

$$1 + 1 > 2?$$

Getting More Out of Multi-Modality Imaging

Matthias J. Ehrhardt

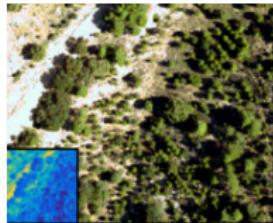
September 26, 2019

Institute for
Mathematical Innovation

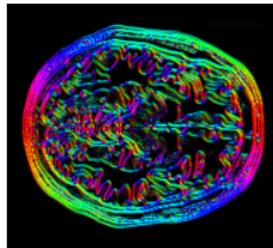


Outline

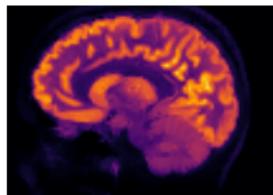
1) Motivation: Examples of Multi-Modality Imaging (Why?**)**



2) Mathematical Models for Multi-Modality Imaging (How?**)**



3) Application Examples: Remote Sensing and Medical Imaging ($1 + 1 > 2?$)



Motivation: Examples of Multi-Modality Imaging

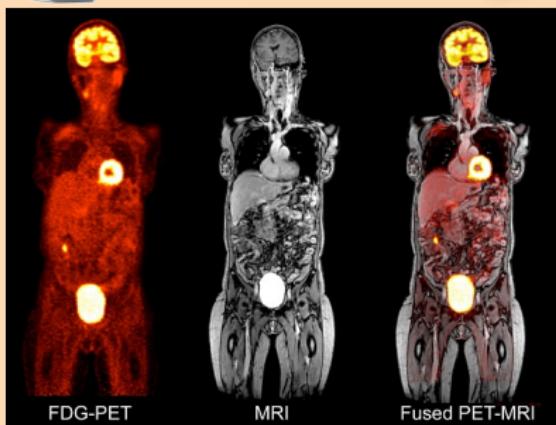
Multi-Modality Imaging Examples

PET-MR

PET-MR (and PET-CT, SPECT-MR, SPECT-CT)



Combine **anatomical (MRI)** and **functional (PET)** information



7 **clinical scanners** in UK

Currently images are just
overlaid

Challenge: Reduce scanning time, increase image quality, lower dose

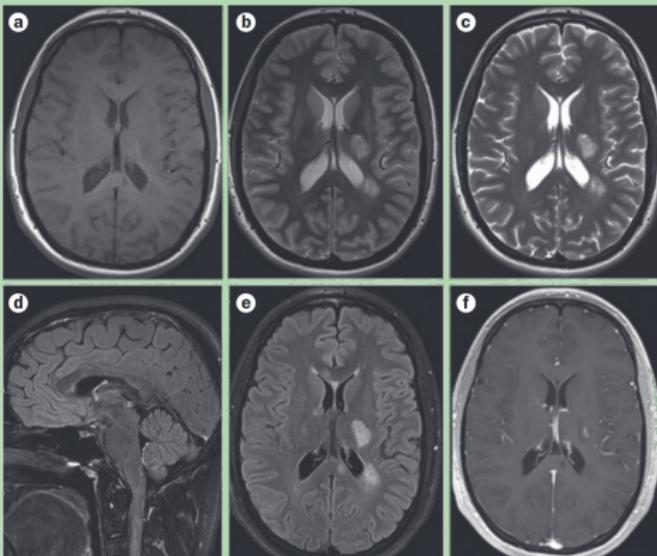
image: Sheth and Gee, 2012

Multi-Modality Imaging Examples

PET-MR

Multi MRI

Multi-Sequence MRI



pre-contrast

T_1 -weighted (a),
dual-echo T_2 (b, c)

post-contrast

2D T_2 FLAIR (d, e),
 T_1 -weighted (f)

**Standardized
MRI protocol**
for multiple sclerosis

6 scans, total 30 min

Rovira et al., Nature Reviews Neurology, 2015

Challenge: Reduce scanning time

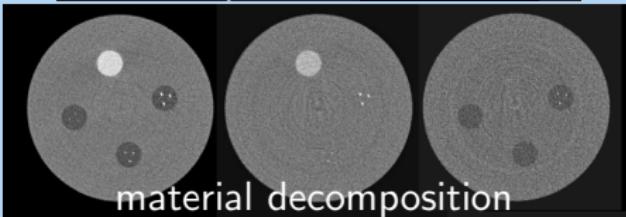
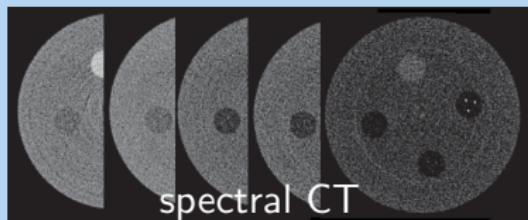
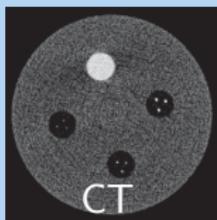
Multi-Modality Imaging Examples

PET-MR

Multi MRI

Spectral CT

Spectral CT



images:

[Shikhaliev and Fritz, 2011](#)

Acquisition: energy resolved measurements

Combination: material information

Challenge: Low dose / high noise in some channels

Multi-Modality Imaging Examples

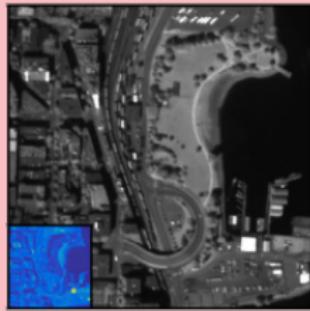
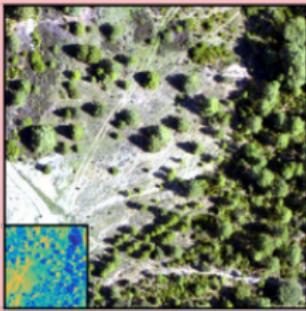
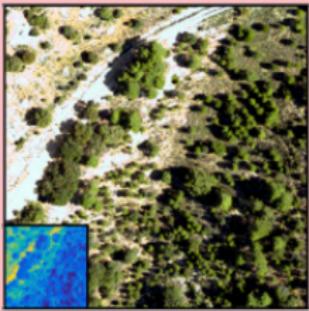
PET-MR

