Exersise 3.7

Given $x_0 \in \mathbb{R}^n$ and any $\epsilon > 0$, let $\delta = \epsilon$. For any $x \in B(x_0, \delta)$ we have $\|x - x_0\| < \delta = \epsilon$. We also have $|f(x) - f(x_0)| = |\|x\| - \|x_0\||$.
We know that we have $|\|x\| - \|x_0\|| = \|x\| - \|x_0\|$ or $-(\|x\| - \|x_0\|) = \|x_0\| - \|x\|$. By the triangle inequality we know both $\|x\| - \|x_0\| \leq \|x - x_0\|$ and $\|x_0\| - \|x\| \leq \|x_0 - x\| = \|x - x_0\|$.
And so $|f(x) - f(x_0)| = |\|x\| - \|x_0\|| \leq \|x - x_0\| < \epsilon$
Thus f is continuous

Exersise 3.9

a. If there exists some N such that $x_j = x_k$ for all $j, k > N$ then $\delta(x_j, x_k) = 0 < \epsilon$ for all $\epsilon > 0$, and so by definition x_n converges. Conversly if x_n converges, let $\epsilon = 1/2$. We have that for some N, $\delta(x_j, x_k) < 1/2$ for all $j, k > N$. Since $\delta(x_j, x_k) > \epsilon$ if and only if $x_j \neq x_k$, we have that $x_j = x_k$ for all $j, k > N$

b. For any $x_0 \in X$ and any $\epsilon > 0$, let $\delta = 1/2$. We have that $\delta(x, x_0) < \delta$ if and only if $x = x_0$, by definition of the discrete metric. Therefore $B_(x_0, \delta) = \{x_0\}$ and as one of the properties of the metric, we have $d(f(x_0), f(x_0)) = 0 < \epsilon$. Therefore by definition f is continuous

Exersise 3.11

For a given $\epsilon > 0$, f continuous means for that given $\epsilon > 0$ there exists a $\delta > 0$ such that $d(x, x_i) < \delta$ implies $\rho(f(x), f(x_i)) < \epsilon$. $x_n \to x$ means there is a $N > 0$ such that for $k > N$, $d(x_k, x) < \delta$ and therefore for $k > N$, $\rho(f(x), f(x_k)) < \epsilon$. Thus $f(x_n) \to f(x)$

Exersise 3.14

For any $\theta_0 \in [0, 2\pi)$, given $\epsilon > 0$ let $\delta =$. For any $\theta \in [0, 2\pi)$ with $|\theta - \theta_0| < \delta$ we have

$$\|f(\theta) - f(\theta_0)\| = \sqrt{(\cos(\theta) - \cos(\theta_0))^2 + (\sin(\theta) - \sin(\theta_0)^2}$$

$$= \sqrt{\cos^2(\theta) - 2\cos(\theta_0)\cos(\theta) + \cos^2(\theta_0) + \sin^2(\theta) - 2\sin(\theta_0)\sin(\theta) + \sin^2(\theta_0)}$$

$$= \sqrt{2(1 - (\cos(\theta_0)\cos(\theta) + \sin(\theta_0)\sin(\theta)))}$$

Using the sum formula ($\cos(a - b) = \cos a \cos b + \sin a \sin b$) we have:

$$= \sqrt{2(1 - \cos(\theta - \theta_0))}$$

A common property of sin is that $|\sin x| < |x|$ since $|x|$ is the arc length while sin is the vertical length of point on the unit circle. Therefore $\sin^2(\theta - \theta_0) = 1 - \cos^2(\theta - \theta_0) < (\theta - \theta_0)^2 < \delta^2$. And so we have

$$\|f(\theta) - f(\theta_0)\| <$$

Exersise 3.17

Exersise §13, 3

Exersise §13, 4