DecernsMCDA DE

User Manual

Contents

What is *DECERNS*?

DECERNS (*Decision Evaluation in Complex Risk Network Systems*) is a family of web-based and desktop Decision Support Systems (DSSs).

DECERNS as a *project* provides a methodology and software tools which will facilitate decision-making support in the field of alternative choice, multi-criteria decision analysis of various [spatial] alternatives, including land-use planning, environmental protection and risk management.

DECERNS systems/software may be subdivided into the following three categories:

DecernsMCDA DE: desktop system for Multi-Criteria Decision Analysis (MCDA), which includes several MCDA methods and tools;

DecernsGIS DE: desktop software (GIS – Geographic Information System) for spatial data representation, processing and analysis; and

DecernsSDSS: distributed web-based software/SDSS (Spatial DSS) which comprises functions and tools of *DecernsGIS* and *DecernsMCDA* (along with the specific tools for effective integrating spatial data into multicriteria decision analysis). The *MCDA*- and *GIS*-subsystems within the *DecernsSDSS* may also be used as independent web systems. An extension of *DecernsSDSS* can also involve the tools (*ModelsProvider, ModelsManager*) for including into *DecernsSDSS* various math models (risk analysis, dynamics of values, etc).

DECERNS systems/software may be used for practical needs and scientific investigations as well as for education and training within the courses of decision analysis/decision support, GIS and spatial analysis, land-use planning, environmental and risk management, etc.

DECERNS software is developed upon open source technologies.

Introduction

A key component of the ***DECERNS*** project and ***DecernsSDSS*** is the decision support module.

This Manual presents description of the ***DecernsMCDA DE*** (***D***esktop ***E***dition) and includes MCDA methods (models) and tools implemented within the system, corresponding requirements to hardware and software, and several case studies.

MCDA methods, implemented in the ***DecernsMCDA DE***, include the following models/methods for analysis of different categories of multicriteria problems:

basic MADM methods for *choice* and *ranking* alternatives from "best" to "worst":

- MAVT (Multi-Attribute Value Theory);
- AHP (Analytic Hierarchy Process);
- TOPSIS (Technique for Order Preference by Similarity to the Ideal Solution); and
- PROMETHEE (Preference Ranking Organization METHod for Enrichment Evaluations);

advanced MADM methods:

- MAUT (Multi-Attribute Utility Theory) for *ranking* alternatives;

and several original methods preferably for *choice* alternatives with the possibilities of uncertainty treatment and analysis:

- ProMAA (Probabilistic Multi-criteria Acceptability Analysis); and

some extensions of MADM methods based on fuzzy set (fuzzy numbers) approaches:

- FMAVT (Fuzzy MAVT),
- FMAA (Fuzzy Multi-criteria Acceptability Analysis); and

- FlowSort - for *sorting* alternatives into classes/categories (*e.g.*, "unacceptable," "possibly acceptable," "definitely acceptable," etc.).

The structuring of a multicriteria problem is carried out with the use of Value Tree and Performance Table.

DecernsMCDA DE contains tools for sensitivity analysis: weight sensitivity analysis (two approaches – walking weights and line weights – to changing weight coefficients in methods MAVT, MAUT, AHP, PROMETHEE, TOPSIS) and value function sensitivity analysis (changing partial value functions in the methods MAVT, MAUT, ProMAA, FMAA, FMAVT).

Uncertainty/imprecision treatment and analysis can be implemented with the use of *probabilistic approaches* (ProMAA – uncertain criterion values and uncertain weight coefficients; MAUT – uncertain criterion values) and *fuzzy numbers* (FMAA and FMAVT - fuzzy criterion values and fuzzy weights).

DecernsMCDA DE contains also additional tools for data analysis – Value Path and Scatter Plot as well as toolkit for input/output specific settings.

