Active management is not zero-sum

Pedersen contra Sharpe, Buffett y Bogle

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> *Let's imagine the market consists of 100 fund managers, each of whom initially owns one-hundredth of all the companies in the world. Now imagine that 50 of them do nothing (they're index managers), while the rest trade shares among themselves and pay high commissions to advisors and managers, say 2% per year (they're active managers).*
>
> *Now let's wait a year. Both groups will have the same returns before fees since, as a group, they start with the same portfolio and end with the same portfolio, but the indexed group will have an average of 2% higher return after fees. If we wait 10 years, then on average the indexed group will have performed 20% better (actually more because fees are compounded). Therefore, over 10 years, there will be a smaller percentage of active funds that perform better than the index than over 1 year.*

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"This is pure mathematics, this is arithmetic." @uansejo : youtu.be/-q1fXxdaI6w?t=...

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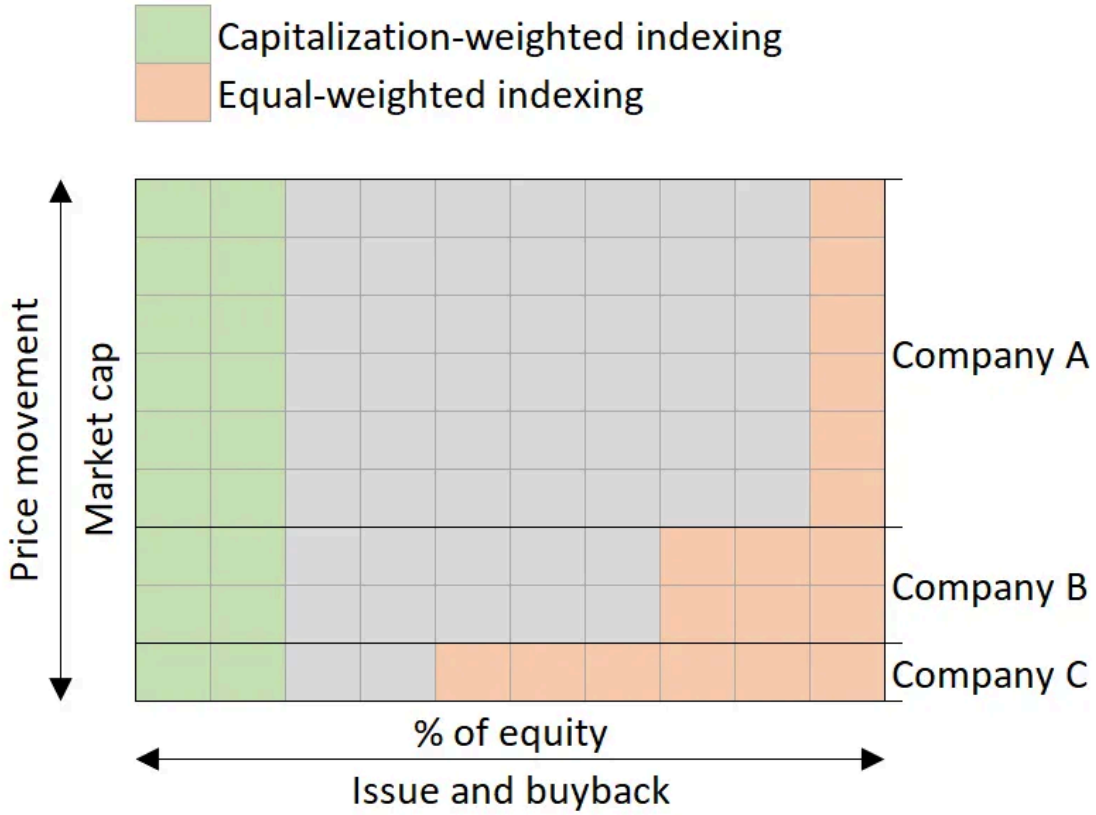
I admit that when I heard this excerpt from Unai, I barely understood the dynamics of passive management. Reflecting on it helped me understand the implications of replicating a capitalization-weighted index. The passive investor is immune to price movements; their movements don't force them to trade: if a company rises, it will weigh more heavily on both the index and their portfolio. Furthermore, it is epistemologically neutral: if an active investor deviates from the index in a certain direction, it is because another is deviating in the opposite direction, so that together they form the index. The capitalization-weighted index reflects the consensus of all active investors, and replicating it is assuming you don't know who is right.



Every portfolio has an opposite to fill the market. If one is right, the other is wrong. Capitalization-weighted is the only symmetric criteria, the only one that matches its opposite.

However, although I found it to be a very useful first approximation, I always felt that Unai's argument was flawed. It's robust because it's simple and elegant, but intuitively, something was wrong, though I wasn't quite sure what.

