Title of the Thesis

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I hereby declare that this thesis was formulated by myself and that no sources or tools other than those cited were used.

Bonn, ...

Date Signature

1. Gutachter: Prof. Dr. John Smith
2. Gutachterin: Prof. Dr. Anne Jones

Acknowledgements

I would like to thank ...

You should probably use \chapter* for acknowledgements at the beginning of a thesis and \chapter for the end.

Contents

CHAPTER 1

Introduction

The introduction usually gives a few pages of introduction to the whole subject, maybe even starting with the Greeks.

For more information on LaTeX and the packages that are available see for example the books of Kopka [1] and Goossens et al [2].

A lot of useful information on particle physics can be found in the “Particle Data Book” [3].

I have resisted the temptation to put a lot of definitions into the file `thesis_defs.sty`, as everyone has their own taste as to what scheme they want to use for names. However, a few examples are included to help you get started:

- cross-sections are measured in pb and integrated luminosity in pb^{-1};
- the K_S^0 is an interesting particle;
- the missing transverse momentum, $p_{\mathrm{T}}^{\mathrm{miss}}$, is often called missing transverse energy, even though it is calculated using a vector sum.

Note that the examples of units assume that you are using the siunitx package.

It also is probably a good idea to include a few well formatted references in the thesis skeleton. More detailed suggestions on what citation types to use can be found in the “Thesis Guide” [4]:

- articles in refereed journals [3, 5];
- a book [6];
- a PhD thesis [7] and a Diplom thesis [8];
- a collection of articles [9];
- a conference note [10];
- a preprint [11] (you can also use `@online` or `@booklet` for such things);
- something that is only available online [4].

At the end of the introduction it is normal to say briefly what comes in the following chapters.

The line at the beginning of this file is used by TeXstudio etc. to specify which is the master LaTeX file, so that you can compile your thesis directly from this file. If your thesis is called something other than `mythesis`, adjust it as appropriate.

CHAPTER 2

Symmetries

2.1 Correlation functions

2.1.1 One-particle correlation function

We start with correlator $\langle phh^\dagger p^\dagger \rangle$

$$C_{Q=0,S=1,S^3=+1;k,q,r,s} = \begin{bmatrix} p_+h_+h_+^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_-^\dagger \\ p_+h_-h_+^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_-^\dagger \\ p_-h_+h_+^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_-^\dagger \\ p_-h_-h_+^\dagger p_+^\dagger & p_-h_-h_+^\dagger p_-^\dagger & p_-h_-h_-^\dagger p_+^\dagger & p_-h_-h_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau)$$

Now, we apply different symmetry operator to the two particle correlation function. When we apply them, we first start with the thermal trace and then switch to matrix form, so that it is easier to see the transformations needed in order to get another two particle correlator.

We obviously start with the trace cyclicity.

$$C_{Q=0,S=1,S^3=+1;k,q,r,s} = \frac{1}{Z} tr \left[p_k h_q e^{-H\tau} h_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} \right] = \frac{1}{Z} tr \left[h_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} p_k h_q e^{-H\tau} \right]$$

Now we apply time reverse

$$\beta \rightarrow \beta - \tau$$

and use the anti-commutation relations of the operators to get

$$\frac{1}{Z} tr \left[h_r^\dagger p_s^\dagger e^{-H(\beta-(\beta-\tau))} p_k h_q e^{-H(\beta-\tau)} \right] = \frac{1}{Z} tr \left[p_s^\dagger h_r^\dagger e^{-H\tau} h_q p_k e^{-H(\beta-\tau)} \right] \tag{2.1}$$

Now we switch to matrix form to see better what happens.

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=+1;k,q,r,s} \\
&= \begin{bmatrix} p_+h_+h_+^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_-^\dagger \\ p_+h_-h_+^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_-^\dagger \\ p_-h_+h_+^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_-^\dagger \\ p_-h_-h_+^\dagger p_+^\dagger & p_-h_-h_+^\dagger p_-^\dagger & p_-h_-h_-^\dagger p_+^\dagger & p_-h_-h_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau) \\
&= \begin{bmatrix} h_+^\dagger p_+^\dagger p_+h_+ & h_+^\dagger p_-^\dagger p_+h_+ & h_-^\dagger p_+^\dagger p_+h_+ & h_-^\dagger p_-^\dagger p_+h_+ \\ h_+^\dagger p_+^\dagger p_+h_- & h_+^\dagger p_-^\dagger p_+h_- & h_-^\dagger p_+^\dagger p_+h_- & h_-^\dagger p_-^\dagger p_+h_- \\ h_+^\dagger p_+^\dagger p_-h_+ & h_+^\dagger p_-^\dagger p_-h_+ & h_-^\dagger p_+^\dagger p_-h_+ & h_-^\dagger p_-^\dagger p_-h_+ \\ h_+^\dagger p_+^\dagger p_-h_- & h_+^\dagger p_-^\dagger p_-h_- & h_-^\dagger p_+^\dagger p_-h_- & h_-^\dagger p_-^\dagger p_-h_- \end{bmatrix}_{r,s,k,q} (\beta-\tau)
\end{aligned}
$$

$$
= \begin{bmatrix} p_+^\dagger h_+^\dagger h_+p_+ & p_-^\dagger h_+^\dagger h_+p_+ & p_+^\dagger h_-^\dagger h_+p_+ & p_-^\dagger h_-^\dagger h_+p_+ \\ p_+^\dagger h_+^\dagger h_-p_+ & p_-^\dagger h_+^\dagger h_-p_+ & p_+^\dagger h_-^\dagger h_-p_+ & p_-^\dagger h_-^\dagger h_-p_+ \\ p_+^\dagger h_+^\dagger h_+p_- & p_-^\dagger h_+^\dagger h_+p_- & p_+^\dagger h_-^\dagger h_+p_- & p_-^\dagger h_-^\dagger h_+p_- \\ p_+^\dagger h_+^\dagger h_-p_- & p_-^\dagger h_+^\dagger h_-p_- & p_+^\dagger h_-^\dagger h_-p_- & p_-^\dagger h_-^\dagger h_-p_- \end{bmatrix}_{s,r,q,k} (\beta-\tau) = C_{Q=0,S=1,S^3=-1;s,r,q,k}
$$

We can notice that order reverse is actually a composed of two transformations – particle-hole momentum switch and time reverse. After we stare at the matrix for an hour, we can figure out what the transformation between the correlation functions is.

$$
C_{Q=0,S=1,S^3=+1;k,q,r,s}(\tau) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} C_{Q=0,S=1,S^3=-1;s,r,q,k}{}^\top(\beta-\tau) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

We repeat the steps again for each symmetry operator. First we start with the thermal trace and then switch to matrix for easy transformations.

$I : H - \vec{\mu} \cdot \vec{q}$

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=+1;k,q,r,s} \\
&= \frac{1}{Z} tr \left[p_k h_q e^{-H\tau} h_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} \right] \\
&= \frac{1}{Z} tr \left[I^{-1} I p_k I^{-1} I h_q I^{-1} I e^{-H\tau} I^{-1} I h_r^\dagger I^{-1} I p_s^\dagger I^{-1} I e^{-H(\beta-\tau)} \right] \\
&= \frac{1}{Z} tr \left[I p_k I^{-1} I h_q I^{-1} I e^{-H\tau} I^{-1} I h_r^\dagger I^{-1} I p_s^\dagger I^{-1} I e^{-H(\beta-\tau) I^{-1}} \right] \\
&= \frac{1}{Z} tr \left[(\Sigma h)_k^\dagger (\Sigma p)_q^\dagger e^{-H\tau} (\Sigma p)_r (\Sigma h)_s e^{-H(\beta-\tau)} \right] \\
&= \frac{1}{Z} tr \left[(\Sigma p)_q^\dagger (\Sigma h)_k^\dagger e^{-H\tau} (\Sigma h)_s (\Sigma p)_r e^{-H(\beta-\tau)} \right]
\end{aligned}
$$

