Complex Numbers

AI24BTECH11034 - Tanush Sri Sai Petla*

Section-A

Fill in the blanks

1. If the expression

$\frac{[\sin(\frac{x}{2})+\cos(\frac{x}{2})+i\tan(x)]}{[1+2i\sin\frac{x}{2}]}$ (1987 - 2 Marks)

is real,then the set of all possible values of x is.....

2. For any two complex numbers z_1, z_2 and any real number a and b.

$|az_1 - bz_2|^2 + |bz_1 + az_2|^2$ =..... (1988 - 2 Marks)

3. If a,b,c are the numbers between 0 and 1 such that the points $z_1 = a + i$, $z_2 = 1 + bi$ and $z_3 = 0$ form an equilateral triangle,then $a =$ .... and $b =$ ..... (1989 - 2 Marks)

4. ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy $BD = 2AC$. If the points D and M represent the complex numbers $1 + i$ and $2 - i$ respectively, then A represents the complex number..... or..... (1993 - 2 Marks)

5. Suppose Z_1,Z_2,Z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$.If $Z_1 = 1+i\sqrt{3}$ then $Z_2 =$ .... ,$Z_3 =$ .... (1994 - 2 Marks)

B True/False

1. For complex number $z_1 = x_1+iy_1$ and $z_2 = x_2+iy_2$, we write $z_1 \cap z_2$, if $x_1 \leq x_2$and $y_1 \leq y_2$ then for all complex numbers z with $1 \cap z$, we have $\frac{1-z}{1+z} \cap 0$ (1981 - 2 Marks)

2. If the complex numbers z_1, z_2 and z_3 represent the vertices of an equilateral triangle such that $|z_1| = |z_2| = |z_3|$ then $z_1 + z_2 + z_3 = 0$ (1984 - 1 Mark)

3. If three complex numbers are in A.P. then they lie on a circle on the complex plane. (1985 - 1 Mark)

4. The cube roots of unity when represented on Argand diagram form the vertices of an equilateral triangle. (1988 - 1 Mark)

C MCQs with One Correct Answer

1) If the cube roots of unity are 1,ω, ω^2 , then the roots of the equation $(x + 1)^8 = 0$ are (1979)
   a) $-1,\ 1 + 2\omega$,$1 + 2\omega^2$
   b) -1 ,$1 - 2\omega$,$1 - 2\omega^2$
   c) $-1, -1, -1$
   d) None of these

2) The smallest positive integer for which $(\frac{1+i}{1-i})^n =$ 1 is (1980)
   a) $n = 8$
   b) $n = 16$
   c) $n = 12$
   d) None of these

3) The complex number $z = x + iy$ which satisfy the equation (1981 - 2 Marks) $\left|\frac{z-5i}{z+5i}\right| = 1$lie on
   a) the x-axis
   b) the straight line $y = 5$
   c) a circle passing through the origin
   d) None of these

4) If $z = (\frac{\sqrt{3}}{2} + \frac{i}{2})^5 + (\frac{\sqrt{3}}{2} - \frac{i}{2})^5$, then (1982 - 2 Marks)
   a) $Re(z) = 0$
   b) $Im(z) = 0$
   c) $Re(z) > 0, Im(z) > 0$
   d) $Re(z) > 0, Im(z) < 0$

5) The inequality $|z - 4| < |z - 2|$ represents the region given by (1982 - 2 Marks)
   a) $Re(z) \geq 0$
   b) $Re(z) < 0$
   c) $Re(z) > 0$
   d) None of these