RELATIONAL ALGEBRA

The foundation of relational algebra

An *algebra* is a set of elements together with the operations that can be executed over those elements.

- example: set of integer numbers together with operators {+,*}
- the foundation is in the *set theory*

Relational algebra is the basic set of operations for the relational model

- consists of **relational algebra expression**
    - composition of relational algebra operations
- used as a basis for implementing and optimizing SQL queries
    - query trees

Basic Operations in Codd's papers

– Selection
– Projection
– Union
– Intersection
– Difference
– Cross Join
– Join

Basic operations

Unary operations:

- Select σ
- Project π
- Rename ρ

Binary operations:

- Union $\cup$
- Intersection $\cap$
- Set Difference $-$
- Cartesian product (Cross Join) $\times$
- Join $\bowtie$

The Select operation (σ)

Select operation denoted by σ is a subset of tuples from a relation that satisfies a selection condition.

- *restricts* the tuples in a relation to only those that satisfy the condition.
- general form:

$$\sigma_{<selection-condition>}(R)$$

- one selection condition is a Boolean expression of the following two types:
  $< attribute\_name >< comparison\_op >< constant\_value >$ or
  $< attribute\_name >< comparison\_op >< attribute\_name >$
- can be considered as a *filter* that keeps only those tuples that satisfy a condition

The resulting relation has the same attributes as R

- $\sigma_{Class=\text{"3"}}(Student)$
- $\sigma_{Salary>40000}(Employee)$

Select - example

Student

SSN	MATR_NUM	STUDENT_NAME	CLASS
123-45-6789	9240006	John Brown	1
050-42-3729	5765763	Christine Smith	2
527-42-1289	1069362	Leslie Connor	1
103-42-4789	2795741	John Viener	1
416-41-1298	3761763	Leslie Connor	3

$$\sigma_{Class=”1”}(Student)$$

Student

SSN	MATR_NUM	STUDENT_NAME	CLASS
123-45-6789	9240006	John Brown	1
527-42-1289	1069362	Leslie Connor	1
103-42-4789	2795741	John Viener	1

Properties of the select operation

- Select operation is commutative
  $\sigma_{<cond1>}(\sigma_{<cond2>}(R)) = \sigma_{<cond2>}(\sigma_{<cond1>}(R))$
- and it holds
  $\sigma_{<cond1>}(\sigma_{<cond2>}(R)) = \sigma_{<cond1> \wedge <cond2>}(R)$

in SQL, *select operation* is typically specified in the WHERE clause (**not** in the SELECT clause):

```
SELECT *
FROM EMPLOYEE
WHERE DN0=4 AND SALARY > 30000
```

$$\sigma_{Dno=\text{"}4\text{"} \wedge Salary>30000}(Employee)$$

The Project operation(π)

Project operation denoted by π is a subset of columns from a relation

- it selects certain columns and discards the other columns
- general form:

$$\pi_{<attribute\_list>}(R)$$

The resulting relation has

- attributes which are a subset of attributes in R
- tuples in the resulting relation cannot be duplicated which is different from SQL (SQL uses DISTINCT to eliminate duplicates)

Examples:

- $\pi_{Student\_name}(Student)$
- $\pi_{LName,FName,Salary}(Employee)$

Project - example

Student

SSN	MATR_NUM	STUDENT_NAME	CLASS
123-45-6789	9240006	John Brown	1
050-42-3729	5765763	Christine Smith	2
527-42-1289	1069362	Leslie Connor	1
103-42-4789	2795741	John Viener	1
416-41-1298	3761763	Leslie Connor	3

Project-result

STUDENT_NAME	CLASS
John Brown	1
Christine Smith	2
Leslie Connor	1
John Viener	1
Leslie Connor	3

$$ProjectResult \leftarrow \pi_{Student\_name, Class}(Student)$$

The PROJECT operation(π)

If a list of attributes $< list2 >$ contains attributes in $< list1 >$

$$\pi_{<list1>}(\pi_{<list2>}(R)) = \pi_{<list1>}(R))$$

Relational algebra has relational expressions always as sets

- by contrast, SQL considers *multisets* or *bags*
- that's why corresponding SQL has to use DISTINCT to be equivalent to the project operation

```
SELECT DISTINCT FName, LName, Salary
FROM EMPLOYEE
```

$$\pi_{FName,LName,Salary}(Employee)$$

If the keyword DISTINCT is removed we got all duplicates included

The RENAME operation(ρ)

The **RENAME** operation renames either relation name or the attribute names or both.

