# A Novel Matrix-Encoding Method for Privacy-Preserving Neural Networks (Inference)

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# **ABSTRACT**

Homomorphic Encryption has advantages in addressing privacy and security issues that involve sensitive data in medical, financial, or other types, enabling the cloud server to deal with private data by using cloud-based machine learning solutions like convolutional neural networks, without the concern about privacy leakage. In this work, we present Volley Revolver, a novel matrix-encoding method that is particularly convenient for privacy-preserving neural networks to make predictions, and use it to implement a CNN for handwritten image classification. Based on this encoding method, we develop several additional operations for putting into practice the secure matrix multiplication over encrypted data matrices. For two matrices A and B to perform multiplication  $A \times B$ , the main idea is, in a simple version, to encrypt matrix A and the transposition of the matrix B into two ciphertexts respectively. Along with the additional operations, the homomorphic matrix multiplication  $A \times B$ can be calculated over encrypted data matrices efficiently. For the convolution operation in CNN, on the basis of the Volley Revolver encoding method, we develop a feasible and efficient evaluation strategy for performing the convolution operation. We in advance span each convolution kernel of CNN to a matrix space of the same size as the input image so as to generate several ciphertexts, each of which is later used together with the input image for calculating some part of the final convolution result. We accumulate all these part results of convolution operation and thus obtain the final convolution result.

Our encoding method Volley Revolver is applicable to most of the current homomorphic encryption schemes. In addition, our solution is flexible in the sense that it can compute the multiplication of matrices of arbitrary shapes, including the one between two square matrices of the same order, and that it could be used to build a convolutional neural network as deep as it needs to be. What is more, by the natural features of the design itself, our methods of calculating the matrix multiplication and convolution operation can make full use of the multithreading and multiple vCPUs, which is an advantage to adopting a high-performance cloud to achieve reasonable performance for real-world application.

On a google cloud server with 16 vCPUs, our implementation of convolutional neural networks (with one convolutional layer) takes less than half an hour ( $\sim$  17 mins) to compute ten probabilities of 32 input images of size  $28\times28$  simultaneously. Experiments show that cloud server with more vCPUs tends to achieve a better performance in response.

Keywords Privacy-Preserving · Homomorphic Encryption · Convolutional Neural Networks · Matrix-Encoding

# 1 Introduction

# 1.1 Background

Machine learning used in some specific domains such as health and finance should preserve privacy while processing private or confidential data to make accurate predictions. In this study, we focus on privacy-preserving neural networks inference, which is popularized by the work on CryptoNets [1]. This problem aims to outsource a well-trained inference model to a cloud computing service, in order to make predictions on private data. For this purpose, the data should be encrypted first and then sent to the cloud computing service, which should not be capable of having access to the raw

data. Compared to other cryptology technologies such as Secure Multi-Party Computation and hardware enclaves(eg., Intel's Software Guard Extensions), Homomorphic Encryption (HE) provides the most stringent security for this task.

#### 1.2 Related Work

Privacy-preserving deep learning inference model based on HE has been receiving more and more attention in recent years since Gilad-Bachrach et al. [1] proposed a framework called Cryptonets which was the first demonstration of the feasibility of combining HE with convolutional neural networks (CNN). Cryptonets applies neural networks to make accurate inferences on encrypted data with high throughput. However, Cryptonets has a limitation that it can easily create a memory bottleneck while handling networks with many nodes due to its encoding scheme. Chanranne et al. [2] extended this work to deeper CNN using a different underlying software library called HElib [3] and leveraged batch normalization and training process to develop better quality polynomial approximations of the ReLU function for stability and accuracy. Chou et al. [4] developed a pruning and quantization approach with other deep-learning optimization techniques and presented a method for encrypted neural networks inference, Faster CryptoNets. Brutzkus et al. [5] developed new encoding methods for representing the data other than the one used in Cryptonets during the computation, and presented the Low-Latency CryptoNets (LoLa) solution which eliminated the memory bottleneck that existed in the above three works to a great extent, allowing for privacy-preserving inferences on wider networks. Both Faster CryptoNets and LoLa applied transfer learning to private inference in parallel to each other's work. Jiang et al. [6] proposed an efficient evaluation strategy for secure outsourced matrix multiplication, which is the main operation for neural networks, with the help of a novel matrix encoding method. This work achieved an even more reduction in ciphertext message sizes, and a rather reasonable latency. However, its solution had only one convolutional layer and might be difficult in practical application for CNN with over two convolutional layers.

#### 1.3 Contributions

This work is most closely related to [6] but different in many ways. In this study, our contributions lie in four main parts:

- 1. We introduce a novel data-encoding method for matrix multiplications on encrypted matrices, Volley Revolver, which is so flexible that it can be used to multiply matrices of arbitrary shape with the help of a few HE-friendly operations such as RowsRevolver.
- 2. We propose a feasible and efficient evaluation strategy for convolution operation, which is compatible with Volley Revolver, by designing an efficient homomorphic operation function called SumForConv to sum some part results of convolution operations.
- 3. We develop some simulated operations on the packed ciphertext encrypting an image dataset to provide a compelling new perspective of viewing the dataset as a three-dimensional structure. This is crucial for our solution to perform highly parallel computations of convolution operations.
- 4. We adopt different polynomials for each activation layer, whose coefficients are determined by the training process rather than fixed in advance to approximate the ReLU function over a given range.

Using Volley Revolver as the underlying encoding method in the HE environment, both matrix multiplications and convolution operations can make full use of a high-performance cloud server with a large number of vCPUs, with the help of the multithreaded programming, yielding a reasonable amortized performance for real-world application.

# 2 Preliminaries

Let " $\oplus$ " and " $\otimes$ " denote the component-wise addition and multiplication respectively between ciphertexts encrypting a matrix or vector, and the ciphertext ct.P the encryption of a matrix or vector P. Let both " $\circ$ " and " $\circ$ " denote the Hadamard product between two matrices. Let  $I_{[i][j]}^{(m)}$  represent the single pixel of the j-th element in the i-th row of the m-th image (example) from the dataset.

# 2.1 Homomorphic Encryption

Homomorphic Encryption is one kind of encryptions, but has its characteristic in that under this encryption scheme operations on encrypted data generate ciphertexts encrypting the right results of corresponding operations on plaintext without decrypting the data nor requiring access to the (private) secret key. Since Gentry [7] presented the first fully homomorphic encryption scheme, tackling the over three decades problem, much progress has been made on an efficient

data encoding scheme for the application of machine learning to HE. Cheon et al. [8] constructed an HE scheme (CKKS) that can deal with this technique problem efficiently, coming up with a new procedure called rescaling for approximate arithmetic in order to manage the magnitude of plaintext. Their open-source library, HEAAN, like other HE libraries, also supports a Single Instruction Multiple Data (SIMD) manner [9] to encrypt multiple values into a single ciphertext. We adopted HEAAN in our implementation of this study.

Given the security parameter, HEAAN outputs a secret key sk, a public key pk, and whatever needed public keys used for other operations such as rotation. For a vector  $\mathbf{m}$  of size n, where n is a power of two and also the number of slots. For simplicity, we will ignore the rescale operation and deem the following operations to deal with the magnitude of plaintext automatedly. HEAAN has the following functions to support the HE scheme:

 $\operatorname{Enc}_{pk}(\mathbf{m})$ : For the public key pk and a message vector  $\mathbf{m}$ , HEAAN encrypts a message  $\mathbf{m}$  into a ciphertext ct.

 $Dec_{sk}(ct)$ : Using the secret key, this algorithm returns the message vector encrypted by the ciphertext.

Add(ct<sub>1</sub>, ct<sub>2</sub>): this operation returns a new ciphertext that encrypts the message  $Dec_{sk}(ct_1) \oplus Dec_{sk}(ct_2)$ .

