Gate PH-2010

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27) The solution of the differential equation for $y(t)$: $\frac{d^2y}{dt^2} - y = 2\cosh(t)$, subject to the initial conditions $y(0) = 0$ and $\frac{dy}{dt}|_{t=0} = 0$, is:

a) $\frac{1}{2}\cosh(t) + t\sinh(t)$ c) $t\cosh(t)$

b) $-\sinh(t) + t\cosh(t)$ d) $t\sinh(t)$

28) Given the recurrence relation for the Legendre polynomials:

$$(2n+1)P_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$$

which of the following integrals has a non-zero value?

a) $\int_{-1}^{1} x^2 P_n(x) P_{n+1}(x)\, dx$ c) $\int_{-1}^{1} x\,[P_n(x)]^2\, dx$

b) $\int_{-1}^{1} x P_n(x) P_{n+2}(x)\, dx$ d) $\int_{-1}^{1} x^2 P_n(x) P_{n+2}(x)\, dx$

29) For a two-dimensional free electron gas, the electronic density n and the Fermi energy E_f are related by:

a) $n = \frac{(2mE_f)^{\frac{3}{2}}}{3\pi^2\hbar^3}$ c) $n = \frac{mE_f}{2\pi\hbar^2}$

b) $n = \frac{mE_f}{\pi\hbar^2}$ d) $n = \frac{2^{\frac{3}{2}}(mE_f)^{\frac{1}{2}}}{\pi\hbar}$

30) Far away from any of the resonance frequencies of a medium, the real part of the dielectric permittivity is

a) Always independent of frequency

b) Monotonically decreasing with frequency

c) Monotonically increasing with frequency

d) A non-monotonic function of frequency

31) The ground state wavefunction of a deuteron is in a superposition of s and d states. Which of the following is NOT true as a consequence?

a) It has a non-zero quadrupole moment

b) The neutron-proton potential is non-central

c) The orbital wavefunction is not spherically symmetric

d) The Hamiltonian does not conserve the total angular momentum

32) The first three energy levels of ^{228}Th$_{90}$ are shown below:

4^+	————————	187 keV
2^+	————————	57.5 keV
0^+	————————	0 keV

The expected spin-parity and energy of the next level are given by:

a) $(6^+, 400 \text{ keV})$ c) $(2^+, 400 \text{ keV})$

b) $(6^+, 300 \text{ keV})$ d) $(4^+, 300 \text{ keV})$

33) The quark content of Σ^+, K, π and p is indicated:

$$|\Sigma^+\rangle = |uus\rangle; \quad |K^+\rangle = |us\rangle; \quad |\pi^+\rangle = |ud\rangle; \quad |p\rangle = |uud\rangle.$$

In the process, $\pi^- + p \rightarrow K^- + \Sigma'$, considering strong interactions only, which of the following statements is true?

a) The process is allowed because $\Delta S = 0$.
b) The process is allowed because $\Delta I_y = 0$.
c) The process is not allowed because $\Delta S \neq 0$ and $\Delta I_z \neq 0$.
d) The process is not allowed because the baryon number is violated.

34) The three principal moments of inertia of a methanol (CH_3OH) molecule have the property $I_x = I_y = I$ and $I_z \neq I$. The rotational energy eigenvalues are

a) $\frac{\hbar^2}{2I} l(l+1) + \frac{\hbar^2 m_i^2}{2} \left(\frac{1}{I_i} - \frac{1}{I} \right)$

c) $\frac{\hbar^2 m_i^2}{2} \left(\frac{1}{I_i} - \frac{1}{I} \right)$

b) $\frac{\hbar^2}{2I} l(l+1)$

d) $\frac{\hbar^2}{2I} l(l+1) + \frac{\hbar^2 m_i^2}{2} \left(\frac{1}{I_i} + \frac{1}{I} \right)$

35) A particle of mass m is confined in the potential

$$V(x) = \begin{cases} \frac{1}{2} m\omega^2 x^2 & \text{for } x > 0, \\ \infty & \text{for } x \leq 0. \end{cases}$$

Let the wavefunction of the particle is given by

$$\psi(x) = -\frac{1}{\sqrt{5}} \psi_0 + \frac{2}{\sqrt{5}} \psi_1,$$

where ψ_0 and ψ_1 are the eigenfunctions of the ground state and the first excited state, respectively. The expectation value of the energy is

a) $\frac{31}{10}\hbar\omega$ b) $\frac{25}{10}\hbar\omega$ c) $\frac{13}{10}\hbar\omega$ d) $\frac{11}{10}\hbar\omega$

36) Match the typical spectra of stable molecules with the corresponding wave-number range:

(A) Electronic spectra   (1) $10^6 cm^{-1}$ and above
(B) Rotational spectra   (2) $10^5 - 10^6 cm^{-1}$
(C) Molecular dissociation   (3) $10^0 - 10^2 cm^{-1}$

a) $A - 2, B - 1, C - 3$ c) $A - 3, B - 2, C - 1$
b) $A - 2, B - 3, C - 1$ d) $A - 1, B - 2, C - 3$

37) Consider the operations $P : \mathbf{r} \to -\mathbf{r}$ (parity) and $T : t \to -t$ (time-reversal). For the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$, which of the following set of transformations is correct?

a) $P : \mathbf{E} \to -\mathbf{E}, \mathbf{B} \to \mathbf{B}$;
    $T : \mathbf{E} \to \mathbf{E}, \mathbf{B} \to -\mathbf{B}$

b) $P : \mathbf{E} \to \mathbf{E}, \mathbf{B} \to \mathbf{B}$;
    $T : \mathbf{E} \to \mathbf{E}, \mathbf{B} \to \mathbf{B}$

c) $P : \mathbf{E} \to -\mathbf{E}, \mathbf{B} \to \mathbf{B}$;
    $T : \mathbf{E} \to -\mathbf{E}, \mathbf{B} \to -\mathbf{B}$

d) $P : \mathbf{E} \to \mathbf{E}, \mathbf{B} \to -\mathbf{B}$;
 $T : \mathbf{E} \to -\mathbf{E}, \mathbf{B} \to \mathbf{B}$

38) Two magnetic dipoles of magnitude m each are placed in a plane as shown.
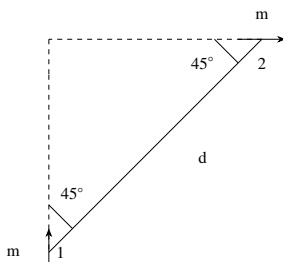
The energy of interaction is given by:

a) Zero

b) $\dfrac{\mu_0}{4\pi} \dfrac{m^2}{d^3}$

c) $\dfrac{3\mu_0}{2\pi} \dfrac{m^2}{d^3}$

d) $-\dfrac{3\mu_0}{8\pi} \dfrac{m^2}{d^3}$

39) Consider a conducting loop of radius a and total loop resistance R placed in a region with a magnetic field B, thereby enclosing a flux Φ_0. The loop is connected to an electronic circuit as shown, the capacitor being initially uncharged.
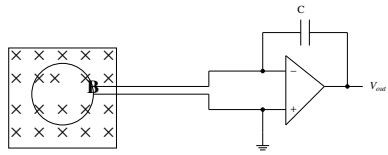
If the loop is pulled out of the region of the magnetic field at a constant speed v, the final output voltage V_{out} is independent of:

a) Φ_0

b) R

c) u

d) C