Multi MRI

Spectral CT

Hyper
+ optical

Image fusion in remote sensing



Acquisition: low resolution hyperspectral data (127 channels, $1m \times 1m$) and high resolution photograph ($0.25m \times 0.25m$) acquired **on plane or satellite**, e.g. by NERC Airborne Research & Survey Facility

Challenge: get best of both worlds—high spatial and spectral resolution

Multi-Modality Imaging Examples

PET-MR

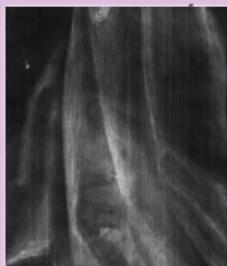
Multi MRI

Spectral CT

Hyper
+ optical

X-ray
+ optical

X-ray separation for art restauration [Deligiannis et al. 2017](#)



Acquisition: photographs and x-ray images

Challenge: separate the x-rays of the doors

Fairly Large Field

- Regular sessions at major conferences: Applied Inverse Problems, SIAM Imaging
- Symposium in Manchester in 3-6 Nov 2019
- Special Issue in IOP Inverse Problems

The image shows the top navigation bar of the IOPscience website. It includes links for "IOPscience", "Journals", "Books", "Publishing Support", and "Login". There is also a search bar labeled "Search IOPscience content" and a "Search" button. On the right, there are links for "Article Lookup" and "Journal Links".

Inverse Problems

Special issue on Joint Reconstruction and Multi-Modality/Multi-Spectral Imaging

Inverse Problems is pleased to announce the following upcoming special issue, which is now open for submissions via our submissions [page](#). We also kindly ask you to distribute this call among all colleagues who might be interested in submitting their work.

Guest editors

- **Simon Arridge** University College London, UK
- **Martin Burger** Universität Münster, Germany
- **Matthias Ehrhardt** University of Cambridge, UK

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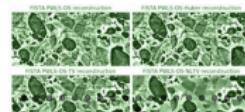
[Journal collections](#)

- Collaborative Software Projects: CCPi (Phil Withers) and CCP PETMR

CCPi
Tomographic Imaging

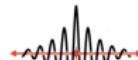


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Collaborative Computational Project in Tomographic Imaging aims to provide the UK tomography community with a toolbox of algorithms that increases the quality and level of information that can be extracted by computed tomography. Chaired by Prof Philip Withers (University of Manchester), co-ordinated by staff in the Scientific Computing Department, STFC, led by a working group of experimental and theoretical academics with links to Diamond Light Source, DSI and Industry Read more here.

NEWS FLASH: CCPi 10th Annual Event; at University of Southampton - 11-13 September website



Synergistic PET-MR Reconstruction

[Welcome](#) [Reports](#) [Software](#) > [Events](#) > [Funded Exchange](#) [Educational Resources](#) [Contacts](#)

CCP PET-MR

Collaborative Computational Project in Positron Emission Tomography and Magnetic Resonance Imaging

Home > Synergistic Reconstruction Symposium

Synergistic Reconstruction Symposium

Date:

Sunday, November 3, 2019 - 08:00 to Wednesday, November 6, 2019 - 18:00

Update

Future Events

► Bi-monthly software t-con Fri, 04/10/2019 - 10:30 to 12:00

► Bi-monthly software t-con Fri, 08/11/2019 - 10:30 to 12:00

► CCP User and Developers Meeting at VET 2019.

Mathematical Models for Multi-Modality Imaging

Image Reconstruction

Variational Approach:

$$u^* \in \arg \min_u \left\{ \mathcal{D}(\mathbf{A}u, b) + \alpha \mathcal{J}(u) + \varphi_C(u) \right\}$$

A forward operator (often but not always linear),

e.g. Radon transform

D data fit, e.g. least-squares $\mathcal{D}(\mathbf{A}u, b) = \frac{1}{2} \|\mathbf{A}u - b\|^2$,
Kullback–Leibler divergence

$$\mathcal{D}(\mathbf{A}u, b) = \int \mathbf{A}u - b + b \log(b/\mathbf{A}y)$$

J regularizer, e.g. total variation

$$\mathcal{J}(u) = \text{TV}(u) := \sum_i |\nabla u_i| \quad \text{Rudin et al., 1992}$$

φ_C constraints, e.g. nonnegativity

Image Reconstruction

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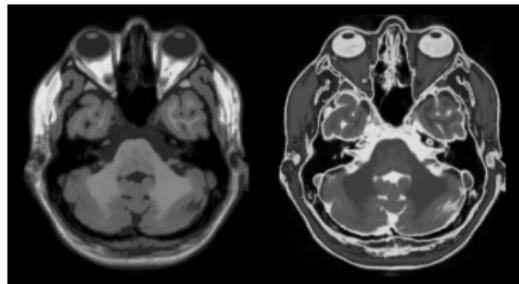
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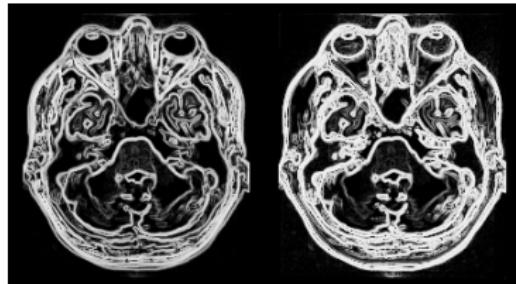
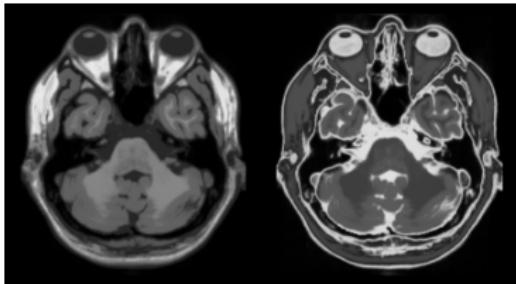
ι_C constraints, e.g. nonnegativity

How to include information from other modalities?