 $Mul(ct_1, ct_2)$ : this procedure returns a new ciphertext that encrypts the message  $Dec_{sk}(ct_1) \otimes Dec_{sk}(ct_2)$ .

Rot(ct, l): this procedure generates a ciphertext encrypting a new plaintext vector obtained by rotating the vector m to the left by l positions, where m is the original message encrypted by ct.

### 2.2 Database Encoding Method

For most machine learning applications, the training dataset is organized into a matrix, in which each row represents an example. Kim et al. [10] propose an efficient database encoding method to encrypt this matrix into a single ciphertext. If the training dataset is too large to be encrypted into one ciphertext, in which case the mini-batch method usually should be adopted, the training dataset could be divided into multiple same-size submatrices. The size of those submatrices should be proper and no bigger than the number of slots so as to match the maximum capability of a ciphertext. If the last submatrix does not have enough data to fully utilize the slots, the remaining available slots can be filled with zeros.

For brevity and clarity, assuming that the training dataset has n samples and each sample has f features and that its structure matrix  $Z \in \mathbb{R}^{n \times f}$  can be encrypted into a single ciphertext in a row-by-row manner, in which case the numbers n and f should be power-of-two integers due to the underlying algebra theory. If in the practical application they are not, again, zeros can be padding at the end of each row of the matrix or at the bottom of the matrix. Kim et al. [10] provide two basic but important shifting operations for the management of data in the matrix using the rotation operation: the incomplete column shifting and the row shifting. The two matrices obtained from the matrix Z by shifting 1 and f positions respectively are shown as follows:

$$Z = \begin{bmatrix} z_{[1][1]} & z_{[1][2]} & \cdots & z_{[1][f]} \\ z_{[2][1]} & z_{[2][2]} & \cdots & z_{[2][f]} \\ \vdots & \vdots & \ddots & \vdots \\ z_{[n][1]} & z_{[n][2]} & \cdots & z_{[n][f]} \end{bmatrix} \longmapsto \begin{bmatrix} z_{[1][2]} & z_{[1][3]} & \cdots & z_{[2][1]} \\ z_{[2][2]} & z_{[2][3]} & \cdots & z_{[3][1]} \\ \vdots & \vdots & \ddots & \vdots \\ z_{[n][2]} & z_{[n][3]} & \cdots & z_{[1][1]} \end{bmatrix}, \begin{bmatrix} z_{[2][1]} & z_{[2][2]} & \cdots & z_{[2][f]} \\ \vdots & \vdots & \ddots & \vdots \\ z_{[n][1]} & z_{[n][2]} & \cdots & z_{[n][f]} \end{bmatrix}.$$

Using the same encoding method, Han et al. [11] summarize another two procedures, SumRowVec and SumColVec, to compute the summation of each row and column. These two algorithms are important in computing the inner product. The results of two procedures on Z are as follows:

$$\begin{aligned} & \operatorname{SumRowVec}(Z) = \begin{bmatrix} \sum_{i=1}^{n} z_{[i][1]} & \sum_{i=1}^{n} z_{[i][2]} & \dots & \sum_{i=1}^{n} z_{[i][f]} \\ \sum_{i=1}^{n} z_{[i][1]} & \sum_{i=1}^{n} z_{[i][2]} & \dots & \sum_{i=1}^{n} z_{[i][f]} \\ \vdots & & \vdots & \ddots & \vdots \\ \sum_{i=1}^{n} z_{[i][1]} & \sum_{i=1}^{n} z_{[i][2]} & \dots & \sum_{i=1}^{n} z_{[i][f]} \end{bmatrix}, \\ & \operatorname{SumColVec}(Z) = \begin{bmatrix} \sum_{j=1}^{f} z_{[1][j]} & \sum_{j=1}^{f} z_{[1][j]} & \dots & \sum_{j=1}^{f} z_{[1][j]} \\ \sum_{j=1}^{f} z_{[2][j]} & \sum_{j=1}^{f} z_{[2][j]} & \dots & \sum_{j=1}^{f} z_{[2][j]} \\ \vdots & & \vdots & \ddots & \vdots \\ \sum_{j=1}^{f} z_{[n][j]} & \sum_{j=1}^{f} z_{[n][j]} & \dots & \sum_{j=1}^{f} z_{[n][j]} \end{bmatrix}. \end{aligned}$$

Refer to [11] for a detailed algorithm description.

We propose a new useful procedure called SumForConv to facilitate the convolution operations for every image, as shown in algorithm 1. Below we illustrate the result of SumForConv taking the example that n and f are both 4 and

that the kernel size is  $3 \times 3$ :

where  $s_{[i][j]} = \sum_{p=i}^{i+2} \sum_{q=j}^{j+2} z_{[p][q]}$  for  $1 \le i \le 2$  and  $1 \le j \le 2$ . While being used in the convolutional layer, the function SumForConv can help to compute some part results of the convolution operation for an image simultaneously, which is very convenient for efficiency and parallelism.

# Algorithm 1 SumForConv: sum some part results of convolution operation after one element-wise multiplication

**Input:** a ciphertect ct. I encrypting a (convolved) image I of size  $h \times w$ , the size  $k \times k$  of some kernal K with its bias  $k_0$ , and a stride of (1,1)

```
Output: a ciphertext ct.I_s encrypting a resulting image I_s of the same size as I
                                                                                                                         \triangleright I_s \in \mathbb{R}^{(h-k+1)\times(w-k+1)}
 1: Set I_s \leftarrow \mathbf{0}
 2: for i := 1 to (h - k + 1) do
          for j := 1 to (w - k + 1) do
 3:
               I_s[i][j] \leftarrow k_0
 4:
 5:
          end for
 6: end for
 7: \mathsf{ct}.I_s \leftarrow \mathsf{Enc}_{pk}(I_s)
     ▶ Accumulate columns (could be computed in parallel)
 8: for pos := 0 to k - 1 do
          \mathsf{ct}.T \leftarrow \mathsf{Rot}(\mathsf{ct}.I,pos)
10:
          \mathsf{ct}.I_s \leftarrow \mathsf{Add}(\mathsf{ct}.I_s, \mathsf{ct}.T)
11: end for
     12: for pos := 1 to k - 1 do
13:
          \mathsf{ct}.T \leftarrow \mathsf{Rot}(\mathsf{ct}.I, pos \times w)
14:
          \mathsf{ct}.I_s \leftarrow \mathsf{Add}(\mathsf{ct}.I_s, \mathsf{ct}.T)
15: end for
     ▶ Build a new designed matrix to filter out the garbage values
                                                                                                                                          \triangleright\,M\in\mathbb{R}^{h\times w}
16: Set M \leftarrow \mathbf{0}
17: for hth := 0 to (h-1) do
18:
          for wth := 0 to (w - 1) do
               if wth \mod k = 0 and wth + k \le width and hth \mod k = 0 and hth + k \le height then
19:
20:
                     M[hth][wth] \leftarrow 1
21:
               end if
          end for
22:
23: end for
24: \operatorname{ct}.M \leftarrow \operatorname{Enc}_{pk}(M)
25: \mathsf{ct}.I_s \leftarrow \mathsf{Mul}(\mathsf{ct}.M, \mathsf{ct}.I_s)
26: return ct.I_s
```

# 2.3 Convolutional Neural Networks

Convolutional Neural Networks are neural networks particularly tailored for image recognition, equipped with two distinct kinds of layers: Convolutional layer (CONV) and Pooling layer (POOL) in addition to another two basic kinds of layers: Fully Connected layer (FC) and Activation layer (ACT). A CNN for image classification has a common architecture:  $[[CONV \to ACT]^p \to POOL]^q \to [CONV \to ACT] \to [FC \to ACT]^r \to FC$ , where p, q and r are integers usually greater than 1. In our implementation, we use the same CNN architecture as [6]:  $[CONV \to ACT] \to [FC \to ACT] \to FC$ .

