Probability and Applied Statistics Formula Sheet
Alexis Petito

Definition 1.1 – Mean of n Measured Responses.
The mean of a sample *n* measured responses.
$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

Definition 1.2 – Variance of Sample Measurements
The variance of a sample of measurements.
$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$$

Definition 1.3 – Standard Deviation of Sample Measurements
Standard deviation of a sample measurements.
$$s = \sqrt{s^2}$$

Definition 2.7 - Permutations
Ordered arrangement of *r* distinct objects called a *permutation***.**
$$P_r^n = \frac{n!}{(n-r)!}$$

Definition 2.8 - Combinations
The number of *combinations* of *n* objects taken *r* at a time
$$C_r^n = \frac{P_r^n}{r!} = \frac{n!}{r!\,(n-r)!}$$

Theorem 2.3 – n Objects into K Groups
The number of ways partitioning *n* distinct objects into *k* distinct groups containing *n* objects, respectively where each object appears in exactly one group and $\sum_{i=1}^{k} n_i = n$, is
$$N = \binom{n}{n_1\ n_2\ \ldots n_k} = \frac{n!}{n_1!\ n_2\ !\ldots n_k!}$$

Definition 2.9 – A Given B
The *conditional probability of an event* A, given that an event B has occurred, is equal to
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Definition 2.10 – Determining Independence
Two events A and B are said to be *independent* if any one of the following holds:
$P(A|B) = P(A)$
$P(B|A) = P(B)$
$P(A \cap B) = P(A)P(B)$
Otherwise, the events are said to be *dependent*.

Theorem 2.5 - The Multiplicative Law of Probability
The probability of the intersection of two events A and B is
$P(A \cap B) = P(A)P(B|A)$
$= P(B)P(A|B)$
If A and B are independent, then
$P(A \cap B) = P(A)P(B)$

Theorem 2.6 - The Additive Law of Probability
The probability of the union of two events A and B is
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
If A and B are mutually exclusive events, $P(A \cap B) = 0$ and
$P(A \cup B) = P(A) + P(B)$

Theorem 2.7 – Determine A from $\overline{A}$
If A is an event, then
$P(A) = 1 - P(\overline{A})$

Definition 2.11 – Partition of S
For some positive integer k, let the sets $B_1 B_2 B_3 \ldots, B_k$ be such that
1. $S = B_1 \cup B_2 \cup \ldots \cup B_k$
2. $B_i \cap B_j = \emptyset$, for $i \neq j$

Then the collection of sets $\{B_1 B_2, \ldots, B_k\}$ is said to be a *partition* of S

Theorem 2.8
Assume that $\{B_1 B_2, \ldots, B_k\}$ is a partition of S (see definition 2.11) such that $P(B_i) > 0$, for $i = 1,2, \ldots, k$. Then for any event A

$$P(A) = \sum_{i=1}^{k} P(A|B_i)P(B_i)$$

Theorem 2.9 - Bayes' Rule
Assume that $\{B_1 B_2, \ldots, B_k\}$ is a partition of S (see definition 2.11) such that $P(B_i) > 0$, for $i = 1,2, \ldots, k$. Then

$$P(B_j|A) = \frac{P(A|B_j)P(B_j)}{\sum_{i=1}^{k} P(A|B_i)P(B_i)}$$

Bayes' Theorem
Bayes Theorem: For two events A and B in sample space S, with $P(A) > 0$ and $P(B) > 0$,

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

If $0 < P(B) < 1$, we may write by the *Theorem of Total Probability*.

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|\overline{B})P(\overline{B})}$$

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Conditional Probability

$\mathrm{P(A|B)} = \frac{\mathrm{P(A \cap B)}}{\mathrm{P(B)}}$

Probability Mass Function

$p(y) = P(Y = y)$

Expected Value

Let Y be a discrete random variable with the probability function $p(y)$. Then the *expected value* of Y, $E(Y)$, is defined to be,

$\mathrm{E}(Y) = \sum_{y \epsilon Y} yp(y)$

Theorem 3.2 – Binomial Distribution

Let Y be a discrete random variable with probability function $p(y)$ and $g(y)$ be a real-valued function of Y. Then the expected value of $g(Y)$
Is given by,

$\mathrm{E}[g(Y)] = \sum_{all\ y} g(y)p(y)$

Definition 3.5 – Expected Variance of a Random Variable

If Y is a random variable with mean $E(Y) = \mu$, the variance of a random varible Y is defined to be the expected value of $(Y - \mu)^2$. That is,

$V(Y) = E[(Y - \mu)^2]$

The Standard Deviation of Y

$\sqrt{V[Y]}$

Binomial Distribution

$p(y) = P(Y = y) = \binom{n}{y} p^y q^{n-y} \qquad y \epsilon \{0,1,2, \dots, n\}$

Theorem 3.7 – Binomial Distribution Expected Value and Variance

Let Y be a binomial random variable based on *n* trials and success probability *p*. Then,

$\mu = E(Y) = np$ (Mean/Expected)

$\sigma^2 = V(Y) = npq$ (Standard Deviation)

Geometric Distribution

$p(y) = \mathrm{P}(Y = y) = q^{y-1}p$

Extra Geometric Distribution Formulas

A success occurs on or before the nth trial.

$P(X \leq n) = 1 - (1-p)^n$

A success occurs before the nth trial.

$P(X < n) = 1 - (1-p)^{n-1}$

A success occurs on or after the nth trial.

$P(X \geq n) = (1-p)^{n-1}$

A success occurs after the nth trial.

$P(X > n) = (1-p)^n$

Theorem 3.8 – Geometric Distribution Expected Value and Variance

If Y is a random variable with geometric distribution,

$$\mu = E(Y) = \frac{1}{p} \; and \; \sigma^2 = V(Y) = \frac{1-p}{p^2}$$

Hypergeometric Distribution

$$p(y) = P(Y = y) = P(A) = \frac{n_A}{n_S} = \frac{\binom{r}{y} \, x \binom{N-r}{n-y}}{\binom{N}{n}}$$

Theorem 3.10 – Hypergeometric Distribution Expected Value and Variance

If Y is a random variable with hypergeometric distribution,

$$\mu = E(Y) = \frac{nr}{N} \; and \; \sigma^2 = V(Y) = n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$$

Negative Binomial Distribution

$$p(y) = \binom{y-1}{r-1} p^r q^{y-r}$$

Theorem 3.9 – Negative Binomial Distribution Expected Value and Variance.

If Y is a random variable with negative binomial distribution,

$$\mu = E(Y) = \frac{r}{p} \; and \; \sigma^2 = V(Y) = \left(\frac{r(1-p)}{p^2}\right)$$

Definition 3.11 – Poisson Distribution

A random variable Y is said to have *Poisson probability distribution* if and only if,

$$p(y) = \frac{\lambda^y}{y!} e^{-\lambda}, \quad y = 0, 1, 2, \ldots, \lambda > 0.$$

Theorem 3.11 – Poisson Distribution Expected Value and Variance

If Y is a random variable possessing a Poisson Distribution with parameter λ, then

$\mu = E(Y) = \lambda \; and \; \sigma^2 = V(Y) = \lambda$

Theorem 3.4 – Chebyshev's Theorem

Let Y be a random variable with mean μ and finite variance σ^2. Then, for any constant $k > 0$,

$$P(|Y - \mu| < k\sigma) \geq 1 - \frac{1}{k^2} \text{ or } P(|Y - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$