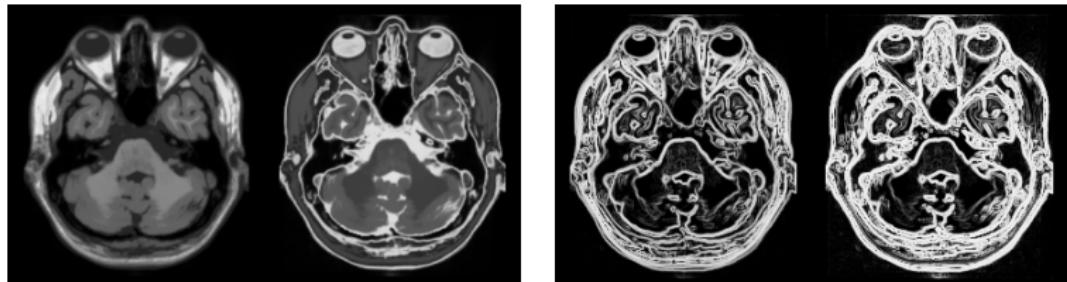
Modelling Structural Similarity



Modelling Structural Similarity



Modelling Structural Similarity



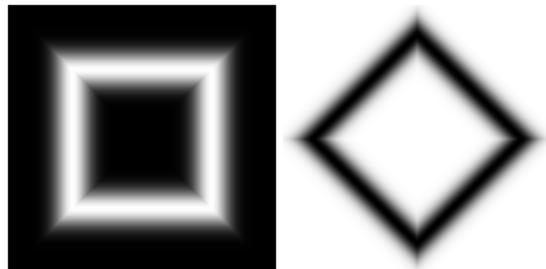
Definition: The **Weighted Total Variation (wTV)** of u is

$$\text{dTV}(u) := \sum_i w_i \|\nabla u_i\|, \quad 0 \leq w_i \leq 1$$

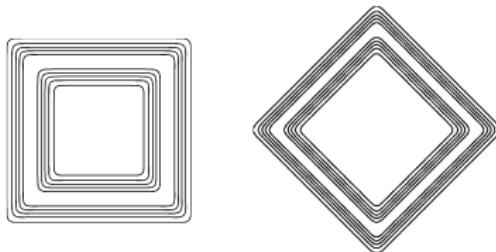
See e.g. [Ehrhardt and Betcke '16](#)

- If $c > 0, c < w_i$, then $c \text{TV} \leq \text{wTV} \leq \text{TV}$.
- If $w_i = 1$, then $\text{wTV} = \text{TV}$.
- $w_i = \frac{\eta}{\|\nabla v_i\|_\eta}, \quad \|\nabla v_i\|_\eta^2 = \|\nabla v_i\|^2 + \eta^2, \quad \eta > 0$

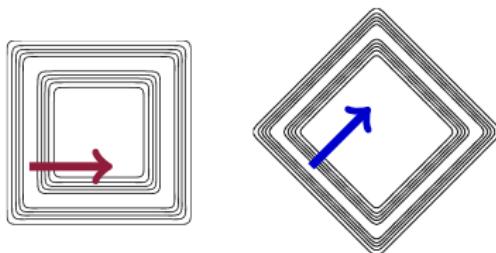
Modelling Structural Similarity



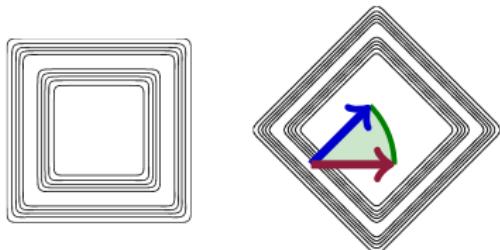
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Modelling Structural Similarity

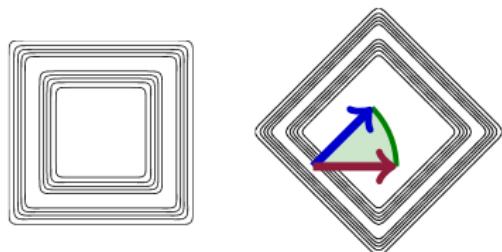


Modelling Structural Similarity



$$\langle \nabla u, \nabla v \rangle = \cos(\theta) |\nabla u| |\nabla v|$$

Modelling Structural Similarity

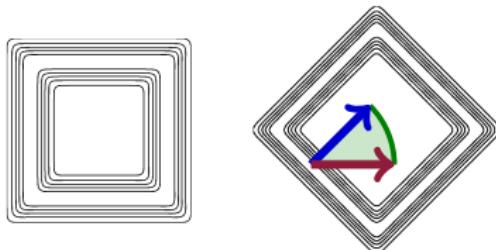


$$\langle \nabla u, \nabla v \rangle = \cos(\theta) |\nabla u| |\nabla v|$$

Definition: Two images u and v are said to have **parallel level sets** or are **structurally similar** (denoted by $u \sim v$) if $\theta = 0$ or $\theta = \pi$, i.e.

