2-4, 10

Calculating the tangent vectors with the parametrization x

$$x_s = ((1 - r\kappa(s)\cos v)t(s) + (-r\theta(s)\cos v)b(s) + (r\theta(s)\sin v)n(s))$$

$$x_v = ((-r\sin v)n(s) + (r\cos v)b(s))$$

After numberous computations we arrive at

$$N(s,v)\cdot x_s = -(n(s)\cos v + b(s)\sin v)\cdot((1-r\kappa(s)\cos v)t(s) + (-r\theta(s)\cos v)b(s) + (r\theta(s)\sin v)n(s)) = 0$$

$$N(s,v)\cdot x_v = -(n(s)\cos v + b(s)\sin v)\cdot((-r\sin v)n(s) + (r\cos v)b(s)) = 0$$

Thus $N(s,v)$ is normal to S since it is normal to both x_s, x_v

2-4 18

By problem 17 we know that if S and P intersect transversally, in other words P is not the tangent space of S at p then P must intersect P by a regular curve. Thus since S intersects P at only a point, it must be the case $P = T_p(S)$

To prove 17, we know that S_1 and S_2 are locally the graphs of some smooth functions $f(x,y,z) = 0, g(x,y,z) = 0$ in a neighborhood of p. 0 is a regular value of f, g. $S_1 \cap S_2$ is precisely the inverse image of 0 of the function $F(x,y,z) = (f(x,y,z), g(x,y,,z))$. Since the normals of S_1, S_2 are not linearly dependent $(f_x, f_y, f_z), (g_x, g_y, g_z)$ are linearly independent. Thus $(0,0)$ is a regular value of F so $S_1 \cap S_2$ is a regular curve

2-4 21

For any $p, q \in S$, since S is connected and regular it is path connected. Thus there exists a path $\alpha : [0,1] \to S$ from p to q. We have that

$$\frac{d}{ds} f(\alpha(s)) = df_{\alpha(s)}(\alpha'(s)) = 0$$

Thus $f(\alpha(s))$ is constant as a function from $\mathbb{R} \to \mathbb{R}^3$ and thus

$$f(p) = f(\alpha(0)) = f(\alpha(1)) = f(q)$$

Thus f is constant

2-4 24

For any curve $\alpha : (a,b) \to S_1$ at p ($\alpha(0) = p$) by the usual chain rule

$$\frac{d}{ds}(\psi \circ \varphi \circ \alpha)$$

$$= d\psi_{\varphi(p)}\ d\varphi_p \alpha'(0)$$

Thus by definition of the matrix multiplied by $\alpha'(0)$ to get the tangent vector:

$$d(\psi \circ \varphi) = d\psi_{\varphi(p)}\ d\varphi_p$$