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=+1;k,q,r,s} \\
&= I^{-1} \begin{bmatrix} p_+h_+h_+^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_-^\dagger \\ p_+h_-h_+^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_-^\dagger \\ p_-h_+h_+^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_-^\dagger \\ p_-h_-h_+^\dagger p_+^\dagger & p_-h_-h_+^\dagger p_-^\dagger & p_-h_-h_-^\dagger p_+^\dagger & p_-h_-h_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau)\ I \\
&= \begin{bmatrix} (\Sigma h)_+^\dagger (\Sigma p)_+^\dagger (\Sigma p)_+ (\Sigma h)_+ & (\Sigma h)_+^\dagger (\Sigma p)_+^\dagger (\Sigma p)_+ (\Sigma h)_- & (\Sigma h)_+^\dagger (\Sigma p)_+^\dagger (\Sigma p)_- (\Sigma h)_+ & (\Sigma h)_+^\dagger (\Sigma p)_+^\dagger (\Sigma p)_- (\Sigma h)_- \\ (\Sigma h)_+^\dagger (\Sigma p)_-^\dagger (\Sigma p)_+ (\Sigma h)_+ & (\Sigma h)_+^\dagger (\Sigma p)_-^\dagger (\Sigma p)_+ (\Sigma h)_- & (\Sigma h)_+^\dagger (\Sigma p)_-^\dagger (\Sigma p)_- (\Sigma h)_+ & (\Sigma h)_+^\dagger (\Sigma p)_-^\dagger (\Sigma p)_- (\Sigma h)_- \\ (\Sigma h)_-^\dagger (\Sigma p)_+^\dagger (\Sigma p)_+ (\Sigma h)_+ & (\Sigma h)_-^\dagger (\Sigma p)_+^\dagger (\Sigma p)_+ (\Sigma h)_- & (\Sigma h)_-^\dagger (\Sigma p)_+^\dagger (\Sigma p)_- (\Sigma h)_+ & (\Sigma h)_-^\dagger (\Sigma p)_+^\dagger (\Sigma p)_- (\Sigma h)_- \\ (\Sigma h)_-^\dagger (\Sigma p)_-^\dagger (\Sigma p)_+ (\Sigma h)_+ & (\Sigma h)_-^\dagger (\Sigma p)_-^\dagger (\Sigma p)_+ (\Sigma h)_- & (\Sigma h)_-^\dagger (\Sigma p)_-^\dagger (\Sigma p)_- (\Sigma h)_+ & (\Sigma h)_-^\dagger (\Sigma p)_-^\dagger (\Sigma p)_- (\Sigma h)_- \end{bmatrix} \\
&= \begin{bmatrix} h_-^\dagger p_-^\dagger p_-h_- & h_-^\dagger p_-^\dagger p_-h_+ & h_-^\dagger p_-^\dagger p_+h_- & h_-^\dagger p_-^\dagger p_+h_+ \\ h_-^\dagger p_+^\dagger p_-h_- & h_-^\dagger p_+^\dagger p_-h_+ & h_-^\dagger p_+^\dagger p_+h_- & h_-^\dagger p_+^\dagger p_+h_+ \\ h_+^\dagger p_-^\dagger p_-h_- & h_+^\dagger p_-^\dagger p_-h_+ & h_+^\dagger p_-^\dagger p_+h_- & h_+^\dagger p_-^\dagger p_+h_+ \\ h_+^\dagger p_+^\dagger p_-h_- & h_+^\dagger p_+^\dagger p_-h_+ & h_+^\dagger p_+^\dagger p_+h_- & h_+^\dagger p_+^\dagger p_+h_+ \end{bmatrix}_{k,q,r,s} (\tau)
\end{aligned}
$$

$$
\begin{aligned}
&= \begin{bmatrix} p_-h_-h_-^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_-^\dagger \\ p_-h_-h_-^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_+^\dagger \\ p_-h_-h_+^\dagger p_-^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_+h_+h_+^\dagger p_-^\dagger \\ p_-h_-h_+^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_+^\dagger \end{bmatrix}_{r,s,k,q} (\beta - \tau) \\
&= \begin{bmatrix} p_-^\dagger h_-^\dagger h_-p_- & p_-^\dagger h_-^\dagger h_+p_- & p_-^\dagger h_-^\dagger h_-p_+ & p_-^\dagger h_-^\dagger h_+p_+ \\ p_+^\dagger h_-^\dagger h_-p_- & p_+^\dagger h_-^\dagger h_+p_- & p_+^\dagger h_-^\dagger h_-p_+ & p_+^\dagger h_-^\dagger h_+p_+ \\ p_-^\dagger h_+^\dagger h_-p_- & p_-^\dagger h_+^\dagger h_+p_- & p_-^\dagger h_+^\dagger h_-p_+ & p_-^\dagger h_+^\dagger h_+p_+ \\ p_+^\dagger h_+^\dagger h_-p_- & p_+^\dagger h_+^\dagger h_+p_- & p_+^\dagger h_+^\dagger h_-p_+ & p_+^\dagger h_+^\dagger h_+p_+ \end{bmatrix}_{q,k,s,r} (\tau)
\end{aligned}
$$

Depending on what transformation we use to get back to $\langle p h h^\dagger p^\dagger \rangle$, we get two transformations which are time reverse to one another. We notice the same behaviour for all operators.

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=+1;k,q,r,s}(\tau) \\
&\qquad = \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} {C_{Q=0,S=1,S^3=+1;r,s,k,q}}^\top(\beta-\tau) \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=+1;k,q,r,s}(\tau) \\
&\qquad = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} C_{Q=0,S=1,S^3=-1;q,k,s,r}(\tau) \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

$C : H$

$$
\begin{aligned}
C&_{Q=0,S=1,S^3=+1;k,q,r,s}\\
&=\frac{1}{Z}tr\left[p_k h_q e^{-H\tau} h_r^\dagger p_s^\dagger e^{-H(\beta-\tau)}\right]\\
&=\frac{1}{Z}tr\left[C^{-1}Cp_kC^{-1}Ch_qC^{-1}Ce^{-H\tau}C^{-1}Ch_r^\dagger C^{-1}Cp_s^\dagger C^{-1}Ce^{-H(\beta-\tau)}\right]\\
&=\frac{1}{Z}tr\left[h_k p_q e^{-H\tau} p_r^\dagger h_s^\dagger e^{-H(\beta-\tau)}\right]\\
&=\frac{1}{Z}tr\left[p_q h_k e^{-H\tau} h_s^\dagger p_r^\dagger e^{-H(\beta-\tau)}\right]
\end{aligned}
$$

$$
\begin{aligned}
C&_{Q=0,S=1,S^3=+1;k,q,r,s}\\
&= C^{-1}\begin{bmatrix}
p_+h_+h_+^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_-^\dagger\\
p_+h_-h_+^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_-^\dagger\\
p_-h_+h_+^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_-^\dagger\\
p_-h_-h_+^\dagger p_+^\dagger & p_-h_-h_+^\dagger p_-^\dagger & p_-h_-h_-^\dagger p_+^\dagger & p_-h_-h_-^\dagger p_-^\dagger
\end{bmatrix}_{k,q,r,s} (\tau)\, C
\end{aligned}
$$

$$
= \begin{bmatrix}
h_+p_+p_+^\dagger h_+^\dagger & h_+p_+p_+^\dagger h_-^\dagger & h_+p_+p_-^\dagger h_+^\dagger & h_+p_+p_-^\dagger h_-^\dagger\\
h_+p_-p_+^\dagger h_+^\dagger & h_+p_-p_+^\dagger h_-^\dagger & h_+p_-p_-^\dagger h_+^\dagger & h_+p_-p_-^\dagger h_-^\dagger\\
h_-p_+p_+^\dagger h_+^\dagger & h_-p_+p_+^\dagger h_-^\dagger & h_-p_+p_-^\dagger h_+^\dagger & h_-p_+p_-^\dagger h_-^\dagger\\
h_-p_-p_+^\dagger h_+^\dagger & h_-p_-p_+^\dagger h_-^\dagger & h_-p_-p_-^\dagger h_+^\dagger & h_-p_-p_-^\dagger h_-^\dagger
\end{bmatrix}_{k,q,r,s} (\tau)
$$

