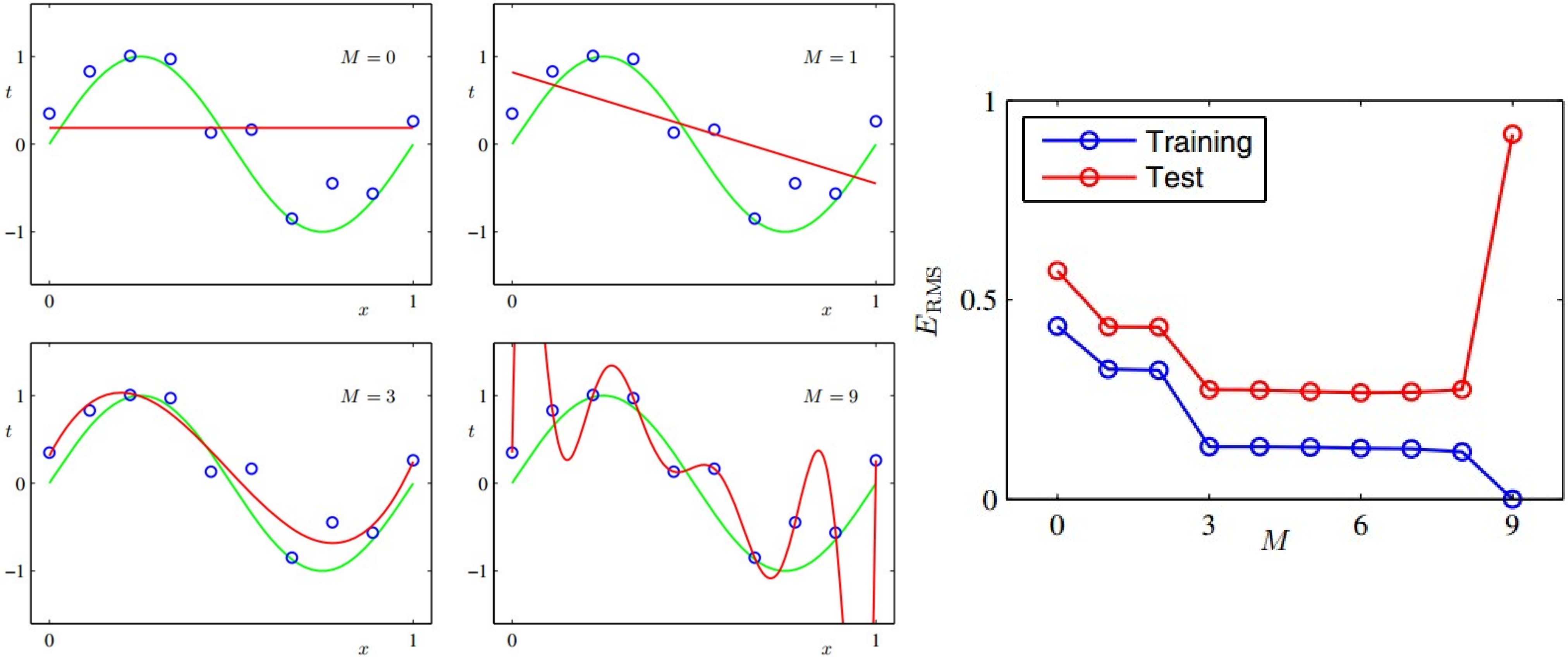
Lecture 4 – Part I
Regularization

- Motivation, definition
- Observation: Large weights and overfitting
- Regularization: closed form in linear regression + intuitions
- Does it work ? A few examples
- The Bayesian interpretation
- Regul during GD: Parameter shrinkage, weight decay
- Lasso

Complexity controlled **explicitly** (rare case)

