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math581

Coursework 1

Coursework 1:

Consider data $\vec{x} = (x_1, x_2, \dots, x_n)$, independent and identically distributed (iid) realisations from a random variable $X$ with probability density function

$$f(x; \beta, \gamma) = \begin{cases} \frac{1}{2}\beta e^{-\beta x} & x \geq 0 \\ \frac{1}{2}\gamma e^{-\gamma x} & x < 0 \end{cases}$$

This is an asymmetric random variable which could be used to model log-daily returns if positive and negative returns behave differently.

1. Write down the log-likelihood for a sample of size $n$.

Hint: Consider positive and negative values of $x$ separately with $m$ denoting the total number of positive $x$'s.

The data $\vec{x} = (x_1, x_2, \ldots, x_n)$ are iid realisations from a random variable $X$ with probability density function

$$f(x;\beta,\gamma) = \begin{cases} \frac{1}{2}\beta e^{-\beta x} & x \geq 0 \\ \frac{1}{2}\gamma e^{\gamma x} & x < 0 \end{cases}$$

Hence $\vec{\theta} = (\beta, \gamma)$. To obtain the likelihood function for a sample of size $n$ we order the data so that:

$(x_1, \ldots, x_m) \geq 0$ and $(x_{m+1}, \ldots, x_n) < 0$

In this way we manage to consider values of $x \geq 0$ and values of $x < 0$ separately with $m$ denoting the total number of $x$'s in the sample of size $n$ that are greater than or equal to zero and $(n-m)$ denoting the total number of $x$'s in the sample that are negative.

The likelihood of the data $\vec{x} = (x_1, \ldots, x_n)$ is therefore

$$L(\vec{\theta}) = \prod_{i=1}^{n} f(x_i \mid \vec{\theta})$$

$$= \prod_{i=1}^{m} \frac{1}{2}\beta e^{-\beta x_i} \prod_{i=m+1}^{n} \frac{1}{2}\gamma e^{\gamma x_i}$$

2. Calculate the MLEs of $\beta$ and $\gamma$

In order to calculate the MLEs of $\beta$ and $\gamma$, we first need to find the log-likelihood, which is defined as

$$\ell(\vec{\theta}) = \sum_{i=1}^{n} \ln(f(x_i \mid \vec{\theta}))$$

$$= \sum_{i=1}^{m} \ln\left(\frac{1}{2}\beta e^{-\beta x_i}\right) + \sum_{i=m+1}^{n} \ln\left(\frac{1}{2}\gamma e^{\gamma x_i}\right)$$

$$= \sum_{i=1}^{m} \left(\ln\left(\frac{1}{2}\right) + \ln(\beta) - \beta x_i\right)$$

$$+ \sum_{i=m+1}^{n} \left(\ln\left(\frac{1}{2}\right) + \ln(\gamma) + \gamma x_i\right)$$

$$= \sum_{i=1}^{m} \left(-\ln(2) + \ln(\beta) - \beta x_i\right)$$

$$+ \sum_{i=m+1}^{n} \left(-\ln(2) + \ln(\gamma) + \gamma x_i\right)$$

$$= -m\ln(2) + m\ln(\beta) - \beta \sum_{i=1}^{m} x_i$$

$$-(n-m)\ln(2) + (n-m)\ln(\gamma) + \gamma \sum_{i=m+1}^{n} x_i$$

Differentiating the log-likelihood with respect to $\beta$ and $\gamma$ yields

$$\frac{\partial}{\partial \beta} \ell(\vec{\theta}) = \frac{m}{\beta} - \sum_{i=1}^{m} x_i$$

and

$$\frac{\partial}{\partial \gamma} \ell(\vec{\theta}) = \frac{(n-m)}{\gamma} + \sum_{i=m+1}^{n} x_i$$

Since $\vec{\theta} = \hat{\theta}$ satisfies

$\frac{\partial}{\partial \beta} \ell(\hat{\theta}) = 0$ and $\frac{\partial}{\partial \gamma} \ell(\hat{\theta}) = 0$ (because $\hat{\theta}$ maximizes the likelihood function)
we solve with respect to $\hat{\beta}$ and $\hat{\gamma}$ as shown below:

$$\frac{\partial}{\partial \beta} \ell(\hat{\theta}) = \frac{m}{\hat{\beta}} - \sum_{i=1}^{m} x_i = 0$$

$$\Rightarrow \frac{m}{\hat{\beta}} = \sum_{i=1}^{m} x_i \Rightarrow \hat{\beta} \sum_{i=1}^{m} x_i = m$$

$$\Rightarrow \hat{\beta} = \frac{m}{\sum_{i=1}^{m} x_i} = \left( \frac{\sum_{i=1}^{m} x_i}{m} \right)^{-1}$$

so that the MLE for $\beta$ ($\hat{\beta}$) is the inverse of the mean of the m $x_i$ values of our sample that are greater than or equal to 0.

