\dagger_+h^\dagger_- & h_-h_+h^\dagger_-h^\dagger_+ & h_-h_+h^\dagger_-h^\dagger_- \\ h_-h_-h^\dagger_+h^\dagger_+ & h_-h_-h^\dagger_+h^\dagger_- & h_-h_-h^\dagger_-h^\dagger_+ & h_-h_-h^\dagger_-h^\dagger_- \end{bmatrix}_{k,q,r,s} (\tau)
$$

$$
= \begin{bmatrix} h_+h_+h^\dagger_+h^\dagger_+ & h_+h_+h^\dagger_-h^\dagger_+ & h_+h_+h^\dagger_+h^\dagger_- & h_+h_+h^\dagger_-h^\dagger_- \\ h_-h_+h^\dagger_+h^\dagger_+ & h_-h_+h^\dagger_-h^\dagger_+ & h_-h_+h^\dagger_+h^\dagger_- & h_-h_+h^\dagger_-h^\dagger_- \\ h_+h_-h^\dagger_+h^\dagger_+ & h_+h_-h^\dagger_-h^\dagger_+ & h_+h_-h^\dagger_+h^\dagger_- & h_+h_-h^\dagger_-h^\dagger_- \\ h_-h_-h^\dagger_+h^\dagger_+ & h_-h_-h^\dagger_-h^\dagger_+ & h_-h_-h^\dagger_+h^\dagger_- & h_-h_-h^\dagger_-h^\dagger_- \end{bmatrix}_{q,k,s,r} (\tau)
$$

$$
= - \begin{bmatrix} h_+h_+h^\dagger_+h^\dagger_+ & h_+h_+h^\dagger_+h^\dagger_- & h_+h_+h^\dagger_-h^\dagger_+ & h_+h_+h^\dagger_-h^\dagger_- \\ h_-h_+h^\dagger_+h^\dagger_+ & h_-h_+h^\dagger_+h^\dagger_- & h_-h_+h^\dagger_-h^\dagger_+ & h_-h_+h^\dagger_-h^\dagger_- \\ h_+h_-h^\dagger_+h^\dagger_+ & h_+h_-h^\dagger_+h^\dagger_- & h_+h_-h^\dagger_-h^\dagger_+ & h_+h_-h^\dagger_-h^\dagger_- \\ h_-h_-h^\dagger_+h^\dagger_+ & h_-h_-h^\dagger_+h^\dagger_- & h_-h_-h^\dagger_-h^\dagger_+ & h_-h_-h^\dagger_-h^\dagger_- \end{bmatrix}_{q,k,r,s} (\tau)
$$

$$
= - \begin{bmatrix} h^\dagger_+h^\dagger_+h_+h_+ & h^\dagger_+h^\dagger_-h_+h_+ & h^\dagger_-h^\dagger_+h_+h_+ & h^\dagger_-h^\dagger_-h_+h_+ \\ h^\dagger_+h^\dagger_+h_-h_+ & h^\dagger_+h^\dagger_-h_-h_+ & h^\dagger_-h^\dagger_+h_-h_+ & h^\dagger_-h^\dagger_-h_-h_+ \\ h^\dagger_+h^\dagger_+h_+h_- & h^\dagger_+h^\dagger_-h_+h_- & h^\dagger_-h^\dagger_+h_+h_- & h^\dagger_-h^\dagger_-h_+h_- \\ h^\dagger_+h^\dagger_+h_-h_- & h^\dagger_+h^\dagger_-h_-h_- & h^\dagger_-h^\dagger_+h_-h_- & h^\dagger_-h^\dagger_-h_-h_- \end{bmatrix}_{k,q,s,r} (\tau)
$$