$$\nabla u \parallel \nabla v \quad \text{i.e. } \exists \alpha \text{ such that } \nabla u = \alpha \nabla v .$$

Modelling Structural Similarity



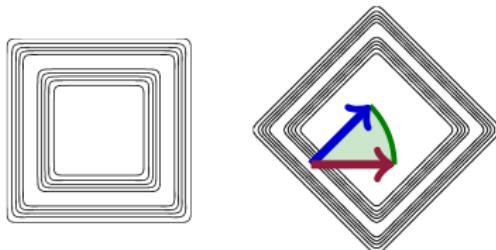
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- ▶ Dominant idea in this field
 - ▶ Parallel Level Set Prior, e.g. [Ehrhardt and Arridge '14](#)
 - ▶ Directional Total Variation, e.g. [Ehrhardt and Betcke '16](#)
 - ▶ Total Nuclear Variation, e.g. [Knoll et al. '16](#)
 - ▶ Coupled Bregman iterations, e.g. [Rasch et al. '18](#)
- ▶ Others are: joint sparsity (e.g. wTV), joint entropy, ...

Modelling Structural Similarity



$$\langle \nabla u, \nabla v \rangle = \cos(\theta) |\nabla u| |\nabla v|$$

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Directional Total Variation

- Note that if $\|\nabla v\| = 1$, then

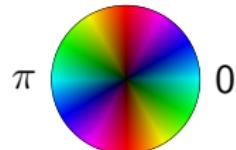
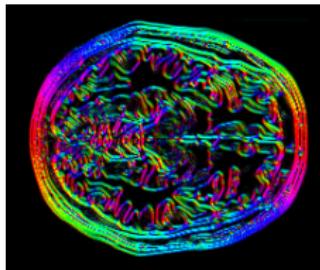
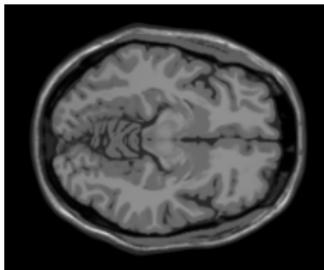
$$u \sim v \Leftrightarrow \nabla u - \langle \nabla u, \nabla v \rangle \nabla v = 0$$

Definition: The **Directional Total Variation (dT V)** of u is

$$\text{dT V}(u) := \sum_i \|[\mathbf{I} - \xi_i \xi_i^T] \nabla u_i\|, \quad 0 \leq \|\xi_i\| \leq 1$$

Ehrhardt and Betcke '16, related to Kaipio et al. '99, Bayram and Kamasak '12

- If $c > 0$, $\|\xi_i\|^2 \leq 1 - c$, then $c \text{TV} \leq \text{dT V} \leq \text{TV}$.
- If $\xi_i = 0$, then $\text{dT V} = \text{TV}$.
- $\xi_i = \frac{\nabla v_i}{\|\nabla v_i\|_\eta}$, $\|\nabla v_i\|_\eta^2 = \|\nabla v_i\|^2 + \eta^2$, $\eta > 0$



Application Examples

Multi-Modality Imaging Examples

PET-MR

Multi MRI

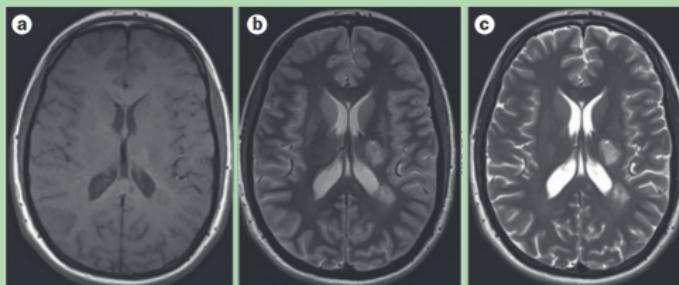
Spectral CT

Hyper
+ optical

X-ray
+ optical

Multi-Sequence MRI

Ehrhardt and Betcke, SIAM J. Imaging Sci., vol. 9, no. 3, pp. 1084–1106, 2016.



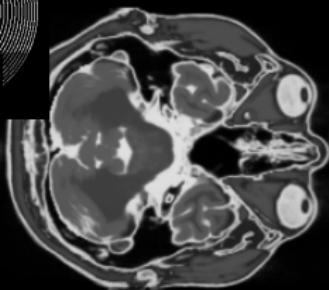
Joint work with:

Computer Science: M. Betcke (UCL)