$$\frac{\partial}{\partial \gamma} \ell(\hat{\theta}) = \frac{(n-m)}{\hat{\gamma}} + \sum_{i=m+1}^{n} x_i = 0.$$

$$\Rightarrow \frac{(n-m)}{\hat{\gamma}} = -\sum_{i=m+1}^{n} x_i$$

$$\Rightarrow -\hat{\gamma} \sum_{i=m+1}^{n} x_i = (n-m)$$

$$\Rightarrow \hat{\gamma} = -\frac{(n-m)}{\sum_{i=m+1}^{n} x_i} = -\left(\frac{\sum_{i=m+1}^{n} x_i}{n-m}\right)^{-1}$$

Similarly the MLE of $\gamma$ ($\hat{\gamma}$) is equal to minus the inverse of the mean of the negative $(n-m)$ $x_i$ values of our sample.

3. Calculate the asymptotic distributions of $\hat{\beta}$ and $\hat{\gamma}$.

To calculate the asymptotic distributions of $\hat{\beta}$ and $\hat{\gamma}$, we first need to calculated the observed information and the expected information of $\vec{\theta} = (\beta, \gamma)$

We know that

$$I_0(\theta) = \begin{bmatrix} -\frac{\partial^2}{\partial \beta^2} \ell(\vec{\theta}) & -\frac{\partial^2}{\partial \beta \partial \gamma} \ell(\vec{\theta}) \\ -\frac{\partial^2}{\partial \beta \partial \gamma} \ell(\vec{\theta}) & -\frac{\partial^2}{\partial \beta \partial \gamma} \ell(\vec{\theta}) \end{bmatrix}$$

Intermediate calculation:

$$-\frac{\partial^2}{\partial \beta^2} \ell(\vec{\theta}) = -\left(m\left(-\beta^{-2}\right)\right) = -\left(-\frac{m}{\beta^2}\right) = \frac{m}{\beta^2}$$

$$-\frac{\partial^2}{\partial \beta \partial \gamma} \ell(\vec{\theta}) = 0 = -\frac{\partial^2}{\partial \beta \partial \gamma} \ell(\vec{\theta})$$

$$-\frac{\partial^2}{\partial \gamma^2} \ell(\vec{\theta}) = -\left((n-m) \cdot (-1)\gamma^{-2}\right)$$

$$= \frac{(n-m)}{\gamma^2}$$

$$\Rightarrow I_0(\vec{\theta}) = \begin{bmatrix} \dfrac{m}{\beta^2} & 0 \\ 0 & \dfrac{(n-m)}{\lambda^2} \end{bmatrix}$$

so

$$I_0(\hat{\theta}) = \begin{bmatrix} \dfrac{m}{\hat{\beta}^2} & 0 \\ 0 & \dfrac{(n-m)}{\hat{\lambda}^2} \end{bmatrix}$$

and $$I_0(\hat{\theta})^{-1} = \begin{bmatrix} \dfrac{\hat{\beta}^2}{m} & 0 \\ 0 & \dfrac{\hat{\lambda}^2}{(n-m)} \end{bmatrix}$$

Furthermore

$$I_E(\vec{\theta}) = \begin{bmatrix} E\left(-\dfrac{\partial^2}{\partial\beta^2} \ell(\vec{\theta})\right) & E\left(-\dfrac{\partial^2}{\partial\beta\partial\lambda} \ell(\vec{\theta})\right) \\ E\left(-\dfrac{\partial^2}{\partial\lambda\partial\beta} \ell(\vec{\theta})\right) & E\left(-\dfrac{\partial^2}{\partial\lambda^2} \ell(\vec{\theta})\right) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{m}{\beta^2} & 0 \\ 0 & \frac{(n-m)}{\lambda^2} \end{bmatrix}$$

and

$$I_E(\Theta)^{-1} = \begin{bmatrix} \frac{\beta^2}{m} & 0 \\ 0 & \frac{\lambda^2}{(n-m)} \end{bmatrix}$$

