2-3, 1

A is continuous since its components are continuous functions. Given any point $p \in S^2$ we can choose parametarizations $x_1 : U_1 \subset \mathbb{R}^2 \to S^2$, $x_2 : U_2 \subset \mathbb{R}^2 \to S^2$ such that $p \in x_1(U_1)$, $A(p) \in x_2(U_2)$.
From class we established there are the following parametarizations we can choose from:

$$x_1, x_2 \in \{(\cos\theta \sin\varphi, \sin\theta \sin\varphi, \cos\varphi), (\cos\theta \sin\varphi + \pi, \sin\theta \sin\varphi + \pi, \cos\varphi + \pi),$$

$$(\cos\theta + \pi \sin\varphi, \sin\theta + \pi \sin\varphi, \cos\varphi) \ldots\}$$

We must show that

$$x_2^{-1} \circ A \circ x_1$$

is differentiable at p. Notice that $A = A^{-1}$ and thus by showing A is differentiable we have shown its inverse to be differentiable. Hence concluding A is a diffeomorphism.
It is clear this composition is differentiable since each component

$$x_2^{-1} \circ A \circ x_1(p) = (x_{2,1}^{-1}(-x_{1,1}(p_1)), x_{2,2}^{-1}(-x_{1,2}(p_2)), x_{2,3}^{-1}(-x_{1,3}(p_3)))$$

Is a composition of differentiable functions and thus differntiable regardless of choice of x_1, x_2.

2-3 3

We have the diffeomorphism $f : \mathbb{R}^2 \to P$ with $f(x, y) = (x, y, z^2)$. To establish f is a diffeomorphism, notice that f is also a parametarization. Thus the conditions of being a diffeomorphism rely on asking whether over any open set if

$$f^{-1} \circ f \circ \mathrm{id}$$

is differntiable. This mapping is the identity mapping so clearly differntiable.

2-3 4

The diffeomorphism $f : S^2 \to E$ is defined by

$$f(x, y, z) = (ax, by, cz)$$

We have that this is a diffeomorphism since notice that for any parametarization $x_1 : U \subset \mathbb{R}^2 \to S^2$ around a point $p \in S^2$ we have a parametarization $x_2 = f \circ x_1$ of E around $f(p)$. Thus we have

$$x_1^{-1} \circ f \circ x_2 = x_1^{-1} \circ f^{-1} \circ f \circ x_1 = \mathrm{id}$$

is differentiable

2-3 6

For diffeomorphism $f : S_1 \to S_2$, suppose we have the parametarizations around a point $p \in S^1$ and $f(p) \in S_2$

$$x_1 : U \subset \mathbb{R}^2 \to S_1, x_2 : V \subset \mathbb{R}^2 \to S_2$$

$$y_1 : U \subset \mathbb{R}^2 \to S_1, y_2 : V \subset \mathbb{R}^2 \to S_2$$

We need to show that if the parametarization with x_1, x_2 establishes f to be differentiable then so does y_1, y_2

2-3 7