M = polynomial order



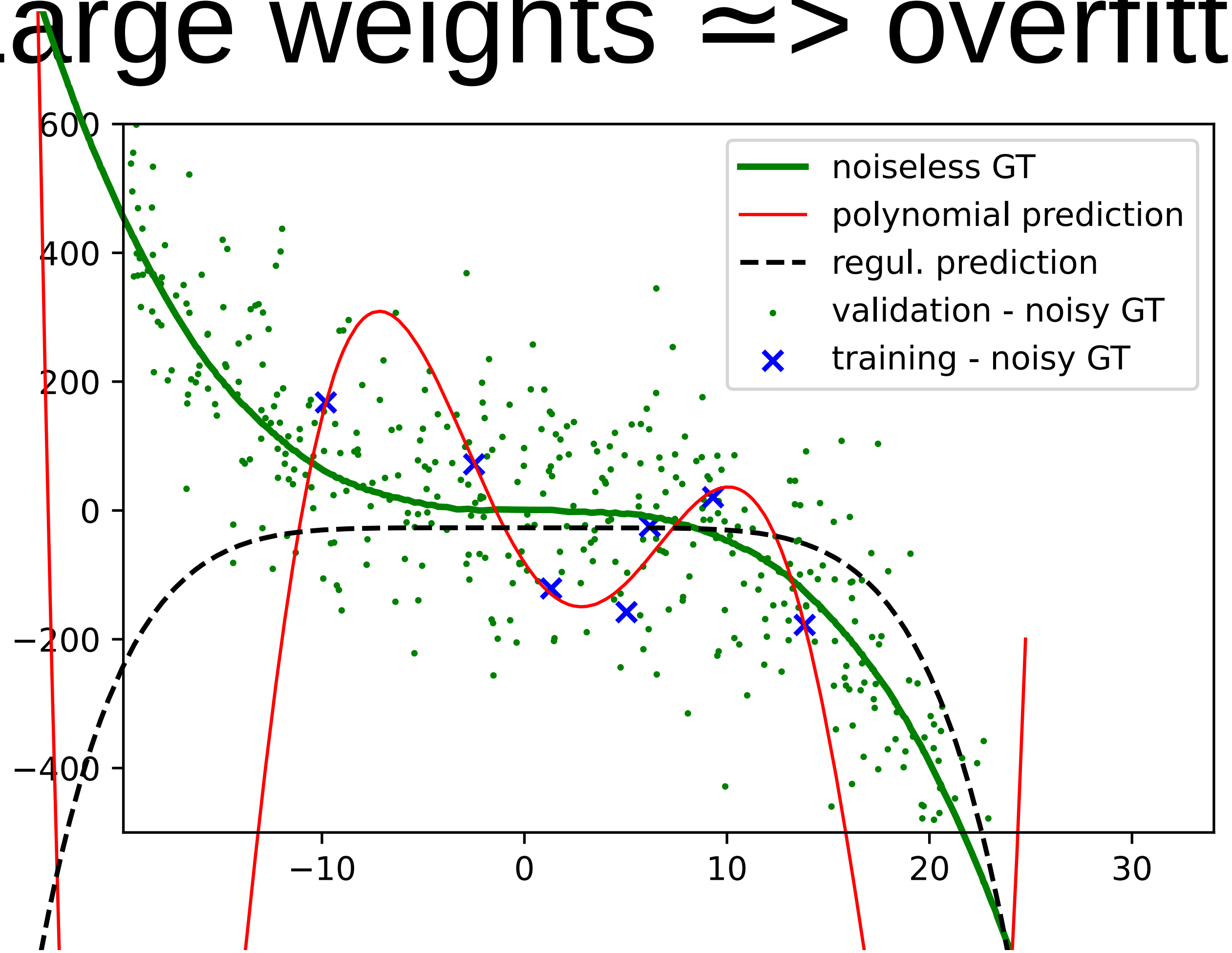
Bishop, 2006

Regularization (general definition)

- A possible def: *“Regularization is any modification we make to a learning algorithm that is intended to reduce its generalization error but not its training error.”* *From Deep Learning*, by Ian Goodfellow and Yoshua Bengio and Aaron Courville

  https://www.deeplearningbook.org/
- Goal: Regularization allows to ***restrain* a model’s complexity, *quantitatively*,** without *explicitly* limiting the model (i.e. order of polynomial fitting, etc)
- Examples:
  - Lasso, Ridge, Elastic-Net
  - Dropout (see DeepNetworks)
  - feature selection procedures
  - ensemble methods
- Here we focus on classics, i.e. **Ridge** and **Lasso**

Empirical Observation
Large weights ≃> overfitting



- No regularization : bad score, typically high weights (esp. coeffs of large order are too high)
- cf `lecture4-unregularized regression has large coefficients.ipynb` .

Intuition: Large weights≃>overfitting

(it’s actually more complicated)

- **Large weights** : output $W.x$ is **very sensitive** to small changes in data x. So, small perturbation of training data $\rightarrow$ big changes in weights $\rightarrow$ big changes in output ($\rightarrow$ overfitting)
- **Small weights** : output $W.x$ is less sensitive, i.e. is **more robust** w.r.t. change in data : not so different output for slightly different data $\rightarrow$ less overfitting (=better generaliztion)
- Remark: actually, the value of weights itself is meaningless. But, that’s the spirit.