Convolutional layer is the fundamental basis of a CNN, which has kernels of size  $k \times k$ , a stride of (s,s), and a channel (mapcount) of c. Each kernel has  $k \times k \times c$  parameters besides a kernel bias  $k_0$ , which are all trainable parameters and tuned during the training. Given a greyscale image  $I \in \mathbb{R}^{h \times w}$  and a kernel  $K \in \mathbb{R}^{k \times k}$ , the result of convolving this input image I with stride of (1,1) is the output image  $I' \in \mathbb{R}^{h' \times w'}$  with  $I'_{[i'][j']} = k_0 + \sum_{i=1}^k \sum_{j=1}^k K_{[i][j]} \times I_{[i'+i][j'+j]}$ 

for  $0 < i' \le h - k + 1$  and  $0 < j' \le w - k + 1$ . It can be extended to a color image or a convolved image with many channels, refer to [9] for a detail. If there are multiple kernels, the convolutional layer stacks all the convolved results of each kernel and outputs a three-dimensional tensor. For a single input sample, FC layer only accepts a unidimensional vector, which is why the output of previous layers ( $[[CONV \to ACT]^p \to POOL]$ ) or  $[CONV \to ACT]$ ) should be flattened before being fed into FC layer.

## 3 Technical details

In an HE-based environment, how to encode the database and represent the data will directly affect the feasibility of the implementation of neural networks and their computational efficiency and memory usage. Gilad-Bachrach et al. [1] use a single encoding representation in their solution CryptoNets, where the same features (pixels) of all labeled images are encrypted into a single ciphertext. CryptoNets can thus manage to operate any pixel of all images by accessing the distinct ciphertexts, and thus has no need to use the rotation operation, hindering its packed ciphertexts at full use. Brutzkus et al. [5] later improved CryptoNets by devising other several different representations of the encrypted messages and alternating among them during the computation, in their solution LoLa. LoLa uses ciphertext packing technique and ciphertext rotation operation, and treats the message as a matrix structure just like in the work of Kim et al. [10]. However, there are many redundant data in the matrix that LoLa used to encode the message, where each row of this matrix stores the same data, and hence one SIMD multiplication results in little useful information. Jiang et al. [6] propose an efficient evaluation algorithm for homomorphic multiplication of two matrices A and B based on a row ordering encoding method that encrypts two matrices into two ciphertexts respectively. They also introduce a method to perform the square-matrix transposition on packed ciphertexts over an HE system. The ability of this encoding method to store data is close to the best use of packed plaintext slots to the great extent. However, it is unnecessary to directly encode the second matrix in one ciphertext in an HE system and there is no need to calculate the homomorphic matrix multiplication just like in the plaintext (naive) environment.

In an attempt to overcome these limitations in the former work, we introduce a novel matrix encoding method called Volley Revolver, which is particularly suitable for secure matrix multiplication — the main computation block in privacy-preserving neural networks. The basic idea is to place each semantically-complete unit information (such as an example in a dataset) into the corresponding each row of the matrix and encrypt this matrix into a single ciphertext in a row-by-row manner just as the database encoding method introduced by Kim et al. [10]. When applying this encoding method to private neural networks, Volley Revolver puts the whole weights of every neural node into the corresponding row of a matrix, organizes all the nodes from the same layer into this matrix, and encrypts this matrix into a single ciphertext. When it comes to the homomorphic multiplication of two matrices  $A \in \mathbb{R}^{m \times n}$  and  $B \in \mathbb{R}^{n \times p}$ , in a general case where  $m \geq p$ , Volley Revolver encodes matrix A in the same way as [6] but encodes the padding forms of the transpose of the matrix B by spanning it to a matrix space of the same size as A.

Volley Revolver is designed as a data-encoding method along with two main operations, RowsRevolver and SumForConv to manipulate the matrices, and several associated algorithms, together to perform matrix multiplication and convolution operation in an HE system.

#### 3.1 Encoding Method for Matrix Multiplication

Suppose that we are given an  $m \times n$  matrix  $\mathbf{A}$  and a  $n \times p$  matrix  $\mathbf{B}$  and suppose to compute the matrix  $\mathbf{C}$  of size  $m \times p$  that is the matrix product  $\mathbf{C} = \mathbf{A} \cdot \mathbf{B}$ :

$$\mathbf{A} = \begin{bmatrix} a_{[1][1]} & a_{[1][2]} & \dots & a_{[1][n]} \\ a_{[2][1]} & a_{[2][2]} & \dots & a_{[2][n]} \\ \vdots & \vdots & \ddots & \vdots \\ a_{[m][1]} & a_{[n][2]} & \dots & a_{[m][n]} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} b_{[1][1]} & b_{[1][2]} & \dots & b_{[1][p]} \\ b_{[2][1]} & b_{[2][2]} & \dots & b_{[2][p]} \\ \vdots & \vdots & \ddots & \vdots \\ b_{[n][1]} & b_{[n][2]} & \dots & b_{[n][p]} \end{bmatrix}, \mathbf{C} = \begin{bmatrix} c_{[1][1]} & c_{[1][2]} & \dots & c_{[1][p]} \\ c_{[2][1]} & c_{[2][2]} & \dots & c_{[2][p]} \\ \vdots & \vdots & \ddots & \vdots \\ c_{[m][1]} & c_{[m][2]} & \dots & c_{[m][p]} \end{bmatrix}$$
 where  $c_{[i][j]} = \sum_{k=1}^{n} a_{[i][k]} \times b_{[k][j]}.$ 

For the simplicity of presentation, we assume that each of the three matrices A, B and C could be encrypted into a single ciphertext. We also make the assumption that m is greater than p, m > p. We will not in detail illustrate the other cases where  $m \le p$ , which is similar to this one.

In the case where m>p: Volley Revolver encodes the matrixes  ${\bf A}$  and  ${\bf B}$  into two ciphertexts respectively by using two row-ordering encoding maps to transform the matrices into two matrices of the same size as matrix A. If m and p are not powers of two, we can pad zeros at the end of each row or at the bottom of the matrix. For the matrix  ${\bf A}$ , we adopt the same encoding method that [6] did by the encoding map  $\tau_a: {\bf A} \mapsto \bar{{\bf A}} = (a_{[k/n][k\%n]})_{1 \le k \le m \times n}$ . For the matrix  ${\bf B}$ , we design a very different encoding method from [6] for Volley Revolver: we transpose the matrix  ${\bf B}$ 

first and then extend the transpose matrix in the vertical direction to the size  $m \times n$ . Therefore Volley Revolver uses the encoding map  $\tau_b : \mathbf{B} \mapsto \bar{\mathbf{B}} = (b_{[k/p][k\%n]})_{1 \le k \le m \times n}$ . The result of the mapping  $\tau_b$  is as follow ( $\tau_a$  is an identity map  $\tau_a : \mathbf{A} \mapsto \mathbf{A}$  when m > p):

$$\mathbf{B} = \begin{bmatrix} b_{[1][1]} & b_{[1][2]} & \cdots & b_{[1][p]} \\ b_{[2][1]} & b_{[2][2]} & \cdots & b_{[2][p]} \\ \vdots & \vdots & \ddots & \vdots \\ b_{[n][1]} & b_{[n][2]} & \cdots & b_{[n][p]} \end{bmatrix} \mapsto \begin{bmatrix} b_{[1][1]} & b_{[2][1]} & \cdots & b_{[n][1]} \\ b_{[1][2]} & b_{[2][2]} & \cdots & b_{[n][2]} \\ \vdots & \vdots & \ddots & \vdots \\ b_{[n][1]} & b_{[n][2]} & \cdots & b_{[n][p]} \end{bmatrix} \mapsto \mathbf{\bar{B}} = \begin{bmatrix} b_{[1][1]} & b_{[2][1]} & \cdots & b_{[n][1]} \\ b_{[1][2]} & b_{[2][2]} & \cdots & b_{[n][2]} \\ \vdots & \vdots & \ddots & \vdots \\ b_{[1][n]} & b_{[2][n]} & \cdots & b_{[n][n]} \\ b_{[1][1]} & b_{[2][1]} & \cdots & b_{[n][n]} \\ \vdots & \vdots & \ddots & \vdots \\ b_{[1][n\%p]} & b_{[2][n\%p]} & \cdots & b_{[n][n\%p]} \end{bmatrix}$$

In the following subsection 3.2, we describe an efficient evaluation strategy for homomorphic matrix multiplication based on the Volley Revolver encoding method. For simplicity, we only describe the case where m is greater than p, other cases are similar to the former one and will not be elaborated on in this paper.