$$
= \begin{bmatrix} h^\dagger_+h^\dagger_+h_+h_+ & h^\dagger_+h^\dagger_-h_+h_+ & h^\dagger_-h^\dagger_+h_+h_+ & h^\dagger_-h^\dagger_-h_+h_+ \\ h^\dagger_+h^\dagger_+h_+h_- & h^\dagger_+h^\dagger_-h_+h_- & h^\dagger_-h^\dagger_+h_+h_- & h^\dagger_-h^\dagger_-h_+h_- \\ h^\dagger_+h^\dagger_+h_-h_+ & h^\dagger_+h^\dagger_-h_-h_+ & h^\dagger_-h^\dagger_+h_-h_+ & h^\dagger_-h^\dagger_-h_-h_+ \\ h^\dagger_+h^\dagger_+h_-h_- & h^\dagger_+h^\dagger_-h_-h_- & h^\dagger_-h^\dagger_+h_-h_- & h^\dagger_-h^\dagger_-h_-h_- \end{bmatrix}_{r,s,k,q} (\beta - \tau)
$$

$$
= \begin{bmatrix} h^\dagger_+h^\dagger_+h_+h_+ & h^\dagger_-h^\dagger_+h_+h_+ & h^\dagger_+h^\dagger_-h_+h_+ & h^\dagger_-h^\dagger_-h_+h_+ \\ h^\dagger_+h^\dagger_+h_-h_+ & h^\dagger_-h^\dagger_+h_-h_+ & h^\dagger_+h^\dagger_-h_-h_+ & h^\dagger_-h^\dagger_-h_-h_+ \\ h^\dagger_+h^\dagger_+h_+h_- & h^\dagger_-h^\dagger_+h_+h_- & h^\dagger_+h^\dagger_-h_+h_- & h^\dagger_-h^\dagger_-h_+h_- \\ h^\dagger_+h^\dagger_+h_-h_- & h^\dagger_-h^\dagger_+h_-h_- & h^\dagger_+h^\dagger_-h_-h_- & h^\dagger_-h^\dagger_-h_-h_- \end{bmatrix}_{s,r,q,k} (\beta - \tau)
$$

$$
= - \begin{bmatrix} h^\dagger_+h^\dagger_+h_+h_+ & h^\dagger_-h^\dagger_+h_+h_+ & h^\dagger_+h^\dagger_-h_+h_+ & h^\dagger_-h^\dagger_-h_+h_+ \\ h^\dagger_+h^\dagger_+h_+h_- & h^\dagger_-h^\dagger_+h_+h_- & h^\dagger_+h^\dagger_-h_+h_- & h^\dagger_-h^\dagger_-h_+h_- \\ h^\dagger_+h^\dagger_+h_-h_+ & h^\dagger_-h^\dagger_+h_-h_+ & h^\dagger_+h^\dagger_-h_-h_+ & h^\dagger_-h^\dagger_-h_-h_+ \\ h^\dagger_+h^\dagger_+h_-h_- & h^\dagger_-h^\dagger_+h_-h_- & h^\dagger_+h^\dagger_-h_-h_- & h^\dagger_-h^\dagger_-h_-h_- \end{bmatrix}_{s,r,k,q} (\beta - \tau)
$$

$$
= - \begin{bmatrix} h^\dagger_+h^\dagger_+h_+h_+ & h^\dagger_+h^\dagger_-h_+h_+ & h^\dagger_-h^\dagger_+h_+h_+ & h^\dagger_-h^\dagger_-h_+h_+ \\ h^\dagger_+h^\dagger_+h_-h_+ & h^\dagger_+h^\dagger_-h_-h_+ & h^\dagger_-h^\dagger_+h_-h_+ & h^\dagger_-h^\dagger_-h_-h_+ \\ h^\dagger_+h^\dagger_+h_+h_- & h^\dagger_+h^\dagger_-h_+h_- & h^\dagger_-h^\dagger_+h_+h_- & h^\dagger_-h^\dagger_-h_+h_- \\ h^\dagger_+h^\dagger_+h_-h_- & h^\dagger_+h^\dagger_-h_-h_- & h^\dagger_-h^\dagger_+h_-h_- & h^\dagger_-h^\dagger_-h_-h_- \end{bmatrix}_{r,s,q,k} (\beta - \tau)
$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= C_{Q=-2,S=1,S^3=-1;k,q,r,s}(\tau)\\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_5 \cdot C_{Q=-2,S=1,S^3=-1;q,k,s,r}(\tau) \cdot \sigma_5\\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_5 \cdot C_{Q=-2,S=1,S^3=-1;q,k,r,s}(\tau)\\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -C_{Q=-2,S=1,S^3=-1;k,q,s,r}(\tau) \cdot \sigma_5\\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= C_{Q=-2,S=1,S^3=+1;r,s,k,q}{}^{\top}(\beta-\tau)\\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_5 \cdot C_{Q=-2,S=1,S^3=+1;s,r,q,k}{}^{\top}(\beta-\tau) \cdot \sigma_5\\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -C_{Q=-2,S=1,S^3=+1;s,r,k,q}{}^{\top}(\beta-\tau) \cdot \sigma_5\\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_5 \cdot C_{Q=-2,S=1,S^3=+1;r,s,q,k}{}^{\top}(\beta-\tau)
\end{aligned}
$$