- general form of the rename operation is

$$\rho_{S(B_1,B_2,\ldots B_n)}(R)$$

  where S is a new relation name and $B_1, B_2, \ldots, B_n$ is a set of attributes

Other two forms are contained in the general form

- $\rho_S(R)$
- $\rho_{(B_1,B_2,\ldots B_n))}(R)$

SQL uses aliasing **AS** to implement the rename operation

```
SELECT S.ssn AS s_ssn, S.matr_num as s_matr_num,
      S.student_name as s_student_name, S.class AS s_class
FROM Student AS S;
```

The RENAME operation - example

Student

SSN	MATR_NUM	STUDENT_NAME	CLASS
123-45-6789	9240006	John Brown	1
050-42-3729	5765763	Christine Smith	2
527-42-1289	1069362	Leslie Connor	1
103-42-4789	2795741	John Viener	1
416-41-1298	3761763	Leslie Connor	3

Student1

S_SSN	S_MATR_NUM	S_STUDENT_NAME	S_CLASS
123-45-6789	9240006	John Brown	1
050-42-3729	5765763	Christine Smith	2
527-42-1289	1069362	Leslie Connor	1
103-42-4789	2795741	John Viener	1
416-41-1298	3761763	Leslie Connor	3

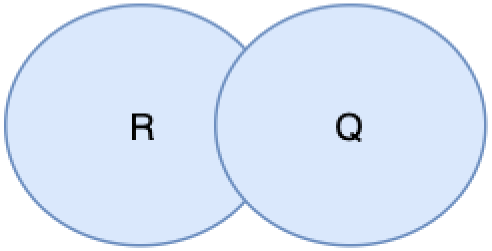
$$\rho_{Student1(s\_ssn,s\_matr\_num,s\_student\_name,s\_class)}(Student(ssn, matr\_num, student\_name, class))$$

The Union operation

The Union operation, denoted by $R \cup Q$

$$R \cup Q := \{r | r \in R \vee r \in Q\}$$

is a relation that includes all tuples that are either in R or in S, or in both R and S.



Duplicates are eliminated

Union - example

R

A	B
a1	b1
a1	b2
a2	b3

Q

A	B
a1	b1
a3	b1

UNION

A	B
a1	b1
a1	b2
a2	b3
a3	b1

Two relations $R(A_1, \ldots, A_n)$ and $Q(B_1, \ldots, B_n)$ are union compatible if they

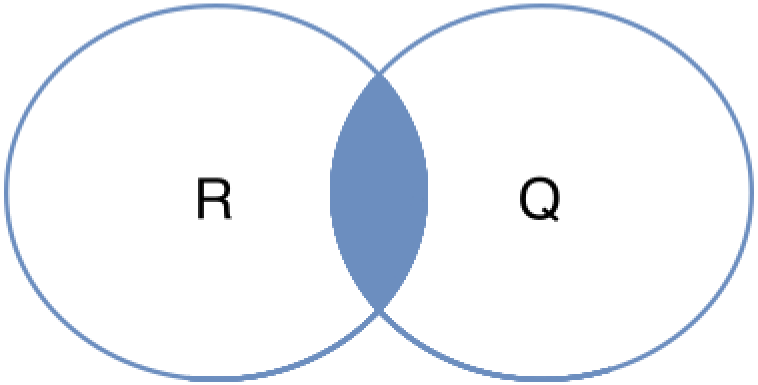
- have the same degree n and
- $dom(A_i) = dom(B_i), \ for \ 1 \leq i \leq n$

The Intersection operation

The Intersection operation, denoted by

$$R \cap Q := \{r | r \in R \wedge r \in Q\}$$

is a relation that includes all tuples that are in both R and S.



