**16.1** This is a special case of thm 16.1 ii:
We have

$$(-1)a + a = (-1 + 1)a = 0 \cdot a = 0$$

and so subtracting $a$ on both sides yields

$$(-1)a = -a$$

**16.7** Since $F$ is a field we know there is $a^{-1} \in F$ such that $aa^{-1} = 1$. Therefore if we let $x = a^{-1}(-b)$ we satisfy the equation:

$$a(a^{-1}(-b)) + b = (aa^{-1})(-b) + b = -b + b = 0$$

We get that first equality since $\cdot$ is associative

**16.11**

a. The only unit is $(1, 1)$ since for any $a, b \in \mathbb{Z}$, $ab = 1 \Leftrightarrow a = 1, b = 1$. The only zero-divisor is $(0, 0)$ since for any $a, b \in \mathbb{Z}$, $ab = 0 \Leftrightarrow a = 0$ and/or $b = 0$. Since the set of nilpotents elements is a subset of zero-divisors, it follows that the only nilpotent is also $(0, 0)$.

b. From previous knowledge of groups we know every element in $\mathbb{Z}_3$ has an inverse under the group operation of multiplication modulo 3, therefore we know for any $(a, b) \in \mathbb{Z}_3 \oplus \mathbb{Z}_3$ there is a $(a^{-1}, b^{-1}) \in \mathbb{Z}_3 \oplus \mathbb{Z}_3$ such that $(a, b)(a^{-1}, b^{-1}) = (1, 1)$ and so every element in $\mathbb{Z}_3 \oplus \mathbb{Z}_3$ is a unit. Since 3 is prime there is no two numbers that can multiply together to be a multiple of 3 unless one of the two numbers is already a multiple of 3, only $(0, 0)$ is a zero-divisor and from that it follows (since the set of nilpotents is a subset of zero-divisors) that $(0, 0)$ is the only nilpotent

c. The units are $(1, 1), (1, 5), (3, 1), (3, 5)$ with respective inverses $(1, 1), (1, 5), (3, 1), (3, 5)$. The zero-divisors are all the rest of the elements: $(0, 2), (0, 3), (0, 4), (2, 2), (2, 3), (2, 4)$. The nilpotents are $(0, 0), (2, 0)$

**16.13**

a. If there were two multiplicative identities: $1 \neq 1'$ we would have by definition of the multiplicative identity

$$1 = 1 \cdot 1' = 1'$$

and so $1 = 1'$

b. If there were two multiplicative inverses, let $\beta$ and $\alpha$ be multiplicative inverses of $a$. We have

$$\beta = \beta(a\alpha) = (\beta a)\alpha = \alpha$$

And so $\beta = \alpha$

**A**

**B**

**C**

**D**

**E**