Regularized PET reconstruction with a stochastic algorithm: user-friendly step-sizes lead to faster convergence

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What?

$$\min_{x \geq 0} \sum_{i=1}^{n} \underbrace{\mathrm{KL}(L_i x + b_i, d_i)}_{\text{data fit over subsets}} + \underbrace{\lambda \|\nabla x\|_{2,1}}_{\text{TV prior}} \qquad (*)$$

How?

- **OSEM** Ordered Subsets Expectation Maximization [1]
- **PDHG** Primal-Dual Hybrid Gradient algorithm [4]
- **SPDHG** Stochastic Primal-Dual Hybrid Gradient [2,3]

	OSEM	PDHG	SPDHG
Use subsets	Yes	No	Yes
Convergence guarantee	No	Yes	Yes
Non-smooth prior	No	Yes	Yes

Primal-dual problem

Generalization of problem $(*)$ for convex functions f_i with convex conjugate f_i^* and convex function g:

$$\min_x \sum_{i=1}^{n+1} f_i(A_i x) + g(x) \quad = \min_x \sup_{y_i} \sum_{i=1}^{n+1} \langle A_i x, y_i \rangle - f_i^*(y_i) + g(x)$$

SPDHG

Input: Primal step-size τ and dual $(\sigma_i)_{1 \leq i \leq n+1}$
Probabilities $p_i, 1 \leq i \leq n$
$x = 0,\ y = 0,\ z = \bar{z} = A^T y = 0;$
for $m = 1, \ldots,$ **do**
$x = \mathrm{prox}_{\tau g}(x - \tau \bar{z});$
Pick $i \in \{1, \ldots n+1\}$ with prob. p_i;
$y_i^+ = \mathrm{prox}_{\sigma_i f_i^*}(y_i + \sigma_i A_i x);$
$dz = A_i^T(y_i^+ - y_i), \quad z = z + dz, \quad y_i = y_i^+;$
$\bar{z} = z + (1/p_i) \cdot dz$
end

Contribution #1: new admissible step-sizes

Theorem: SPDHG converges for any $\gamma > 0$ and step-sizes as follows:

$$\sigma_i = \frac{0.99}{\gamma \cdot \|A_i\|}, \quad 1 \leq i \leq n+1;$$

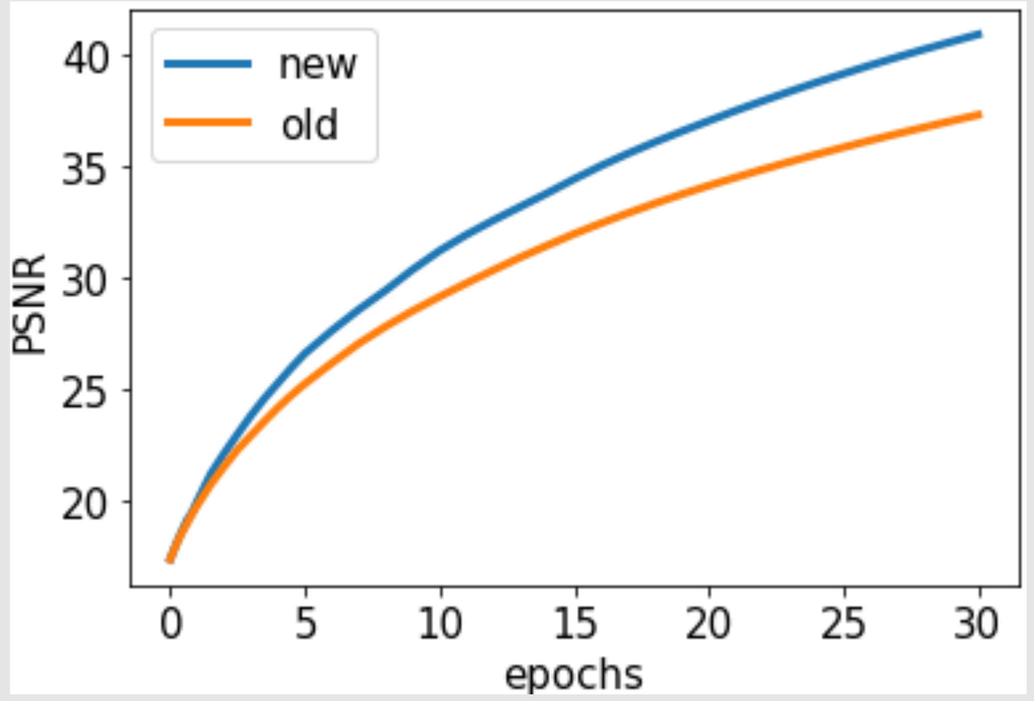
$$\tau_{\text{new}} = \frac{0.99 \cdot \gamma}{\sum_{i=1}^{n+1} \|A_i\|}$$