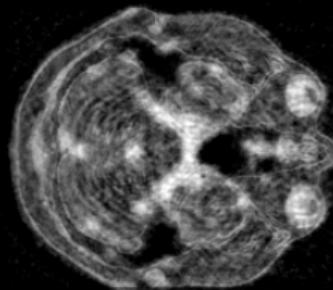
Multi-Sequence MRI Results



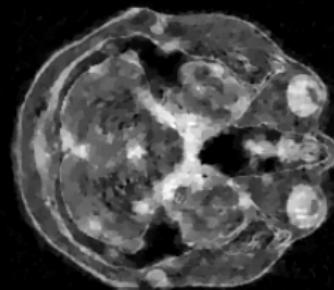
sampling



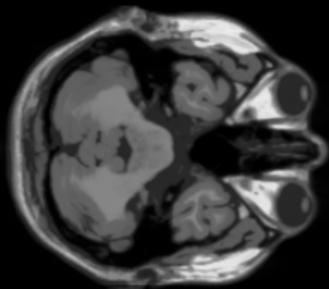
gr. truth



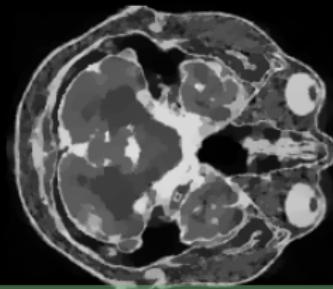
no prior



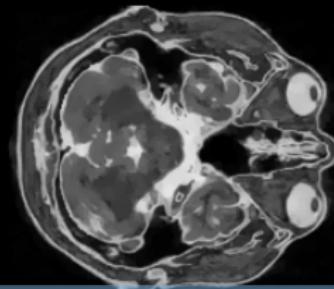
TV



side info



wTV

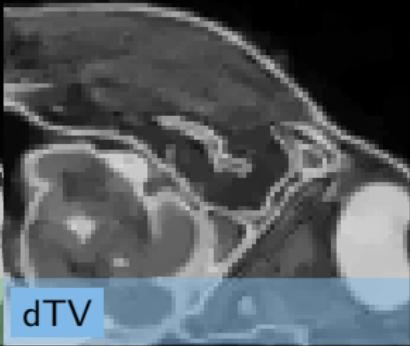
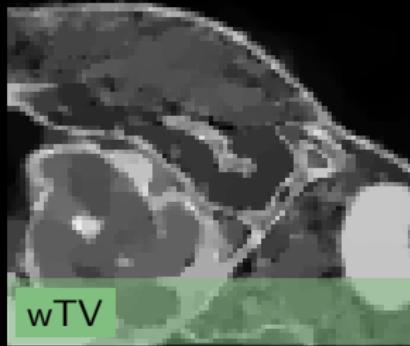
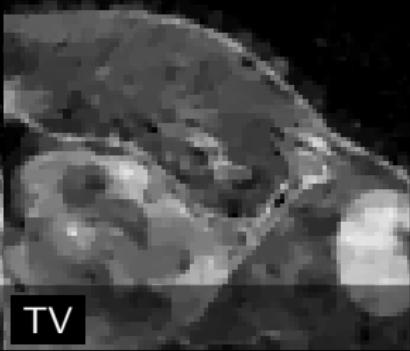
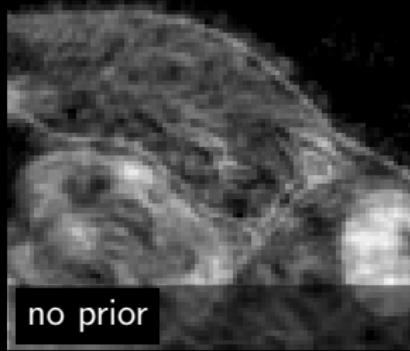


dTV

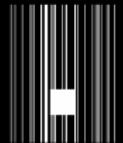
Multi-Sequence MRI Results



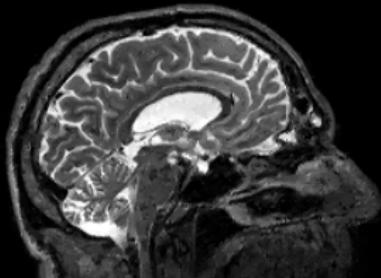
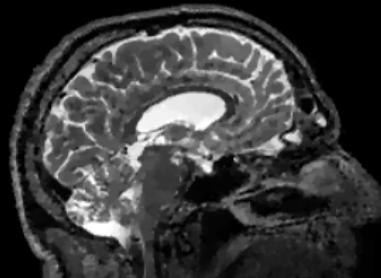
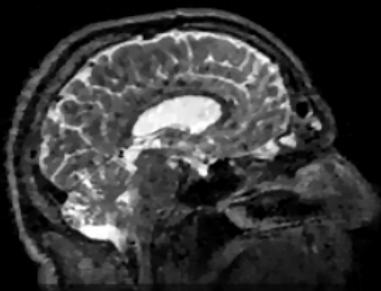
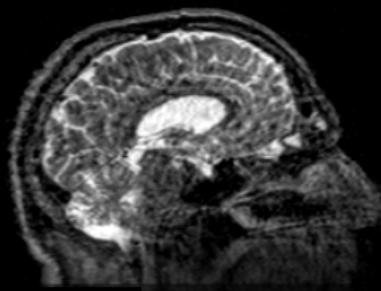
sampling



