Math 441, Fall 2017 ***HOMEWORK #1*** ***Peter Gylys-Colwell***

Exersise §1, 9

For any $s \in S = A - (B \cup C)$ we have $s \in A$ and $s \notin B$, as well as $s \in A$ and $s \notin B$. Therefore $s \in R = (A - B) \cap (A - C)$ and so $S \subseteq R$. For any $r \in R$ we have $r \in A - B$ as well as $r \in A - C$ so r must be in A. Also since $r \in A - B$, $r \notin B$ and similarly since $r \in A - C$, $r \notin C$. Therefore $R \subseteq S$, and so $R = S$

For the other law we have for any $s \in S = A - (B \cap C)$ we have $s \in A$ and s is not in both B and C. Therefore s must not be in either B or C so $s \in A - B$ or $s \in A - C$ whcih means $s \in R = (A - B) \cup (A - C)$. Therefore $S \subseteq R$. We also have for any $r \in R$, r is in $A - B$ or $A - C$ which means $r \in A$ and r is not in both B and C which means $r \in S$. Therefore $R \subseteq S$ and so $R = S$

Exersise §2, 1

a. For any $a \in A_0$, by definition we have $f(a) \in f(A_0)$ and therefore

$$a \in f^{-1}(f(A_0))$$

which means $A_0 \subseteq f^{-1}(f(A_0))$. If f is injective then if there exists $b \notin A_0$ with $b \in A_0 - f^{-1}(f(A_0))$ then $f(b) \in f(A_0)$ which means there exists $a \in A_0$ such that $f(b) = f(a)$ which contradicts injectivity. Therefore $A_0 - f^{-1}(f(A_0)) = \emptyset$ and so $A_0 = f^{-1}(f(A_0))$

b. For any $b \in B_0$ we have by definition $f(f^{-1}(b)) \subseteq B_0$ and so $f(f^{-1}(B_0)) \subseteq B_0$. If f is surjective then for any $b \in B_0$ there is a $a \in A$ such that $f(a) = b$ and therefore $a \in f^{-1}(b)$ and so $b \in f(f^{-1}(b)) \subseteq f(f^{-1}(B_0))$ and therefore $B_0 \subseteq f(f^{-1}(B_0))$. This means that $B_0 = f(f^{-1}(B_0))$

Exersise §2, 2

a. Given any $b \in B_0$, since $B_0 \subseteq B_1$ we know $b \in B_1$. By the definition of $f^{-1}(B_1)$ we have that $f^{-1}(b) \subseteq f^{-1}(B_1)$ since $b \in B_1$. And since $f^{-1}(B_0)$ is a unioun of these preimages which are contained in B_1, we know $f^{-1}(B_0) \subseteq f^{-1}(B_1)$

b. Given any $a \in A$ with $a \in f^{-1}(B_0 \cup B_1)$ or equivalently $f(a) \in B_0 \cup B_1$ we know that $f(a)$ must be in either B_0 or B_1 and so a is in either $f^{-1}(B_0)$ or $f^{-1}(B_1)$. Therefore $a \in f^{-1}(B_0) \cup f^{-1}(B_1)$ and so $f^{-1}(B_0 \cup B_1) \subseteq f^{-1}(B_0) \cup f^{-1}(B_1)$. Conversly if $f(a)$ is in B_0 or in B_1 then $f(a) \in B_0 \cup B_1$ and so $f^{-1}(B_0) \cup f^{-1}(B_1) \subseteq f^{-1}(B_0 \cup B_1)$. Therefore we have equality

c. Given any $a \in A$ with $a \in f^{-1}(B_0 \cap B_1)$ or equivalently $f(a) \in B_0 \cap B_1$ we know that $f(a)$ must be in both B_0 and B_1 and so a is in $f^{-1}(B_0)$ and $f^{-1}(B_1)$. Therefore $a \in f^{-1}(B_0) \cap f^{-1}(B_1)$ and so $f^{-1}(B_0 \cap B_1) \subseteq f^{-1}(B_0) \cap f^{-1}(B_1)$. Conversly if $f(a)$ is in B_0 and in B_1 then $f(a) \in B_0 \cap B_1$ and so $f^{-1}(B_0) \cap f^{-1}(B_1) \subseteq f^{-1}(B_0 \cap B_1)$. Therefore we have equality