XF* : *H

$$C_{Q=2,S=1,S^3=-1;k,q,r,s} = \frac{1}{Z} tr\left[p_k p_q e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)}\right] =$$

$$= \frac{1}{Z} tr\left[(XF)^{-1}(XF)p_k(XF)^{-1}(XF)p_q(XF)^{-1}(XF)e^{-H\tau}(XF)^{-1}(XF)p_r^\dagger(XF)^{-1}(XF)p_s^\dagger(XF)^{-1}(XF)e^{-H(\beta-}\right.$$

$$= \frac{1}{Z} tr\left[(XF)p_k(XF)^{-1}(XF)p_q(XF)^{-1}(XF)e^{-H\tau}(XF)^{-1}(XF)p_r^\dagger(XF)^{-1}(XF)p_s^\dagger(XF)^{-1}(XF)e^{-H(\beta-\tau)}(XF)\right.$$

$$= \frac{1}{Z} tr\left[(\Sigma p)_k^\dagger(\Sigma p)_q^\dagger e^{-H\tau}(\Sigma p)_r(\Sigma p)_s e^{-H(\beta-\tau)}\right]$$

$$= \frac{1}{Z} tr\left[(\Sigma p)_q^\dagger(\Sigma p)_k^\dagger e^{-H\tau}(\Sigma p)_s(\Sigma p)_r e^{-H(\beta-\tau)}\right]$$

$$= -\frac{1}{Z} tr\left[(\Sigma p)_q^\dagger(\Sigma p)_k^\dagger e^{-H\tau}(\Sigma p)_r(\Sigma p)_s e^{-H(\beta-\tau)}\right]$$

$$= -\frac{1}{Z} tr\left[(\Sigma p)_k^\dagger(\Sigma p)_q^\dagger e^{-H\tau}(\Sigma p)_s(\Sigma p)_r e^{-H(\beta-\tau)}\right]$$

$$= \frac{1}{Z} tr\left[(\Sigma p)_r(\Sigma p)_s e^{-H(\beta-\tau)}(\Sigma p)_k^\dagger(\Sigma p)_q^\dagger e^{-H\tau}\right]$$

$$= \frac{1}{Z} tr\left[(\Sigma p)_s(\Sigma p)_r e^{-H(\beta-\tau)}(\Sigma p)_q^\dagger(\Sigma p)_k^\dagger e^{-H\tau}\right]$$

$$= -\frac{1}{Z} tr\left[(\Sigma p)_s(\Sigma p)_r e^{-H(\beta-\tau)}(\Sigma p)_k^\dagger(\Sigma p)_q^\dagger e^{-H\tau}\right]$$

$$= -\frac{1}{Z} tr\left[(\Sigma p)_r(\Sigma p)_s e^{-H(\beta-\tau)}(\Sigma p)_q^\dagger(\Sigma p)_k^\dagger e^{-H\tau}\right]$$

$$C_{Q=2,S=1,S^3=-1;k,q,r,s} = (XF)^{-1} \begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau)(XF) =$$

