Gate PH-2010

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27) The solution of the differential equation for $y(t)$: $\frac{d^2y}{dt^2} - y = 2\cosh(t)$, subject to the initial conditions $y(0) = 0$ and $\frac{dy}{dt}|_{t=0} = 0$, is:

a) $\frac{1}{2}\cosh(t) + t\sinh(t)$
b) $-\sinh(t) + t\cosh(t)$
c) $t\cosh(t)$
d) $t\sinh(t)$

28) Given the recurrence relation for the Legendre polynomials:

$$(2n+1)P_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$$

which of the following integrals has a non-zero value?

a) $\int_{-1}^{1} x^2 P_n(x) P_{n+1}(x)\,dx$
b) $\int_{-1}^{1} xP_n(x) P_{n+2}(x)\,dx$
c) $\int_{-1}^{1} x[P_n(x)]^2\,dx$
d) $\int_{-1}^{1} x^2 P_n(x) P_{n+2}(x)\,dx$

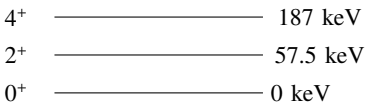
29) For a two-dimensional free electron gas, the electronic density n and the Fermi energy E_f are related by:

a) $\mathrm{n} = \frac{(2mE_f)^{\frac{3}{2}}}{3\pi^2\hbar^3}$
b) $\mathrm{n} = \frac{mE_f}{\pi\hbar^2}$
c) $\mathrm{n} = \frac{mE_f}{2\pi\hbar^2}$
d) $\mathrm{n} = \frac{2^{\frac{3}{2}}(mE_f)^{\frac{1}{2}}}{\pi\hbar}$

30) Far away from any of the resonance frequencies of a medium, the real part of the dielectric permittivity is
a) Always independent of frequency
b) Monotonically decreasing with frequency
c) Monotonically increasing with frequency
d) A non-monotonic function of frequency

31) The ground state wavefunction of a deuteron is in a superposition of s and d states. Which of the following is NOT true as a consequence?
a) It has a non-zero quadrupole moment
b) The neutron-proton potential is non-central
c) The orbital wavefunction is not spherically symmetric
d) The Hamiltonian does not conserve the total angular momentum

32) The first three energy levels of $^{228}\mathrm{Th}_{90}$ are shown below:



The expected spin-parity and energy of the next level are given by:

a) $(6^+, 400\ \text{keV})$
b) $(6^+, 300\ \text{keV})$
c) $(2^+, 400\ \text{keV})$
d) $(4^+, 300\ \text{keV})$

33) The quark content of Σ^+, K, π and p is indicated:

$$|\Sigma^+\rangle = |uus\rangle; \quad |K^+\rangle = |us\rangle; \quad |\pi^+\rangle = |ud\rangle; \quad |p\rangle = |uud\rangle.$$

In the process, $\pi^- + p \to K^- + \Sigma'$, considering strong interactions only, which of the following statements is true?

a) The process is allowed because $\Delta S = 0$.
b) The process is allowed because $\Delta I_y = 0$.
c) The process is not allowed because $\Delta S \neq 0$ and $\Delta I_z \neq 0$.
d) The process is not allowed because the baryon number is violated.

34) The three principal moments of inertia of a methanol (CH_3OH) molecule have the property $I_x = I_y = I$ and $I_z \neq I$. The rotational energy eigenvalues are

a) $\frac{\hbar^2}{2I} l(l+1) + \frac{\hbar^2 m_i^2}{2}\left(\frac{1}{I_i} - \frac{1}{I}\right)$
b) $\frac{\hbar^2}{2I} l(l+1)$
c) $\frac{\hbar^2 m_i^2}{2}\left(\frac{1}{I_i} - \frac{1}{I}\right)$
d) $\frac{\hbar^2}{2I} l(l+1) + \frac{\hbar^2 m_i^2}{2}\left(\frac{1}{I_i} + \frac{1}{I}\right)$