#### 3.2 Homomorphic Matrix Multiplication

In this subsection, we report an efficient evaluation algorithm for homomorphic matrix multiplication based on the Volley Revolver encoding method. The algorithm uses a ciphertext ct.R encrypting a plaintext of zero vector as an accumulator and a ciphertext operation RowsRevolver to perform a specific kind of row shifting on the encrypted matrix. For the matrix  $\mathbf{B}$ , RowsRevolver pops up the first row of  $\mathbf{B}$  and appends another corresponding already exist row at the end of  $\mathbf{B}$ :

$$\bar{\mathbf{B}} = \begin{bmatrix} b_{[1][1]} & b_{[2][1]} & \dots & b_{[n][1]} \\ b_{[1][2]} & b_{[2][2]} & \dots & b_{[n][2]} \\ \vdots & \vdots & \ddots & \vdots \\ b_{[1][p]} & b_{[2][p]} & \dots & b_{[n][p]} \\ b_{[1][1]} & b_{[2][1]} & \dots & b_{[n][1]} \\ \vdots & \vdots & \ddots & \vdots \\ b_{[1][p]} & b_{[2][p]} & \dots & b_{[n][p]} \end{bmatrix} \mapsto \begin{bmatrix} b_{[1][2]} & b_{[2][2]} & \dots & b_{[n][2]} \\ \vdots & \vdots & \ddots & \vdots \\ b_{[1][p]} & b_{[2][p]} & \dots & b_{[n][p]} \\ b_{[1][1]} & b_{[2][1]} & \dots & b_{[n][1]} \\ \vdots & \vdots & \ddots & \vdots \\ b_{[1][r]} & b_{[2][r]} & \dots & b_{[n][r]} \\ b_{[1][(r+1)\%p]} & b_{[2][(r+1)\%p]} & \dots & b_{[n][(r+1)\%p]} \end{bmatrix}$$

Algorithm 2 describes how the procedure RowsRevolver generates a new ciphertext from  $ct.\overline{B}$ .

For two ciphertexts ct. A and ct.  $\bar{B}$ , the algorithm for homomorphic matrix multiplication has p iterations. For the k-th iteration where  $1 \le k \le p$ :

- Step 1. this step use RowRevolver on the ct. $\bar{B}$  to generate a a new ciphertext ct. $\bar{B}_1$ , and then computes the homomorphic multiplication between the ciphertexts ct.A and ct. $\bar{B}_1$ , to get the resulting product ct. $A\bar{B}_1$ . When k=1, in this case RowRevolver just return a copy of the ciphertext ct. $\bar{B}$ .
- Step 2. this step use SumColVec on ct. $A\bar{B}_1$  to collect the summation of the data in each row of  $A\bar{B}_1$  for the some inner products of the final result.
- Step 3. this step designs a special plaintext matrix F and uses it to generate a ciphertext  $\operatorname{ct.} F$  for performing one SIMD multiplication between  $\operatorname{ct.} F$  and  $\operatorname{ct.} D$ , in order to filter the redundancy element in each row of  $\operatorname{ct.} D$ , resulting the ciphertext  $\operatorname{ct.} D_1$ .
- Step 4. this step use a global variable ciphertext ct.C initially encrypting the zero matrix to accumulate the intermediate resulting ciphertext  $ct.D_1$ . The matrix C could be initialized by zero or a given value such as the weight bias of the fully connected layer.

The algorithm will repeat step 1 to step 4 for p times and finally aggregates all the intermediate ciphertexts, returning the matrix product  $\mathtt{ct.}C$ .

Algorithm 3 shows how to perform the matrix multiplication operation in an HE environment based on the Volley Revolver encoding method.

Figure 1 describes a simple case for the algorithm 3 where m=2, n=4 and p=2.

The calculation process of this method, especially for the simple case where m=p, is intuitively similar to a special kind of revolver that can fire multiple bullets at once (The first matrix A is settled still while the second matrix  $\mathbf B$  is revolved). That is why we term our encoding method "Volley Revolver".

## **Algorithm 2** RowsRevolver: To shift row like a revolver

**Input:** a ciphertext ct.M encrypting a matrix M of size  $m \times n$ , the number p, and the number idx that is determined in the Algorithm 3

```
Output: a ciphertext ct.R encrypting the resulting matrix R of the same size as M
                                                                                                                                                                          \triangleright R \in \mathbb{R}^{m \times n}
 1: Set R \leftarrow \mathbf{0}
 2: \mathsf{ct}.R \leftarrow \mathsf{Enc}_{pk}(R)
      [Step 1]: Rotate the ciphertext first and then filter out the last row
 3: \mathsf{ct}.T \leftarrow \mathsf{Rot}(\mathsf{ct}.M, n)
      ▶ Build a specially designed matrix to filter out the last row
                                                                                                                                                                         \triangleright F_1 \in \mathbb{R}^{m \times n}
 4: Set F_1 \leftarrow \mathbf{1}
 5: for j := 1 to n do
            F_1[m][j] \leftarrow 0
 6:
 7: end for
 8: \operatorname{ct.} F_1 \leftarrow \operatorname{Enc}_{pk}(F_1)
 9: \mathsf{ct}.T_1 \leftarrow \mathsf{Mul}(\mathsf{ct}.F_1, \mathsf{ct}.T)
      [Step 2]: Rotate the ciphertext first and then filter out the last row
10: \operatorname{ct}.\hat{P} \leftarrow \operatorname{Rot}(\operatorname{ct}.M, n \times ((m\%p + idx + 1)\%p - idx))
      ▶ Build a specially designed matrix to filter out the last row
                                                                                                                                                                         \triangleright F_2 \in \mathbb{R}^{m \times n}
11: Set F_2 \leftarrow \mathbf{0}
12: for j := 1 to n do
            F_2[m][j] \leftarrow 1
13:
14: end for
15: \operatorname{ct.} F_2 \leftarrow \operatorname{Enc}_{pk}(F_2)
16: \operatorname{ct.} T_2 \leftarrow \operatorname{Mul}(\operatorname{ct.} F_2, \operatorname{ct.} P)
      ▷ concatenate
17: \mathsf{ct}.R \leftarrow \mathsf{Add}(\mathsf{ct}.T_1, \mathsf{ct}.T_2)
18: return ct.R
```