Therefore

$$I_E(\hat{\Theta}) = \begin{bmatrix} \frac{m}{\hat{\beta}^2} & 0 \\ 0 & \frac{(n-m)}{\hat{\lambda}^2} \end{bmatrix} = \begin{bmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{bmatrix}$$

since $I_{12} = E\left( \frac{\partial^2}{\partial\beta\partial\lambda} \ell(\vec{\Theta}) \right) = 0$

then $\beta$ and $\gamma$ are orthogonal and

$$I_E(\hat{\Theta})^{-1} = \begin{bmatrix} \frac{1}{I_{11}} & 0 \\ 0 & \frac{1}{I_{22}} \end{bmatrix} = \begin{bmatrix} \frac{\hat{\beta}^2}{m} & 0 \\ 0 & \frac{\hat{\gamma}^2}{(n-m)} \end{bmatrix}$$

this means that $\beta$ and $\gamma$ are asymptotically independent which implies that we can summarise the joint inferences of $\beta$ and $\gamma$ just by marginal inferences.

Since the asymptotic distribution of $\hat{\Theta}$ (when $\Theta$ is a single parameter) is normal with mean $\Theta_0$ and $V(\hat{\Theta}) = I_E(\Theta_0)^{-1}$, we can conclude that

$$\hat{\beta} \sim N\left(\beta, \frac{\beta^2}{m}\right) \text{ and } \hat{\gamma} \sim N\left(\gamma, \frac{\gamma^2}{(n-m)}\right)$$

Intermediate calculations:

$$V(\hat{\beta}) = I_E(\beta)^{-1} = \frac{1}{I_{11}} = \frac{\beta^2}{m}$$

$$V(\hat{\gamma}) = I_E(\gamma)^{-1} = \frac{1}{I_{22}} = \frac{\gamma^2}{(n-m)}$$

Consider time-series ws $\alpha$ from workshop 1
4. Calculate the MLEs for $\hat{\beta}$ and $\hat{\lambda}$ for these data.

By running the R code for task 4 in "Coursework1.R", the MLE for $\beta$ is $\hat{\beta} \approx 1.5562$ and the MLE for $\lambda$ is $\hat{\lambda} \approx 2.0675$

5. Using the asymptotic distributions for the MLEs construct 95% confidence intervals for $\hat{\beta}$ and $\hat{\eta}$.

Since $\beta$ and $\eta$ are asymptotically independent and the approximation to the asymptotic distribution is

$$\hat{\beta} \sim N\left(\beta, \frac{\hat{\beta}^2}{m}\right) \quad \text{and} \quad \hat{\eta} \sim N\left(\eta, \frac{\hat{\eta}^2}{n-m}\right),$$

by standardizing $\hat{\beta}$ and $\hat{\eta}$ we obtain:

$$P\left(-z_{\alpha/2} \leq \frac{\hat{\beta}-\beta}{\sqrt{\frac{\hat{\beta}^2}{m}}} \leq z_{\alpha/2}\right) = 1-\alpha$$

and

$$P\left(-z_{\alpha/2} \leq \frac{\hat{\eta}-\eta}{\sqrt{\frac{\hat{\eta}^2}{n-m}}} \leq z_{\alpha/2}\right) = 1-\alpha$$

where $z_{\alpha/2}$ is the $1-\alpha/2$ quantile of the standard normal $N(0,1)$ distribution

By rearranging,

we obtain

$$(\tilde{\beta}_e, \tilde{\beta}_u) = \hat{\beta} \pm z_{\alpha/2} \left( \frac{\hat{\beta}^2}{m} \right)^{1/2}$$

$$= \hat{\beta} \pm z_{\alpha/2} \frac{\hat{\beta}}{m^{1/2}}$$

and

$$(\tilde{\gamma}_e, \tilde{\gamma}_u) = \hat{\gamma} \pm z_{\alpha/2} \left( \frac{\hat{\gamma}^2}{(n-m)} \right)^{1/2}$$

$$= \hat{\gamma} \pm z_{\alpha/2} \frac{\hat{\gamma}}{(n-m)^{1/2}}$$

which are approximate $(1-\alpha) \cdot 100\%$ confidence intervals for $\beta$ and $\gamma$ respectively.

To construct a $95\% = 0.95 = 1 - 0.05$ confidence interval, $\alpha = 0.05$ and therefore $z_{0.05/2} = z_{0.025} \approx 1.96$. (where 0.025 is the area to the left of $z_{0.025}$).

