Gate MA-2016

AI24BTECH11034 Tanush Sri Sai Petla

1) Let X be a random variable with the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & x < 0 \\ x^2 & 0 \le x < \frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} \le x < 1 \\ 1 & x \ge 1. \end{cases}$$

Then $P\left(\frac{1}{4} < X < 1\right)$ is equal to ___________

2) Let y be the curve which passes through $(0,1)$ and intersects each curve of the family $y = cx^2$ orthogonally. Then y also passes through the point

a) $(\sqrt{2}, 0)$
b) $(0, \sqrt{2})$
c) $(1, 1)$
d) $(-1, 1)$

3) Let $S(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$ be the Fourier series of the 2π periodic function defined by $f(x) = x^2 + 4\sin(x)\cos(x), -\pi \le x \le \pi$. Then

$$\left| \sum_{n=0}^{\infty} a_n - \sum_{n=1}^{\infty} b_n \right|$$

is equal to ___________

4) Let $y(t)$ be a continuous function on $[0, \infty)$. If

$$y(t) = t\left(1 - 4\int_0^t y(x)\,dx\right) + 4\int_0^t xy(x)\,dx$$

then $\int_0^{\pi/2} y(t)\,dt$ is equal to ___________

5) Let $S_n = \sum_{k=1}^{n} \frac{1}{k}$ and $I_n = \int_1^n \frac{x-[x]}{x^2} dx$. Then $S_{10} + I_{10}$ is equal to

a) $\ln 10 + 1$
b) $\ln 10 - 1$
c) $\ln 10 - \frac{1}{10}$
d) $\ln 10 + \frac{1}{10}$

6) For any $(x, y) \in \mathbb{R}^2 \setminus \overline{B(0,1)}$, let

$$\begin{aligned} f(x,y) &= \text{distance}\left((x,y), \overline{B(0,1)}\right) \\ &= \text{infimum}\left\{ \sqrt{(x-x_1)^2 + (y-y_1)^2} : (x_1, y_1) \in \overline{B(0,1)} \right\}. \end{aligned}$$

Then, $||\nabla f(3,4)||$ is equal to ___________

7) If $f(x) = \left(\int_0^x e^{-t^2}\,dt\right)^2$ and $g(x) = \int_0^1 \frac{e^{-x^2\left(1+t^2\right)}}{1+t^2}\,dt$. Then $f'\left(\sqrt{\pi}\right) + g'\left(\sqrt{\pi}\right)$ is equal to ___________.

8) Let $M = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$ be a real matrix with eigenvalues 1, 0, and 3. If the eigenvectors corresponding to 1 and 0 are $(1,1,1)^T$ and $(1,-1,0)^T$ respectively, then the value of $3f$ is equal to ___________.

9) Let $M = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $e^M = Id + M + \frac{1}{2!}M^2 + \frac{1}{3!}M^3 + \ldots$. If $e^M = [b_{ij}]$, then

$$\frac{1}{e}\sum_{i=1}^{3}\sum_{j=1}^{3} b_{ij}$$

is equal to ___________

10) Let the integral $I = \int_0^4 f(x)\,dx$, where
$f(x) = \begin{cases} x & 0 \le x \le 2 \\ 4 - x & 2 \le x \le 4 \end{cases}$
Consider the following statements P and Q:
(P): The integral values I_2 and I_3 are exact for any function.
(Q): The integral values I_2 and I_3 are exact only for certain types of functions.

Which of the above statements hold TRUE?

a) Both P and Q
b) Only P
c) Only Q
d) Neither P nor Q

11) The difference between the least two eigenvalues of the boundary value problem

$$y'' + \lambda y = 0, \quad 0 < x < \pi$$

$$y(0) = 0, y'(\pi) = 0$$

is equal to ___________.

12) The number of roots of the equation $x^2 - \cos(x) = 0$ in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is equal to ___________

13) For the fixed point iteration $x_{k+1} = g(x_k)$, $k = 0, 1, 2, \ldots$, consider the following statements P and Q:

(P): If $g(x) = 1 + \frac{2}{x}$, then the fixed point iteration converges to 2 for all $x_0 \in [1, 100]$.

(Q): If $g(x) = \sqrt{2 + x}$, then the fixed point iteration converges to 2 for all $x_0 \in [0, 100]$.

Which of the above statements hold TRUE?

a) Both P and Q

b) Only P

c) Only Q

d) Neither P nor Q