$$
= \begin{bmatrix}
p_+^\dagger h_+^\dagger h_+p_+ & p_+^\dagger h_-^\dagger h_+p_+ & p_-^\dagger h_+^\dagger h_+p_+ & p_-^\dagger h_-^\dagger h_+p_+\\
p_+^\dagger h_+^\dagger h_+p_- & p_+^\dagger h_-^\dagger h_+p_- & p_-^\dagger h_+^\dagger h_+p_- & p_-^\dagger h_-^\dagger h_+p_-\\
p_+^\dagger h_+^\dagger h_-p_+ & p_+^\dagger h_-^\dagger h_-p_+ & p_-^\dagger h_+^\dagger h_-p_+ & p_-^\dagger h_-^\dagger h_-p_+\\
p_+^\dagger h_+^\dagger h_-p_- & p_+^\dagger h_-^\dagger h_-p_- & p_-^\dagger h_+^\dagger h_-p_- & p_-^\dagger h_-^\dagger h_-p_-
\end{bmatrix}_{r,s,k,q} (\beta-\tau)
$$

$$
= \begin{bmatrix}
p_+h_+h_+^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_-^\dagger\\
p_-h_+h_+^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_-^\dagger\\
p_+h_-h_+^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_-^\dagger\\
p_-h_-h_+^\dagger p_+^\dagger & p_-h_-h_-^\dagger p_+^\dagger & p_-h_-h_+^\dagger p_-^\dagger & p_-h_-h_-^\dagger p_-^\dagger
\end{bmatrix}_{q,k,s,r} (\tau)
$$

$$
C_{Q=0,S=1,S^3=+1;k,q,r,s}(\tau) = C_{Q=0,S=1,S^3=-1;r,s,k,q}{}^{\top}(\beta-\tau)
$$

$$
C_{Q=0,S=1,S^3=+1;k,q,r,s}(\tau) = \begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix} C_{Q=0,S=1,S^3=+1;q,k,s,r}(\tau) \begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$XF : H$

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=+1;k,q,r,s} \\
&= \frac{1}{Z} tr \left[p_k h_q e^{-H\tau} h_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} \right] \\
&= \frac{1}{Z} tr \left[(XF)^{-1}(XF) p_k (XF)^{-1}(XF) h_q (XF)^{-1}(XF) e^{-H\tau} (XF)^{-1}(XF) h_r^\dagger (XF)^{-1}(XF) p_s^\dagger (XF)^{-1}(XF) e^{-H(\beta-\tau)} \right] \\
&= \frac{1}{Z} tr \left[(\Sigma p)_k^\dagger (\Sigma h)_q^\dagger e^{-H\tau} (\Sigma h)_r (\Sigma p)_s e^{-H(\beta-\tau)} \right]
\end{aligned}
$$

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=+1;k,q,r,s} \\
&= (XF)^{-1} \begin{bmatrix} p_+h_+h_+^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_-^\dagger \\ p_+h_-h_+^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_-^\dagger \\ p_-h_+h_+^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_-^\dagger \\ p_-h_-h_+^\dagger p_+^\dagger & p_-h_-h_+^\dagger p_-^\dagger & p_-h_-h_-^\dagger p_+^\dagger & p_-h_-h_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau)\ (XF) \\
&= \begin{bmatrix} (\Sigma p)_+^\dagger (\Sigma h)_+^\dagger (\Sigma h)_+ (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma h)_+^\dagger (\Sigma h)_+ (\Sigma p)_- & (\Sigma p)_+^\dagger (\Sigma h)_+^\dagger (\Sigma h)_- (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma h)_+^\dagger (\Sigma h)_- (\Sigma p)_- \\ (\Sigma p)_+^\dagger (\Sigma h)_-^\dagger (\Sigma h)_+ (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma h)_-^\dagger (\Sigma h)_+ (\Sigma p)_- & (\Sigma p)_+^\dagger (\Sigma h)_-^\dagger (\Sigma h)_- (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma h)_-^\dagger (\Sigma h)_- (\Sigma p)_- \\ (\Sigma p)_-^\dagger (\Sigma h)_+^\dagger (\Sigma h)_+ (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma h)_+^\dagger (\Sigma h)_+ (\Sigma p)_- & (\Sigma p)_-^\dagger (\Sigma h)_+^\dagger (\Sigma h)_- (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma h)_+^\dagger (\Sigma h)_- (\Sigma p)_- \\ (\Sigma p)_-^\dagger (\Sigma h)_-^\dagger (\Sigma h)_+ (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma h)_-^\dagger (\Sigma h)_+ (\Sigma p)_- & (\Sigma p)_-^\dagger (\Sigma h)_-^\dagger (\Sigma h)_- (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma h)_-^\dagger (\Sigma h)_- (\Sigma p)_- \end{bmatrix} \\
&= \begin{bmatrix} p_-^\dagger h_-^\dagger h_- p_- & p_-^\dagger h_-^\dagger h_- p_+ & p_-^\dagger h_-^\dagger h_+ p_- & p_-^\dagger h_-^\dagger h_+ p_+ \\ p_-^\dagger h_+^\dagger h_- p_- & p_-^\dagger h_+^\dagger h_- p_+ & p_-^\dagger h_+^\dagger h_+ p_- & p_-^\dagger h_+^\dagger h_+ p_+ \\ p_+^\dagger h_-^\dagger h_- p_- & p_+^\dagger h_-^\dagger h_- p_+ & p_+^\dagger h_-^\dagger h_+ p_- & p_+^\dagger h_-^\dagger h_+ p_+ \\ p_+^\dagger h_+^\dagger h_- p_- & p_+^\dagger h_+^\dagger h_- p_+ & p_+^\dagger h_+^\dagger h_+ p_- & p_+^\dagger h_+^\dagger h_+ p_+ \end{bmatrix}_{k,q,r,s} (\tau)
\end{aligned}
$$

$$
= \begin{bmatrix} p_-h_-h_-^\dagger p_-^\dagger & p_+h_-h_-^\dagger p_-^\dagger & p_-h_+h_-^\dagger p_-^\dagger & p_+h_+h_-^\dagger p_-^\dagger \\ p_-h_-h_+^\dagger p_-^\dagger & p_+h_-h_+^\dagger p_-^\dagger & p_-h_+h_+^\dagger p_-^\dagger & p_+h_+h_+^\dagger p_-^\dagger \\ p_-h_-h_-^\dagger p_+^\dagger & p_+h_-h_-^\dagger p_+^\dagger & p_-h_+h_-^\dagger p_+^\dagger & p_+h_+h_-^\dagger p_+^\dagger \\ p_-h_-h_+^\dagger p_+^\dagger & p_+h_-h_+^\dagger p_+^\dagger & p_-h_+h_+^\dagger p_+^\dagger & p_+h_+h_+^\dagger p_+^\dagger \end{bmatrix}_{s,r,k,q} (\beta-\tau)
$$

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=+1;k,q,r,s}(\tau) \\
&\qquad = \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} C_{Q=0,S=1,S^3=-1;k,q,r,s}(\tau) \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&C_{Q=0,S=1,S^3=+1;k,q,r,s}(\tau) \\
&\qquad = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} C_{Q=0,S=1,S^3=+1;s,r,q,k}{}^\top(\beta-\tau) \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

The table below shows the summary of all symmetries that we calculated.
For more compact form, we write

	Q	S	S^3	trafo	$kqrs$	τ
Start with	0	1	+1	1	1234	τ
trace cycl	0	1	-1	$\sigma_5 \top \sigma_5$	4321	$\beta - \tau$
I	0	1	+1	$\sigma_1 \top \sigma_1$	3412	$\beta - \tau$
	0	1	-1	$\sigma_5 \sigma_1 \cdot \sigma_1 \sigma_5$	2143	τ
C	0	1	-1	$\top$	3412	$\beta - \tau$
	0	1	+1	$\sigma_5 \cdot \sigma_5$	2143	τ
XF	0	1	-1	$\sigma_1 \cdot \sigma_1$	1234	τ
	0	1	+1	$\sigma_5 \sigma_1 \top \sigma_1 \sigma_5$	4321	$\beta - \tau$

$$\sigma_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2.1.2 Two-particle correlation function

We start with correlator $\langle p p p^\dagger p^\dagger \rangle$. In trace and matrix forms it look like this.