Adding a Regularization term

There are two standard regularization terms. For a ML problem with a given Loss L :

- **Ridge** regul.:

- **Lasso** regul.:

- Elastic-Net: a mix of them both:

Linear Regression: **One-shot solution**

- (without regularization)

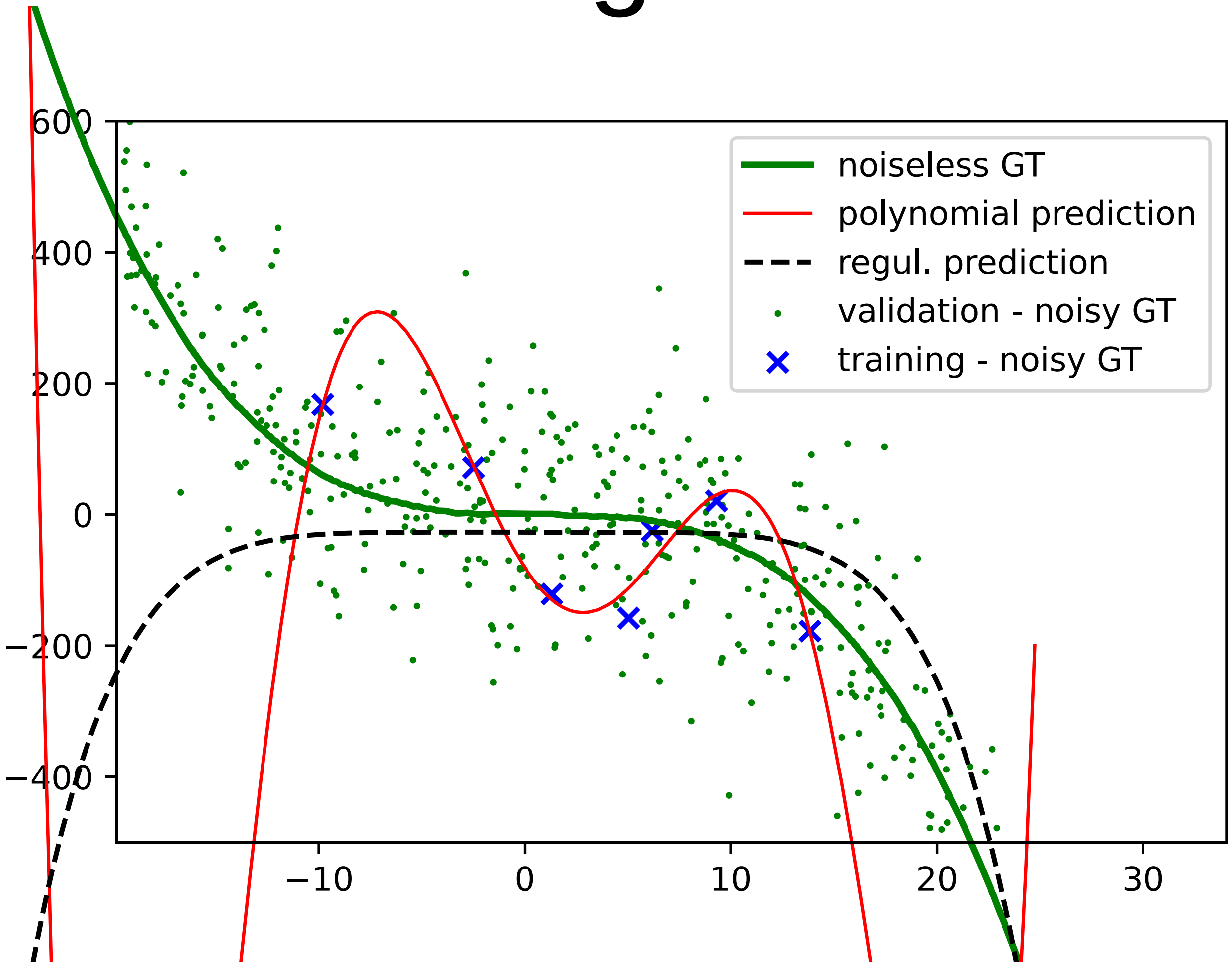
Linear Regression:
One-shot solution

- (**with** regularization)

Linear Regression:
One-shot solution

- (**with** regularization, in D=1 – even more intuitive)