d. Given any $a \in A$ with $a \in f^{-1}(B_0 - B_1)$ or equivalently $f(a) \in B_0 - B_1$ we know that $f(a)$ must be in B_0 and not B_1 and so a is in $f^{-1}(B_0)$ and $f^{-1}(B_1)$. Therefore $a \in f^{-1}(B_0) - f^{-1}(B_1)$ and so $f^{-1}(B_0 - B_1) \subseteq f^{-1}(B_0) - f^{-1}(B_1)$. Conversly if $f(a)$ is in B_0 and not in B_1 then $f(a) \in B_0 - B_1$ and so $f^{-1}(B_0) - f^{-1}(B_1) \subseteq f^{-1}(B_0 - B_1)$. Therefore we have equality

e. Given any $f(a) \in f(A_0)$ for some $a \in A$, since $a \in A_0 \subseteq A_1$ we have that $a \in A_1$ and so $f(a) \in f(A_1)$. Therefore $f(A_0) \subseteq f(A_1)$

f. Given any $f(a) \in f(A_0 \cup A_1)$ for some $a \in A$ we have that $a \in A_0 \cup A_1$ and so a is either in A_0 or A_1 so $f(a) \in f(A_0) \cup f(A_1)$ and so $f(A_0 \cup A_1) \subseteq f(A_1) \cup f(A_0)$. Conversly for any $f(a) \in f(A_0) \cup f(A_1)$ we know $f(a)$ is in either $f(A_0)$ or $f(A_1)$ and so $a \in A_0$ or $a \in A_1$ therefore $a \in A_0 \cup A_1$ and therefore $f(a) \in f(A_0 \cup A_1)$. Therefore we have equality

g. For any $f(a) \in f(A_0 \cap A_1)$ we know $a \in A_0 \cap A_1$ and therefore since a is in A_0 and A_1, $f(a) \in f(A_0)$ and $f(a) \in f(A_1)$. Therefore $f(A_0 \cap A_1) \subseteq f(A_0) \cap f(A_1)$. If f is injective, for any $f(a) \in f(A_0) \cap f(A_1)$ we know $f(a)$ is in both $f(A_0)$ and in $f(A_1)$. Therefore there exists elements $a_0 \in A_0, a_1 \in A_1$ such that $f(a_0) = f(a) \in f(A_0)$ and $f(a_1) = f(a) \in f(A_1)$. Since f is injective however $a = a_0 = a_1$ and we know a is in both A_0 and A_1. Therefore $f(a) \in f(A_0 \cap A_1)$, and thus we have equality of the sets

h. For any $f(a) \in f(A_0 \cap A_1)$ we know $a \in A_0 \cap A_1$ and therefore since a is in A_0 and A_1, $f(a) \in f(A_0)$ and $f(a) \in f(A_1)$. Therefore $f(A_0 \cap A_1) \subseteq f(A_0) \cap f(A_1)$. If f is injective, for any $f(a) \in f(A_0) \cap f(A_1)$ we know $f(a)$ is in both $f(A_0)$ and in $f(A_1)$. Therefore there exists elements $a_0 \in A_0, a_1 \in A_1$ such that $f(a_0) = f(a) \in f(A_0)$ and $f(a_1) = f(a) \in f(A_1)$. Since f is injective however $a = a_0 = a_1$ and we know a is in both A_0 and A_1. Therefore $f(a) \in f(A_0 \cap A_1)$, and thus we have equality of the sets

Exersise §2, 4

a.

b.

c.

d.

Exersise §2, 5

a.

b.

c.

d.

Exersise §3, 4