## Algorithm 3 Homomorphic matrix multiplication

```
\textbf{Input:} \ \ \text{Two encrypted matrixs ct.} A \ \text{and ct.} \\ \bar{\mathbf{B}} \ \text{for} \ A \in \mathbb{R}^{m \times n}, B \in \mathbb{R}^{n \times p} \ \text{and} \ B \xrightarrow{\texttt{Volley Revolver Encoding}} \bar{\mathbf{B}} \in \mathbb{R}^{m \times n}
Output: The encrypted resulting matrixs ct. C for C \in \mathbb{R}^{m \times p} of the matrix product A \cdot B
  1: Set C \leftarrow \mathbf{0}
                                                \triangleright C: Accumulate intermediate matrices, could be the weight bias of the activation layer
 2: \mathsf{ct}.C \leftarrow \mathsf{Enc}_{pk}(C)
       b the outer loop (could be in parallel)
  3: for idx := 0 to p - 1 do
             \mathsf{ct}.T \leftarrow \mathsf{RowsRevolver}(\mathsf{ct}.\bar{\mathbf{B}}, p, idx)
             \mathsf{ct}.T \leftarrow \mathsf{Mul}(\mathsf{ct}.A, \mathsf{ct}.T)
  5:
             \mathsf{ct}.T \leftarrow \mathsf{SumColVec}(\mathsf{ct}.T)
  6:
             ▷ Build a specifically designed matrix to clean up the redundant values
                                                                                                                                                                          \triangleright\,F\in\mathbb{R}^{m\times n}
             Set F \leftarrow \mathbf{0}
  7:
             for i := 1 to m do
  8:
                   F[i][(i+idx)\%n] \leftarrow 1
 9:
10:
             end for
             \mathsf{ct}.F \leftarrow \mathsf{Enc}_{pk}(F)
11:
12:
             \mathsf{ct}.T \leftarrow \mathsf{Mul}(\mathsf{ct}.F, \mathsf{ct}.T)
             > To accumulate the intermediate
             \mathsf{ct}.C \leftarrow \mathsf{Add}(\mathsf{ct}.C, \mathsf{ct}.T)
13:
14: end for
15: return ct.C
```

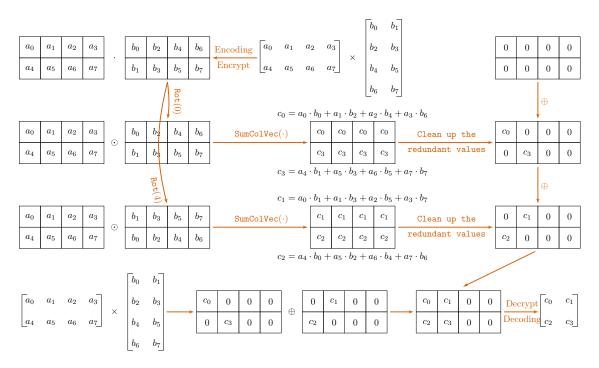


Figure 1: The whole pipeline of homomorphic matrix multiplication

## 3.3 Homomorphic Convolution Operation

In this subsection, we first introduce a novel but impractical algorithm to calculate the convolution operation for a single grayscale image of size  $h \times w$  based on the assumption that this single image can happen to be encrypted into a single ciphertext without vacant slots left, meaning the number n of slots in a packed ciphertext chance to be  $n = h \times w$ . We then illustrate how to use this method to compute the convolution operation of several images of  $any\ size$  at the same time in parallel for a convolutional layer after these images have been encrypted into a ciphertext and been viewed as several virtual ciphertexts inhabiting this real ciphertext. For the simplicity of presentation, we assume that the image is grayscale and the image dataset can be encrypted into a single ciphertext.

#### 3.3.1 An impractical algorithm

Assume that we have a grayscale image I of size  $h \times w$  that is encrypted to a ciphertext  $\operatorname{ct} I$  and a kernel K of size  $k \times k$  with its bias  $k_0$  such that h and w are both greater than k. Based on the assumption that this image  $\operatorname{can} happen$  to be encrypted into a ciphertext  $\operatorname{ct} I$  with no more or less vacant slots, we present an efficient algorithm to compute the convolution operation. For simplicity and practical efficiency, we also set the stride size to the usual default value 1 and adopt no padding technique in this algorithm — these are not musts.

Before the algorithm starts, the kernel K should be called by an operation that we term Kernelspanner to generate  $k \times k$  ciphertexts in advance, each of which encrypts a plaintext matrix  $P_i$  for  $1 \le i \le k \times k$ , using the map to span the  $k \times k$  kernel to a  $h \times w$  matrix space of the same size as the image. For a simple example that h = 4, w = 4 and k = 2, Kernelspanner generates 4 ciphertexts as follow:

$$\begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix} \xrightarrow[\mathbb{R}^{k \times k} \mapsto k^2 \cdot \mathbb{R}^{h \times w}]{} \begin{bmatrix} k_1 & k_2 & k_1 & k_2 \\ k_3 & k_4 & k_3 & k_4 \\ k_1 & k_2 & k_1 & k_2 \\ k_3 & k_4 & k_3 & k_4 \end{bmatrix}, \begin{bmatrix} 0 & k_1 & k_2 & 0 \\ 0 & k_3 & k_4 & 0 \\ 0 & k_1 & k_2 & 0 \\ 0 & k_3 & k_4 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ k_1 & k_2 & k_1 & k_2 \\ k_3 & k_4 & k_3 & k_4 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & k_1 & k_2 & 0 \\ 0 & k_3 & k_4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & k_1 & k_2 & 0 \\ 0 & k_3 & k_4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & k_1 & k_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & k_1 & k_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & k_1 & k_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & k_1 & k_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & k_1 & k_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Algorithm 4 describes this transformation Kernelspanner in detail.

Like the homomorphic algorithm for matrix multiplication, our method needs an accumulator ciphertext ct.R to accumulate the intermediate ciphertexts, which should be initially encrypted by the kernel bias  $k_0$ . Following the above

## **Algorithm 4** Kernelspanner: Span the kernel to a matrix space

**Input:** A kernel K of the size  $k \times k$  with its bias  $k_0$ , the size  $k \times k$  of a image (matrix), and a stride of (1,1)**Output:** The  $k^2$  ciphertexts ct. $S_{[i]}$  for  $1 \le i \le k^2$ , each of which encrypts a resulting matrix (spanned kernel) of the  $\triangleright$  To generate  $k^2$  ciphertexts (could be in parallel) 1: **for** i := 0 to k - 1 **do for** j := 0 to k - 1 **do** 2:  $\triangleright \, M \in \mathbb{R}^{h \times w}$ 3: Set  $M \leftarrow \mathbf{0}$ **for** hth := 0 to h - 1 **do** 4: 5: for wth := 0 to w - 1 do if  $(wth-i) \mod k = 0$  and  $wth+k \le w$  and  $(hth-j) \mod k = 0$  and  $hth+k \le h$  then 6: **for** kri := 0 to k - 1 **do** 7: 8: **for** kci := 0 to k - 1 **do** 9:  $M[hth + kri + 1][wth + kci + 1] \leftarrow K[kri + 1][kci + 1]$ 10: end for end for 11: 12: end if end for 13: end for 14: 15:  $\mathsf{ct.}S_{[i\times k+j+1]} \leftarrow \mathsf{Enc}_{pk}(M)$ 16:

example, the kernel bias  $k_0$  will be used to generate a ciphertext as follow:

17: **end for** 

18: **return** ct. $S_{[i]}$  for  $1 \le i \le k^2$ 

$$[k_0] \mapsto Enc \begin{bmatrix} k_0 & k_0 & k_0 & 0 \\ k_0 & k_0 & k_0 & 0 \\ k_0 & k_0 & k_0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Our impractical homomorphic algorithm for convolution operation proceeds through  $k \times k$  iterations, and the *i*-th iteration consists of the following four steps for  $1 \le i \le k^2$ :

- Step 1. For the ciphertexts ct.I and  $ct.P_i$ , this step computes their SIMD multiplication and outputs the ciphertext  $ct.R_i$ .
- Step 2. To aggregate the values of some blocks of size  $k \times k$ , this step applies the procedure SumForConv to the ciphertext ct. $R_i$  in order to generate ciphertext ct. $\bar{R}_i$ . This step can obtain some of the final convolution results.
- Step 3. This step generates a ciphertext encrypting a specially-designed matrix and then uses it to filter out the garbage data in  $\mathtt{ct}.\bar{R}_i$  by one ciphertext multiplication, and outputs a ciphertext  $\mathtt{ct}.\tilde{R}_i$ .
- Step 4. At this step, the homomorphic convolution-operation algorithm updates the accumulator ciphertext ct.R by homomorphically adding ct. $\tilde{R}_i$  to it.