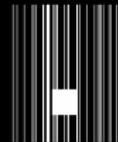
Multi-Sequence MRI Results



sampling



Multi-Sequence MRI Results



sampling



gr. truth



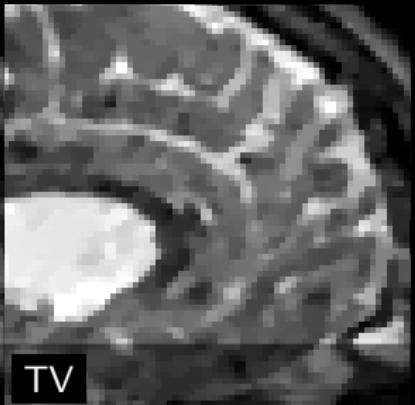
side info



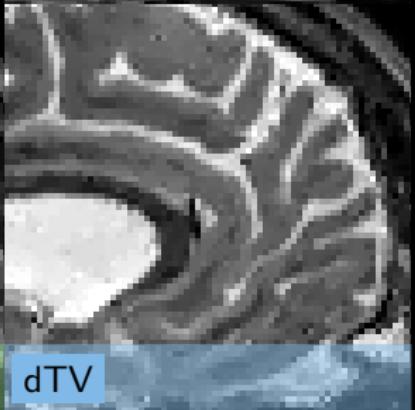
no prior



wTV

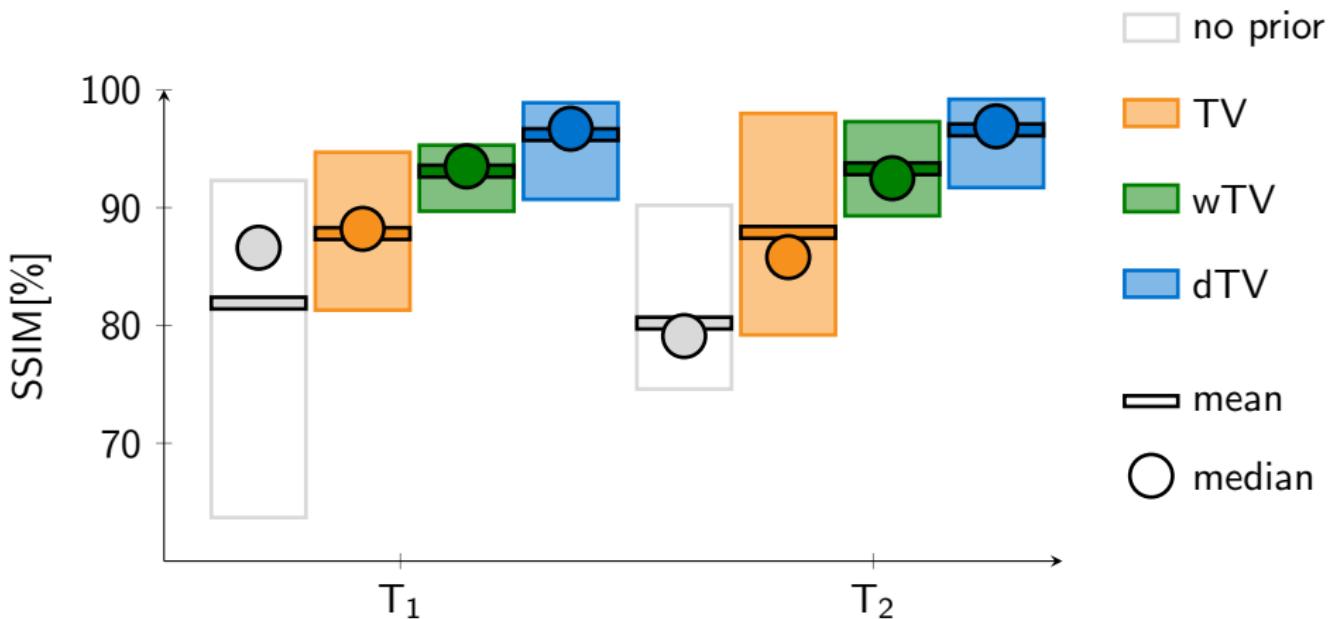


TV



dTV

Quantitative Results



- Range (min, max), mean and median over 12 data sets

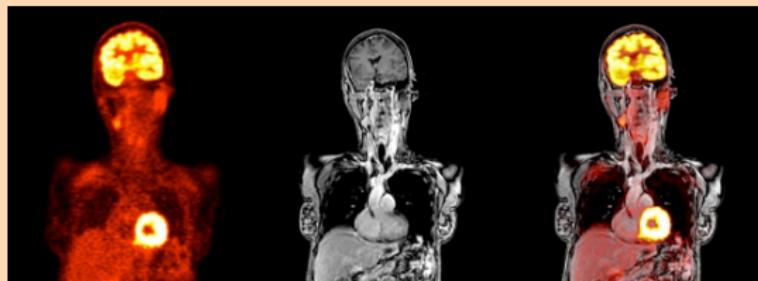
Multi-Modality Imaging Examples

PET-MR

PET-MR

Ehrhardt et al., Phys. Med. Biol. (in press), 2019

Ehrhardt et al., Proceedings of SPIE, vol. 10394, pp. 1–12, 2017



Joint work with:

Mathematics: A. Chambolle (École Polytechnique, France), P. Richtárik (KAUST, Saudi Arabia), C. Schönlieb (Cambridge)

Medical Physics: P. Markiewicz (UCL),

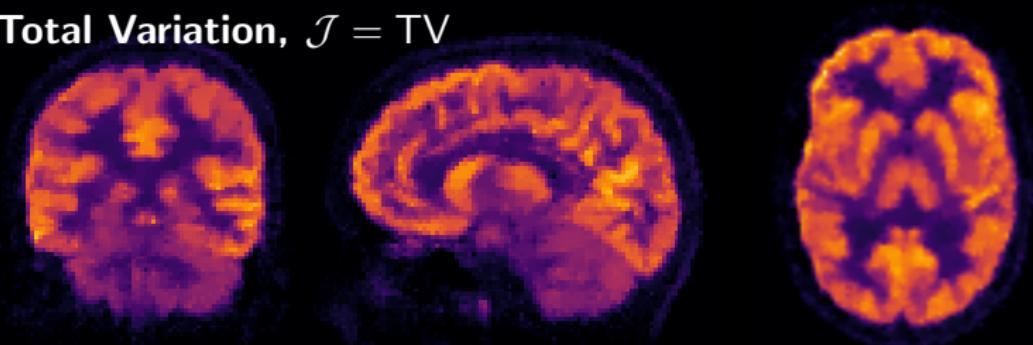
Neurology: J. Schott (UCL)

PET-MR Results

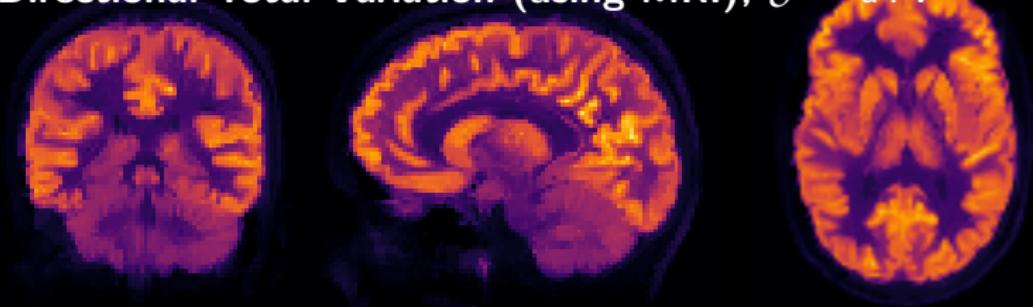
Reconstruction model:

$$\min_u \left\{ \text{KL}(\mathbf{A}u + r; b) + \lambda \mathcal{J}(u) + \iota_{\geq 0}(u) \right\}$$

