Gate PH-2010

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1) The solution of the differential equation for $y(t)$: $\frac{d^2y}{dt^2} - y = 2\cosh(t)$, subject to the initial conditions $y(0) = 0$ and $\frac{dy}{dt}|_{t=0} = 0$, is:

   a) $\frac{1}{2}\cosh(t) + t\sinh(t)$
   b) $-\sinh(t) + t\cosh(t)$
   c) $t\cosh(t)$
   d) $t\sinh(t)$

2) Given the recurrence relation for the Legendre polynomials:

$$(2n+1) P_n(x) = (n+1) P_{n+1}(x) + nP_{n-1}(x)$$

   which of the following integrals has a non-zero value?

   a) $\int_{-1}^{1} x^2 P_n(x) P_{n+1}(x)\, dx$
   b) $\int_{-1}^{1} xP_n(x) P_{n+2}(x)\, dx$
   c) $\int_{-1}^{1} x [P_n(x)]^2\, dx$
   d) $\int_{-1}^{1} x^2 P_n(x) P_{n+2}(x)\, dx$

3) For a two-dimensional free electron gas, the electronic density n and the Fermi energy E_f are related by:

   a) n $= \frac{(2mE_f)^{\frac{3}{2}}}{3\pi^2\hbar^3}$
   b) n $= \frac{mE_f}{\pi\hbar^2}$
   c) n $= \frac{mE_f}{2\pi\hbar^2}$
   d) n $= \frac{2^{\frac{3}{2}}(mE_f)^{\frac{1}{2}}}{\pi\hbar}$

4) Far away from any of the resonance frequencies of a medium, the real part of the dielectric permittivity is

   a) Always independent of frequency
   b) Monotonically decreasing with frequency
   c) Monotonically increasing with frequency
   d) A non-monotonic function of frequency

5) The ground state wavefunction of a deuteron is in a superposition of s and d states. Which of the following is NOT true as a consequence?

   a) It has a non-zero quadrupole moment
   b) The neutron-proton potential is non-central
   c) The orbital wavefunction is not spherically symmetric
   d) The Hamiltonian does not conserve the total angular momentum

6) The first three energy levels of $^{228}Th_{90}$ are shown below:

4^+ ———————— 187 keV

2^+ ———————— 57.5 keV

0^+ ———————— 0 keV

The expected spin-parity and energy of the next level are given by:

a) $(6^+, 400\text{ keV})$
b) $(6^+, 300\text{ keV})$
c) $(2^+, 400\text{ keV})$
d) $(4^+, 300\text{ keV})$

7) The quark content of Σ^+, K, π and p is indicated:

$$|\Sigma^+\rangle = |uus\rangle; \quad |K^+\rangle = |u\bar{s}\rangle; \quad |\pi^+\rangle = |u\bar{d}\rangle; \quad |p\rangle = |uud\rangle.$$

In the process, $\pi^- + p \to K^- + \Sigma'$, considering strong interactions only, which of the following statements is true?

a) The process is allowed because $\Delta S = 0$.
b) The process is allowed because $\Delta I_y = 0$.
c) The process is not allowed because $\Delta S \neq 0$ and $\Delta I_z \neq 0$.
d) The process is not allowed because the baryon number is violated.

8) The three principal moments of inertia of a methanol (CH_3OH) molecule have the property $I_x = I_y = I$ and $I_z \neq I$. The rotational energy eigenvalues are

a) $\frac{\hbar^2}{2I} l(l+1) + \frac{\hbar^2 m_i^2}{2}\left(\frac{1}{I_i} - \frac{1}{I}\right)$
b) $\frac{\hbar^2}{2I} l(l+1)$
c) $\frac{\hbar^2 m_i^2}{2}\left(\frac{1}{I_i} - \frac{1}{I}\right)$
d) $\frac{\hbar^2}{2I} l(l+1) + \frac{\hbar^2 m_i^2}{2}\left(\frac{1}{I_i} + \frac{1}{I}\right)$