Compare to primal step-size of [8]:

$$\tau_{\text{old}} = \min_{1 \leq i \leq n+1} \frac{0.99 \cdot \gamma \cdot p_i}{\|A_i\|}$$

Test on real data: brain amyloid scan [6,7], Siemens Biograph mMR. The projections are obtained with package NiftyPET [5].

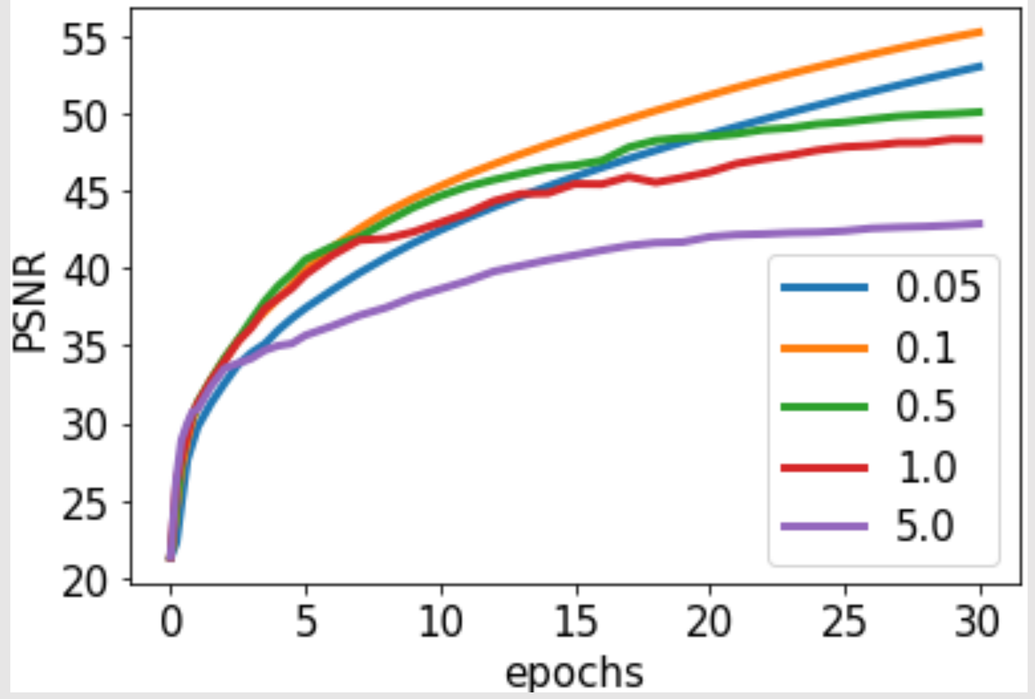
A reference reconstruction is obtained by 2000 iterations of PDHG, and the PSNR to the reference is measured for SPDHG under old and new step-sizes.

