HIGH-FIDELITY, MULTIQUBIT GENERALIZED MEASUREMENTS WITH DYNAMIC CIRCUITS

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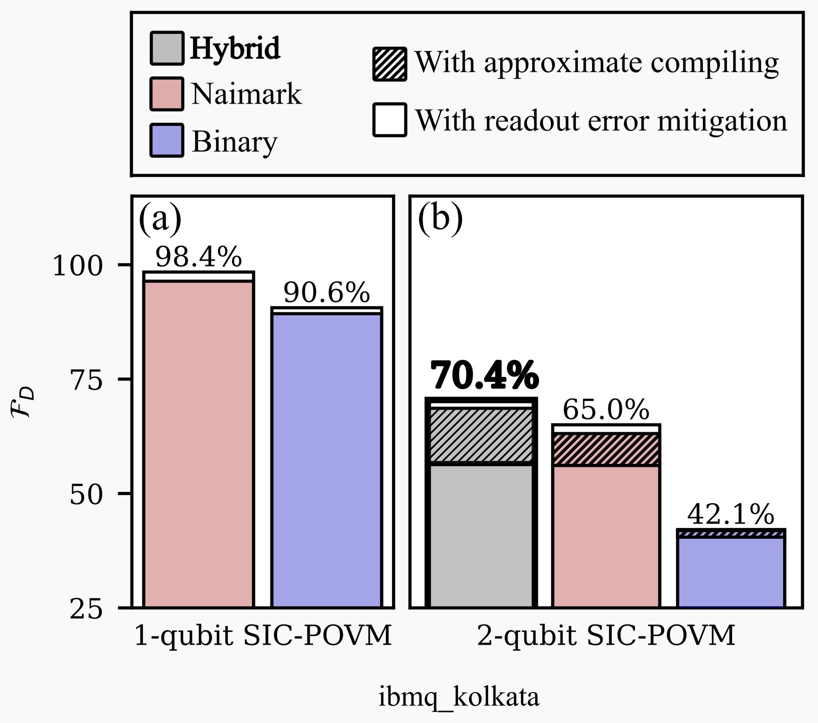
RESULTS



We present a new scalable approach that requires only a single auxiliary qubit and uses dynamic circuits to realize POVMs. Our approach hybridizes binary search with Naimark's dilation and scales to multi-qubit POVMs. We combine our hybrid POVM method, approximate compiling, and a novel readout error mitigation technique to realize a two-qubit informationally complete POVM on superconducting qubits with **70% fidelity**, improving from the bare Naimark's dilation by 18%. With that, **we demonstrate the feasibility of generalized measurements on near-term quantum systems.**

Main contributions:

- New hybrid method for POVM implementation using dynamic circuits
- Using approximate compiling to improve POVM fidelity
- Novel conditional readout error mitigation
- Hardware demonstration of one- and two-qubit SIC-POVMs with high-fidelity

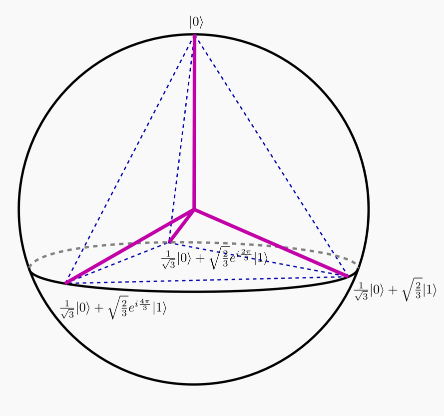
Future work:

- Scaling to more qubits with faster feed-forward operations and optimized compilation
- Development of hardware-efficient, parametric POVMs

BACKGROUND

Positive operator-valued measures (POVMs) define the most general framework for quantum measurements and are central in many quantum information protocols, including, e.g., optimal state tomography [1]. However, realizing multi-qubit POVMs is challenging because conventional methods require many additional qubits and general unitary operations in high-dimensional Hilbert space.