As a result, a 95% confidence interval for $\beta$ is: $(\tilde{\beta}_e, \tilde{\beta}_u) = \hat{\beta} \pm z_{0.025} \frac{\hat{\beta}}{m^{1/2}}$

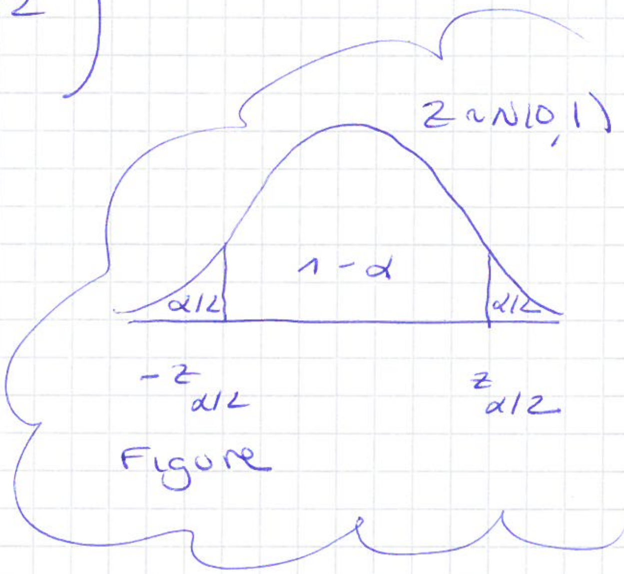
$\approx (1.2512, 1.8612)$

and a 95 % confidence interval for $\lambda$ is

$$(\tilde{\lambda}_e, \tilde{\lambda}_u) = \hat{\lambda} \pm z_{0.025} \frac{\hat{\lambda}}{(n-m)^{1/2}} = (1.7845, 2.3505)$$

Calculations: $\Rightarrow \hat{\theta} \sim N\left(\theta_0, I_E(\hat{\theta})^{-1}\right)$

$$P\left(-z_{\alpha/2} \leq \frac{\hat{\theta} - \theta_0}{\left(I_E(\hat{\theta})^{-1}\right)^{1/2}} \leq z_{\alpha/2}\right) = 1 - \alpha.$$




$$\Rightarrow -z_{\alpha/2} \leq \frac{\hat{\theta} - \theta_0}{I_E(\hat{\theta})^{-\frac{1}{2}}} \leq z_{\alpha/2}$$

$$\Rightarrow -z_{\alpha/2} \cdot I_E(\hat{\theta})^{-1/2} \leq \hat{\theta} - \theta_0 \leq z_{\alpha/2} I_E(\hat{\theta})^{-1/2}$$

$$\Rightarrow -z_{\alpha/2} \cdot I_E(\hat{\theta})^{-1/2} - \hat{\theta} \leq -\theta_0 \leq z_{\alpha/2} I_E(\hat{\theta})^{-1/2} - \hat{\theta}$$

$$\Rightarrow z_{\alpha/2} \cdot I_E(\hat{\theta})^{-1/2} + \hat{\theta} \geq \theta_0 \geq -z_{\alpha/2} I_E(\hat{\theta})^{-1/2} + \hat{\theta}$$

$$\Rightarrow -z_{\alpha/2} I_E(\hat{\theta})^{-1/2} + \hat{\theta} \leq \theta_0 \leq z_{\alpha/2} I_E(\hat{\theta})^{-1/2} + \hat{\theta}$$

$$\Rightarrow \theta_0 \in \left[\hat{\theta} - z_{\alpha/2} I_E(\hat{\theta})^{-1/2}, \hat{\theta} + z_{\alpha/2} I_E(\hat{\theta})^{-1/2}\right]$$

$$= \left[\tilde{\theta}_e, \tilde{\theta}_u\right] \quad \text{and} \quad I_E(\hat{\theta})^{-1/2} = \left(I_E(\hat{\theta})^{-1}\right)^{1/2}$$