$$= \begin{bmatrix} (\Sigma p)_+^\dagger (\Sigma p)_+^\dagger (\Sigma p)_+ (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma p)_+^\dagger (\Sigma p)_+ (\Sigma p)_- & (\Sigma p)_+^\dagger (\Sigma p)_+^\dagger (\Sigma p)_- (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma p)_+^\dagger (\Sigma p)_- (\Sigma p)_- \\ (\Sigma p)_+^\dagger (\Sigma p)_-^\dagger (\Sigma p)_+ (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma p)_-^\dagger (\Sigma p)_+ (\Sigma p)_- & (\Sigma p)_+^\dagger (\Sigma p)_-^\dagger (\Sigma p)_- (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma p)_-^\dagger (\Sigma p)_- (\Sigma p)_- \\ (\Sigma p)_-^\dagger (\Sigma p)_+^\dagger (\Sigma p)_+ (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma p)_+^\dagger (\Sigma p)_+ (\Sigma p)_- & (\Sigma p)_-^\dagger (\Sigma p)_+^\dagger (\Sigma p)_- (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma p)_+^\dagger (\Sigma p)_- (\Sigma p)_- \\ (\Sigma p)_-^\dagger (\Sigma p)_-^\dagger (\Sigma p)_+ (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma p)_-^\dagger (\Sigma p)_+ (\Sigma p)_- & (\Sigma p)_-^\dagger (\Sigma p)_-^\dagger (\Sigma p)_- (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma p)_-^\dagger (\Sigma p)_- (\Sigma p)_- \end{bmatrix}$$

$$= \begin{bmatrix} p_-^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_+p_+ \\ p_-^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_+p_+ \\ p_+^\dagger p_-^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_+^\dagger p_+p_+ \end{bmatrix}_{k,q,r,s} (\tau)$$

$$= \begin{bmatrix} p_-^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_-^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_+p_+ \\ p_-^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_+^\dagger p_+p_+ \end{bmatrix}_{q,k,s,r} (\tau)$$

$$= - \begin{bmatrix} p_-^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_-^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_+p_+ \\ p_-^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_+^\dagger p_+p_+ \end{bmatrix}_{q,k,r,s} (\tau)$$

$$= - \begin{bmatrix} p_-^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_+p_+ \\ p_-^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_+p_+ \\ p_+^\dagger p_-^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_+^\dagger p_+p_+ \end{bmatrix}_{k,q,s,r} (\tau)$$

$$= \begin{bmatrix} p_-p_-p_-^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_+^\dagger \\ p_-p_-p_+^\dagger p_-^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_+p_+^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_+^\dagger \end{bmatrix}_{r,s,k,q} (\beta - \tau)$$

$$= \begin{bmatrix} p_-p_-p_-^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_-^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_+p_+p_+^\dagger p_-^\dagger \\ p_-p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_+^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_+^\dagger \end{bmatrix}_{s,r,q,k} (\beta - \tau)$$

$$= - \begin{bmatrix} p_-p_-p_-^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_+^\dagger \\ p_-p_-p_+^\dagger p_-^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_+p_+p_+^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_+^\dagger \end{bmatrix}_{s,r,k,q} (\beta - \tau)$$

$$= - \begin{bmatrix} p_-p_-p_-^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_-^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_+p_+^\dagger p_-^\dagger \\ p_-p_-p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_+^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_+^\dagger \end{bmatrix}_{r,s,q,k} (\beta - \tau)$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_1 \cdot C_{Q=2,S=1,S^3=+1;k,q,r,s}(\tau) \cdot \sigma_1 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_5 \cdot \sigma_1 \cdot C_{Q=2,S=1,S^3=+1;q,k,s,r}(\tau) \cdot \sigma_1 \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_5 \cdot \sigma_1 \cdot C_{Q=2,S=1,S^3=+1;q,k,r,s}(\tau) \cdot \sigma_1 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_1 \cdot C_{Q=2,S=1,S^3=+1;k,q,s,r}(\tau) \cdot \sigma_1 \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_1 \cdot C_{Q=2,S=1,S^3=-1;r,s,k,q}{}^{\top}(\beta-\tau) \cdot \sigma_1 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_5 \cdot \sigma_1 \cdot C_{Q=2,S=1,S^3=-1;s,r,q,k}{}^{\top}(\beta-\tau) \cdot \sigma_1 \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_1 \cdot C_{Q=2,S=1,S^3=-1;s,r,k,q}{}^{\top}(\beta-\tau) \cdot \sigma_1 \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_5 \cdot \sigma_1 \cdot C_{Q=2,S=1,S^3=-1;r,s,q,k}{}^{\top}(\beta-\tau) \cdot \sigma_1
\end{aligned}
$$

