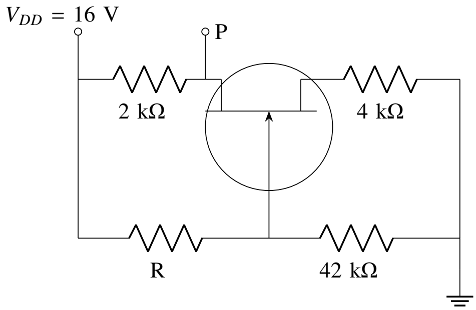
# Gate PH-2007

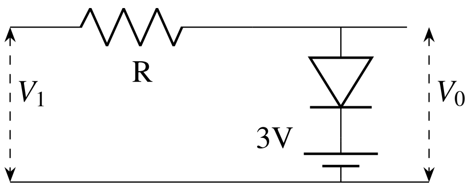
AI24BTECH11034 Tanush Sri Sai Petla

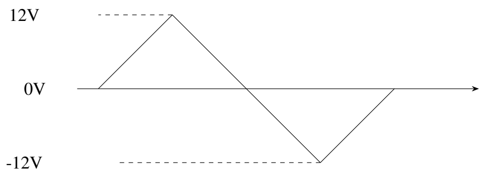
69) In the circuit shown, the voltage at test point P is 12 V and the voltage between gate and source is -2 V. The value of $R$ (in kΩ) is

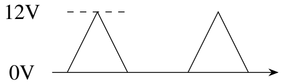



a) 42 b) 48 c) 56 d) 70

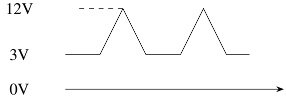
70) When an input voltage $V_i$, of the form shown, is applied to the circuit given below, the output voltage $V_o$ is of the form:



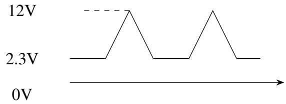



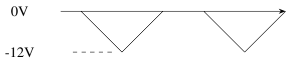

a)




b)




c)




d)

## Common Data Questions

**Common Data for Questions 71, 72, 73:**
A particle of mass $m$ is confined in the ground state of a one-dimensional box, extending from $x = -2L$ to $x = +2L$. The wavefunction of the particle in this state is

$$\psi(x) = \psi_0 \cos\left(\frac{\pi x}{4L}\right)$$

where $\psi_0$ is a constant.

71) The normalization factor $\psi_0$ of this wavefunction is

a) $\frac{\sqrt{2}}{L}$ b) $\frac{1}{4L}$ c) $\frac{1}{\sqrt{2L}}$ d) $\frac{1}{L}$

72) The energy eigenvalue corresponding to this state is

a) $\frac{\hbar^2\pi^2}{2mL^2}$ b) $\frac{\hbar^2\pi^2}{4mL^2}$ c) $\frac{\hbar^2\pi^2}{16mL^2}$ d) $\frac{\hbar^2\pi^2}{32mL^2}$

73) The expectation value of $p^2$ ($p$ is the momentum operator) in this state is

a) 0 b) $\frac{\hbar^2\pi^2}{32L^2}$ c) $\frac{\hbar^2\pi^2}{16L^2}$ d) $\frac{\hbar^2\pi^2}{8L^2}$

**Common Data for Questions 74, 75:**
The Fresnel relations between the amplitudes of incident and reflected electromagnetic waves at an interface between air and a dielectric of refractive index $\mu_r$ are:

$$\frac{E_{\parallel}^{\text{reflected}}}{E_{\parallel}^{\text{incident}}} = \frac{\mu_r \cos r - \cos i}{\mu_r \cos r + \cos i} \quad \text{and} \quad \frac{E_{\perp}^{\text{reflected}}}{E_{\perp}^{\text{incident}}} = \frac{\cos i - \mu_r \cos r}{\cos i + \mu_r \cos r}$$

The subscripts $\|$ and $\perp$ refer to polarization, parallel and normal to the plane of incidence respectively. Here, $i$ and $r$ are the angles of incidence and refraction respectively.

74) The condition for the reflected ray to be completely polarized is

a) $\mu \cos i = \cos r$ b) $\cos i = \mu \cos r$ c) $\mu \cos i = -\cos r$ d) $\cos i = -\mu \cos r$

75) For normal incidence at an air-glass interface with $\mu = 1.5$, the fraction of energy reflected is given by

a) 0.40 b) 0.20 c) 0.16 d) 0.04

**Linked Answer Questions: Q.76 to Q.85 carry two marks each.**

**Statement for Linked Answer Questions 76 and 77:**

In the laboratory frame, a particle P of rest mass $m_e$ is moving in the positive x direction with a speed of $\frac{5c}{19}$. It approaches an identical particle Q, moving in the negative x direction with a speed of $\frac{2c}{5}$.