6. Is there evidence for setting $\gamma = \beta$? justify your answer.

Since the 95% confidence interval for $\beta$ is $(\tilde{\beta}_e, \tilde{\beta}_u) = (1.2512, 1.8612)$ and the 95% confidence level for $\gamma$ is, $(\hat{\gamma}_e, \hat{\gamma}_u) = (1.7845, 2.3505)$, we observe that the 95% confidence interval for $\gamma$ is not a subset of the 95% confidence interval for $\beta$ and that the 95% confidence interval for $\gamma$ ranges over an interval containing mostly values greater than the values in the 95% confidence interval for $\beta$. Furthermore, these confidence intervals barely overlap eachother and their respective range are very similar meaning that the probability of obtaining a true value for $\beta$ equal to the true value of $\gamma$ is very small and therefore unlikely. Finally, the level of confidence (95%) that truly $\beta \in (\tilde{\beta}_e, \tilde{\beta}_u)$ and $\gamma \in (\tilde{\gamma}_e, \tilde{\gamma}_u)$ is relatively high and we can therefore conclude that setting $\gamma = \beta$ is not justified because most likely the true value of $\gamma$ will be greater than the true value of $\beta$.

7. Using a qq-plot, evaluate how reasonable is the assumption that the underlying model is X.

Hint: to get a sample w of size 2N from X, we can use the fact that the positive values follow Exp(β) distribution and (minus) the negative values follow Exp(γ) distribution.

Thus R code:

w = c(rexp(N, betahat), -rexp(N, gammahat))

By plotting a QQ-plot representing the sample quantiles (on the y-axis) against the theoretical quantiles (on the x-axis), we observe that the plotted points follow the 45° line y = x (in red) except for the extreme points on the far left and right of the plot. This indicates that the X distribution must be similar to the distribution of ws2 (which we will call Y) except for the lower and upper quantiles which will differ in value.

The points on the left extreme are above the line y = x while the points on the left extreme are below the line y = x. Furthermore, the distance between y = x and the right extreme points is greater than the distance between the left extreme points and the line y = x.

To understand how X differs from Y, we plot the densities of $x_i \sim X$ (simulated using the fact that the positive values follow Exp(β)

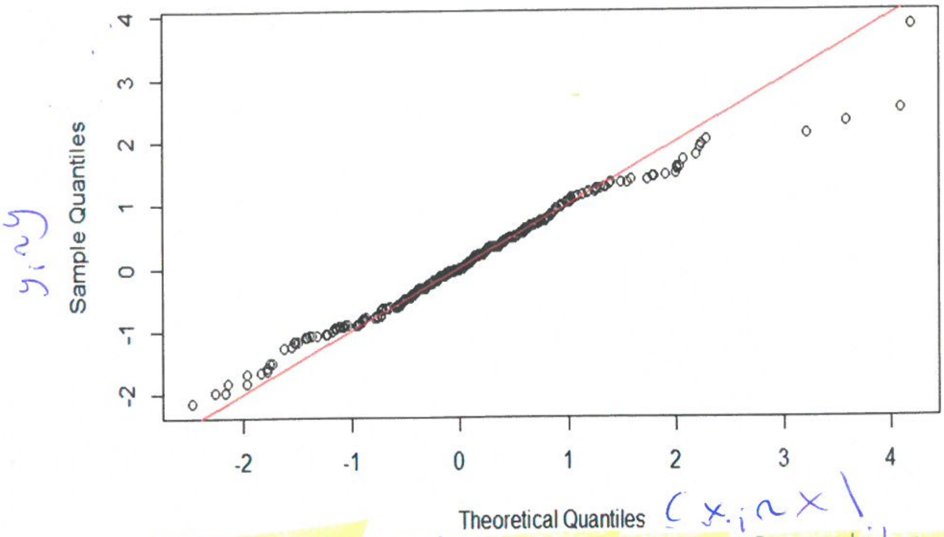



Fig1: QQ-plot of sample Quantiles against theoretical quantiles

distribution and (minus) the negative values follow $Exp(\lambda)$ distribution) and $y_i \sim y$ (our data, ws2). We obtain the following density plots:

Fig2: Plot of the pdf of the y distribution (distribution of our sample)