Definition: A POVM is a set $F = \{F_i\}$ of M positive semi-definite Hermitian operators, called POVM elements. Each element corresponds to a measurement outcome i with probability $P(i) = \mathrm{Tr}(F_i\rho)$ where ρ is the state of the system.

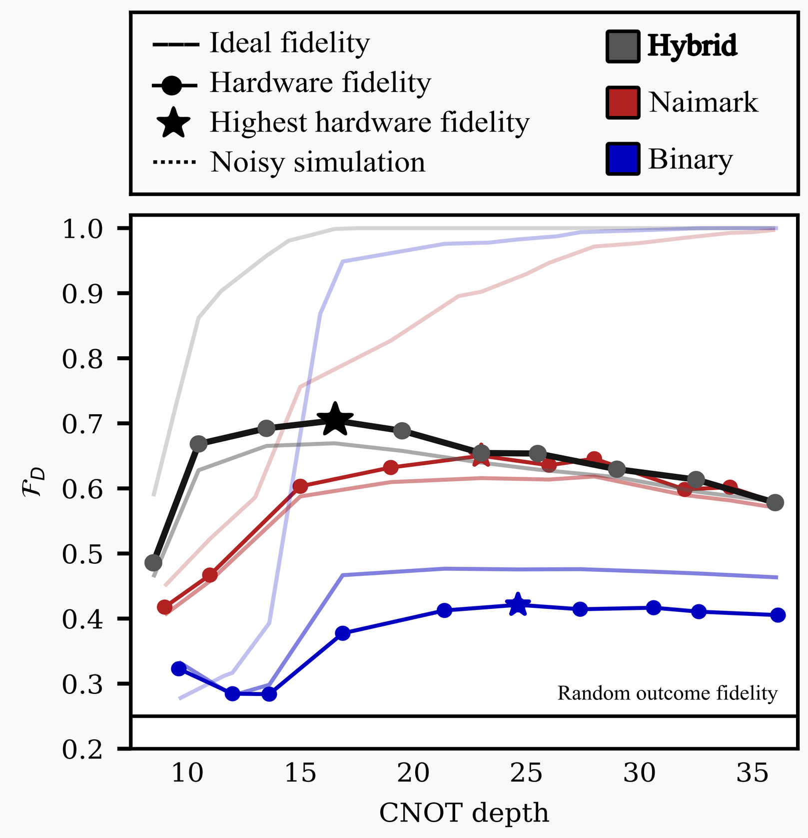


1-qubit SIC-POVM with M=4 elements

SIC-POVM. (2024, February 1). In Wikipedia. https://en.wikipedia.org/wiki/SIC-POVM

[1] A. J. Scott, J. Phys. A **39**, 13507 (2006)