Note that the steps 1–3 in this algorithm can be computed in parallel with  $k \times k$  threads. Figure 2 describes a simple case for the algorithm where h=3, w=4 and k=3.

In a nutshell, we describe how to compute homomorphic convolution operation in Algorithm 5 in detail.

In the following subsection, we will show how to make this impractical homomorphic algorithm work efficiently in real-world cases.

## 3.3.2 Encoding Method for Convolution Operation

For simplification, we assume that the dataset  $X \in \mathbb{R}^{m \times f}$  can be encrypted into a single ciphertext ct.X, m is a power of two, all the images are grayscale and have the size  $h \times w$ . The first layer of a CNN is the convolutional layer, whose input is the dataset. The encoding method for convolution operation has to deal with the images from a dataset X and the convolved images from the previous convolutional layer. Volley Revolver can deal with both situations with a single formation and encodes the dataset as a matrix using the database encoding method [10]. In most cases,  $h \times w < f$ , if this happened, zeros could be used for padding:

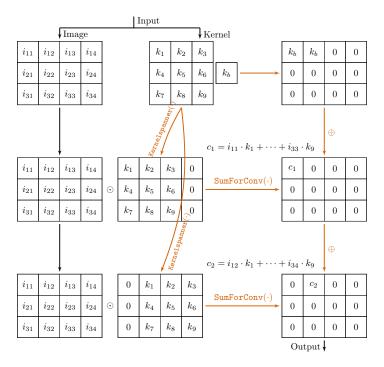


Figure 2: The whole pipeline of homomorphic convolution operation

$$X = \begin{bmatrix} I_{[1][1]}^{(1)} & I_{[1][2]}^{(1)} & \dots & I_{[h][w]}^{(1)} \\ I_{[1][1]}^{(2)} & I_{[1][2]}^{(2)} & \dots & I_{[h][w]}^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ I_{[1][1]}^{(m)} & I_{[1][2]}^{(m)} & \dots & I_{[h][w]}^{(m)} \end{bmatrix} \mapsto \mathsf{ct}. \\ X = Enc \begin{bmatrix} I_{[1][1]}^{(1)} & I_{[1][2]}^{(1)} & \dots & I_{[h][w]}^{(1)} & 0 & \dots & 0 \\ I_{[1][1]}^{(2)} & I_{[1][2]}^{(2)} & \dots & I_{[h][w]}^{(2)} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ I_{[1][1]}^{(m)} & I_{[1][2]}^{(m)} & \dots & I_{[h][w]}^{(m)} & 0 & \dots & 0 \end{bmatrix}.$$

What is more, Volley Revolver extends this database encoding method with some additional operations to view the dataset matrix X as a three-dimensional structure.

Algorithm 5 is a feasible and efficient way to calculate the secure convolution operation in an HE encryption domain. However, its working-environment assumption that the size of an image is exactly the length of the plaintext, which rarely happens, is too strict to make it a practical algorithm, leaving this algorithm directly useless. In addition, algorithm 5 can only deal with one image at a time due to the assumption that a single ciphertext only encrypts only one image, which is too inefficient for real-world applications.

To solve this problem, Volley Revolver needs some simulated operations on the ciphertext  $\operatorname{ct} X$  to treat the two-dimensional dataset as a three-dimensional structure, which is the true structure for a grayscale image dataset. These simulated operations together could simulate the first continual space of the same size as an image of each row of the matrix encrypted in a real ciphertext as a virtual ciphertext that can perform all the HE operations. Moreover, the number of plaintext slots is usually set to a large number so that a single ciphertext could encrypt several images. Since machine learning algorithms do the same calculation for every example (image) from the dataset, we can thus perform the same simulated HE operations on these virtual ciphertexts simultaneously by operating the real ciphertext. For example, the ciphertext encrypting the dataset  $X \in \mathbb{R}^{m \times f}$  could be used to simulate m virtual ciphertexts  $\operatorname{vct}_i$  for

# Algorithm 5 Homomorphic convolution operation

```
Input: An encrypted Image ct.I for I \in \mathbb{R}^{h \times w} and a kernel K of size k \times k with its bias k_0
Output: The encrypted resulting image ct.I_s where I_s has the same size as I
       > The Third Party owing the kernel perform Kernelspanner and prepares the ciphertext encrypting the kernel bias
                                                                                                                                                                    \triangleright for where 1 \le i \le k^2
  1: ct.S_{[i]} \leftarrow \texttt{Kernelspanner}(K, h, w)
                                                                                                                                                                                       \triangleright I_s \in \mathbb{R}^{h \times w}
  2: Set I_s \leftarrow \mathbf{0}
 3: for i := 1 to h - k + 1 do
 4:
             for j := 1 to w - k + 1 do
                    I_s[i][j] \leftarrow k_0
 5:
              end for
 6:
 7: end for
 8: \mathsf{ct}.I_s \leftarrow \mathsf{Enc}_{pk}(I_s)
       So begins the Cloud its work
 9: for i := 0 to k - 1 do
              for j := 0 to k - 1 do
10:
11:
                    \mathsf{ct}.T \leftarrow \mathsf{Mul}(\mathsf{ct}.I, \mathsf{ct}.S_{[i \times k + j + 1]})
12:
                    \mathtt{ct}.T \leftarrow \mathtt{SumForConv}(\mathtt{ct}.T)
                    ▷ Design a matrix to filter out redundant values
                    ▶ Build a specifically designed matrix to clean up the redundant values
                                                                                                                                                                                       \triangleright F \in \mathbb{R}^{m \times n}
13:
                    Set F \leftarrow \mathbf{0}
                    for hth := 0 to h - 1 do
14:
                           for wth := 0 to w - 1 do
15:
                                  if (wth-i) \mod k = 0 and wth+k \le w and (hth-j) \mod k = 0 and hth+k \le h then
16:
                                        F[hth][wth] \leftarrow 1
17:
                                  end if
18:
                           end for
19:
                    end for
20:
21:
                    \mathsf{ct}.F \leftarrow \mathsf{Enc}_{pk}(F)
                    \mathsf{ct}.T \leftarrow \mathsf{Mul}(\mathsf{ct}.F, \mathsf{ct}.T)
22:
                    > To accumulate the intermediate
23:
                    \mathsf{ct}.I_s \leftarrow \mathsf{Add}(\mathsf{ct}.I_s, \mathsf{ct}.T)
24:
              end for
25: end for
26: return ct.I_s
1 \le i \le m, as shown below:
\mathtt{ct}.X = \!Enc \begin{bmatrix} I_{[1][1]}^{(1)} & I_{[1][2]}^{(1)} & \dots & I_{[h][w]}^{(1)} & 0 & \dots & 0 \\ I_{[1][1]}^{(2)} & I_{[1][2]}^{(2)} & \dots & I_{[h][w]}^{(2)} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ I_{[1][1]}^{(m)} & I_{[1][2]}^{(m)} & \dots & I_{[h][w]}^{(m)} & 0 & \dots & 0 \end{bmatrix} \\ \to Enc \begin{bmatrix} \mathtt{vEnc} \left[ I_{[1][1]}^{(1)} & I_{[1][2]}^{(1)} & \dots & I_{[h][w]}^{(1)} \right] & 0 & \dots & 0 \\ \mathtt{vEnc} \left[ I_{[1][1]}^{(2)} & I_{[1][2]}^{(2)} & \dots & I_{[h][w]}^{(2)} \right] & 0 & \dots & 0 \\ \mathtt{vEnc} \left[ I_{[1][1]}^{(m)} & I_{[1][2]}^{(m)} & \dots & I_{[h][w]}^{(m)} \right] & 0 & \dots & 0 \end{bmatrix},
```

Similar to a real ciphertext, a virtual ciphertext has the following virtual HE operations: vEnc, vDec, vAdd, vMul, vRescale, vBootstrapping, vRot.

vEnc and vDec: A virtual ciphertext doesn't need to perform the virtual operations vEnc and vDec due to the fact that we already assume the virtual ciphertext has inhabited a real ciphertext and the encoding method will take care of them.

