Exersise 7

a. For any even integer n we can write it as the product $n = 2k$ for some $k \in \mathbb{Z}$. Therefore $n^2 = (2k)^2 = 4k^2$ and therefore 4 divides n^2

b. For any even integer n we can write it as the product $n = 4k$ for some $k \in \mathbb{Z}$. Therefore $n^3 = (2k)^3 = 8k^3$ and therefore 8 divides n^3

c. In the prime factorization of twice and odd cube, $2k^3$ where k odd, we know 2 does not divide k and therefore does not divide k^3 and so there is only 2^1 in the prime factorization of $2k^3$. Therefore 8 cannot divide $2k^3$ since $8 = 2^3$ does not divide the powers of 2 in the prime factorization of $2k^3$

d. Suppose for contradiction $\sqrt[3]{2} = \frac{a}{b}$ where a, b are relatively prime. Then we have $2b^3 = a^3$. Since $2b^3$ is even, a^3 is even. The only way it is possible for a^3 to be divisible by 2 is if 2 divides a. Therefore a must be even, which means $a = 2n$ for some $n \in F$ so $a^3 = 8n^3 = 2b^3$, So $b^3 = 4n^3$. Therefore b^3 is even which means b must be even

Exersise 10

Exersise 13

Exersise 1

a.
$$\{x \in \mathbb{Q} : x^2 = 2\} = \emptyset$$

b. If $x \in \mathbb{Q}$ and $x > 0$ then $\exists\, n \in \mathbb{N}$ such that $\frac{1}{n} < x$

Exersise 2