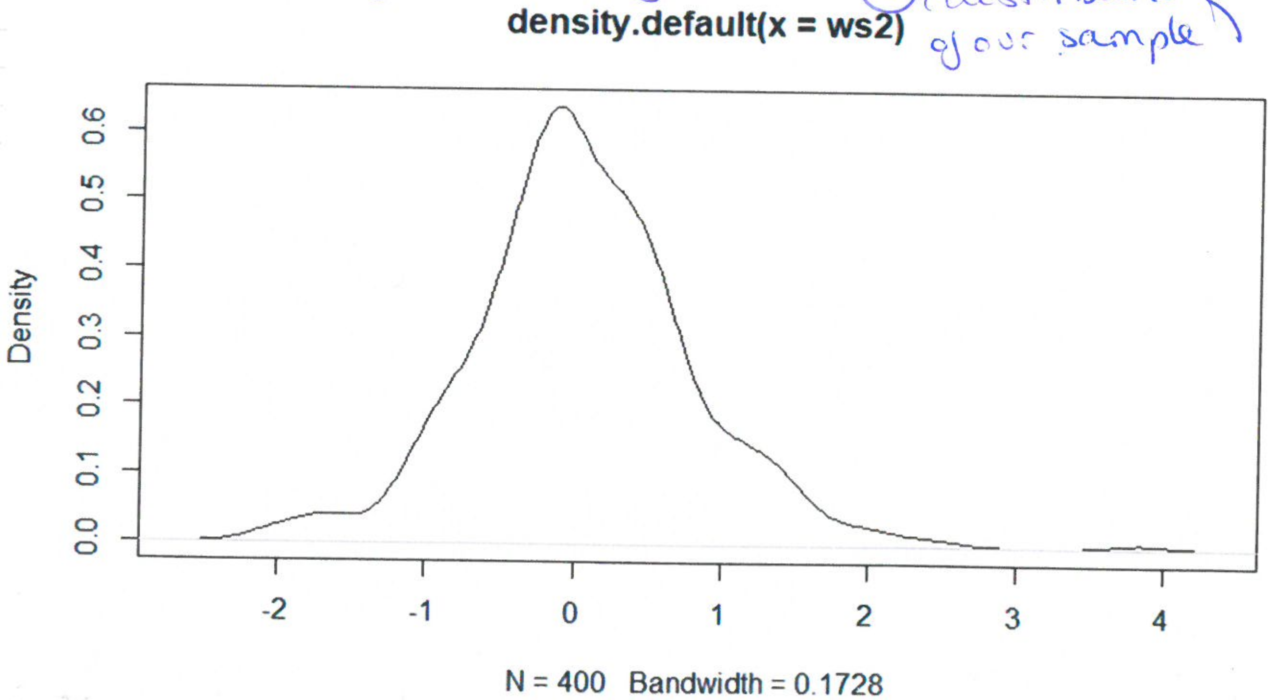
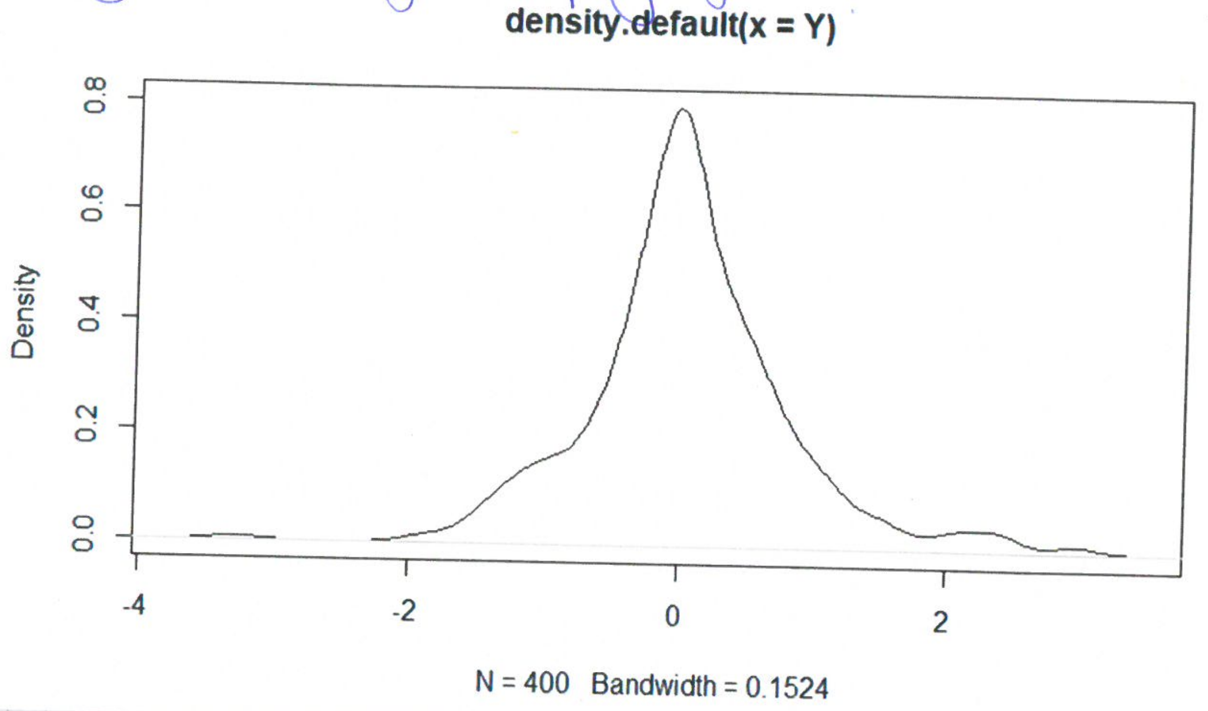


