Math 461, Fall 2016 ***HOMEWORK #3*** ***Peter Gylys-Colwell***

1. In order for X_i to divide 2^{20} X_i must be a power of 2. We now have the equivalent problem, how many ways can we choose these nonegative powers for each X_i so that they add up to 20.
As established in class we know the number of ways to add up 5 ordered nonzero numbers so that they add up to 20 is

$$\binom{20+5-1}{5-1} = \binom{24}{4}$$

Which is the answer.

2. As established in lecture, there are $\binom{n+k-1}{k}$ ways to put k objects in n ordered objects, Therefore we have

$$\binom{15}{7}\binom{10}{2}$$

ways to put the 7 white and then 2 black billards in 9 distinguishable pockets

3. This is equivalent to counting the following way:
There are 7 unchosen chairs, now we have 8 spots inbetween and on the sides of these unchosen chairs to put the 5 chosen chairs, No two chosen chairs can occupy the same spot since that would mean they are next to each other. This is equivalent to the count

$$\binom{8}{5}$$

4. We can calculate this by

$$|A| - |S| - |C| + |S \cap C|$$

Where A is the set of integers from 1 to 1000, S is the set of integers that are perfect squares ≤ 1000, and C is the set of integers that are perfect cubes ≤ 1000
It is clear $|A| = 1000$.
$32^2 = 1024$, and $31^2 = 961$, and so $1 \leq n \leq 31 \Leftrightarrow n^2 \in S$ so $|S| = 31$.
$10^3 = 1000$, and so $|C| = 10$.
looking at $S \cap C$, we can look through each term of C, and find that only $1, 4^3, 9^3 \in S$, and so $|S \cap C|$
And so the count totals to

$$1000 - 31 - 10 + 3 = 962$$

5. We can count this by

$$|S| - |T_A \cup T_B \cup T_C|$$

Where S is the set of all possible rearrangments, and T_X is the set of all rearrangments that contain three consecutive letters X.
Using the inclustion exclusion principle, this is equivalent to

$$|S| - (|T_A| + |T_B| + |T_C|) + (|T_A \cap T_B| + |T_A \cap T_C| + |T_B \cap T_C|) - |T_A \cap T_B \cap T_C|$$

For $|S|$ we have 9 spots to put 3 As, then 6 spots for 3 Bs, and the Cs will take whats left. So

$$|S| = \binom{9}{3}\binom{6}{3}$$

As for the other sets, if we treat each triplet as one letter, there is a one to one correspondence between the number of strings with one of the triplets and the string treating the triplet as one letter. So we have

$$|T_A| = |T_B| = |T_C| = \binom{7}{3}\binom{4}{3}$$

Similarly for the intersections we treat each triplet as a seperate letter:

$$|T_A \cap T_B| = |T_A \cap T_C| = |T_B \cap T_C| = 3\binom{5}{3}$$

And for the intersection off all three sets we are taking the permutations of three different letters so we have

$$|T_A \cap T_B \cap T_C| = 3!$$

So the total count is

$$\binom{9}{3}\binom{6}{3} - 3\binom{7}{3}\binom{4}{3} + 9\binom{5}{3} - 3!$$