The suspicions arose from two sources. First, it depicted a non-equilibrium model. If the argument were true, we could only expect that in a few years, decades, or centuries, everything would be passive and active management would be nonexistent. And second, gathering information and analyzing it to make better decisions is a useful

function that the market usually rewards in any micro context, but which it wouldn't be rewarding in this case.

So we have two self-evident assumptions that are incompatible with simple and seemingly robust arithmetic. How do we fix this? Well, let's start from the beginning.

Sharpe arithmetic

Unai's fragment is a very concise summary of the Gotrocks family parable. The Gotrocks family is a story invented by Warren Buffett in his 2005 annual letter (page 18), whose conclusion, in Buffett's own words, is *"For investors as a whole, returns decrease as motion increases* ." John Bogle later included this parable in chapter 1 of his book *The Little Book of Common Sense Investing* . However, and although these are the popular versions that have become popular, the foundations of this parable are found in a 1991 article by William Sharpe entitled *The Arithmetic of Active Management.* [1] In this article Sharpe explains that:

1. Before costs, the profitability of active and passive management is the same since both groups have the same portfolio in aggregate.
2. After costs, the profitability of active management is lower than passive management because they have incurred higher costs.

That is, active management is a zero-sum game and becomes negative-sum after costs. Furthermore, Sharpe makes some statements in the article that I find quite shocking in retrospect. For example, he says that if the data doesn't support him, it's because they're wrong: *"Empirical analyses that appear to refute this principle are guilty of improper measurement* ." He also says in footnote 3 [2] that *"some corporate events require more complex calculations but do not affect the basic principles* . "

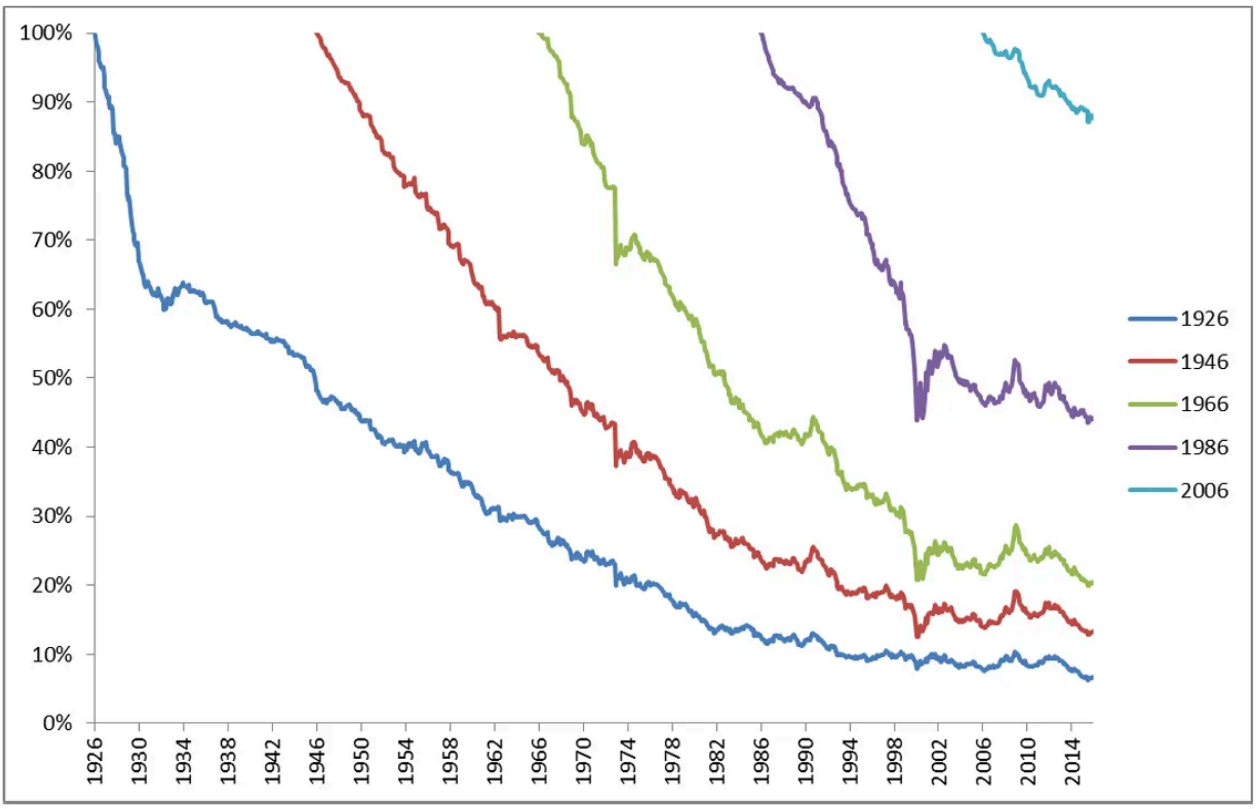
And it's precisely this footnote that destroys his argument. I suppose Sharpe thought it was a minor detail—how can a few corporate events change such a robust conclusion? Well, it may be a detail, but it's by no means minor; it's the heart of what this argument analyzes. Sharpe is comparing whether the costs of collecting and analyzing information make sense. However, his model doesn't include the potential positive effects that incurring those costs can generate. It's like comparing whether it's worth taking out comprehensive insurance in a world without accidents, or assessing the utility of R&D spending in a world where nothing remains to be invented. It doesn't make any sense. The effect of accidents or inventing something new must be incorporated into the model.