$$C_{Q=2,S=1,S^3=-1;k,q,r,s} = \frac{1}{Z} tr \left[p_k p_q e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} \right]$$

$$C_{Q=2,S=1,S^3=-1;k,q,r,s} = \begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau)$$

First, we implement the anti-commutation relations which will give us the identities of this correlation funciton. Again, we use derive the the results through traces and then continue with the matrix form which will give us a better insight on the transformation of all labels that are included.

$$\begin{aligned} C_{Q=2,S=1,S^3=-1;k,q,r,s} &= \frac{1}{Z} tr \left[p_k p_q e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} \right] = \\ &= \frac{1}{Z} tr \left[p_q p_k e^{-H\tau} p_s^\dagger p_r^\dagger e^{-H(\beta-\tau)} \right] \\ &= -\frac{1}{Z} tr \left[p_q p_k e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} \right] \\ &= -\frac{1}{Z} tr \left[p_k p_q e^{-H\tau} p_s^\dagger p_r^\dagger e^{-H(\beta-\tau)} \right] \end{aligned}$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s} &= \begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau) \\
&= \begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{q,k,s,r} (\tau) \\
&= -\begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{q,k,r,s} (\tau) \\
&= -\begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,s,r} (\tau)
\end{aligned}
$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_5 \cdot C_{Q=2,S=1,S^3=-1;q,k,s,r}(\tau) \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_5 \cdot C_{Q=2,S=1,S^3=-1;q,k,s,r}(\tau) \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -C_{Q=2,S=1,S^3=-1;q,k,s,r}(\tau) \cdot \sigma_5
\end{aligned}
$$

These identities show us that we can, average our data four fold only because we are using this kind of correlator.

Using the cyclicity of the trace, we find how the correlation funciton transforms when time is reversed. This will further increase our statistics two times.

Trace ciclicity

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s} &= \frac{1}{Z} tr \left[p_k p_q e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} \right] = \\
&= \frac{1}{Z} tr \left[p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} p_k p_q e^{-H\tau} \right] \\
&= \frac{1}{Z} tr \left[p_s^\dagger p_r^\dagger e^{-H(\beta-\tau)} p_q p_k e^{-H\tau} \right] \\
&= -\frac{1}{Z} tr \left[p_s^\dagger p_r^\dagger e^{-H(\beta-\tau)} p_k p_q e^{-H\tau} \right] \\
&= -\frac{1}{Z} tr \left[p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)} p_q p_k e^{-H\tau} \right]
\end{aligned}
$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s} &= \begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau) = \\
&= \begin{bmatrix} p_+^\dagger p_+^\dagger p_+p_+ & p_+^\dagger p_-^\dagger p_+p_+ & p_-^\dagger p_+^\dagger p_+p_+ & p_-^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_+p_- \\ p_+^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_-p_+ \\ p_+^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_-p_- \end{bmatrix}_{r,s,k,q} (\beta-\tau) \\
&= \begin{bmatrix} p_+^\dagger p_+^\dagger p_+p_+ & p_-^\dagger p_+^\dagger p_+p_+ & p_+^\dagger p_-^\dagger p_+p_+ & p_-^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_-p_+ \\ p_+^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_+p_- \\ p_+^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_-p_- \end{bmatrix}_{s,r,q,k} (\beta-\tau) \\
&= -\begin{bmatrix} p_+^\dagger p_+^\dagger p_+p_+ & p_-^\dagger p_+^\dagger p_+p_+ & p_+^\dagger p_-^\dagger p_+p_+ & p_-^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_+p_- \\ p_+^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_-p_+ \\ p_+^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_-p_- \end{bmatrix}_{s,r,k,q} (\beta-\tau) \\
&= -\begin{bmatrix} p_+^\dagger p_+^\dagger p_+p_+ & p_+^\dagger p_-^\dagger p_+p_+ & p_-^\dagger p_+^\dagger p_+p_+ & p_-^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_-p_+ \\ p_+^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_+p_- \\ p_+^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_-p_- \end{bmatrix}_{r,s,q,k} (\beta-\tau)
\end{aligned}
$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= C_{Q=2,S=1,S^3=+1;r,s,k,q}{}^\top(\beta-\tau) \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_5 \cdot C_{Q=2,S=1,S^3=+1;s,r,q,k}{}^\top(\beta-\tau) \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -C_{Q=2,S=1,S^3=+1;s,r,k,q}{}^\top(\beta-\tau) \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_5 \cdot C_{Q=2,S=1,S^3=+1;r,s,q,k}{}^\top(\beta-\tau)
\end{aligned}
$$

$I : H - \vec{\mu} \cdot \vec{q}$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s} &= \frac{1}{Z} tr\left[p_k p_q e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)}\right] = \\
&= \frac{1}{Z} tr\left[I^{-1} I p_k I^{-1} I p_q I^{-1} I e^{-H\tau} I^{-1} I p_r^\dagger I^{-1} I p_s^\dagger I^{-1} I e^{-H(\beta-\tau)}\right] = \\
&= \frac{1}{Z} tr\left[I p_k I^{-1} I p_q I^{-1} I e^{-H\tau} I^{-1} I p_r^\dagger I^{-1} I p_s^\dagger I^{-1} I e^{-H(\beta-\tau)} I^{-1}\right] = \\
&= \frac{1}{Z} tr\left[(\Sigma h)_k^\dagger (\Sigma h)_q^\dagger e^{-H\tau} (\Sigma h)_r (\Sigma h)_s e^{-H(\beta-\tau)}\right] \\
&= \frac{1}{Z} tr\left[(\Sigma h)_q^\dagger (\Sigma h)_k^\dagger e^{-H\tau} (\Sigma h)_s (\Sigma h)_r e^{-H(\beta-\tau)}\right] \\
&= -\frac{1}{Z} tr\left[(\Sigma h)_q^\dagger (\Sigma h)_k^\dagger e^{-H\tau} (\Sigma h)_r (\Sigma h)_s e^{-H(\beta-\tau)}\right] \\
&= -\frac{1}{Z} tr\left[(\Sigma h)_k^\dagger (\Sigma h)_q^\dagger e^{-H\tau} (\Sigma h)_s (\Sigma h)_r e^{-H(\beta-\tau)}\right] \\
&= \frac{1}{Z} tr\left[(\Sigma h)_r (\Sigma h)_s e^{-H(\beta-\tau)} (\Sigma h)_k^\dagger (\Sigma h)_q^\dagger e^{-H\tau}\right] \\
&= \frac{1}{Z} tr\left[(\Sigma h)_s (\Sigma h)_r e^{-H(\beta-\tau)} (\Sigma h)_q^\dagger (\Sigma h)_k^\dagger e^{-H\tau}\right] \\
&= -\frac{1}{Z} tr\left[(\Sigma h)_s (\Sigma h)_r e^{-H(\beta-\tau)} (\Sigma h)_k^\dagger (\Sigma h)_q^\dagger e^{-H\tau}\right] \\
&= -\frac{1}{Z} tr\left[(\Sigma h)_r (\Sigma h)_s e^{-H(\beta-\tau)} (\Sigma h)_q^\dagger (\Sigma h)_k^\dagger e^{-H\tau}\right]
\end{aligned}
$$

$$C_{Q=2,S=1,S^3=-1;k,q,r,s} = I^{-1} \begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau) I =$$