Goals and objectives

One of the key goals of the ***DECERNS*** project is creation of original desktop and web-based software for effective cross-platform multi-criteria decision analysis with the use of all the basic MCDA methods and tools.
The architecture of systems and developed modules should allow forming different versions of software depending on the methods/tools incorporated in, including customized versions with implemented or new methods/tools.

System Requirements

DecernsMCDA DE is standalone application build according to Java EE 5 specifications and requires installed Java Runtime Environment (JRE) v.1.6.
JRE is free and may be downloaded from http://www.java.com. ***DecernsMCDA DE*** was tested on Windows and Linux platforms with the following hardware:
Processor P4 2.8GHz, 1 GB of RAM, video board with the latest drivers.

License (MCDA DE)

Three categories of licenses are accessible for our clients:

1. ***Individual*** – this type of license is for individual researches who wants to use ***DecernsMCDA DE*** software in own research/case studies. License is applied to client name and can`t be shared between clients.
2. ***Academic*** – this type of license is for academic organisations (schools, universities, etc who use MCDA in educational process). License is applied to organisation name and could be used on limited computers/laptops (depends on how many licenses was bought).
3. ***Commercial*** – this type of license is for companies and organisations who apply MCDA methods for real problem solving. License is applied to organisation name and could be used on limited computers/laptops (depends on how many licenses was bought).

Unregistered version contains different limitations (by number of criteria/alternatives and available methods).

Releases

- v1.0 (build 20140324) – first version of standalone ***DecernsMCDA DE*** software.
- wide range of earlier versions (alpha, beta) that was used for testing purposes.

Contacts

See the list of releases and details on purchasing and support: www.deesoft.ru
Write your questions, suggestions, and remarks/comments to
yatsalo@gmail.com, s.gritsyuk@gmail.com, or llcdeesoft@gmail.com,

Project management (*DecernsMCDA DE*)

Main menu

The main menu includes the following elements:

- **File** – includes options for file/project management
    - **New project** – create a new project. If some project was already opened and contains unsaved data the Save dialog will be shown.
    - **Open project…** – open a project. Project file must have *.dcm extension.
    - **Save project** – save changes to the same project. If it is a first project saving the SaveAs dialog will be shown.
    - **Save project as…** – save changes to new file.
    - **Exit** – exit from the application

- **Samples** – several sample projects are listed under this menu item. If there are no samples "Samples" menu item disappears.

- **Help** – information about application
    - **Help contents** – link to this manual
    - **About** – some brief information about application, application version and developers
    - **Register** – allows you to enter your name (in case of individual license) or the name of your company/organization and license key to register application. If application was already registered this menu item will be disabled.

Toolbar

The main window toolbar provides user interface for all main application functions (see Fig.1).

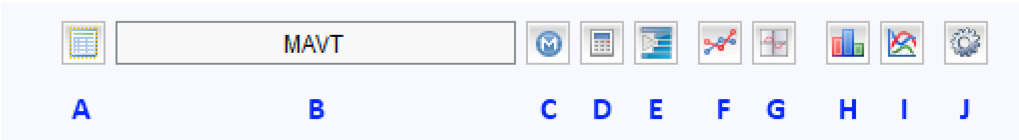


Fig.1 *DecernsMCDA*: Main window toolbar

A - MCDA mode switching button. Switch main working panel from Value Tree display mode to Performance Table display mode.

B - Scenario name area: indicates the current/chosen MCDA model.

C - Scenario dialog button: selection a current MCDA model.

D - Model calculation button. Causes model computation and calls report window.

E - Domination button. Calls Domination tool`s dialog (for more information see Tools-> Domination).

F - Value path button. Call Value path tool`s dialog (for more information see Tools->Value path).

G - Scatter plot button. Call Scatter plot tool`s dialog (for more information see Tools-> Scatter plot).

H - Walking Weights analysis button. Call Walking Weights analysis dialog (for more information see Analysis-> Weights sensitivity).

I - Value function analysis button. Call Value function analysis dialog (for more information see Analysis-> Value function).

J - Settings button opens settings dialog (see below).