The table below shows the summary of all symmetries that we derived. From all these symmetries, with our data we can use only the anti-commutation and trace cyclicity, and if we calculate at half-filling, we could use also the XF symmetry. In the worst case, we can make our statistics 8 times better and 16 times better if we work at $\mu = 0$.

	Q	S	S^3	trafo	$kqrs$	τ
Start with	2	1	-1	1	1234	τ
Anti-commutation	2	1	-1	$\sigma_5 \cdot \sigma_5$	2143	τ
	2	1	-1	$-\sigma_5 \cdot$	2134	τ
	2	1	-1	$- \cdot \sigma_5$	1243	τ
trace cycl	2	1	+1	$\top$	3412	$\beta - \tau$
	2	1	+1	$\sigma_5 \top \sigma_5$	4321	$\beta - \tau$
	2	1	+1	$-\top \sigma_5$	4312	$\beta - \tau$
	2	1	+1	$-\sigma_5 \top$	3421	$\beta - \tau$
I	-2	1	+1	$\sigma_1 \cdot \sigma_1$	1234	τ
	-2	1	+1	$\sigma_5\sigma_1 \cdot \sigma_1\sigma_5$	2143	τ
	-2	1	+1	$-\sigma_5\sigma_1 \cdot \sigma_1$	2134	τ
	-2	1	+1	$-\sigma_1 \cdot \sigma_1\sigma_5$	1243	τ
	-2	1	-1	$\sigma_1 \top \sigma_1$	3412	$\beta - \tau$
	-2	1	-1	$\sigma_5\sigma_1 \top \sigma_1\sigma_5$	4321	$\beta - \tau$
	-2	1	-1	$-\sigma_1 \top \sigma_1\sigma_5$	4312	$\beta - \tau$
	-2	1	-1	$-\sigma_5\sigma_1 \top \sigma_1$	3421	$\beta - \tau$
C	-2	1	-1	1	1234	τ
	-2	1	-1	$\sigma_5 \cdot \sigma_5$	2143	τ
	-2	1	-1	$-\sigma_5 \cdot$	2134	τ
	-2	1	-1	$- \cdot \sigma_5$	1243	τ
	-2	1	+1	$\top$	3412	$\beta - \tau$
	-2	1	+1	$\sigma_5 \top \sigma_5$	4321	$\beta - \tau$
	-2	1	+1	$-\top \sigma_5$	4312	$\beta - \tau$
	-2	1	+1	$-\sigma_5 \top$	3421	$\beta - \tau$
XF	2	1	+1	$\sigma_1 \cdot \sigma_1$	1234	τ
	2	1	+1	$\sigma_5\sigma_1 \cdot \sigma_1\sigma_5$	2143	τ
	2	1	+1	$-\sigma_5\sigma_1 \cdot \sigma_1$	2134	τ
	2	1	+1	$-\sigma_1 \cdot \sigma_1\sigma_5$	1243	τ
	2	1	-1	$\sigma_1 \top \sigma_1$	3412	$\beta - \tau$
	2	1	-1	$\sigma_5\sigma_1 \top \sigma_1\sigma_5$	4321	$\beta - \tau$
	2	1	-1	$-\sigma_1 \top \sigma_1\sigma_5$	4312	$\beta - \tau$
	2	1	-1	$-\sigma_5\sigma_1 \top \sigma_1$	3421	$\beta - \tau$

CHAPTER 4

Spectre of Excitons

4.1 Data Generating

4.2 Fitting Data

APPENDIX A

Useful information

In the appendix you usually include extra information that should be documented in your thesis, but not interrupt the flow.

The LaTeX WikiBook [12] is a useful source of information on LaTeX.

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