*vAdd*, *vMul*, *vRescale* and *vBootstrapping*: These four virtual HE operations are all extended from the real ciphertext, meaning that the same operations for the four corresponding real ciphertext operations, namely Add, Mul, Rescale and Bootstrapping, result in the same corresponding virtual operations on the virtual ciphertexts.

vRot: The rotation operation on the virtual ciphertexts is much different from other virtual operations that are inherently the same along with the real ciphertext operations and that can extend from the real ciphertext. vRot needs two rotation operations of the real ciphertext, as described in algorithm 6. We only need to simulate the rotation operation on these virtual ciphertexts to complete the simulation. The virtual rotation operation vRot(ct, r), to rotate the virtual ciphertexts inside the real ciphertext ct to the left by r positions, has the following simulation result:

Given two sets of virtual ciphertexts  $vct_{[i]}$  and  $vct_{[j]}$  that inhabit two ciphertexts  $ct_1$  and  $ct_2$  respectively, for  $1 \le i \le m$  and  $1 \le j \le m$ , the corresponding virtual HE operation  $vAdd(vct_{[i]}, vct_{[j]})$  results:

$$\begin{split} \operatorname{ct}_1 &= Enc \begin{bmatrix} I_{[1][1]}^{(1)} & \dots & I_{[1][w]}^{(1)} \\ \vdots & \ddots & \vdots \\ I_{[h][1]}^{(1)} & \dots & I_{[h][w]}^{(1)} \end{bmatrix} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_{[h][1]}^{(1)} & \dots & I_{[h][w]}^{(1)} \end{bmatrix} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_{[h][1]}^{(1)} & \dots & I_{[h][w]}^{(1)} \end{bmatrix} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_{[h][1]}^{(1)} & \dots & I_{[h][w]}^{(1)} \end{bmatrix} & 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ I_{[h][1]}^{(m)} & \dots & I_{[h][w]}^{(m)} \end{bmatrix} & 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ I_{[h][1]}^{(m)} & \dots & I_{[h][w]}^{(m)} \end{bmatrix} & 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ I_{[h][1]}^{(m)} & \dots & I_{[h][w]}^{(m)} \end{bmatrix} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_{[h][1]}^{(m)} & \dots & I_{[h][w]}^{(m)} + G_{[h][w]}^{(1)} \end{bmatrix} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_{[h][1]}^{(m)} & \dots & I_{[h][w]}^{(m)} + G_{[h][w]}^{(m)} \end{bmatrix} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_{[h][1]}^{(m)} & \dots & I_{[h][w]}^{(m)} + G_{[h][w]}^{(m)} \end{bmatrix} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_{[h][1]}^{(m)} & \dots & I_{[h][w]}^{(m)} + G_{[h][w]}^{(m)} \end{bmatrix} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_{[h][1]}^{(m)} & \dots & I_{[h][w]}^{(m)} + G_{[h][w]}^{(m)} \end{bmatrix} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_{[h][1]}^{(m)} & \dots & I_{[h][w]}^{(m)} + G_{[h][w]}^{(m)} \end{bmatrix} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_{[h][1]}^{(m)} & \dots & I_{[h][w]}^{(m)} + G_{[h][w]}^{(m)} \end{bmatrix} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_{[h][1]}^{(m)} & \dots & I_{[h][w]}^{(m)} + G_{[h][w]}^{(m)} \end{bmatrix} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_{[h][1]}^{(m)} & \dots & I_{[h][w]}^{(m)} + G_{[h][w]}^{(m)} \end{bmatrix} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_{[h][1]}^{(m)} & \dots & I_{[h][w]}^{(m)} + G_{[h][w]}^{(m)} \end{bmatrix} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots$$

The virtual HE operation vMul obtains a similar result.

To bring all the pieces together, we can use the algorithm 5 to perform convolution operations for several images in parallel based on the simulation virtual ciphertexts and the simulation virtual operations.

The most interesting and efficient part of this simulated operation is that a sequence of operations on a real ciphertext results in the same corresponding operations on the multiple (m in this case) virtual ciphertexts, which would suffice the real-world applications. (How could I come up with such a bizarre idea!)

## Algorithm 6 vRot: virtual rotation operations on a set of virtual ciphertexts

**Input:** a ciphertext ct.X encrypting a dataset matrix X of size  $m \times n$ , each row of which encrypts an image of size  $h \times w$  such that  $h \times w \le n$ , and the number r of rotations to the left **Output:** a ciphertext ct.R encrypting the resulting matrix R of the same size as X

```
\triangleright R \in \mathbb{R}^{m \times n}
 1: Set R \leftarrow \mathbf{0}
 2: \mathsf{ct}.R \leftarrow \mathsf{Enc}_{nk}(R)
      [Step 1]: Rotate the real ciphertext to the left by r positions first and then clean up the garbages values
 3: \mathsf{ct}.T \leftarrow \mathsf{Rot}(\mathsf{ct}.X, r)
      ▶ Build a specially designed matrix
 4: Set F_1 \leftarrow \mathbf{0}
                                                                                                                                                             \triangleright F_1 \in \mathbb{R}^{m \times n}
  5: for i := 1 to m do
            for j := 1 to h \times w - r do
  7:
                  F_1[i][j] \leftarrow 1
 8:
            end for
 9: end for
10: \mathsf{ct}.F_1 \leftarrow \mathsf{Enc}_{pk}(F_1)
11: \mathsf{ct}.T_1 \leftarrow \mathsf{Mul}(\mathsf{ct}.F_1, \mathsf{ct}.T)
      [Step 2]: Rotate the real ciphertext to the right by h \times w - r positions (the same as to the left by m \times n - h \times w + r
      positions) first and then clean up the garbage values
12: \mathsf{ct}.P \leftarrow \mathsf{Rot}(\mathsf{ct}.X, m \times n - h \times w + r)
      ⊳ Build a specilal designed matrix
13: Set F_2 \leftarrow \mathbf{0}
                                                                                                                                                             \triangleright F_2 \in \mathbb{R}^{m \times n}
14: for i := 1 to m do
15:
            for j := h \times w - r + 1 to h \times w do
16:
                  F_2[m][j] \leftarrow 1
17:
            end for
18: end for
19: \mathsf{ct}.F_2 \leftarrow \mathsf{Enc}_{pk}(F_2)
20: \mathsf{ct}.T_2 \leftarrow \mathsf{Mul}(\mathsf{ct}.F_2, \mathsf{ct}.P)
      21: \mathsf{ct}.R \leftarrow \mathsf{Add}(\mathsf{ct}.T_1, \mathsf{ct}.T_2)
22: return ct.R
```

# 4 Privacy-preserving CNN Inference

## 4.1 Limitations on applying CNN to HE

Homomorphic Encryption has a main limitation when used for machine learning applications like CNN: it cannot directly compute functions such as the Relu activation function. The common solution is to replace the undirectly-computed function with a polynomial approximation over a small range domain, by using the "least square" method that has been implemented by several softwares such as the function polyfit in Octave, MatLab and Python.

We use Octave to generate a degree-three polynomial over some domain, and just initialize all the activation layers with this polynomial and leave the training process to determine the coefficients of polynomials for every activation Layer, without paying too much care to it.

Other computation operations, such as matrix multiplication in the fully connected layer and convolutional operation in the convolutional layer, can also be done by the algorithms we proposed above.