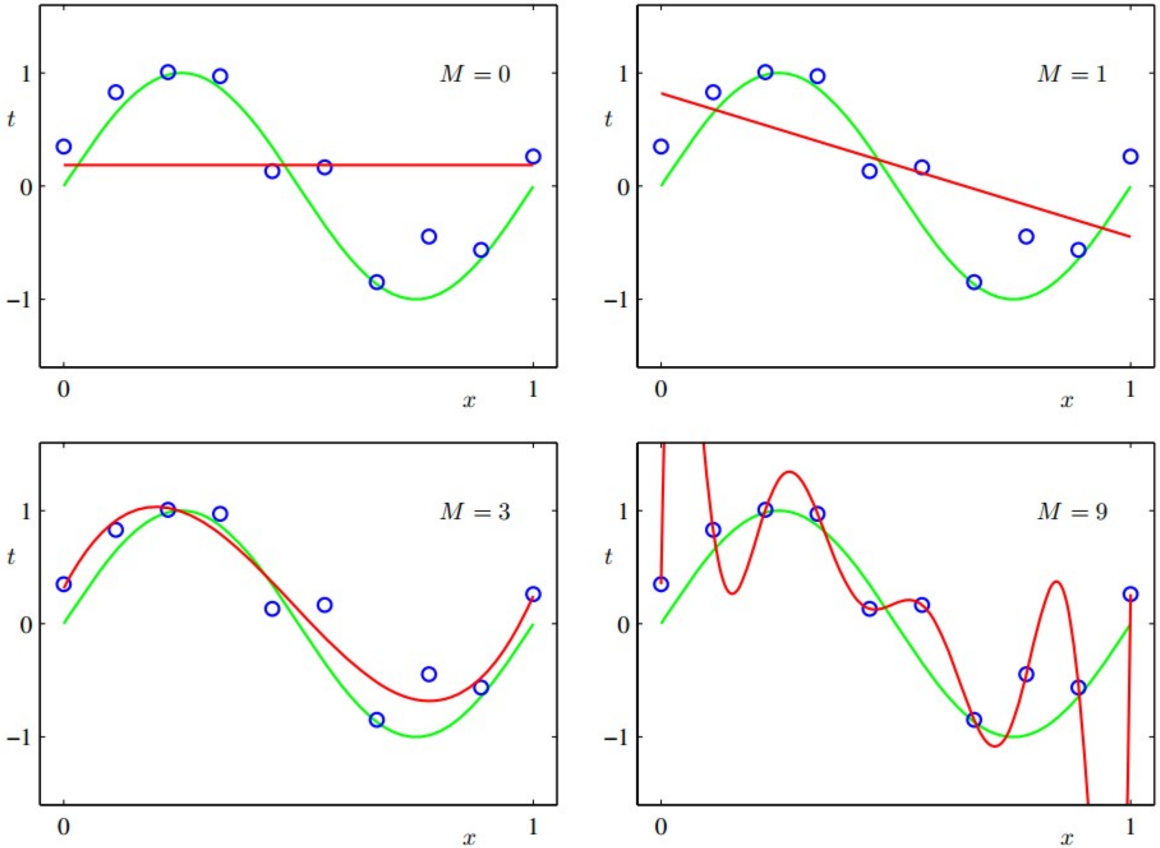
Does regul. work ?

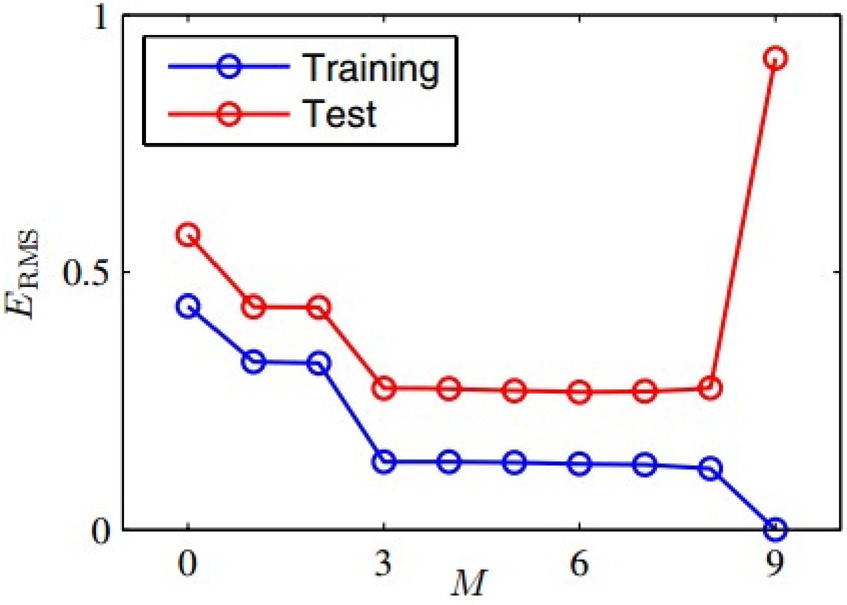


- No regularization : bad score, typically high weights (esp. coeffs of large order are too high)
- **With regularization: better score, all coeffs. shrink a lot (towards 0)**