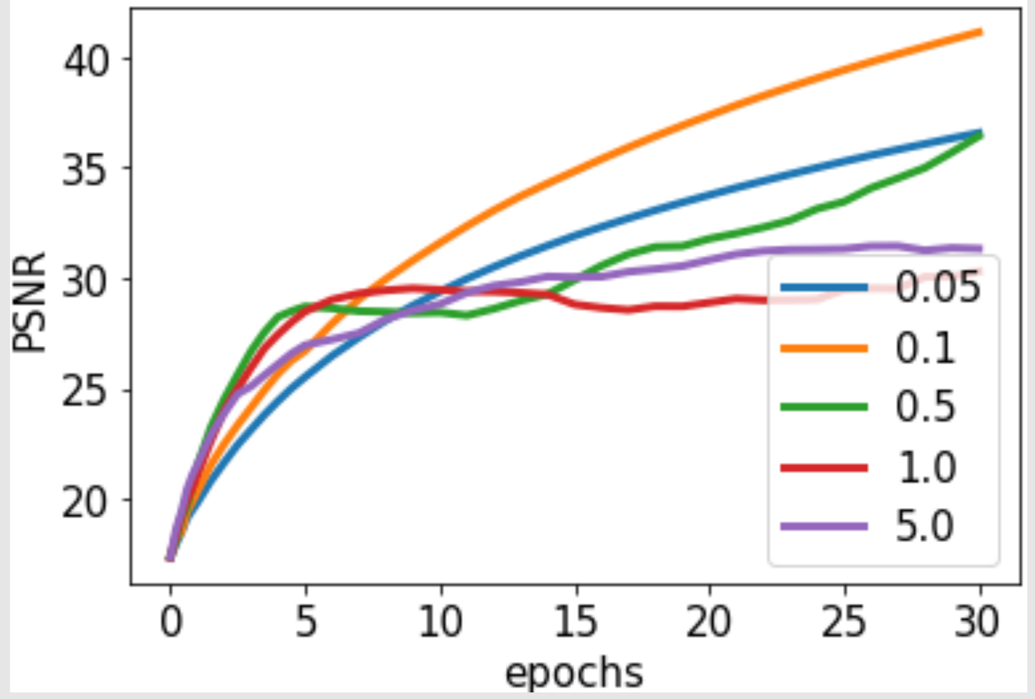


New step-sizes improve convergence speed.

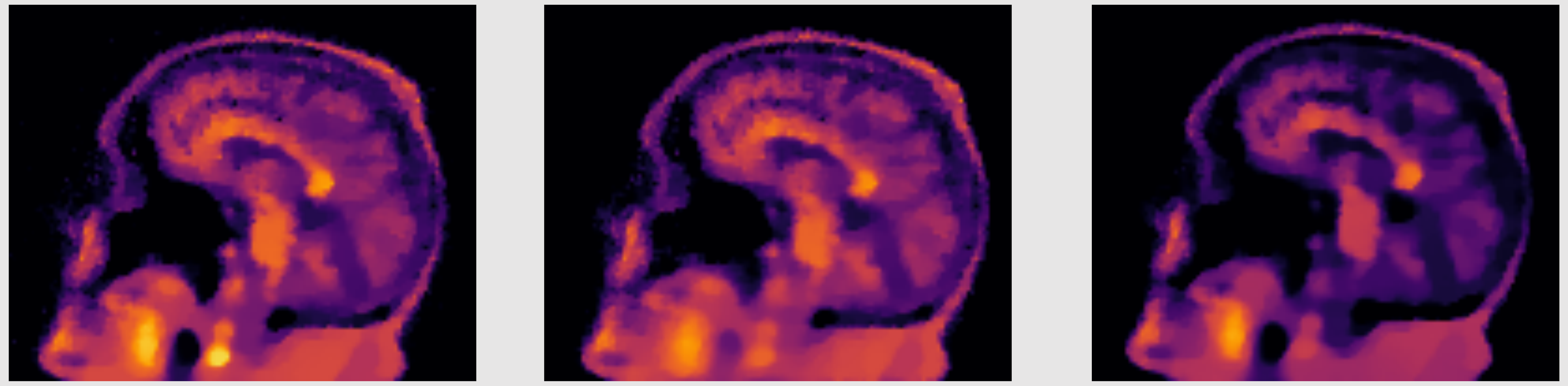
Contribution #2: tune the free parameter

Test on synthetic phantom data (left) vs real data (right): for each dataset, a reference image is created, and the PSNR to the reference is measured for various values of γ.
Fastest reconstruction is obtained with same value $\gamma = 0.1$ for both datasets.





Test on real data: reference image (left),reconstruction with 20 epochs and $\gamma = 0.1$ (middle), reconstruction with 20 epochs and $\gamma = 1.0$ (right).
SPDHG result with calibrated $\gamma = 0.1$ looks closer to reference than with $\gamma = 1.0$.



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