Intersection - example

R

A	B
a1	b1
a1	b2
a2	b3

Q

A	B
a1	b1
a3	b1

INTERSECTION

A	B
a1	b1

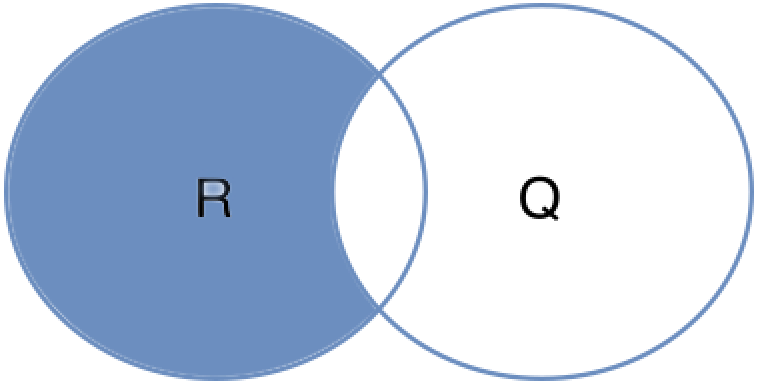
$$INTERSECTION \leftarrow R \cap Q$$

The Set Difference (Minus) operation

The Set Difference (Minus) of relations R and Q, denoted by

$$R - Q := \{r | r \in R \land r \notin Q\}$$

is a relation that includes all tuples that are in R but not in Q.



Set Difference - example

R

A	B
a1	b1
a1	b2
a2	b3
a3	b1

Q

A	B
a1	b1
a3	b2
a2	b3

R - Q

A	B
a1	b2
a3	b1

Question: Does it hold in general?

$$|R - S| \geq |R| - |S|$$

The Cartesian Product (Cross product)

Given relations $R(A_1, A_2, \dots, A_n)$ and $S(B_1, B_2, \dots, B_m)$.
The **Cartesian Product (Cross product)**, denoted by

$$R \times S$$

is a relation $Q(A_1, A_2, \dots, A_n, B_1, B_2, \dots, B_m)$ with the degree $n + m$ and has all combinations of each tuple from the relation R with all tuples from the relation S

Cardinality of the Cartesian product:

$$|R \times S| = |R| \cdot |S|$$

Cross product - example

R

A	B
a1	b1
a1	b2
a2	b3

Q

A	B
a1	b1
a3	b1

CROSS PRODUCT

A	B	A	B
a1	b1	a1	b1
a1	b1	a3	b1
a1	b2	a1	b1
a1	b2	a3	b1
a2	b3	a1	b1
a2	b3	a3	b1

In SQL, the cross product is implemented by putting two (or more tables) in the FROM clause without a join condition in the WHERE clause or by using *cross join*

```
SELECT *
FROM Employee, Dependent
```

```
select *
from employee cross join dependent
```

To extract related tuples cross product is often combined with the SELECT operation and then it's called **JOIN**

The JOIN Operation

Given relations $R(A_1, A_2, \ldots, A_n)$ and $S(B_1, B_2, \ldots, B_m)$.
The **JOIN Operation**, denoted by

$$R \bowtie_{<join\_condition>} S$$

is a relation $Q(A_1, A_2, \ldots, A_n, B_1, B_2, \ldots, B_n)$ that has $n + m$ attributes and has one tuple for each combination of tuples (one from R and one from S) whenever the combination satisfies the join condition.

A general join condition is

- $< condition > AND < condition > AND \ldots AND < condition >$

$< condition >$ is in the form $A\ \theta\ B$ where
$\theta \in \{=, <, \leq, >, \geq, \neq\}$

- because of this general condition it’s called also **THETA_JOIN**

The JOIN operation - example

Formally JOIN can be expressed as a combination of cross product and select operations:

$$R \bowtie_{<join\_condition>} S = \sigma_{<join\_condition>} R \times S$$

R

A	B
a1	b1
a1	b2
a2	b3

S

A	B
a1	b1
a3	b1

RESULT

A	B	A	B
a1	b1	a1	b1
a1	b2	a1	b1

$$RESULT \leftarrow R \bowtie_{R.A=S.A} S$$

JOIN doesn't necessarily preserve all of the information in participating tables

Variations of JOIN: EQUIJOIN and NATURAL JOIN

EQUIJOIN is a JOIN operation whose conditions has only comparison operator =

NATURAL JOIN, denoted by (*) is an EQUIJOIN operation with removed duplicated attributes

R

A	B
a1	b1
a1	b2
a2	b3

S

A	C
a1	c1
a3	c1

RESULT

A	B	C
a1	b1	c1
a1	b2	c1

$$RESULT \leftarrow R *_{R.A=S.A} S$$

Complete Set of Relational algebra operations

Set of the following relational algebra operations is **complete**:

- Selection (σ)
- Projection (π)
- Union ($\cup$)
- Rename (ρ)
- Set difference ($-$)
- Cartesian product ($\times$)

Any other relational algebra expression can be expressed using previous operations