In the case of financial markets, what active management pursues with the costs it incurs is to find assets that are more profitable than the index. If this happens simply by exchanging slips of paper between active managers, the net effect is zero: one active manager thinks one thing and another thinks the opposite. In these cases, we could say that both active managers have wasted money, incurred analysis costs, and in aggregate, we know it was a zero-sum game. The market as a whole will have only achieved a more accurate price or greater certainty about the value of assets (which could be useful), but there has been no real change in the economy that could generate additional profits to justify the costs incurred.

However, when the opinions of active managers do not fully offset each other, there is a net effect that moves asset prices and this ends up translating into changes in the composition of the index. [3]That is to say, active management as a whole, beyond the fact that some of its conclusions may cancel each other out, serves the function of finding and maintaining the productive organization that best satisfies our consumption desires. And the specific mechanism through which these changes in the index manifest themselves are precisely corporate events: expansions, buybacks, inclusions, and exclusions.

And this is precisely what Lasse Heje Pedersen points out in his 2018 article titled " *Sharpening the Arithmetic of Active Management* ." Pedersen is a Danish financial economist, professor at Copenhagen Business School, and partner at AQR Capital, known for his research on liquidity risk and asset pricing.

Pedersen explains that Sharpe arithmetic is based on the implicit assumption that the market never changes. But the real world isn't like that; the set of market values isn't fixed. The average annual turnover reaches 7.6% for all US listed stocks from 1926 to 2015 (and for bonds it would be even higher at ~20%). Because of this, even passive investors must trade regularly to maintain their market portfolio. This chart shows how a passive investor who never trades would separate himself from the market.



Pedersen doesn't just tell a story; he also cites papers, crunches numbers, and attempts to estimate the impact of these corporate events. He concludes that active management plays a crucial economic role, aiding efficient capital allocation, and that active and passive investors coexist in an "efficiently inefficient" market. This is, in fact, the title of his latest book (*Efficiently Inefficient: How Smart Money Invests and Market Prices Are Determined*), which leaves us with quotes like these:

> *If markets are completely efficient, active investing is pointless, since prices already reflect all possible information. But without active investors, who would make the market efficient?*
>
> *Furthermore, given that investors pay billions in fees to active managers, either the stock markets are inefficient (and can be beaten) or the asset management market is (because they pay for nothing). Neither can be fully efficient.*
>
> *Efficiently inefficient: Sufficiently inefficient that active managers can be compensated for their costs, and sufficiently efficient that those rewards, after costs, do not incentivize the entry of new managers or additional capital.*

Unlike Sharpe, Pedersen's model does offer an equilibrium: there is an optimal point in the amount of resources the market should dedicate to analysis; below that point, active managers will obtain extraordinary benefits, and above that point, they will fail

to compensate for their costs. It is also a stable equilibrium: any disturbance endogenously generates incentives to return to equilibrium. [4].

The market must balance planned production (companies) and desired consumption (investors). Active management can create value by influencing production flows (corporate events) and consumption flows (subscriptions and redemptions).

It took us 30 years, but we can finally sleep soundly.

The true story of the Gotrocks family

Let's imagine for a moment that all the companies in the market are owned by a single family: the Gotrocks. Generation after generation, this family grows richer thanks to the total profits earned by their companies. In the Gotrocks' household, everyone grows richer at the same rate, and everything runs smoothly. Each family member owns, proportionally, an identical share of all the companies in the market.

Now suppose an active manager approaches a family member, let's call him Mr. Buffit Gotrocks. The manager convinces Mr. Buffit to try to outsmart his relatives by buying some shares and selling others. The manager, in exchange for a commission, of course, kindly agrees to handle these transactions.

But who is he going to trade with? The entire market (except him) is passive. We might think there would be no prices or transactions, but there are, for at least three reasons:

1. The active manager, even if he or she finds no sellers or buyers, will have a mental purchase and sale price: a bid and an ask that reflect his or her internal valuation of each company.
2. Passive family members, even if they replicate the index, will have net flows from subscriptions and redemptions, and will therefore need to go to the market.
3. And above all, companies themselves also have a bid and an ask: they are willing to buy back their own shares or issue new ones depending on the price and expected profitability of their projects.