Scenario management

DecernsMCDA DE application supports working with scenarios. Every scenario has its own name, description and method used for calculations. When you create new project only one default scenario created – MAVT method scenario, but at any moment you can add more scenarios with help of simple dialog windows (see Fig.2 and Fig.3).

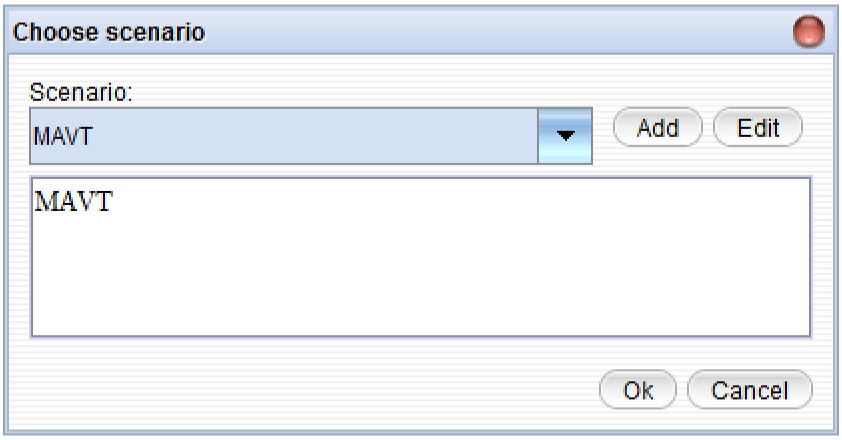


Fig.2 *DecernsMCDA*: The given Scenario and Scenario (MCDA model) choosing

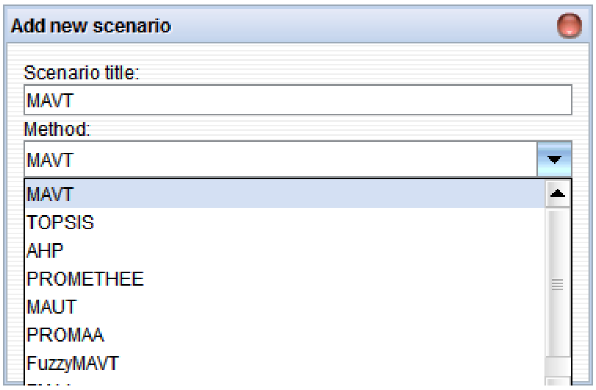


Fig.3 *DecernsMCDA*: Selection of MCDA model

The current version of ***DecernsMCDA DE*** supports the following MCDA methods:

- MAVT
- TOPSIS
- AHP
- PROMETHEE I, II
- MAUT
- Fuzzy MAVT
- ProMAA
- FMAA
- FlowSort

For detailed description of all methods see “Methods” chapter.

Settings

Settings dialog (see Fig.4) allows you to change some project specific settings, which include:

- Significance level - Parameter for confidence intervals calculation used within probabilistic analysis (for probability distributions).
- Accuracy - Number of digits after comma that must be shown (affect all numbers in project; see Pattern as example).
- Number of alpha-cuts - Parameter used for fuzzy numbers and fuzzy calculations.
- Best alternatives #. Number of best alternatives that must be shown in results if there are too many alternatives.