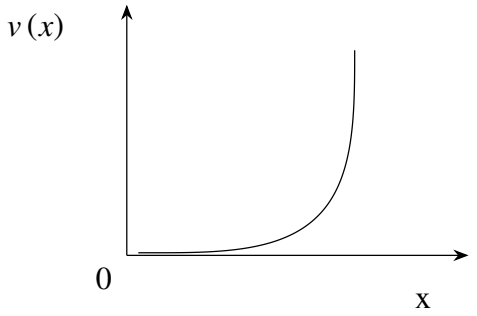
35) A particle of mass m is confined in the potential

$$V(x) = \begin{cases} \frac{1}{2} m\omega^2 x^2 & \text{for } x > 0, \\ \infty & \text{for } x \leq 0. \end{cases}$$

Let the wavefunction of the particle is given by

$$\psi(x) = -\frac{1}{\sqrt{5}}\psi_0 + \frac{2}{\sqrt{5}}\psi_1,$$

where ψ_0 and ψ_1 are the eigenfunctions of the ground state and the first excited state, respectively. The expectation value of the energy is



a) $\frac{31}{10}\hbar\omega$ b) $\frac{25}{10}\hbar\omega$ c) $\frac{13}{10}\hbar\omega$ d) $\frac{11}{10}\hbar\omega$

36) Match the typical spectra of stable molecules with the corresponding wave-number range:

(A) Electronic spectra
(B) Rotational spectra
(C) Molecular dissociation

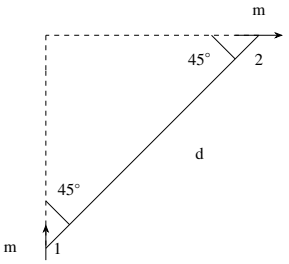
(1) $10^6 cm^{-1}$ and above
(2) $10^5 - 10^6 cm^{-1}$
(3) $10^0 - 10^2 cm^{-1}$

a) $A-2, B-1, C-3$
b) $A-2, B-3, C-1$
c) $A-3, B-2, C-1$
d) $A-1, B-2, C-3$

37) Consider the operations $P : \bar{r} \to -\bar{r}$ (parity) and $T : t \to -t$ (time-reversal). For the electric and magnetic fields $\bar{E}$ and $\bar{B}$, which of the following set of transformations is correct?

a) $P : \bar{E} \to -\bar{E}, \bar{B} \to \bar{B}$;
$T : \bar{E} \to \bar{E}, \bar{B} \to -\bar{B}$
b) $P : \bar{E} \to \bar{E}, \bar{B} \to \bar{B}$;
$T : \bar{E} \to \bar{E}, \bar{B} \to \bar{B}$
c) $P : \bar{E} \to -\bar{E}, \bar{B} \to \bar{B}$;
$T : \bar{E} \to -\bar{E}, \bar{B} \to -\bar{B}$
d) $P : \bar{E} \to \bar{E}, \bar{B} \to -\bar{B}$;
$T : \bar{E} \to -\bar{E}, \bar{B} \to \bar{B}$

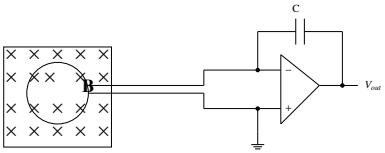
38) Two magnetic dipoles of magnitude m each are placed in a plane as shown.



The energy of interaction is given by:

a) Zero
b) $\frac{\mu_0}{4\pi}\frac{m^2}{d^3}$
c) $\frac{3\mu_0}{2\pi}\frac{m^2}{d^3}$
d) $-\frac{3\mu_0}{8\pi}\frac{m^2}{d^3}$

39) Consider a conducting loop of radius a and total loop resistance R placed in a region with a magnetic field B, thereby enclosing a flux Φ_0. The loop is connected to an electronic circuit as shown, the capacitor being initially uncharged.



If the loop is pulled out of the region of the magnetic field at a constant speed v, the final output voltage V_{out} is independent of:

a) Φ_0
b) R
c) u
d) C