Total Variation, $\mathcal{J} = \text{TV}$



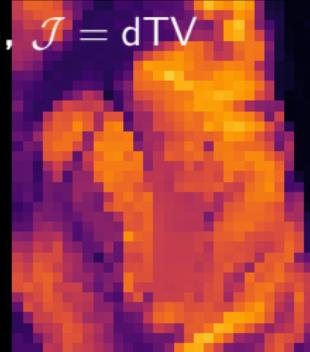
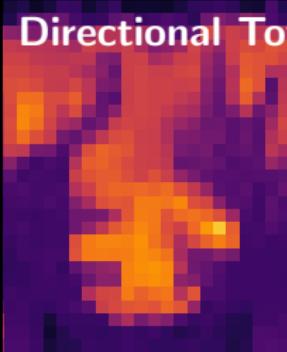
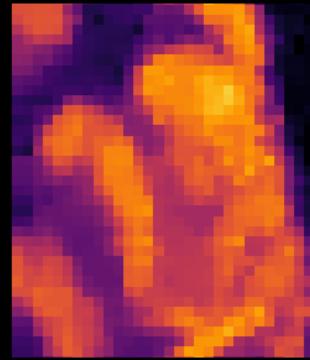
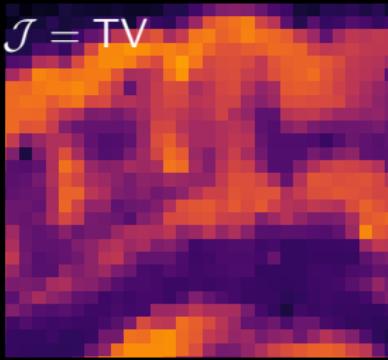
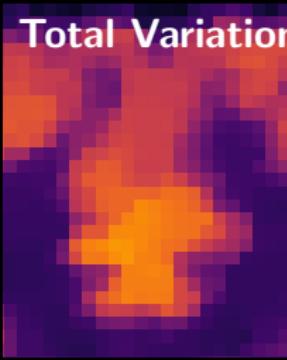
Directional Total Variation (using MRI), $\mathcal{J} = \text{dT}V$



PET-MR Results

Reconstruction model:

$$\min_u \left\{ \text{KL}(\mathbf{A}u + r; b) + \lambda \mathcal{J}(u) + \iota_{\geq 0}(u) \right\}$$



Multi-Modality Imaging Examples

PET-MR

Multi MRI

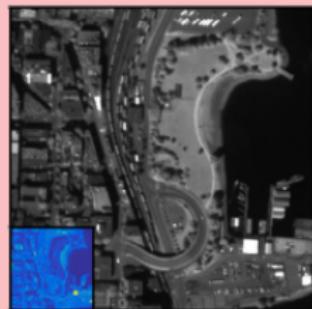
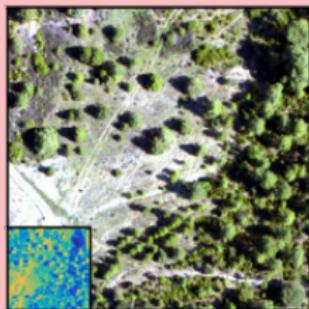
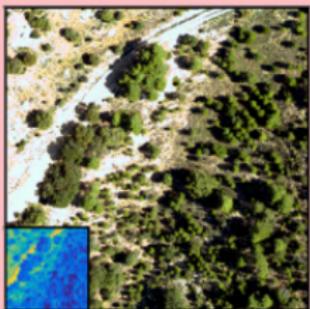
Spectral CT

Hyper
+ optical

X-ray
+ optical

Image fusion in remote sensing

Bungert et al., Inverse Probl., vol. 34, no. 4, p. 044003, 2018



Joint work with:

Mathematics: L. Bungert (Erlangen, Germany), R. Reisenhofer (Vienna, Austria), J. Rasch (Berlin, Germany), C. Schönlieb (Cambridge),

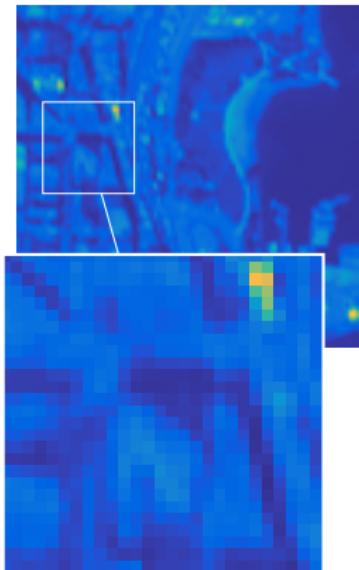
Biology: D. Coomes (Cambridge)

Standard regularization versus image fusion

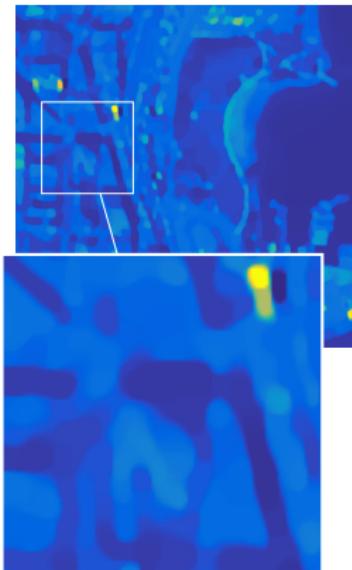
Reconstruction model:

$$\min_u \left\{ \frac{1}{2} \|\mathbf{S}(u * k) - v\|^2 + \lambda \mathcal{J}(u) + \iota_{\geq 0}(u) \right\}$$

