1. If we consider the spanning tree T of the graph G defined with verticies being towns and edges being weather or not it is possible to fly between the two towns. We know that T has 100 verticies, and since it is a tree, it has 99 edges. If we double all edges of T, this new graph T' will have 198 edges, and also every vertex in T' will have even degree since doubling every edge in T would double the degree of each vertex. Therefore T' is eulerian. An eulerian path of T' corresponds to a flight around the country visiting all the towns in 198 flights since every vertex must be visited in an eulerian path of a connected graph and there are exactly 198 edges in a eulerian path of T' and we know every edge and vertex of T' is contained in G. Therefore there is always such a trip with exacly 198 flights.

2. We can consider a spanning tree of G, T. We know that T has a leaf v. Removing v from G would yield a connected graph G'. The reason being that T with v removed is still a tree T' (since T being a tree $\Leftrightarrow |V(T)| = |E(T)| + 1$ and removing v removes 1 from both $|V(T)|$ and $|E(T)|$ so the equality still holds for the resulting graph). And T' is a spanning tree of G' since all verticies in G are present in T' and vice-versa. Therefore since G' has a spanning tree, it is connected.

3.

a. If $(7, 8) \notin E$ then there are 6 verticies with degree 1 that will accept 6 edges total, but 7 and 8 must pair up with 8 edges total which cannot happen
Therefore $(7, 8) \in E$. There are 6 vertices left with degree 1, and 6 edges that must come from 7 and 8. If any of these degree 1 verticies paired up then we would have the problem that 7 and 8 need to pair up with 3 vertices each (for a total of 6 edges), but there are only 4 verticies that will accept 4 edges total.
Therefore each of these degree 1 vertices must share an edge with either 7 or 8. There are $\binom{6}{3}$ ways to choose this.

b. We know that the number of odd degree verticies must be even. For all the possible graphs with 8 vertices of degree 1, we know that none of them are connected since a connected graph must have a spanning tree which would mean the number of edges $\geq$ number of vertices -1, but number of edges $= \frac{1}{2}\sum$ degrees $= 4 <$ number of vertices -1. Part a counts the number with 6 verticies of degree 1, however we must account relabeling verticies so we multiply by $\binom{8}{2}$. And the graphs with number of 4 degree vertices ≥ 4 are not trees, the reason being is that number of edges $= \frac{1}{2}\sum$ degrees $\geq \frac{1}{2}(4 + 4 + 4 + 4 + 1 + 1 + 1 + 1) = 10 >$ the number of vertices, and trees must have the property that number of verticies = number of edges $+1$.
So the total number of trees is

$$\binom{6}{3}\binom{8}{2}$$

4. We can count the number of spanning trees of the K_{n-1} subgraph, and then attatch the nth vertex with an edge to one of the verticies of this spanning tree. The resulting graph would be a spanning tree of K_n with n as a leaf. There are $n-1$ ways to attach n since there are $n-1$ verticies with which n can attach. These methods of coming up with such spanning trees account for all spanning trees of K_n with n as a leaf since for any spanning tree of K_n with n as a leaf, we can remove n to have a spanning tree of K_{n-1}, and so the method of adding n back to this spanning tree of K_{n-1} along with the edge that was removed will yield the original spanning tree. We also know that the images of these mapings dont intersect since the vertex n is attatched is unique to each mapping.
From Cayleys's formula we have $(n-1)^{n-3}$ spanning trees of K_{n-1}, and there are $n-1$ of these mappings, there are

$$(n-1)(n-1)^{n-3} = (n-1)^{n-2}$$

total spanning trees with n as a leaf.

5. We can consider the connected component of 1. If the connected component has k vertices, there are $\binom{n-1}{k-1}$ ways to choose these other verticies. Of these vertices, by Cayley's formula there are k^{k-2} possible trees of this connected component. The other connected component has the rest of verticies, so there are $(n-k)^{n-k-2}$ possible trees of this connected component. Therefore for a forrest with two connected components with the connected component of 1 having size k we have, $k^{k-2}\binom{n-1}{k-1}k^{k-2}(n-k)^{n-k-2}$ possible forrests
To get all possible forrests with two connected components, we can sum k from 1 to $n-1$:

$$\sum_{k=1}^{n-1} \binom{n-1}{k-1} k^{k-2}(n-k)^{n-k-2}$$

6. Every pair $\{i, j\} \subset V$ appears as an edge in the same number of trees. The reason for this is because for any pair i, j there was nothing special about i or j and we can simply relabel the verticies to have any verticy pair take the places of i, j and recount the trees in the same way. We can let the number of times any of these pairs of verticies show up as an edge in a tree be k. We know that the total number of trees is n^{n-2}. Each of these trees has $n-1$ edges, and so there are $(n-1)n^{n-2}$ total edges of these trees. We know that the total number of edges of these trees must equal the total number of times each pair shows up as an edge in a tree multiplied by the total number of pairs. We know the number of pairs is $\binom{n}{2}$ and total number of times a pair shows up is k. So we have

$$\binom{n}{2}k = (n-1)n^{n-2}$$

$\frac{n(n-1)}{2}k = (n-1)n^{n-2}$, so $k = 2n^{n-3}$. And k is the number of times $(1, 2)$ shows up as an edge in a tree