Exercise 22

It does follow. For any Cauchy sequence $(a_n)_n$ with the limit a (if it exists), we have that the set $S = (a_n)_n \cup \{a\}$ is closed and bounded (We know that every Cauchy sequence is bounded). From our midterm problem we know that any subsequence of $(a_n)_n$ converges to a. Therefore in order for S to be compact, a must exist in S and so exists in M

Exercise 23

We know that $(0, 1)$ is open in $\mathbb{R}$ as proven in class. For any r, we have that $(0, r/2) \in B_r(0, 0)$ but $(0, r/2) \notin (0, 1) \times \{0\}$ therefore $B_r(0, 0) \not\subseteq (0, 1) \times \{0\}$. Therefore $(0, 1) \times \{0\}$ is not open.

Exercise 28

(a) Not necessarily. Consider the map from the unit circle $f : S^1 \to [0, 2\pi)$ where $f(\cos(\theta), \sin(\theta)) = \theta$. We have that the inverse image of the open set $[0, \epsilon)$ is the closed set $\{(\cos(\theta), \sin(\theta)) : \theta \in [0, \epsilon)\}$
(b) Yes. Since f has a continous inverse mapping f^{-1} for any open set $U \subseteq M$ we have that the pullback of f^{-1} of an open set is open. Thus $f(U) = (f^{-1})^{-1}(U)$ is open.
(c) Yes. Since f is bijective it has an inverse f^{-1}. For any open set U, the pullback of f^{-1} of U is just $f(U)$ which is open. Thus f^{-1} is continous. So f is a homeomorphism.
(d) Not necessarily. Consider the map $f : \mathbb{R}^2 \to [0, 2\pi)$ where $f(\cos(\theta), \sin(\theta)) = \theta$. This is the inverse map as the example used in (a). Thus it is bijective, specifically surjective. However the same open set in the example does not map to an open set

Exercise 32

For any point $p \in \mathbb{N}$ we have that for $r = 1$, by definition the set $B_r(p) = \{p\}$ is open. Therefore singletone points are open in $\mathbb{N}$, so any set $S \subseteq \mathbb{N}$ is open since

$$S = \bigcup_{s \in S} \{s\}$$

And we know arbitrary unions of open sets are open. Therefore we have that S^c is open as well. The complement of an open set is closed so we know that S is closed as well. Therefore every set $S \subseteq \mathbb{N}$ is clopen.
This means that every function $f : \mathbb{N} \to M$ is continous since the inverse image of any open set $U \subseteq M$ will be open.

Exercise 34

For any closed set $L \subset N$ with N closed from the inheritance principle we know $L = C \cap N$ for some closed set $C \subset M$. Intersections of closed sets are closed. Thus L is closed in M. Similarly if $U \subset N$ is open and N is open, then from the inheritance principle $U = V \cap N$ where V is open in M. Finite intersections of open sets are open, thus U is open in M.

Exercise 38

For d_E:
We have that $d_E(p,p') = \sqrt{\langle p,p' \rangle}$ with the inner product $\langle p,p' \rangle =$ For $d_{\max}$:
We have that $d_{\max}(p,p') = \sqrt{\langle p,p' \rangle}$ with the inner product $\langle p,p' \rangle =$ For d_{sum}:
We have that $d_{\text{sum}}(p,p') = \sqrt{\langle p,p' \rangle}$ with the inner product $\langle p,p' \rangle =$

Exercise 52

(a) Letting

Exercise Additional Problem 1