$$= \begin{bmatrix} (\Sigma h)_+^\dagger (\Sigma h)_+^\dagger (\Sigma h)_+ (\Sigma h)_+ & (\Sigma h)_+^\dagger (\Sigma h)_+^\dagger (\Sigma h)_+ (\Sigma h)_- & (\Sigma h)_+^\dagger (\Sigma h)_+^\dagger (\Sigma h)_- (\Sigma h)_+ & (\Sigma h)_+^\dagger (\Sigma h)_+^\dagger (\Sigma h)_- \\ (\Sigma h)_+^\dagger (\Sigma h)_-^\dagger (\Sigma h)_+ (\Sigma h)_+ & (\Sigma h)_+^\dagger (\Sigma h)_-^\dagger (\Sigma h)_+ (\Sigma h)_- & (\Sigma h)_+^\dagger (\Sigma h)_-^\dagger (\Sigma h)_- (\Sigma h)_+ & (\Sigma h)_+^\dagger (\Sigma h)_-^\dagger (\Sigma h)_- \\ (\Sigma h)_-^\dagger (\Sigma h)_+^\dagger (\Sigma h)_+ (\Sigma h)_+ & (\Sigma h)_-^\dagger (\Sigma h)_+^\dagger (\Sigma h)_+ (\Sigma h)_- & (\Sigma h)_-^\dagger (\Sigma h)_+^\dagger (\Sigma h)_- (\Sigma h)_+ & (\Sigma h)_-^\dagger (\Sigma h)_+^\dagger (\Sigma h)_- \\ (\Sigma h)_-^\dagger (\Sigma h)_-^\dagger (\Sigma h)_+ (\Sigma h)_+ & (\Sigma h)_-^\dagger (\Sigma h)_-^\dagger (\Sigma h)_+ (\Sigma h)_- & (\Sigma h)_-^\dagger (\Sigma h)_-^\dagger (\Sigma h)_- (\Sigma h)_+ & (\Sigma h)_-^\dagger (\Sigma h)_-^\dagger (\Sigma h)_- \end{bmatrix}$$

$$= \begin{bmatrix} h_-^\dagger h_-^\dagger h_- h_- & h_-^\dagger h_-^\dagger h_- h_+ & h_-^\dagger h_-^\dagger h_+ h_- & h_-^\dagger h_-^\dagger h_+ h_+ \\ h_-^\dagger h_+^\dagger h_- h_- & h_-^\dagger h_+^\dagger h_- h_+ & h_-^\dagger h_+^\dagger h_+ h_- & h_-^\dagger h_+^\dagger h_+ h_+ \\ h_+^\dagger h_-^\dagger h_- h_- & h_+^\dagger h_-^\dagger h_- h_+ & h_+^\dagger h_-^\dagger h_+ h_- & h_+^\dagger h_-^\dagger h_+ h_+ \\ h_+^\dagger h_+^\dagger h_- h_- & h_+^\dagger h_+^\dagger h_- h_+ & h_+^\dagger h_+^\dagger h_+ h_- & h_+^\dagger h_+^\dagger h_+ h_+ \end{bmatrix}_{k,q,r,s} (\tau)$$

$$= \begin{bmatrix} h_-^\dagger h_-^\dagger h_- h_- & h_-^\dagger h_-^\dagger h_+ h_- & h_-^\dagger h_-^\dagger h_- h_+ & h_-^\dagger h_-^\dagger h_+ h_+ \\ h_+^\dagger h_-^\dagger h_- h_- & h_+^\dagger h_-^\dagger h_+ h_- & h_+^\dagger h_-^\dagger h_- h_+ & h_+^\dagger h_-^\dagger h_+ h_+ \\ h_-^\dagger h_+^\dagger h_- h_- & h_-^\dagger h_+^\dagger h_+ h_- & h_-^\dagger h_+^\dagger h_- h_+ & h_-^\dagger h_+^\dagger h_+ h_+ \\ h_+^\dagger h_+^\dagger h_- h_- & h_+^\dagger h_+^\dagger h_+ h_- & h_+^\dagger h_+^\dagger h_- h_+ & h_+^\dagger h_+^\dagger h_+ h_+ \end{bmatrix}_{q,k,s,r} (\tau)$$

$$= - \begin{bmatrix} h_-^\dagger h_-^\dagger h_- h_- & h_-^\dagger h_-^\dagger h_- h_+ & h_-^\dagger h_-^\dagger h_+ h_- & h_-^\dagger h_-^\dagger h_+ h_+ \\ h_+^\dagger h_-^\dagger h_- h_- & h_+^\dagger h_-^\dagger h_- h_+ & h_+^\dagger h_-^\dagger h_+ h_- & h_+^\dagger h_-^\dagger h_+ h_+ \\ h_-^\dagger h_+^\dagger h_- h_- & h_-^\dagger h_+^\dagger h_- h_+ & h_-^\dagger h_+^\dagger h_+ h_- & h_-^\dagger h_+^\dagger h_+ h_+ \\ h_+^\dagger h_+^\dagger h_- h_- & h_+^\dagger h_+^\dagger h_- h_+ & h_+^\dagger h_+^\dagger h_+ h_- & h_+^\dagger h_+^\dagger h_+ h_+ \end{bmatrix}_{q,k,r,s} (\tau)$$

$$= - \begin{bmatrix} h_-^\dagger h_-^\dagger h_- h_- & h_-^\dagger h_-^\dagger h_+ h_- & h_-^\dagger h_-^\dagger h_- h_+ & h_-^\dagger h_-^\dagger h_+ h_+ \\ h_-^\dagger h_+^\dagger h_- h_- & h_-^\dagger h_+^\dagger h_+ h_- & h_-^\dagger h_+^\dagger h_- h_+ & h_-^\dagger h_+^\dagger h_+ h_+ \\ h_+^\dagger h_-^\dagger h_- h_- & h_+^\dagger h_-^\dagger h_+ h_- & h_+^\dagger h_-^\dagger h_- h_+ & h_+^\dagger h_-^\dagger h_+ h_+ \\ h_+^\dagger h_+^\dagger h_- h_- & h_+^\dagger h_+^\dagger h_+ h_- & h_+^\dagger h_+^\dagger h_- h_+ & h_+^\dagger h_+^\dagger h_+ h_+ \end{bmatrix}_{k,q,s,r} (\tau)$$

$$= \begin{bmatrix} h_- h_- h_-^\dagger h_-^\dagger & h_- h_+ h_-^\dagger h_-^\dagger & h_+ h_- h_-^\dagger h_-^\dagger & h_+ h_+ h_-^\dagger h_-^\dagger \\ h_- h_- h_-^\dagger h_+^\dagger & h_- h_+ h_-^\dagger h_+^\dagger & h_+ h_- h_-^\dagger h_+^\dagger & h_+ h_+ h_-^\dagger h_+^\dagger \\ h_- h_- h_+^\dagger h_-^\dagger & h_- h_+ h_+^\dagger h_-^\dagger & h_+ h_- h_+^\dagger h_-^\dagger & h_+ h_+ h_+^\dagger h_-^\dagger \\ h_- h_- h_+^\dagger h_+^\dagger & h_- h_+ h_+^\dagger h_+^\dagger & h_+ h_- h_+^\dagger h_+^\dagger & h_+ h_+ h_+^\dagger h_+^\dagger \end{bmatrix}_{r,s,k,q} (\beta - \tau)$$

$$= \begin{bmatrix} h_- h_- h_-^\dagger h_-^\dagger & h_+ h_- h_-^\dagger h_-^\dagger & h_- h_+ h_-^\dagger h_-^\dagger & h_+ h_+ h_-^\dagger h_-^\dagger \\ h_- h_- h_+^\dagger h_-^\dagger & h_+ h_- h_+^\dagger h_-^\dagger & h_- h_+ h_+^\dagger h_-^\dagger & h_+ h_+ h_+^\dagger h_-^\dagger \\ h_- h_- h_-^\dagger h_+^\dagger & h_+ h_- h_-^\dagger h_+^\dagger & h_- h_+ h_-^\dagger h_+^\dagger & h_+ h_+ h_-^\dagger h_+^\dagger \\ h_- h_- h_+^\dagger h_+^\dagger & h_+ h_- h_+^\dagger h_+^\dagger & h_- h_+ h_+^\dagger h_+^\dagger & h_+ h_+ h_+^\dagger h_+^\dagger \end{bmatrix}_{s,r,q,k} (\beta - \tau)$$

$$= - \begin{bmatrix} h_- h_- h_-^\dagger h_-^\dagger & h_+ h_- h_-^\dagger h_-^\dagger & h_- h_+ h_-^\dagger h_-^\dagger & h_+ h_+ h_-^\dagger h_-^\dagger \\ h_- h_- h_-^\dagger h_+^\dagger & h_+ h_- h_-^\dagger h_+^\dagger & h_- h_+ h_-^\dagger h_+^\dagger & h_+ h_+ h_-^\dagger h_+^\dagger \\ h_- h_- h_+^\dagger h_-^\dagger & h_+ h_- h_+^\dagger h_-^\dagger & h_- h_+ h_+^\dagger h_-^\dagger & h_+ h_+ h_+^\dagger h_-^\dagger \\ h_- h_- h_+^\dagger h_+^\dagger & h_+ h_- h_+^\dagger h_+^\dagger & h_- h_+ h_+^\dagger h_+^\dagger & h_+ h_+ h_+^\dagger h_+^\dagger \end{bmatrix}_{s,r,k,q} (\beta - \tau)$$