APPROXIMATE COMPILING



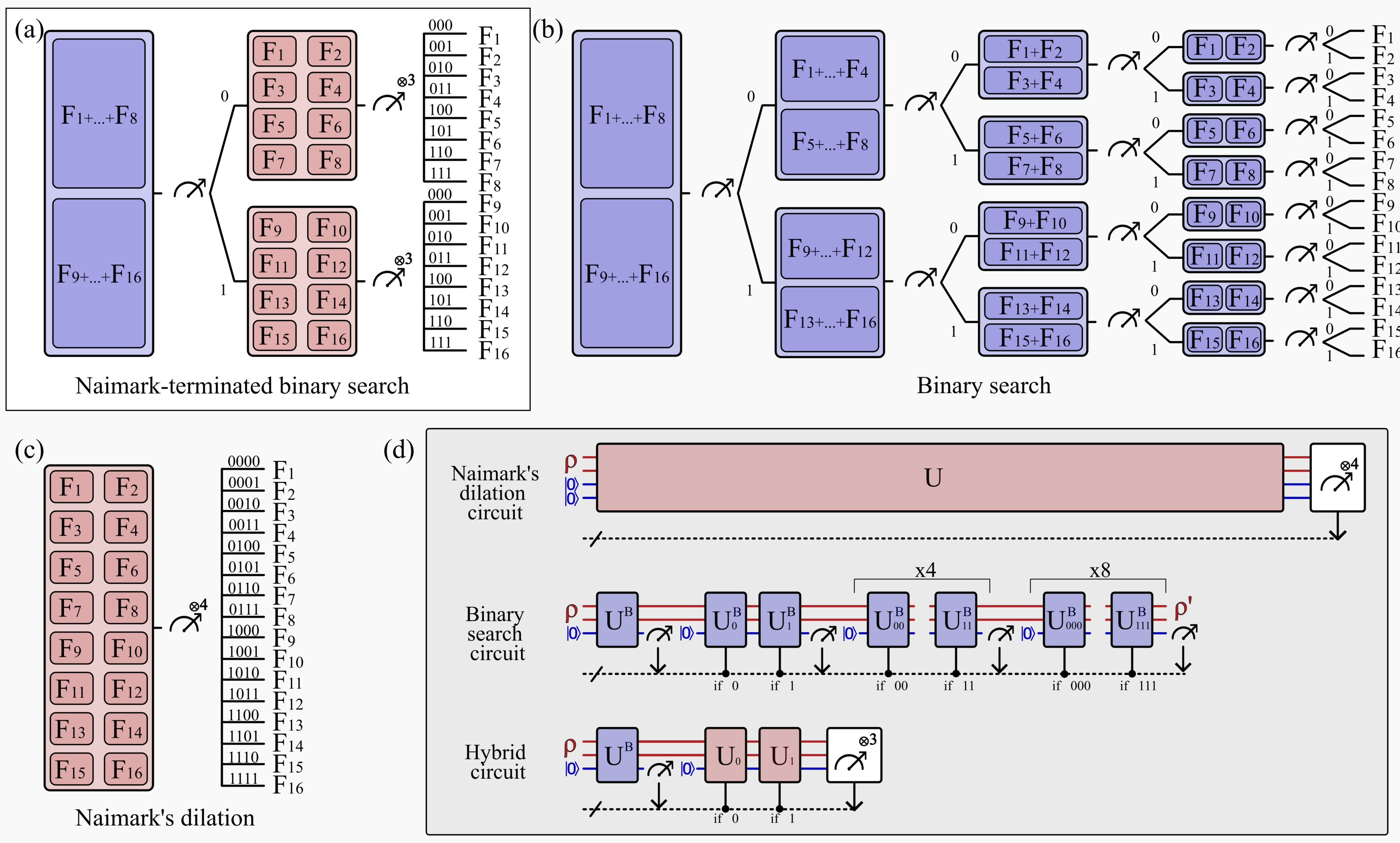
- Current hardware requires efficient decomposition of unitaries in native gates.
- Exact compiling techniques are infeasible due to prohibitively deep circuits.

⇓

- We use approximate compiling [2] to find an optimal tradeoff between the approximation of the target measurement and the noise due to circuit depth, in agreement with the depolarizing noise model.

[2] L.Madden and A.Simonetto, ACM Transactions on Quantum Computing **3**, 2, Article 7 (2022)

NAIMARK-TERMINATED BINARY SEARCH



Two-qubit POVM with M=16 POVM elements realized through **(a)** hybrid scheme (blue and red), **(b)** binary tree (blue), and **(c)** Naimark's dilation (red).

Naimark's dilation: Realizes a POVM as a projective measurement in a higher-dimensional Hilbert space [3,4].
Drawback: Requires auxiliary qubits and complex unitary operations in the extended Hilbert space.

Binary search: Realizes a POVM as a sequence of two-outcome POVMs using a single auxillary qubit and feed-forward operations [5,6].

$$B_0 := \sum_{i=1}^{M/2} F_i \quad \text{and} \quad B_1 := \sum_{i=M/2+1}^{M} F_i$$

Drawback: Requires many feed-forward operations and deep circuits.

