Exersise 6.1

(1) We know that degree 3 polinomials are irreducible iff they have a root. We can use the rational root test to verify which polinomials have roots.
A root of $x^3 + nx + 2$ must be $1, -1, 2$ or -2. We have that $1 + n + 2 = 0 \Rightarrow n = -3$, $-1 - n + 2 = 0 \Rightarrow n = -1$, $2^3 + 2n + 2 = 0 \Rightarrow n = -5$ and $-2^3 - 2n + 2 = 0 \Rightarrow n = -3$. Thus $x^3 + nx + 2$ is irreducible iff $n \neq 1, -3, -5$

(2) Over $\mathbb{Z}$, $x^8 - 1 = (x-1)(x+1)(x^2+1)(x^4+1)$. We know that $x^2 + 1$ is irreducible since it has no roots, $x^4 + 1$ is irreducible since

Exersise 6.2

(1) If $n = m$ it is clear that $R^n \cong R^m$ since they are the same. If $n \neq m$ then we know that isomorphisms preserve the the rank of free modules (which is well defined when R is commutative), and since they have different rank, there cannot exist an isomorphism between the two modules

(2)

Exersise 6.3

Exersise 6.4

Exersise 6.5

Exersise 6.6