$$= - \begin{bmatrix} h_- h_- h_-^\dagger h_-^\dagger & h_- h_+ h_-^\dagger h_-^\dagger & h_+ h_- h_-^\dagger h_-^\dagger & h_+ h_+ h_-^\dagger h_-^\dagger \\ h_- h_- h_+^\dagger h_-^\dagger & h_- h_+ h_+^\dagger h_-^\dagger & h_+ h_- h_+^\dagger h_-^\dagger & h_+ h_+ h_+^\dagger h_-^\dagger \\ h_- h_- h_-^\dagger h_+^\dagger & h_- h_+ h_-^\dagger h_+^\dagger & h_+ h_- h_-^\dagger h_+^\dagger & h_+ h_+ h_-^\dagger h_+^\dagger \\ h_- h_- h_+^\dagger h_+^\dagger & h_- h_+ h_+^\dagger h_+^\dagger & h_+ h_- h_+^\dagger h_+^\dagger & h_+ h_+ h_+^\dagger h_+^\dagger \end{bmatrix}_{r,s,q,k} (\beta - \tau)$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_1 \cdot C_{Q=-2,S=1,S^3=+1;k,q,r,s}(\tau) \cdot \sigma_1 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_5 \cdot \sigma_1 \cdot C_{Q=-2,S=1,S^3=+1;q,k,s,r}(\tau) \cdot \sigma_1 \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_5 \cdot \sigma_1 \cdot C_{Q=-2,S=1,S^3=+1;q,k,r,s}(\tau) \cdot \sigma_1 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_1 \cdot C_{Q=-2,S=1,S^3=+1;k,q,s,r}(\tau) \cdot \sigma_1 \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_1 \cdot C_{Q=-2,S=1,S^3=-1;r,s,k,q}{}^\top(\beta-\tau) \cdot \sigma_1 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_5 \cdot \sigma_1 \cdot C_{Q=-2,S=1,S^3=-1;s,r,q,k}{}^\top(\beta-\tau) \cdot \sigma_1 \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_1 \cdot C_{Q=-2,S=1,S^3=-1;s,r,k,q}{}^\top(\beta-\tau) \cdot \sigma_1 \cdot \sigma_5 \\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_5 \cdot \sigma_1 \cdot C_{Q=-2,S=1,S^3=-1;r,s,q,k}{}^\top(\beta-\tau) \cdot \sigma_1
\end{aligned}
$$

C* : *H

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s} &= \frac{1}{Z} tr\left[p_k p_q e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)}\right] = \\
&= \frac{1}{Z} tr\left[C^{-1} C p_k C^{-1} C p_q C^{-1} C e^{-H\tau} C^{-1} C p_r^\dagger C^{-1} C p_s^\dagger C^{-1} C e^{-H(\beta-\tau)}\right] = \\
&= \frac{1}{Z} tr\left[C p_k C^{-1} C p_q C^{-1} C e^{-H\tau} C^{-1} C p_r^\dagger C^{-1} C p_s^\dagger C^{-1} C e^{-H(\beta-\tau)} C^{-1}\right] = \\
&= \frac{1}{Z} tr\left[h_k h_q e^{-H\tau} h_r^\dagger h_s^\dagger e^{-H(\beta-\tau)}\right] \\
&= \frac{1}{Z} tr\left[h_q h_k - H\tau h_s^\dagger h_r^\dagger e^{-H(\beta-\tau)}\right] \\
&= -\frac{1}{Z} tr\left[h_q h_k e^{-H\tau} h_r^\dagger h_s^\dagger e^{-H(\beta-\tau)}\right] \\
&= -\frac{1}{Z} tr\left[h_k h_q e^{-H\tau} h_s^\dagger h_r^\dagger e^{-H(\beta-\tau)}\right] \\
&= \frac{1}{Z} tr\left[h_s^\dagger h_r^\dagger e^{-H(\beta-\tau)} h_q h_k e^{-H\tau}\right] \\
&= \frac{1}{Z} tr\left[h_r^\dagger h_s^\dagger e^{-H(\beta-\tau)} h_k h_q e^{-H\tau}\right] \\
&= -\frac{1}{Z} tr\left[h_r^\dagger h_s^\dagger e^{-H(\beta-\tau)} h_q h_k e^{-H\tau}\right] \\
&= -\frac{1}{Z} tr\left[h_s^\dagger h_r^\dagger e^{-H(\beta-\tau)} h_k h_q e^{-H\tau}\right]
\end{aligned}
$$

$$
C_{Q=2,S=1,S^3=-1;k,q,r,s} = C^{-1} \begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau) C =
$$

$$
= \begin{bmatrix} h_+h_+h_+^\dagger h_+^\dagger & h_+h_+h_+^\dagger h_-^\dagger & h_+h_+h_-^\dagger h_+^\dagger & h_+h_+h_-^\dagger h_-^\dagger \\ h_+h_-h_+^\dagger h_+^\dagger & h_+h_-h_+^\dagger h_-^\dagger & h_+h_-h_-^\dagger h_+^\dagger & h_+h_-h_-^\dagger h_-^\dagger \\ h_-h_+h_+^\dagger h_+^\dagger & h_-h_+h_+^\dagger h_-^\dagger & h_-h_+h_-^\dagger h_+^\dagger & h_-h_+h_-^\dagger h_-^\dagger \\ h_-h_-h_+^\dagger h_+^\dagger & h_-h_-h_+^\dagger h_-^\dagger & h_-h_-h_-^\dagger h_+^\dagger & h_-h_-h_-^\dagger h_-^\dagger \end{bmatrix}_{k,q,r,s} (\tau)
$$

$$
= \begin{bmatrix} h_+h_+h_+^\dagger h_+^\dagger & h_+h_+h_-^\dagger h_+^\dagger & h_+h_+h_+^\dagger h_-^\dagger & h_+h_+h_-^\dagger h_-^\dagger \\ h_-h_+h_+^\dagger h_+^\dagger & h_-h_+h_-^\dagger h_+^\dagger & h_-h_+h_+^\dagger h_-^\dagger & h_-h_+h_-^\dagger h_-^\dagger \\ h_+h_-h_+^\dagger h_+^\dagger & h_+h_-h_-^\dagger h_+^\dagger & h_+h_-h_+^\dagger h_-^\dagger & h_+h_-h_-^\dagger h_-^\dagger \\ h_-h_-h_+^\dagger h_+^\dagger & h_-h_-h_-^\dagger h_+^\dagger & h_-h_-h_+^\dagger h_-^\dagger & h_-h_-h_-^\dagger h_-^\dagger \end{bmatrix}_{q,k,s,r} (\tau)
$$

$$
= - \begin{bmatrix} h_+h_+h_+^\dagger h_+^\dagger & h_+h_+h_+^\dagger h_-^\dagger & h_+h_+h_-^\dagger h_+^\dagger & h_+h_+h_-^\dagger h_-^\dagger \\ h_-h_+h_+^\dagger h_+^\dagger & h_-h_+h_+^\dagger h_-^\dagger & h_-h_+h_-^\dagger h_+^\dagger & h_-h_+h_-^\dagger h_-^\dagger \\ h_+h_-h_+^\dagger h_+^\dagger & h_+h_-h_+^\dagger h_-^\dagger & h_+h_-h_-^\dagger h_+^\dagger & h_+h_-h_-^\dagger h_-^\dagger \\ h_-h_-h_+^\dagger h_+^\dagger & h_-h_-h_+^\dagger h_-^\dagger & h_-h_-h_-^\dagger h_+^\dagger & h_-h_-h_-^\dagger h_-^\dagger \end{bmatrix}_{q,k,r,s} (\tau)
$$