Complexity controlled **explicitly** (rare case)

M = polynomial order

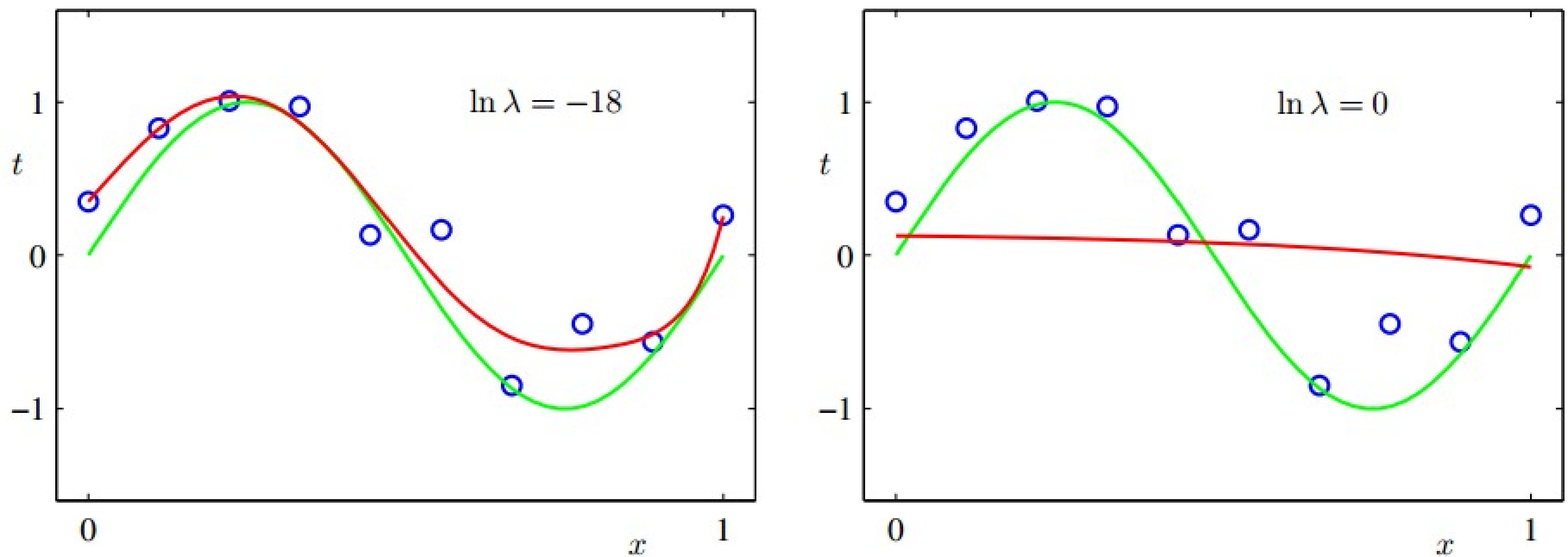




Bishop, 2006

Complexity controlled **indirectly : regularization**

M = polynomial order: still $M=9$
But, with **regularization parameter λ** changed



Important: practice with the **tutorial** to do more tests / play with:

`lecture4-regularization-dependence on lambda.ipynb`

Bayesian computations
The basics (prerequisite)

- MLE + an *a priori* opinion on what things should be = Maximum A Posteriori = MAP
- i.e. estimate a random variable, with the opinion (a priori) that its mean is of order τ :

Regularization
Bayesian **interpretation**

- MLE + an *a priori* opinion on what the model is = MAP → we can get the L2 regul from that !
- Assume model's weights follow a Gaussian distribution

Regularization during GD: Parameter shrinkage, weight decay

- What does regularization do **during a GD ?**

Lasso Regularization

- If we use the L1 norm: (or L0 norm)
- Effect: tends to set some weight to 0 exactly
  $\rightarrow$ it's already feature selection !

References

- **Algebra** reminder: *Bishop,* appendix C, p. 695-701 (only 6 pages !!)
- **Regularization**: *Bishop*, sec. 3.1.4, p. 144-146
  See also Sec. 5.5, p. 256-271, for much much more (Neural Nets).
- Another good **book**: (more recent, 2016): *Deep Learning*, by Ian Goodfellow and Yoshua Bengio and Aaron Courville https://www.deeplearningbook.org/ , in particular the chapter 5, https://www.deeplearningbook.org/contents/ml.html
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