- example: $R \cap S \equiv (R \cup S) - ((R - S) \cup (S - R))$
- example: $R \cap S \equiv R - (R - S)$

Other Relational algebra operations

Relational algebra operations that doesn't belong to the set $\{\sigma, \pi, \cup, \rho, -, \times\}$

- such as different types of JOINS
- doesn't increase expressive power of relational algebra
- make the language more convenient

OUTER JOIN operations

Previous join operators match tuples which satisfy the join condition and are called INNER JOIN

– tuples without a matching or with NULL values are eliminated

OUTER JOIN operations keep all the tuple in left or right relation or in both of them, regardless of whether or not they match tuples in the other relation

The LEFT OUTER JOIN operation

R

A	B
a1	b1
a1	b2
a2	b3

S

A	B
a1	b1
a3	b1

LEFT OUTER JOIN $R ⟕_{R.A=S.A} S$

A	B	A	B
a1	b1	a1	b1
a1	b2	a1	b1
a2	b3	null	null

LEFT OUTER JOIN keeps every tuple in the left relation and when there is no matching in the right relation it fills right relation attributes with NULL values

The RIGHT OUTER operations

R

A	B
a1	b1
a1	b2
a2	b3

S

A	B
a1	b1
a3	b1

RIGHT OUTER JOIN

A	B	A	B
a1	b1	a1	b1
a1	b2	a1	b1
null	null	a3	b1

RIGHT OUTER JOIN keeps every tuple in the right (second) relation and when there is no matching in the left relation it fills left relation attributes with NULL values

The FULL OUTER JOIN operation

FULL OUTER JOIN keeps every tuple in both the left and the right relations and when no matching is found it fills attributes with NULL values as needed

How should full outer join $R ⟗_{R.A=S.A} S$ look like? Fill the last two columns .

R

A	B
a1	b1
a1	b2
a2	b3

S

A	B
a1	b1
a3	b1

RIGHT OUTER JOIN

A	B	A	B
a1	b1	a1	b1
a1	b2	a1	b1

$$R ⟗ S = (R ⟕ S) \cup (R ⟖ S)$$

The DIVISION operation

The **DIVISION operation**, denoted by $R \div S$ is applied to two relations R(Z), and S(X) where $X \subset Z$ and gives as a result relation $T(Y)$ with attributes $Y = Z - X$. A tuple t is in the result relation if tuples appear in R such that $t_R[Y] = t$

$$\forall t, t \in r(T) \iff \forall t_S \in r(S) \implies \exists t_R \in r(R), t_R[Y] = t \wedge t_R[X] =$$

Example: *Retrieve the names of employees who work on all projects that "John Smith" works on.*

– dealing with *universal quantification* in queries

Division operation example

R

X	Y
x1	y1
x2	y1
x3	y1
x4	y1
x1	y2
x3	y2
x2	y3
x3	y3
x4	y3
x1	y4
x2	y4
x3	y4

S

X
x1
x2
x3

D = R ÷ S

D

Y
y1
y4

Can be expressed as a sequence of operations:

- $T1 \leftarrow \pi_Y(R)$
- $T2 \leftarrow \pi_Y((S \times T1) - R)$
- $D \leftarrow T1 - T2$

Aggregate functions

Aggregate functions are mathematical aggregate functions on collections of values

- COUNT
- SUM
- AVERAGE
- MAXIMUM
- MINIMUM

Common type of aggregate functions operation includes grouping attributes and then applies aggregate functions

$$G_1, G_2, \ldots, G_m \; g_{f_1(A_1{}'), f_2(A_2{}'), \ldots, f_k(A_k{}')}(R)$$

- $G_1, G_2, \ldots, G_m$ grouping attributes
- $f_1(A_1{}'), f_2(A_2{}'), \ldots, f_k(A_k{}')$ aggregate functions on attributes in R

Aggregate functions - example

Retrieve each department number, the number of employees in the department and their average salary

$$\rho(DNo, numEmployees, avgSalary)({}_{DNo}g_{COUNT\ ssn, AVERAGE\ salary}(EMPLOYEE))$$

DNo	numOfEmployees	avgSalary
5	4	33250
4	3	31000
1	1	55000

Aggregate functions cannot be expressed in the basic relational algebra

Review questions

- The Intersection operation can be expressed using other operations. Explain how?
- Explain the θ-JOIN operation. Where that name comes from?
- What is the result of NATURAL JOIN over two disjunct relations?
- Explain the cardinality of the cross product of two relations ?