16.24 Only do a,b,c

17.1

17.20

A

a. We have
$$a^2 = a \Rightarrow a^2 - a = 0 \Rightarrow a(a-1) = 0$$
Since R is an integral domain, $a(a-1) = 0$ if and only if either a or $a-1$ is zero, and since the additive inverse is unique, that means a is either 1 or 0.

b. The idempotents are 1, 5, and 6.

c. For any $(a, b) \in \mathbb{Z} \times \mathbb{Z}$ we have
$$(a, b)(a, b) = (a, b) \Rightarrow (a^2 - a, b^2 - b) = (0, 0) \Rightarrow a^2 - a = 0, b^2 - b = 0$$
And since $\mathbb{Z}$ is an integral domain, that means $a, b \in \{0, 1\}$ and so the idempotents are $(0, 0), (1, 1), (1, 0), (0, 1)$

B We can deduce the set of idempotents in S is a subset of the idempotents in R since $s \in S \Rightarrow s \in R$ and the conditions in either set is the same: $s^2 = s$.
As shown in problem Aa, the only idempotents in R are 1_R and 0_R
Subrings of an integral domain is an integral domain as well so S also has the property that the idempotents in S are 1_S and 0_R. Therefore we have.
$$\{0_S, 1_S\} \subseteq \{0_R, 1_R\}$$
From basic group theory we know the identity of a subgroup is equal to the identity of the containing group. Therefore $0_S = 0_R$ since 0 is the identity of the groups R, S over additition. So we have $1_S \neq 0_S \Rightarrow 1_S \neq 0_R$. The only other element in $\{0_R, 0_S\}$ that 1_S can be is 1_R

C

D True:
Consider the subring
$$5\mathbb{Z}_{25} = \{0, 5, 10, 15, 20\}$$

E