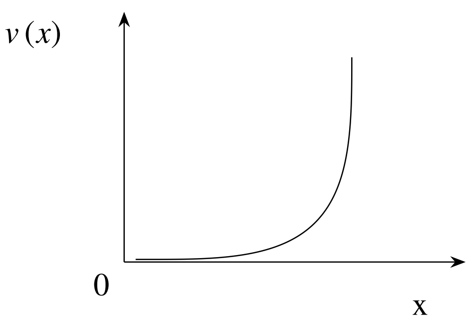
9) A particle of mass m is confined in the potential

$$V(x) = \begin{cases} \frac{1}{2} m\omega^2 x^2 & \text{for } x > 0, \\ \infty & \text{for } x \leq 0. \end{cases}$$

Let the wavefunction of the particle is given by

$$\psi(x) = -\frac{1}{\sqrt{5}}\psi_0 + \frac{2}{\sqrt{5}}\psi_1,$$

where ψ_0 and ψ_1 are the eigenfunctions of the ground state and the first excited state, respectively. The expectation value of the energy is



a) $\frac{31}{10}\hbar\omega$ b) $\frac{25}{10}\hbar\omega$ c) $\frac{13}{10}\hbar\omega$ d) $\frac{11}{10}\hbar\omega$

10) Match the typical spectra of stable molecules with the corresponding wave-number range:

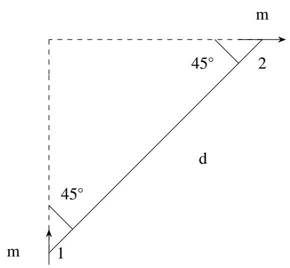
(A) Electronic spectra (1) $10^6 cm^{-1}$ and above
(B) Rotational spectra (2) $10^5 - 10^6 cm^{-1}$
(C) Molecular dissociation (3) $10^0 - 10^2 cm^{-1}$

a) $A-2, B-1, C-3$
b) $A-2, B-3, C-1$
c) $A-3, B-2, C-1$
d) $A-1, B-2, C-3$

11) Consider the operations $P : \mathbf{r} \to -\mathbf{r}$ (parity) and $T : t \to -t$ (time-reversal). For the electric and magnetic fields **E** and **B**, which of the following set of transformations is correct?

a) $P : \mathbf{E} \to -\mathbf{E}, \mathbf{B} \to \mathbf{B}$;
$T : \mathbf{E} \to \mathbf{E}, \mathbf{B} \to -\mathbf{B}$
b) $P : \mathbf{E} \to \mathbf{E}, \mathbf{B} \to \mathbf{B}$;
$T : \mathbf{E} \to \mathbf{E}, \mathbf{B} \to \mathbf{B}$
c) $P : \mathbf{E} \to -\mathbf{E}, \mathbf{B} \to \mathbf{B}$;
$T : \mathbf{E} \to -\mathbf{E}, \mathbf{B} \to -\mathbf{B}$
d) $P : \mathbf{E} \to \mathbf{E}, \mathbf{B} \to -\mathbf{B}$;
$T : \mathbf{E} \to -\mathbf{E}, \mathbf{B} \to \mathbf{B}$

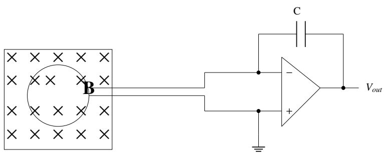
12) Two magnetic dipoles of magnitude m each are placed in a plane as shown.



The energy of interaction is given by:

a) Zero
b) $\frac{\mu_0}{4\pi}\frac{m^2}{d^3}$
c) $\frac{3\mu_0}{2\pi}\frac{m^2}{d^3}$
d) $-\frac{3\mu_0}{8\pi}\frac{m^2}{d^3}$

13) Consider a conducting loop of radius a and total loop resistance R placed in a region with a magnetic field B, thereby enclosing a flux Φ_0. The loop is connected to an electronic circuit as shown, the capacitor being initially uncharged.



If the loop is pulled out of the region of the magnetic field at a constant speed v, the final output voltage V_{out} is independent of:

a) Φ_0
b) R
c) u
d) C