Exersise §1, 9

For any $s \in S = A - (B \cup C)$ we have $s \in A$ and $s \notin B$, as well as $s \in A$ and $s \notin B$. Therefore $s \in R = (A - B) \cap (A - C)$ and so $S \subseteq R$. For any $r \in R$ we have $r \in A - B$ as well as $r \in A - C$ so r must be in A. Also since $r \in A - B$, $r \notin B$ and similarly since $r \in A - C$, $r \notin C$. Therefore $R \subseteq S$, and so $R = S$

For the other law we have for any $s \in S = A - (B \cap C)$ we have $s \in A$ and s is not in both B and C. Therefore s must not be in either B or C so $s \in A - B$ or $s \in A - C$ whcih means $s \in R = (A - B) \cup (A - C)$. Therefore $S \subseteq R$. We also have for any $r \in R$, r is in $A - B$ or $A - C$ which means $r \in A$ and r is not in both B and C which means $r \in S$. Therefore $R \subseteq S$ and so $R = S$

Exersise §2, 1

a. For any $a \in A_0$, by definition we have $f(a) \in f(A_0)$ and therefore

$$a \in f^{-1}(f(A_0))$$

which means $A_0 \subseteq f^{-1}(f(A_0))$. If f is injective then if there exists $b \notin A_0$ with $b \in A_0 - f^{-1}(f(A_0))$ then $f(b) \in f(A_0)$ which means there exists $a \in A_0$ such that $f(b) = f(a)$ which contradicts injectivity. Therefore $A_0 - f^{-1}(f(A_0)) = \emptyset$ and so $A_0 = f^{-1}(f(A_0))$

b. For any $b \in B_0$ we have by definition $f(f^{-1}(b)) \subseteq B_0$ and so $f(f^{-1}(B_0)) \subseteq B_0$. If f is surjective then for any $b \in B_0$ there is a $a \in A$ such that $f(a) = b$ and therefore $a \in f^{-1}(b)$ and so $b \in f(f^{-1}(b)) \subseteq f(f^{-1}(B_0))$ and therefore $B_0 \subseteq f(f^{-1}(B_0))$

Exersise §2, 2

a.

b.

c.

d.

Exersise §2, 4

a.

b.

c.

d.

Exersise §2, 5

a.

b.

c.

d.

Exersise §3, 4