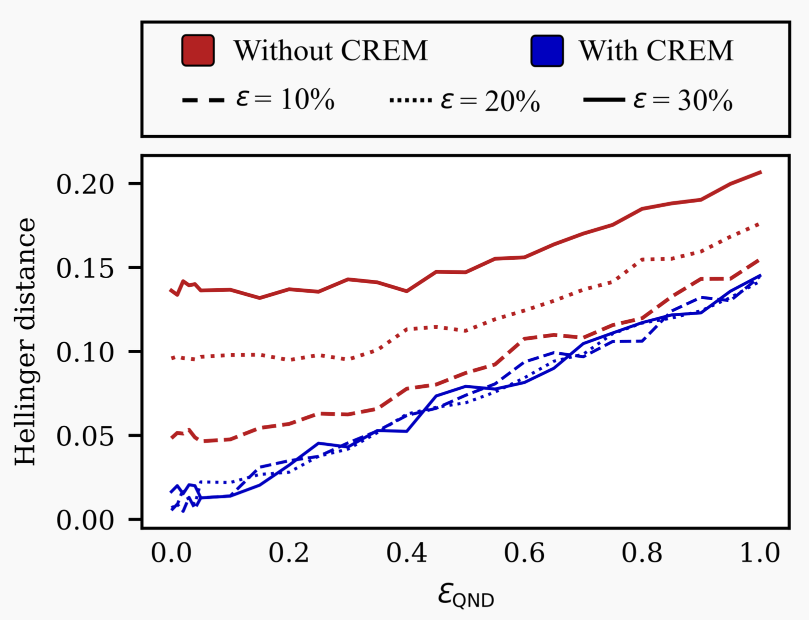
Our hybrid method: First, we narrow down the search range using binary search by repeatedly dividing the set of POVM elements in half. When the number of remaining POVM elements corresponds to the dimension of the compound system, we interrupt the binary search and apply Naimark's dilation. The resulting circuit is shorter than its constituent methods and scales better toward larger systems.

[3] R. Stricker et al., PRX Quantum **3**, 040310 (2022)
[4] L. E. Fischer et al., Phys. Rev. Res. **4**, 033027 (2022)
[5] E. Andersson and D. K. L. Oi, Phys. Rev. A **77**, 052104 (2008)
[6] W. Cai et al., Phys. Rev. Lett. **127**, 090504 (2021)

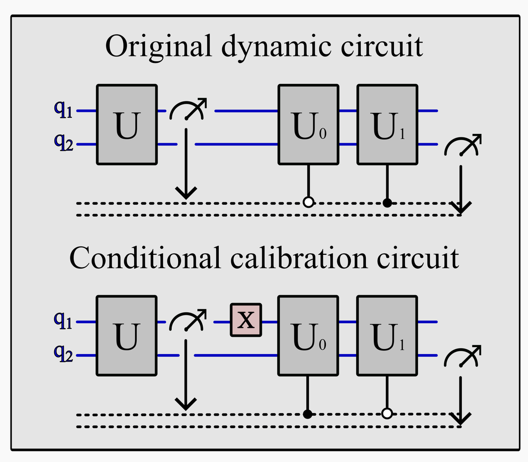
CONDITIONAL READOUT ERROR MITIGATION

Problem: Mitigating readout errors in dynamic circuits is challenging because errors on mid-circuit measurements affect subsequent conditional operations.

Conditional REM: For a circuit with mid-circuit measurements, we run several circuits, each corresponding to a distinct combination of readout errors. In the post-processing, we combine the measurement statistics from each circuit, weighted by the error probabilities obtained from calibration.



$$\begin{pmatrix}\tilde{P}_{00}\\ \tilde{P}_{01}\\ \tilde{P}_{10}\\ \tilde{P}_{11}\end{pmatrix} = \left[\begin{pmatrix}1-\epsilon_0^1 & \epsilon_1^1\\ \epsilon_0^1 & 1-\epsilon_1^1\end{pmatrix} \otimes \begin{pmatrix}1-\epsilon_0^2 & \epsilon_1^2\\ \epsilon_0^2 & 1-\epsilon_1^2\end{pmatrix}\right] \begin{pmatrix}\tilde{Q}_{00}\\ \tilde{Q}_{01}\\ \tilde{Q}_{10}\\ \tilde{Q}_{11}\end{pmatrix}$$



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