$$
= - \begin{bmatrix} h_+^\dagger h_+^\dagger h_+h_+ & h_+^\dagger h_-^\dagger h_+h_+ & h_-^\dagger h_+^\dagger h_+h_+ & h_-^\dagger h_-^\dagger h_+h_+ \\ h_+^\dagger h_+^\dagger h_-h_+ & h_+^\dagger h_-^\dagger h_-h_+ & h_-^\dagger h_+^\dagger h_-h_+ & h_-^\dagger h_-^\dagger h_-h_+ \\ h_+^\dagger h_+^\dagger h_+h_- & h_+^\dagger h_-^\dagger h_+h_- & h_-^\dagger h_+^\dagger h_+h_- & h_-^\dagger h_-^\dagger h_+h_- \\ h_+^\dagger h_+^\dagger h_-h_- & h_+^\dagger h_-^\dagger h_-h_- & h_-^\dagger h_+^\dagger h_-h_- & h_-^\dagger h_-^\dagger h_-h_- \end{bmatrix}_{k,q,s,r} (\tau)
$$

$$
= \begin{bmatrix} h_+^\dagger h_+^\dagger h_+h_+ & h_-^\dagger h_+^\dagger h_+h_+ & h_+^\dagger h_-^\dagger h_+h_+ & h_-^\dagger h_-^\dagger h_+h_+ \\ h_+^\dagger h_+^\dagger h_-h_+ & h_-^\dagger h_+^\dagger h_-h_+ & h_+^\dagger h_-^\dagger h_-h_+ & h_-^\dagger h_-^\dagger h_-h_+ \\ h_+^\dagger h_+^\dagger h_+h_- & h_-^\dagger h_+^\dagger h_+h_- & h_+^\dagger h_-^\dagger h_+h_- & h_-^\dagger h_-^\dagger h_+h_- \\ h_+^\dagger h_+^\dagger h_-h_- & h_-^\dagger h_+^\dagger h_-h_- & h_+^\dagger h_-^\dagger h_-h_- & h_-^\dagger h_-^\dagger h_-h_- \end{bmatrix}_{s,r,q,k} (\beta - \tau)
$$

$$
= \begin{bmatrix} h_+^\dagger h_+^\dagger h_+h_+ & h_+^\dagger h_-^\dagger h_+h_+ & h_-^\dagger h_+^\dagger h_+h_+ & h_-^\dagger h_-^\dagger h_+h_+ \\ h_+^\dagger h_+^\dagger h_+h_- & h_+^\dagger h_-^\dagger h_+h_- & h_-^\dagger h_+^\dagger h_+h_- & h_-^\dagger h_-^\dagger h_+h_- \\ h_+^\dagger h_+^\dagger h_-h_+ & h_+^\dagger h_-^\dagger h_-h_+ & h_-^\dagger h_+^\dagger h_-h_+ & h_-^\dagger h_-^\dagger h_-h_+ \\ h_+^\dagger h_+^\dagger h_-h_- & h_+^\dagger h_-^\dagger h_-h_- & h_-^\dagger h_+^\dagger h_-h_- & h_-^\dagger h_-^\dagger h_-h_- \end{bmatrix}_{r,s,k,q} (\beta - \tau)
$$

$$
= - \begin{bmatrix} h_+^\dagger h_+^\dagger h_+h_+ & h_+^\dagger h_-^\dagger h_+h_+ & h_-^\dagger h_+^\dagger h_+h_+ & h_-^\dagger h_-^\dagger h_+h_+ \\ h_+^\dagger h_+^\dagger h_-h_+ & h_+^\dagger h_-^\dagger h_-h_+ & h_-^\dagger h_+^\dagger h_-h_+ & h_-^\dagger h_-^\dagger h_-h_+ \\ h_+^\dagger h_+^\dagger h_+h_- & h_+^\dagger h_-^\dagger h_+h_- & h_-^\dagger h_+^\dagger h_+h_- & h_-^\dagger h_-^\dagger h_+h_- \\ h_+^\dagger h_+^\dagger h_-h_- & h_+^\dagger h_-^\dagger h_-h_- & h_-^\dagger h_+^\dagger h_-h_- & h_-^\dagger h_-^\dagger h_-h_- \end{bmatrix}_{r,s,q,k} (\beta - \tau)
$$

$$
= - \begin{bmatrix} h_+^\dagger h_+^\dagger h_+h_+ & h_-^\dagger h_+^\dagger h_+h_+ & h_+^\dagger h_-^\dagger h_+h_+ & h_-^\dagger h_-^\dagger h_+h_+ \\ h_+^\dagger h_+^\dagger h_+h_- & h_-^\dagger h_+^\dagger h_+h_- & h_+^\dagger h_-^\dagger h_+h_- & h_-^\dagger h_-^\dagger h_+h_- \\ h_+^\dagger h_+^\dagger h_-h_+ & h_-^\dagger h_+^\dagger h_-h_+ & h_+^\dagger h_-^\dagger h_-h_+ & h_-^\dagger h_-^\dagger h_-h_+ \\ h_+^\dagger h_+^\dagger h_-h_- & h_-^\dagger h_+^\dagger h_-h_- & h_+^\dagger h_-^\dagger h_-h_- & h_-^\dagger h_-^\dagger h_-h_- \end{bmatrix}_{s,r,k,q} (\beta - \tau)
$$

$$
\begin{aligned}
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= C_{Q=-2,S=1,S^3=-1;k,q,r,s}(\tau)\\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_5 \cdot C_{Q=-2,S=1,S^3=-1;q,k,s,r}(\tau) \cdot \sigma_5\\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_5 \cdot C_{Q=-2,S=1,S^3=-1;q,k,r,s}(\tau)\\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -C_{Q=-2,S=1,S^3=-1;k,q,s,r}(\tau) \cdot \sigma_5\\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= \sigma_5 \cdot C_{Q=-2,S=1,S^3=+1;s,r,q,k}{}^{\top}(\beta-\tau) \cdot \sigma_5\\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= C_{Q=-2,S=1,S^3=+1;r,s,k,q}{}^{\top}(\beta-\tau)\\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -\sigma_5 \cdot C_{Q=-2,S=1,S^3=+1;r,s,q,k}{}^{\top}(\beta-\tau)\\
C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) &= -C_{Q=-2,S=1,S^3=+1;s,r,k,q}{}^{\top}(\beta-\tau) \cdot \sigma_5
\end{aligned}
$$

XF : H

$$
\begin{aligned}
&C_{Q=2,S=1,S^3=-1;k,q,r,s} = \frac{1}{Z} tr\left[p_k p_q e^{-H\tau} p_r^\dagger p_s^\dagger e^{-H(\beta-\tau)}\right] =\\
&= \frac{1}{Z} tr\left[(XF)^{-1}(XF)p_k(XF)^{-1}(XF)p_q(XF)^{-1}(XF)e^{-H\tau}(XF)^{-1}(XF)p_r^\dagger(XF)^{-1}(XF)p_s^\dagger(XF)^{-1}(XF)e^{-H(\beta-\tau)}\right] =\\
&= \frac{1}{Z} tr\left[(XF)p_k(XF)^{-1}(XF)p_q(XF)^{-1}(XF)e^{-H\tau}(XF)^{-1}(XF)p_r^\dagger(XF)^{-1}(XF)p_s^\dagger(XF)^{-1}(XF)e^{-H(\beta-\tau)}(XF)^{-1}\right] =\\
&= \frac{1}{Z} tr\left[(\Sigma p)_k^\dagger(\Sigma p)_q^\dagger e^{-H\tau}(\Sigma p)_r(\Sigma p)_s e^{-H(\beta-\tau)}\right]\\
&= \frac{1}{Z} tr\left[(\Sigma p)_q^\dagger(\Sigma p)_k^\dagger e^{-H\tau}(\Sigma p)_s(\Sigma p)_r e^{-H(\beta-\tau)}\right]\\
&= -\frac{1}{Z} tr\left[(\Sigma p)_q^\dagger(\Sigma p)_k^\dagger e^{-H\tau}(\Sigma p)_r(\Sigma p)_s e^{-H(\beta-\tau)}\right]\\
&= -\frac{1}{Z} tr\left[(\Sigma p)_k^\dagger(\Sigma p)_q^\dagger e^{-H\tau}(\Sigma p)_s(\Sigma p)_r e^{-H(\beta-\tau)}\right]\\
&= \frac{1}{Z} tr\left[(\Sigma p)_s(\Sigma p)_r e^{-H(\beta-\tau)}(\Sigma p)_q^\dagger(\Sigma p)_k^\dagger e^{-H\tau}\right]\\
&= \frac{1}{Z} tr\left[(\Sigma p)_r(\Sigma p)_s e^{-H(\beta-\tau)}(\Sigma p)_k^\dagger(\Sigma p)_q^\dagger e^{-H\tau}\right]\\
&= -\frac{1}{Z} tr\left[(\Sigma p)_r(\Sigma p)_s e^{-H(\beta-\tau)}(\Sigma p)_q^\dagger(\Sigma p)_k^\dagger e^{-H\tau}\right]\\
&= -\frac{1}{Z} tr\left[(\Sigma p)_s(\Sigma p)_r e^{-H(\beta-\tau)}(\Sigma p)_k^\dagger(\Sigma p)_q^\dagger e^{-H\tau}\right]
\end{aligned}
$$

