

# Complex Numbers

AI24BTECH11034 - Tanush Sri Sai Petla\*

## Section-A

Fill in the blanks

1. If the expression

$$\frac{[\sin(\frac{x}{2}) + \cos(\frac{x}{2}) + i \tan(x)]}{[1 + 2i \sin \frac{x}{2}]}$$

(1987 – 2Marks)

is real, then the set of all possible values of  $x$  is...

2. For any two complex numbers  $z_1, z_2$  and any real number  $a$  and  $b$ .

$$|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = \dots$$

(1988 – 2Marks)

3. If  $a, b, c$  are the numbers between 0 and 1 such that the points  $z_1 = a + i, z_2 = 1 + bi$  and  $z_3 = 0$  form an equilateral triangle, then  $a = \dots$  and  $b = \dots$  (1989 – 2Marks)

4. ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy  $BD = 2AC$ . If the points D and M represent the complex numbers  $1 + i$  and  $2 - i$  respectively, then A represents the complex number ... or ... (1993 – 2Marks)

5. Suppose  $Z_1, Z_2, Z_3$  are the vertices of an equilateral triangle inscribed in the circle  $|z| = 2$ . If  $Z_1 = 1 + i\sqrt{3}$  then  $Z_2 = \dots, Z_3 = \dots$  (1994 – 2Marks)

B True/False

1. For complex number  $z_1 = x_1 + iy_1$  and  $z_2 = x_2 + iy_2$ , we write  $z_1 \cap z_2$ , if  $x_1 \leq x_2$  and  $y_1 \leq y_2$ . then for all complex numbers  $z$  with  $1 \cap z$ , we have  $\frac{1-z}{1+z} \cap 0$  (1981 – 2Marks)

2. If the complex numbers  $z_1, z_2$  and  $z_3$  represent the vertices of an equilateral triangle such that  $|z_1| = |z_2| = |z_3|$  then  $z_1 + z_2 + z_3 = 0$  (1984 – 1Mark)

3. If three complex numbers are in A.P. then they lie on a circle on the complex plane. (1985 – 1Mark)

4. The cube roots of unity when represented on Argand diagram form the vertices of an equilateral triangle. (1988 – 1Mark)

C MCQs with One Correct Answer

1. If the cube roots of unity are  $1, \omega, \omega^2$ , then the roots of the equation  $(x + 1)^8 = 0$  are (1979)

(a)  $-1, 1 + 2\omega, 1 + 2\omega^2$

(b)  $-1, 1 - 2\omega, 1 - 2\omega^2$

(c)  $-1, -1, -1$

(d) None of these

2. The smallest positive integer for which (1980)

$(\frac{1+i}{1-i})^n = 1$  is

(a)  $n=8$

(b)  $n=16$

(c)  $n=12$

(d) None of these

3. The complex number  $z = x + iy$  which satisfy the equation (1981 – 2Marks)

$|\frac{z-5i}{z+5i}| = 1$  lie on

(a) the  $x$ -axis

(b) the straight line  $y=5$

(c) a circle passing through the origin

(d) None of these

4. If  $z = (\frac{\sqrt{3}}{2} + \frac{i}{2})^5 + (\frac{\sqrt{3}}{2} - \frac{i}{2})^5$ , then (1982 – 2Marks)

(a)  $Re(z)=0$

(b)  $Im(z)=0$

(c)  $Re(z) > 0, Im(z) > 0$

(d)  $Re(z) > 0, Im(z) < 0$

5. The inequality  $|z-4| < |z-2|$  represents the region given by (1982 – 2Marks)

(a)  $Re(z) \geq 0$

(b)  $Re(z) < 0$

(c)  $Re(z) > 0$

(d) None of these