Over time, the active manager discovers that one of the companies is misusing resources and decides to engage in a buyback: he sells all of its shares for 1M. Shortly after, he finds another company with potentially very profitable expansion projects and participates in a capital increase: he buys new shares for 1M.

Some time later, when the results of these corporate actions become public, the passive relatives realize that the market has escaped them. To maintain the index portfolio, they must sell shares of the wasteful company and buy shares of the promising company. The active manager is happy to match them, of course at somewhat less attractive prices than those at which he initially traded. [5].

At the end of the period, the index has achieved whatever profitability it might have (depending on when and how it includes the results of events), but the reality is that Mr. Buffit has been able to beat its peers even after adjusting for costs. And why? The market is now presumably more valuable than at the beginning of the period, and not only because companies may have created value internally, but also because productive factors have been reorganized across companies. [6]The same resources, organized differently, now produce more or more valuable things. The wasteful company, for example, had to sell certain machinery and lay off some workers; with that money, it bought back shares. And where did those factors go? Well, to the promising company that bought them with the money it raised by issuing new shares. This value creation is a direct result of Mr. Buffit and his active manager, and the market rewards their actions with higher returns.

The following year, the active manager tells Mr. Buffit that he sees no more opportunities, that the 'market portfolio' is perfect. Mr. Buffit tells him it doesn't matter, he should keep trying to estimate how much his companies will earn. Over the next few weeks, he meets with his relatives and inquires about their stories: how much they earn, how much they spend, when they retire... He discovers that next Monday his brother-in-law will receive a 400k bonus that he predictably wants to invest. But if no one offers him any compensation, prices will skyrocket. The following month, his cousin plans to buy a house and needs to repay 1M. But if no one offers any compensation, prices will drop to zero. He decides, in exchange for an acceptable spread, to cover the gap himself. [7]Sell shares to your brother-in-law next Monday and buy them from your cousin in a month. And the remaining 600,000? The manager has informed you that dividends will be paid next month, and that's exactly the amount he estimates will cover.

In the following years, Mr. Buffit's extraordinary profits attract more relatives to active management, which, due to the usual dynamics of any sector, erodes its profits. The opportunities to be found (both wasteful and promising companies) and the liquidity gaps to be filled (net subscriptions and redemptions) are the same, but more and more people are competing for them. [8]There comes a point where the profit they earn above

the index falls to equal their costs. At that point, no family member has any incentive to change their strategy. And Buffett's law finally comes true: whoever moves loses.

P.S. 1. You can see all my posts grouped by topic in the Index .

Pd2. Related posts: The Gotrocks Family's Deceptive Story , 3 Bad Arguments for Passive Management , Passive Management in 4 Charts .

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1 Fama and French praising Sharpe's article: Why active investing is a negative sum game .

2 Sharpe's footnote 3 points out the first gap, and footnote 4 the second: *"We assume here that passive managers buy their securities before the start of the period in question and do not sell them until its end. When they buy or sell, they may be forced to negotiate with active managers, due to the latter's willingness to provide the desired liquidity (at a price)."*

3 How our consumption preferences influence our portfolio and how our portfolio influences the productive structure .

4 Although they do not get down in the mud with corporate events as Pedersen does, Grossman and Stiglitz had already written about the information paradox and partially efficient markets 10 years before Sharpe published his arithmetic: On the Impossibility of Informationally Efficient Markets .

5 We could provide some context for this story: how many family members and how many companies there are, at what prices the capital increases and buybacks are made, at what prices the passive family members then trade... I'll leave the practical case here for the curious to enrich as they wish: Can active management increase its weight in the index without any money coming in or out of either active or passive management? For homework: if company A sells its own shares at t=1 for €1, why doesn't it come in at t=2 to provide the counterpart to the passive investor who is buying them at €1.50? Hint: marginal returns.

6 Why can't companies themselves carry out this value creation by shifting factors between companies? They can, and in fact, do so through their M&A departments, but *only* companies doing so exposes investors to agency problems that will ultimately push them toward activism. In any case, analyzing these opportunities is valuable, and investors will bear the cost of this function, whether it's performed within the company, outside of it, or a combination of both.

7 The Treynor model used by Perry Mehrling seems very suggestive to me to illustrate the situation: active management would be acting as a dealer, creating a market for liabilities

within the outside spread generated by the true dealer of last resort, the companies themselves.

8 Competition forces each active relative to try to specialize in a specific knowledge (different strategies that build on each other): some in certain companies or sectors, others in discovering large opportunities that are difficult to see, others in small opportunities that are easy to detect, others specialize in asset classes, others in estimating seasonal or short-term subscription/repayment flows...