Fig 3: Plot of the pdf of the X distribution




From our QQ plot, we know that the lower quantiles of X are lower in value than the lower quantiles of Y . since the left extreme points fall above the line y = x in the QQ-plot (fig. 1). This means that the distribution of X ; (x) has a heavier left tail than the left tail of the y distribution. Similarly we can conclude that the upper quantiles of X are greater in value than the upper quantiles of y because the right extreme points in the QQ plot (Fig. 1) fall below the line y = x. This implies that the distribution X has a lighter right tail than the right tail of y.. we can also observe from the density plot of the X distribution that its pdf is more pointed than the pdf of the y distribution. However since our sample can contain values that can differ from the values obtained by simulation (such as outliers) and that the general trend of the QQ plot is that the quantiles of the distribution of X and Y are similar except for the extreme upper and lower quantiles, then assuming that $X = Y$ (the distribution X is identical to the distribution Y) seems reasonable.

8. Estimate the $\alpha = 0.01$ and $\alpha = 0.1$ Value at Risk (VaR) for the data ws2 (modelling log returns) for an investment of £1000 assuming the data come from X.

Hint: letting S denoting the investment, the VaR, $v_\alpha$ satisfies $P(X < \ln(1 - \frac{v_\alpha}{S}))$

We assume our data are iid with $x_i = ws2_i$ a realisation of a random variable $X_i$ where

$$X_i \sim X(\beta, \gamma)$$

we let $S = 1000$ £ denote our investment. the value of our investment in 24-hours will be $\frac{S y_{n+1}}{y_n}$. we want a level $v_\alpha$ such that

$$P\left(\frac{S y_{n+1}}{y_n} < S - v_\alpha\right) = \alpha$$

we can re-write this as

$$P\left(\frac{y_{n+1}}{y_n} < 1 - \frac{v_\alpha}{S}\right) \stackrel{!}{=} P\left(e^{X_{n+1}} < 1 - \frac{v_\alpha}{S}\right)$$

$$X_{n+1} = \ln\left(\frac{y_{n+1}}{y_n}\right)$$

$$\Rightarrow e^{X_{n+1}} = \frac{y_{n+1}}{y_n}$$

$$= P\left( X_{n+1} < \ln\left( 1 - \frac{v_\alpha}{S} \right) \right) = \alpha$$

Now given $X_{n+1} \sim X(\beta, p)$, we have that

$$P(X_{n+1} < x_\alpha) = \alpha$$

where $x_\alpha$ is the $\alpha$ quantile of the distribution $X$.

Hence

$$x_\alpha = \ln\left( 1 - \frac{v_\alpha}{S} \right)$$

$$\Rightarrow e^{x_\alpha} = 1 - \frac{v_\alpha}{S}$$

$$\Rightarrow 1 - \frac{v_\alpha}{S} = e^{x_\alpha} \Rightarrow - \frac{v_\alpha}{S} = e^{x} - 1$$

$$\Rightarrow \frac{v_\alpha}{S} = 1 - e^{x_\alpha} \Rightarrow v_\alpha = S(1 - e^{x_\alpha})$$

