Complex Numbers

AI24BTECH11034 - Tanush Sri Sai Petla*

Section-A

Fill in the blanks

1. If the expression $\frac{\left[\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)+i\tan(x)\right]}{1+2i\sin\left(\frac{x}{2}\right)}$ $(1987-2Marks)$ is real,then the set of all possible values of x is.....

2. For any two complex numbers z_1, z_2 and any real number a and b.
$|az_1 - bz_2|^2 + |bz_1 + az_2|^2$ =..... $(1988-2Marks)$

3. If a,b,c are the numbers between 0 and 1 such that the points $z_1 = a+i$, $z_2 = 1+bi$ and $z_3 = 0$ form an equilateral triangle,then $a =$ .... and $b =$ ..... $(1989-2Marks)$

4. $ABCD$ is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy $BD = 2AC$. If the points D and M represent the complex numbers $1+i$ and $2-i$ respectively, then A represents the complex number..... or..... $(1993-2Marks)$

5. Suppose Z_1,Z_2,Z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$.If $Z_1 = 1+i\sqrt{3}$ then $Z_2 =$ .... ,$Z_3 =$ .... $(1994-2Marks)$

B True/False

1. For complex number $z_1 = x_1+iy_1$ and $z_2 = x_2+iy_2$, we write $z_1 \cap z_2$, if $x_1 \leq x_2$and $y_1 \leq y_2$ then for all complex numbers z with $1 \cap z$, we have $\frac{1-z}{1+z} \cap 0$ $(1981-2Marks)$

2. If the complex numbers z_1, z_2 and z_3 represent the vertices of an equilateral triangle such that $|z_1| = |z_2| = |z_3|$ then $z_1 + z_2 + z_3 = 0$ brak1984 - 1 Mark

3. If three complex numbers are in A.P. then they lie on a circle on the complex plane. $(1985-1Mark)$

4. The cube roots of unity when represented on Argand diagram form the vertices of an equilateral triangle. $(1988-1Mark)$

C MCQs with One Correct Answer

1. If the cube roots of unity are 1,ω, ω^2 , then the roots of the equation $(x+1)^8 = 0$ are (1979)

(a) $-1,\ i+2\omega,1+2\omega^2$ (c) $-1,-1,-1$
(b) $-1\ ,1-2\omega\ ,1-2\omega^2$ d None of these

2. The smallest positive integer for which $\left(\frac{1+i}{1-i}\right)^n =$ 1 is (1980)

(a) $n = 8$ (c) $n = 12$
(b) $n = 16$ (d) None of these

3. The complex number $z = x+iy$ which satisfy the equation $(1981-2Marks)$ $\left|\frac{z-5i}{z+5i}\right| = 1$lie on

(a) the x-axis the origin
(b) the straight line $y = 5$
(c) a circle passing through(d) None of these

4. If $z = (\frac{\sqrt{3}}{2}+\frac{i}{2})^5 + \left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$, then $(1982-2Marks)$

$(a)\,Re(z) = 0$ $(c)\,Re(z) > 0, Im(z) > 0$
$(b)\,Im(z) = 0$ $(d)\,Re(z) > 0, Im(z) < 0$

5. The inequality $|z-4| < |z-2|$ represents the region given by $(1982-2Marks)$

$(a)\,Re(z) \geq 0$ $(c)\,Re(z) > 0$
$(b)\,Re(z) < 0$ $(d)\,None of these$