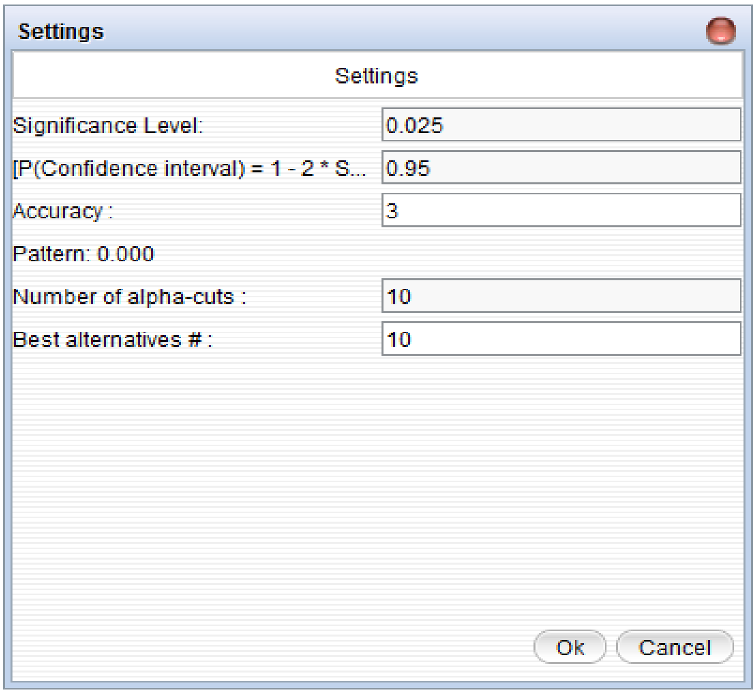


Fig.4 *DecernsMCDA*:Application settings

Multi-Criteria Decision Analysis (MCDA)

Main concepts

MCDA methods aim to evaluate alternatives based on multiple criteria using systematic analyses which overcome the limitations of unstructured individual or group decision-making. The aim of MCDA in a broad sense is to facilitate a decision maker's learning and understanding of the problem. Furthermore, MCDA enhances a decision maker's understanding about their own, other parties' and organizational preferences, values and objectives through exploring these in the context of a structured decision analysis framework.

The following are main categories of problems which are considered to be the basis of MCDA:

- *screening* alternatives – a process of eliminating those alternatives that do not appear to warrant further attention, i.e., selecting a smaller set of alternatives that likely contains the "best"/trade-off alternative;
- *ranking* alternatives (from "best" to "worst" according to a chosen algorithm);
- *selecting/choice* "the most preferred alternative" from a given set of alternatives;
- *sorting* alternatives into classes/categories (*e.g.*, "unacceptable," "possibly acceptable," "definitely acceptable," etc.); and
- *designing* (searching, identifying, creating) a new action/alternative to meet goals.

Some other categories of problems include description or learning problems which involves an analysis of actions to gain greater understanding of what may or may not be achievable. There also exists a portfolio problem where a choice of a subset of alternatives is curried out (here it is necessary to take into account not only individual characteristics of each alternative, but also their positive and negative interrelations).

Three dichotomies within MCDA problems can be distinguished [Malczewski, 1999]:

- multi-attribute decision making (MADM - a finite number of alternatives which are defined explicitly) versus multi-objective decision making (MODM - infinite or large number of alternatives which are defined, as a rule, implicitly);
- individual versus group decision making; and
- decisions under certainty versus decisions under uncertainty.

Within the ***DecernsMCDA DE*** the MADM problems on ranking, choice, and sorting alternatives are considered in the condition of both certainty and uncertainty. The screening problems in the *DecernsMCDA* may be analyzed with the use of the tools for dominance analysis, value path, or implementation of a ranking method for selecting several the "most appropriate" alternatives with subsequent additional analysis of the selected alternatives.

The system for supporting a group decision analysis is not a part of the current ***DecernsMCDA DE*** version.

DecernsMCDA contains all the steps within the decision-making process necessary for the multiple criteria decision analysis of various alternatives.