$$C_{Q=2,S=1,S^3=-1;k,q,r,s} = (XF)^{-1}\begin{bmatrix} p_+p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_+p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_-^\dagger \\ p_-p_+p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_-p_+^\dagger p_-^\dagger & p_-p_-p_-^\dagger p_+^\dagger & p_-p_-p_-^\dagger p_-^\dagger \end{bmatrix}_{k,q,r,s}(\tau)(XF) =$$

$$= \begin{bmatrix} (\Sigma p)_+^\dagger (\Sigma p)_+^\dagger (\Sigma p)_+ (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma p)_+^\dagger (\Sigma p)_+ (\Sigma p)_- & (\Sigma p)_+^\dagger (\Sigma p)_+^\dagger (\Sigma p)_- (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma p)_+^\dagger (\Sigma p)_- \\ (\Sigma p)_+^\dagger (\Sigma p)_-^\dagger (\Sigma p)_+ (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma p)_-^\dagger (\Sigma p)_+ (\Sigma p)_- & (\Sigma p)_+^\dagger (\Sigma p)_-^\dagger (\Sigma p)_- (\Sigma p)_+ & (\Sigma p)_+^\dagger (\Sigma p)_-^\dagger (\Sigma p)_- \\ (\Sigma p)_-^\dagger (\Sigma p)_+^\dagger (\Sigma p)_+ (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma p)_+^\dagger (\Sigma p)_+ (\Sigma p)_- & (\Sigma p)_-^\dagger (\Sigma p)_+^\dagger (\Sigma p)_- (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma p)_+^\dagger (\Sigma p)_- \\ (\Sigma p)_-^\dagger (\Sigma p)_-^\dagger (\Sigma p)_+ (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma p)_-^\dagger (\Sigma p)_+ (\Sigma p)_- & (\Sigma p)_-^\dagger (\Sigma p)_-^\dagger (\Sigma p)_- (\Sigma p)_+ & (\Sigma p)_-^\dagger (\Sigma p)_-^\dagger (\Sigma p)_- \end{bmatrix}$$

$$= \begin{bmatrix} p_-^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_+p_+ \\ p_-^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_+p_+ \\ p_+^\dagger p_-^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_+^\dagger p_+p_+ \end{bmatrix}_{k,q,r,s} (\tau)$$

$$= \begin{bmatrix} p_-^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_-^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_+p_+ \\ p_-^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_+^\dagger p_+p_+ \end{bmatrix}_{q,k,s,r} (\tau)$$

$$= -\begin{bmatrix} p_-^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_-^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_+p_+ \\ p_-^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_+^\dagger p_+p_+ \end{bmatrix}_{q,k,r,s} (\tau)$$

$$= -\begin{bmatrix} p_-^\dagger p_-^\dagger p_-p_- & p_-^\dagger p_-^\dagger p_+p_- & p_-^\dagger p_-^\dagger p_-p_+ & p_-^\dagger p_-^\dagger p_+p_+ \\ p_-^\dagger p_+^\dagger p_-p_- & p_-^\dagger p_+^\dagger p_+p_- & p_-^\dagger p_+^\dagger p_-p_+ & p_-^\dagger p_+^\dagger p_+p_+ \\ p_+^\dagger p_-^\dagger p_-p_- & p_+^\dagger p_-^\dagger p_+p_- & p_+^\dagger p_-^\dagger p_-p_+ & p_+^\dagger p_-^\dagger p_+p_+ \\ p_+^\dagger p_+^\dagger p_-p_- & p_+^\dagger p_+^\dagger p_+p_- & p_+^\dagger p_+^\dagger p_-p_+ & p_+^\dagger p_+^\dagger p_+p_+ \end{bmatrix}_{k,q,s,r} (\tau)$$

$$= \begin{bmatrix} p_-p_-p_-^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_-^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_+p_+p_+^\dagger p_-^\dagger \\ p_-p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_+^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_+^\dagger \end{bmatrix}_{s,r,q,k} (\beta-\tau)$$

$$= \begin{bmatrix} p_-p_-p_-^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_+^\dagger \\ p_-p_-p_+^\dagger p_-^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_+p_+^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_+^\dagger \end{bmatrix}_{r,s,k,q} (\beta-\tau)$$

$$= -\begin{bmatrix} p_-p_-p_-^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_-^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_+p_+p_+^\dagger p_-^\dagger \\ p_-p_-p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_+^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_+^\dagger \end{bmatrix}_{r,s,q,k} (\beta-\tau)$$

$$= -\begin{bmatrix} p_-p_-p_-^\dagger p_-^\dagger & p_+p_-p_-^\dagger p_-^\dagger & p_-p_+p_-^\dagger p_-^\dagger & p_+p_+p_-^\dagger p_-^\dagger \\ p_-p_-p_-^\dagger p_+^\dagger & p_+p_-p_-^\dagger p_+^\dagger & p_-p_+p_-^\dagger p_+^\dagger & p_+p_+p_-^\dagger p_+^\dagger \\ p_-p_-p_+^\dagger p_-^\dagger & p_+p_-p_+^\dagger p_-^\dagger & p_-p_+p_+^\dagger p_-^\dagger & p_+p_+p_+^\dagger p_-^\dagger \\ p_-p_-p_+^\dagger p_+^\dagger & p_+p_-p_+^\dagger p_+^\dagger & p_-p_+p_+^\dagger p_+^\dagger & p_+p_+p_+^\dagger p_+^\dagger \end{bmatrix}_{s,r,q,k} (\beta-\tau)$$

$$C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) = \sigma_1 \cdot C_{Q=2,S=1,S^3=+1;k,q,r,s}(\tau) \cdot \sigma_1$$
$$C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) = \sigma_5 \cdot \sigma_1 \cdot C_{Q=2,S=1,S^3=+1;q,k,s,r}(\tau) \cdot \sigma_1 \cdot \sigma_5$$
$$C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) = -\sigma_5 \cdot \sigma_1 \cdot C_{Q=2,S=1,S^3=+1;q,k,r,s}(\tau) \cdot \sigma_1$$
$$C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) = -\sigma_1 \cdot C_{Q=2,S=1,S^3=+1;k,q,s,r}(\tau) \cdot \sigma_1 \cdot \sigma_5$$
$$C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) = \sigma_5 \cdot \sigma_1 \cdot C_{Q=2,S=1,S^3=-1;s,r,q,k}{}^{\top}(\beta - \tau) \cdot \sigma_1 \cdot \sigma_5$$
$$C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) = \sigma_1 \cdot C_{Q=2,S=1,S^3=-1;r,s,k,q}{}^{\top}(\beta - \tau) \cdot \sigma_1$$
$$C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) = -\sigma_5 \cdot \sigma_1 \cdot C_{Q=2,S=1,S^3=-1;r,s,q,k}{}^{\top}(\beta - \tau) \cdot \sigma_1$$
$$C_{Q=2,S=1,S^3=-1;k,q,r,s}(\tau) = -\sigma_1 \cdot C_{Q=2,S=1,S^3=-1;s,r,k,q}{}^{\top}(\beta - \tau) \cdot \sigma_1 \cdot \sigma_5$$

APPENDIX A

Useful information

In the appendix you usually include extra information that should be documented in your thesis, but not interrupt the flow.

The LaTeX WikiBook [12] is a useful source of information on LaTeX.

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