To make it possible to implement privacy-preserving CNN inference, we first tailor the naive CNN so as to be compatible with HE: we just replace the ReLU function in all the activation layers with the same polynomial of degree 3; Next, we train this tailored CNN on the MNIST dataset, which will tune all the trainable parameters such as the coefficients of the polynomials of each activation layer, the parameters of each kernel of convolutional layer and the weight matrix of the fully connected layer. Finally, after the training process is finished and the parameters are well tuned, the tailored CNN model is ready to prepare for private inference. We can then use this CNN model to perform privacy-preserving CNN inference on the MNIST testing dataset for evaluating the performance of this tailored model.

Table 1: The CNN description of [6] and ours

Layer	[6]	Our CNN
CONV	64 input images of size $28 \times 28$ ,	32 input images of size $28 \times 28$ ,
	4 kernels of size $7 \times 7$ (4 channels),	4 kernels of size $3 \times 3$ (4 channels),
	stride size of (3, 3)	stride size of $(1, 1)$
ACT-1	Squaring 256 input values $x \mapsto x^2$	$x \mapsto -0.00015120704 + 0.4610149 \cdot x$
		$+2.0225089 \cdot x^2 - 1.4511951 \cdot x^3$
FC-1	Fully connecting with $8 \times 8 \times 4 = 256$ inputs	Fully connecting with $26 \times 26 \times 4 = 2704$
	and 64 (neural nodes) outputs	inputs and 64 outputs
ACT-2	Squaring 64 input values $x \mapsto x^2$	$x \mapsto -1.5650465 - 0.9943767 \cdot x$
		$+1.6794522 \cdot x^2 + 0.5350255 \cdot x^3$
FC-2	Fully connecting with 64 inputs and 10 outputs	Fully connecting with 64 inputs and 10 outputs

## 4.2 Neural Networks Architecture

We adopt the same CNN architecture as [6] but with some different hyperparameters, even though our method based on Volley Revolver can use a convolutional neural network as deep as possible. However, in this case, the computation time will therefore increase and the bootstrapping operation will have to be used to refresh the ciphertext, resulting in more time-consuming. For the MNIST database, this CNN architecture of [6] is good enough, achieving an accuracy of 98.65%. Table 1 gives a description of the two neural networks architectures between [6] and ours to the MNIST dataset.

## 4.3 Usage Model and Preparations

Our approach can be used in several usage scenarios as illustrated in [11]. Whatever the scenario is, the data to be privacy-preserving and the tailored CNN model have to be encrypted before being outsourced to the cloud for its service. In a reasonable usage scenario, there are a few different roles including data owner, the model provider, and the cloud server. In some special scenarios, the first two roles can be the same one who would like to get the service from the cloud server.

Based on this usage model, we can make some plausible assignment preparations for the three roles:

- 1. Data Owner: Data owner should prepare its dataset, such as cleaning its data and normalizing its data, and even partitioning the dataset into multiple mini-batches of a suitable size if it is too large. Finally, the data owner encrypts its dataset using the database encoding method introduced by [10] and then sends the ciphertext to the cloud.
- 2. Model Provider: Model provider needs to use Kernelspanner to generate several ciphertexts that encrypt the kernel information, and the Volley Revolver method to encode the weights matrix of fully connected layer and encrypt the resulting matrix later. For the activation layer, the polynomial approximation of ReLU function could be just sent to the cloud as some public parameters without the need for encrypting.
- 3. Cloud Server: Cloud Server provides the cloud service. To this end, the application (program) to implement the homomorphic CNN algorithm should be deployed on the cloud in advance. Some public keys that are essential for HE operations should also be sent to the cloud. Such preparations could be done with the coordination and cooperation of the model provider and cloud server.

Now, we can put the private-preserving CNN inference into practice.

# 5 Experimental Results

We use the C++ programming language to implement our homomorphic CNN inference. Our complete and runnable source code is publicly available at https://github.com/petitioner/HE.CNNinfer.

## 5.1 Database

We evaluate our implementation of the homomorphic CNN model on the MNIST dataset to each time calculate ten possibilities for 32 images of handwritten digits. Having been a typical benchmark for machine learning systems, the MNIST database includes a training dataset of 60 000 images and a testing dataset of 10 000, each image of which is of

size  $28 \times 28$ . For such an image, each pixel is represented by a 256-level grayscale. Moreover, each image depicts a digit from zero to nine and is labeled with it.

# 5.2 Building a model in the clear

In order to build a homomorphic model, we follow the normal approach for the machine-learning training in the clear—except that we replace the normal ReLU function with a polynomial approximation: we (1) train the CNN model described in subsection 4.2 with the MNIST training dataset being normalized into domain [0, 1], just like the normal routine in the clear, and then we (2) implement the well-trained resulting CNN model from step (1) using the HE library and HE programming.

For step (1) we adopt the highly customizable library keras with Tensorflow, which provides us with a simple framework for defining our own model layers such as the activation layer to enact the polynomial activation function. Having obtained a well-trained CNN model, we store the model weights into a CSV file. In step (2) we use the HE programming to implement the CNN model, accessing its weights from the CSV file generated by step (1).

# 5.3 Classifying plaintext inputs

During the training for the homomorphic CNN model, we set the Tensorflow to a precision of 32-bit floating-point type. We normalize the MNIST training dataset by dividing each pixel by the float value 255.

After many attempts to obtain a decent CNN model with higher inference precision on the testing dataset, we finally get a CNN model that could reach a precision of 98.66% in the clear testing dataset.

## 5.4 Classifying encrypted inputs

Implementing our CNN model inference over HE encrypted image dataset was more than a mere complex coding exercise, but also collaboration in cooperating with each layer of the CNN model and arranging consistent. We implement the homomorphic CNN inference with the library HEAAN by [8], which is openly available on GitHub [12, 13] and which was used to efficiently support approximate arithmetic.

Note that before encrypting the testing dataset of images, we also normalize the MNIST testing dataset by dividing each pixel by the float value 255, just like the normal procedure on the training dataset in the clear.

**Parameters**. We follow the notation of [10] and set the HE scheme parameters for our implementment:  $\Delta=2^{45}$  and  $\Delta_c=2^{20}$  (wBits  $=\log_2\Delta=45$  and pBits  $=\log_2\Delta_c=20$ ); slots =32768;  $\log Q=1200$  and  $\log N=16$  to achieve a security level of 80-bits. (see [11, 6] for more details on these parameters).

**Result**. We evaluate the performance of our implementation on the MNIST testing dataset of 10 000 images. Since in this case Volley Revolver encoding method can only deal with 32 MNIST images at one time, we thus partition the 10 000 MNIST testing images into 313 blocks with the last block padded zeros to make it full. We then test the homomorphic CNN inference on these 313 ciphertexts each of which encrypts a block of 32 images. We finally obtain a classification accuracy of 98.65% over the 313 ciphertexts. The processing of each ciphertext outputs 32 digits with the highest probability of each image, and it takes less than 30 minutes on a google cloud server with 16 vCPUs.

**Analysis**. There is a slight difference in the accuracy between the clear and the encryption, which is due to the fact that the accuracy under the ciphertext is not the same as that under the plaintext.

In order to save the modulus, a TensorFlow Lite model could be used to reduce the accuracy under the plaintext from float 32 to float 16.

#### 6 Conclusion

The encoding method we proposed in this work, Volley Revolver, is particularly tailored for privacy-preserving neural networks and probably can be used to assist the private neural networks training, in which case for the backpropagation algorithm of the fully connected layer the first matrix is revolved while the second matrix is settled to be still. We leave it as an opening future study.

We shifted some work related to the CNN model to the model provider and some data preparation processes to the data owner so as to complete the homomorphic CNN inference. We believe it is all right for privacy-preserving inference due to no sensitive information leaking.

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