76) The speed of the particle P in the rest frame of the particle Q is

a) $\frac{7c}{95}$ b) $\frac{13c}{85}$ c) $\frac{3c}{5}$ d) $\frac{63c}{95}$

77) The energy of the particle P in the rest frame of the particle Q is

a) $\frac{1}{2} m_g c^2$ b) $\frac{5}{4} m_0 c^2$ c) $\frac{19}{13} m_0 c^2$ d) $\frac{11}{2} m$

**Statement for Linked Answer Questions 78 and 79**

The atomic density of a solid is $5.85 \times 10^{32} m^{-3}$. Its electrical resistivity is $1.6 \times 10^{-2} \Omega\ m$.

Assume that electrical conduction is described by the Drude model (classical theory), and that each atom contributes one conduction electron.

78) The drift mobility (in $m^2 V^{-1} s.^{-1}$) of the conduction electrons is

a) $6.67 \times 10^{-3}$ b) $6.67 \times 10^{-6}$ c) $7.63 \times 10^{-1}$ d) $7.63 \times 10^{-9}$

79) The relaxation time (mean free time), in seconds, of the conduction electrons is

a) $3.98 \times 10^{-13}$ b) $3.79 \times 10^{-14}$ c) $2.84 \times 10^{-12}$ d) $2.64 \times 10^{-11}$

**Statement for Linked Answer Questions 80 and 81:**

A sphere of radius R carries a polarization $\vec{P} = kr$, where k is a constant and $\vec{r}$ is measured from the centre of the sphere.

80) The bound surface and volume charge densities are given, respectively, by

a) $-k\left|\vec{r}\right|$ and $3k$

b) $k\left|\vec{r}\right|$ and $-3k$

c) $k\left|\vec{r}\right|$ and $4\pi kR$

d) $-k\left|\vec{r}\right|$ and $-4\pi kR$

81) The electric field $\vec{E}$ at a point outside the sphere is given by

a) $\vec{E} = 0$ b) $\vec{E} = \frac{kR\left(R^2-r^2\right)}{4\pi\epsilon_0 r^3}\hat{r}$ c) $\vec{E} = \frac{kR\left(R^2-r^2\right)}{4\pi\epsilon_0 r^3}\hat{r}$ d) $\vec{E} = \frac{3k(r-R)}{4\pi\epsilon_0 r^4}\hat{r}$

**Statement for Linked Answer Questions 82 and 83:**
An ensemble of quantum harmonic oscillators is kept at a finite temperature $T = \frac{1}{k_B\beta}$.

82) The partition function of a single oscillator with energy levels $\left(n+\frac{1}{2}\right)\hbar\omega$ is given by

a) $Z = \frac{e^{-\beta\hbar\omega/2}}{1-e^{-\beta\hbar\omega}}$ b) $Z = \frac{e^{-\beta\hbar\omega}}{1-e^{-\beta\hbar\omega}}$ c) $Z = \frac{1}{1-e^{-\beta\hbar\omega}}$ d) $Z = \frac{1}{1-e^{-2\beta\hbar\omega-\mu}}$

83) The average number of energy quanta of the oscillators is given by

a) $\langle n\rangle = \frac{1}{e^{\beta\hbar\omega}-1}$ b) $\langle n\rangle = \frac{e^{-\beta\hbar\omega}}{e^{\beta\hbar\omega}-1}$ c) $\langle n\rangle = \frac{e^{-2\beta\hbar\omega}}{e^{2\beta\hbar\omega}+1}$ d) $\langle n\rangle = \frac{1}{e^{\beta\hbar\omega}+1}$

**Statement for Linked Answer Questions 84 and 85:**
A 16 $\mu A$ beam of alpha particles, having cross-sectional area $10^{-4}m^2$, is incident on a rhodium target of thickness 1 $\mu m$. This produces neutrons through the reaction $^{103}Rh + \alpha \rightarrow ^{106} Pd + 3n$.

84) The number of alpha particles hitting the target per second is

a) $0.5\times10^{14}$ b) $1.0\times10^{14}$ c) $2.0\times10^{20}$ d) $4.0\times10^{19}$

85) The neutrons are observed at the rate of $1.806\times10^{8}s^{-1}$. If the density of rhodium is approximated as $10^4 kg\ m^{-3}$, the cross-section for the reaction (in barns) is

a) 0.1 b) 0.2 c) 0.4 d) 0.8