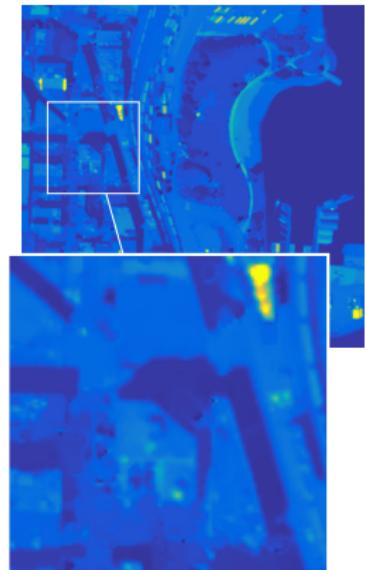
data



standard, $\mathcal{J} = \text{TV}$



fusion, $\mathcal{J} = \text{dTV}$

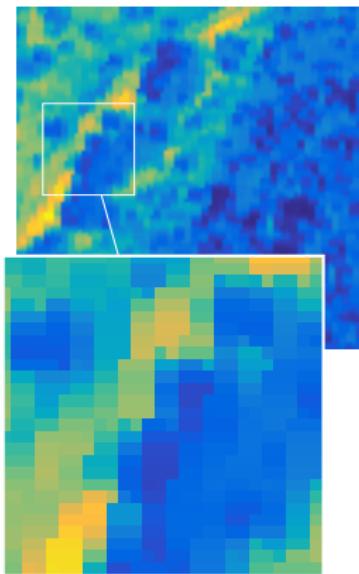


Blind versus non-blind image fusion

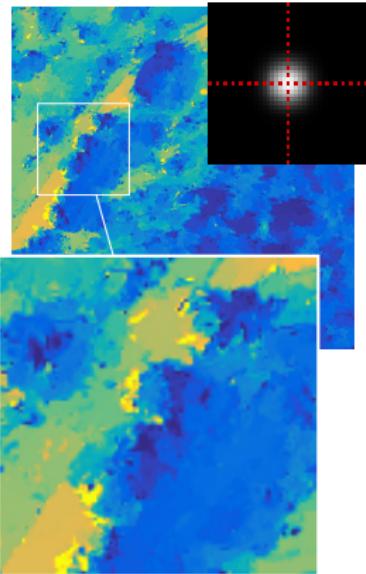
reconstruction model:

$$\min_u \left\{ \frac{1}{2} \|\mathbf{S}(u * k) - v\|^2 + \lambda \mathcal{J}(u) + \iota_{\geq 0}(u) \right\}$$

data



fusion

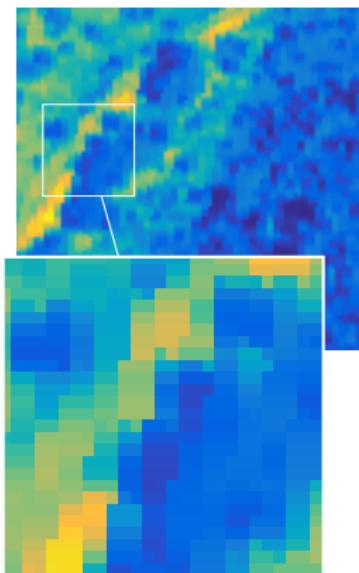


Blind versus non-blind image fusion

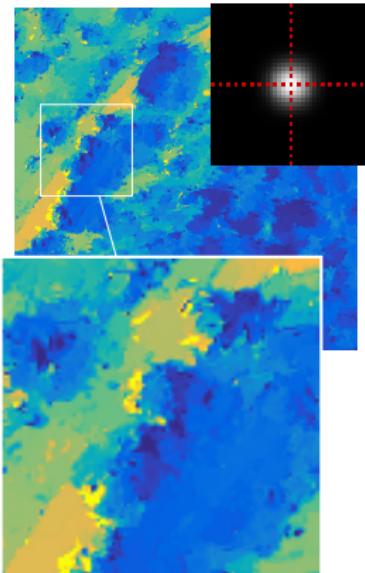
Blind reconstruction model:

$$\min_{u,k} \left\{ \frac{1}{2} \| \mathbf{S}(u * k) - v \|^2 + \lambda \mathcal{J}(u) + i_{\geq 0}(u) + i_S(k) \right\}$$

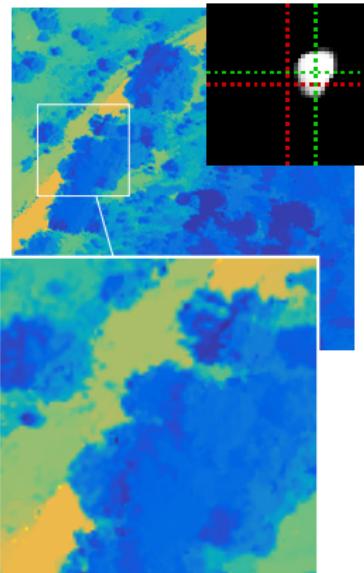
data



fusion



blind fusion



Conclusions and Outlook

Summary:

- ▶ **Multi-Modality Imaging** examples:
PET-MR, multi-sequence MRI, spectral
CT, Hyper + optical,
X-ray + optical
- ▶ **Mathematical Models** to exploit
synergies between modalities
- ▶ **Examples:** indeed often $1 + 1 > 2!$

Future:

- ▶ Which modalities **complement** each other best?
- ▶ Multi-modality imaging can help to **lower dose, increase resolution** ...
- ▶ **Expertise** in image / video processing,
compressed sensing, machine learning ...