To find the $\alpha$, $c_\alpha$ quantile of the distribution, for $\alpha < 0.5$ we have

$$F_X(x_\alpha; \lambda, \beta) = \int_{-\infty}^{x_\alpha} f_X(t; \lambda, \beta)\, dt$$

$$\overset{t<0}{=} \int_{-\infty}^{x_\alpha} \frac{1}{2}\lambda e^{\lambda t}\, dt =$$

$$= \frac{1}{2}\lambda \int_{-\infty}^{x_\alpha} e^{\lambda t}\, dt = \frac{\cancel{\lambda}}{2} \cdot \left[\frac{1}{\cancel{\lambda}} e^{\lambda x}\right]_{-\infty}^{x_\alpha}$$

$$= \frac{1}{2}\left(e^{\lambda x_\alpha} - \lim_{x \to -\infty} e^{x}\right) = \frac{1}{2}\left(e^{\lambda x_\alpha} - 0\right)$$

$$= \frac{1}{2} e^{\lambda x_\alpha} = \alpha.$$

$$\Rightarrow e^{\lambda x_\alpha} = 2\alpha$$

$$\Rightarrow \lambda x_\alpha = \ln(2\alpha)$$

$$\Rightarrow x_\alpha = \frac{\ln(2\alpha)}{\lambda}$$

As a result we have that $\mathrm{VaR}(\alpha)$ is

$$v_\alpha = S\left(1 - e^{x_\alpha}\right) = S\left(1 - e^{\frac{\ln(2\alpha)}{\lambda}}\right)$$

Since $\hat{\lambda}$ denotes the MLE for $\lambda$ we obtain the MLE of $\mathrm{VaR}(\alpha)$ which is defined as

$$\hat{v}_\alpha = S\left(1 - e^{\frac{\ln(2\alpha)}{\hat{\lambda}}}\right)$$

Using the R code for task 7 yields

$$\hat{v}_{0.01} \approx 849.25 \text{ £}$$

and

$$\hat{v}_{0.1} \approx 540.88 \text{ £}$$

Summarising, the MLEs for $\mathrm{VaR}(0.01)$ and $\mathrm{VaR}(0.1)$ are respectively

$$\hat{v}_{0.01} \approx 849.25 \text{ £} \quad \text{and} \quad \hat{v}_{0.1} \approx 540.88 \text{ £}.$$

9. Compute the corresponding standard errors of the $\alpha = 0.01$ and $\alpha = 0.1$ VaR.

To compute the corresponding standard error for our estimate for the VaR($\alpha$), we can first apply the delta method to estimate the variance of $\hat{v}_\alpha$:

we observe that $v_\alpha = s\left(1 - e^{\frac{\ln(2\alpha)}{\lambda}}\right) = g(\lambda)$

and we know from task 4 that $\lambda$ and $\beta$ are asymptotically independent.

As a result:

$$V(\hat{v}_\alpha) = \left(\frac{\partial g(\lambda)}{\partial \lambda}\right)^2 \cdot I_E(\lambda)^{-1}$$

From task $I_E(\lambda)^{-1} = \frac{\lambda^2}{(n-m)}$

and $$\frac{\partial g(\lambda)}{\partial \lambda} = -Se^{\frac{\ln(2\alpha)}{\lambda}} \ln(2\alpha) \cdot (-\lambda^{-2})$$

$$= \frac{Se^{\frac{\ln(2\alpha)}{\lambda}} \ln(2\alpha)}{\lambda^2}$$

Therefore,

$$V(\hat{v}_\alpha) = \left(\frac{Se^{\frac{\ln(2\alpha)}{\hat{\lambda}}} \ln(2\alpha)}{\hat{\lambda}^2}\right)^2 \frac{\hat{\lambda}^2}{(n-m)}$$

$$= \frac{S e^{\frac{2\ln(2\alpha)}{\hat{\gamma}}} \left(\ln(2\alpha)\right)^{\alpha}}{(n-m)\,\hat{\gamma}^{2}}$$

By applying the R code for task 8, we obtain that the variance for $\hat{v}_{0.01}$ and the variance for $\hat{v}_{0.1}$ are respeetively:

$$V(\hat{v}_{0.01}) \approx 396.89$$

and $V(\hat{v}_{0.1}) \approx 623.09$

Furthermore we know that the standard error for $\hat{v}_{\alpha}$ is $\sqrt{V(\hat{v}_{\alpha})}$.

Hence, the standard error for $\hat{v}_{0.01}$ is

$$\sqrt{V(\hat{v}_{0.01})} \approx 19.92$$

and the standard error for $\hat{v}_{0.1}$ is

$$\sqrt{V(\hat{v}_{0.1})} \approx 24.96$$

As a conclusion, the corresponding standard errors of the $\alpha = 0.01$ and $\alpha = 0.1$ VaR are respeetively:

$$\sqrt{V(\hat{v}_{0.01})} \approx 19.92$$

and $\sqrt{V(\hat{v}_{0.1})} \approx 24.96$.