Exersise §16, 1

We have that the topology $\mathcal{T}_X$ of A inherited as a subspace of X is the set of open sets of X intersected with A. The topology T_Y of A inherited as a subspace of Y is the set of open sets of Y intersected with A. We have that each open set of Y is an open set of X intersected with Y. Therefore for any open set $V \in \mathcal{T}_X$ we have that $V = U \cap A$ where U is an open set of X. Since $A \subseteq Y$ we have that $U \cap A \subseteq Y$ so $V = U \cap A = Y \cap (U \cap A) = (Y \cap U) \cap A$. We have that $Y \cap U$ is an open set of Y so $V \in \mathcal{T}_Y$. Thus $\mathcal{T}_X \subseteq \mathcal{T}_Y$. Conversly if $V \in \mathcal{T}_Y$ we have that $V = U \cap A$ for an open set U of Y. By definition we have that $U = Y \cap W$ where W is an open set of X, thus $V = (Y \cap W) \cap A = W \cap (Y \cap A)$ and since $A \subseteq Y$ we have $Y \cap A = A$ so $V = W \cap A$, and thus $V \in \mathcal{T}_X$. So $\mathcal{T}_Y \subseteq \mathcal{T}_X$. And so $\mathcal{T}_X = \mathcal{T}_Y$

Exersise §16, 6

We will call $\mathcal{T}$ the collection described in the problem. For any $x = (x_1, x_2) \in \mathbb{R}^2$, we have that

$$x \in (\lfloor x_1 \rfloor - 1, \lceil x_1 \rceil + 1) \times (\lfloor x_2 \rfloor - 1, \lceil x_2 \rceil + 1) \in \mathcal{T}$$

Thus there is an elt in $\mathcal{T}$ that contains x for any $x \in \mathbb{R}^2$.
Letting $A = (a, b) \times (c, d)$ and $B = (e, f) \times (g, h)$ with $A, B \in \mathcal{T}$ we have that

$$A \cap B = \{(x, y) : \max\{a, e\} < x < \min\{b, f\}, \max\{c, g\} < y < \min\{d, h\}\}$$

Since $a, b, c, d, e, f, g, h \in \mathbb{Q}$ we know that each of these max and mins are in $\mathbb{Q}$ thus we have that $A \cap B \in \mathcal{T}$, therefore for any $x \in A \cap B$ we have $x \in A \cap B \in \mathcal{T}$ with $A \cap B \subseteq A \cap B$. Thus all the conditions of being a basis are satisfied, so $\mathcal{T}$ is a basis.

Exersise §17, 1

We have $X - X = \emptyset, X - \emptyset = X$.
We can use demorgans law, for any arbitrary union:

$$\bigcup (X - C_i) = X - \bigcap C_i = X - C \in \mathcal{T}$$

for some $C \in \mathcal{C}$. For a finite intersection:

$$\bigcap_{i \in [n]} (X - C_i) = X - \bigcup_{i \in [n]} C_i = X - C \in \mathcal{T}$$

for some $C \in \mathcal{C}$.
Thus all the axioms of a topology are satisfied

Exersise §17, 2

We have that $Y - A$ is an open set in Y and thus $Y - A = U \cap Y$ where U is an open set in X. Since Y is closed, $X - Y$ is open, we have that

$$X - A = (Y - A) \cup (X - Y) = U \cup (X - Y)$$

and thus $X - A$ is the union of open sets in X and so A is a closed in X.