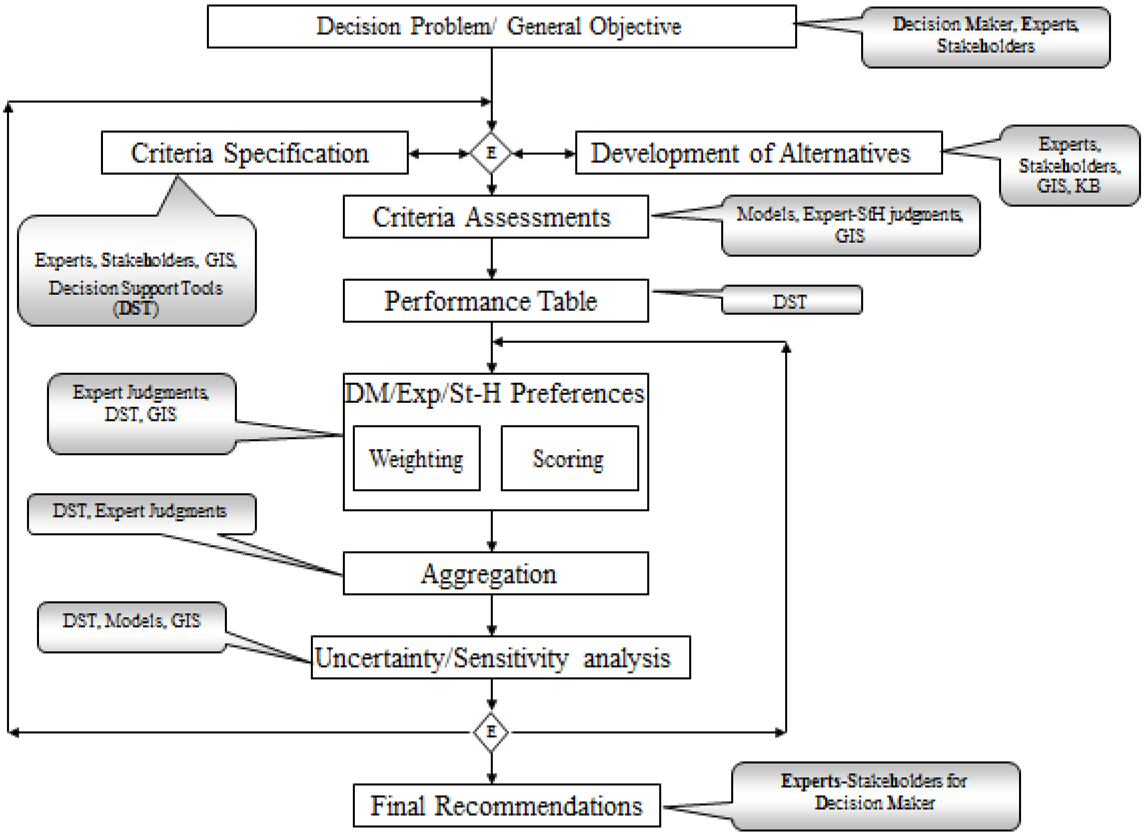


Fig.5 *DecernsMCDA*: Decision process flow chart

Fig.5 presents a decision process flow chart for typical MADM problems. This example highlights the effort of the participants in this process, including experts, decision makers, and a wider range of stakeholders and the involved software decision support tools. The process includes input from stakeholders, decision makers, and scientists and involves:

- problem definition;
- development of alternatives and criteria specification;
- generation of a performance table based on the results of the criteria assessments;
- determination of the preferences and weighting/scaling criteria by the stakeholder community;
- assessments of the alternatives against the different criteria which are conducted using models, GIS tools, and expert/stakeholder judgments; the MCDA tools take this information and perform sensitivity/uncertainty analysis;
- stakeholder review of the resulting selecting/ranking/screening/sorting of alternatives;
- recommendations which are made for the decision makers; a process which can be repeated iteratively to refine any of the steps.

Problem structuring and Model Building

Value Tree

Value Tree (VT) is one of the basic components in any decision support system. ***Decerns*** VT represents multi-level criteria tree and alternatives. Every criterion is subdivided to left (properties) part and right (scoring) part. You can make double-click to activate properties or scoring dialog respectively and right-click to activate pop-up menu with additional operations: add child criterion, delete this criterion, properties, and weighting/scoring method choosing. You can use also right-click on empty tree space to activate tree pop-up menu with the options: add alternative, arrange tree. When you created a new project, you have only a root entry (criterion) in the tree.

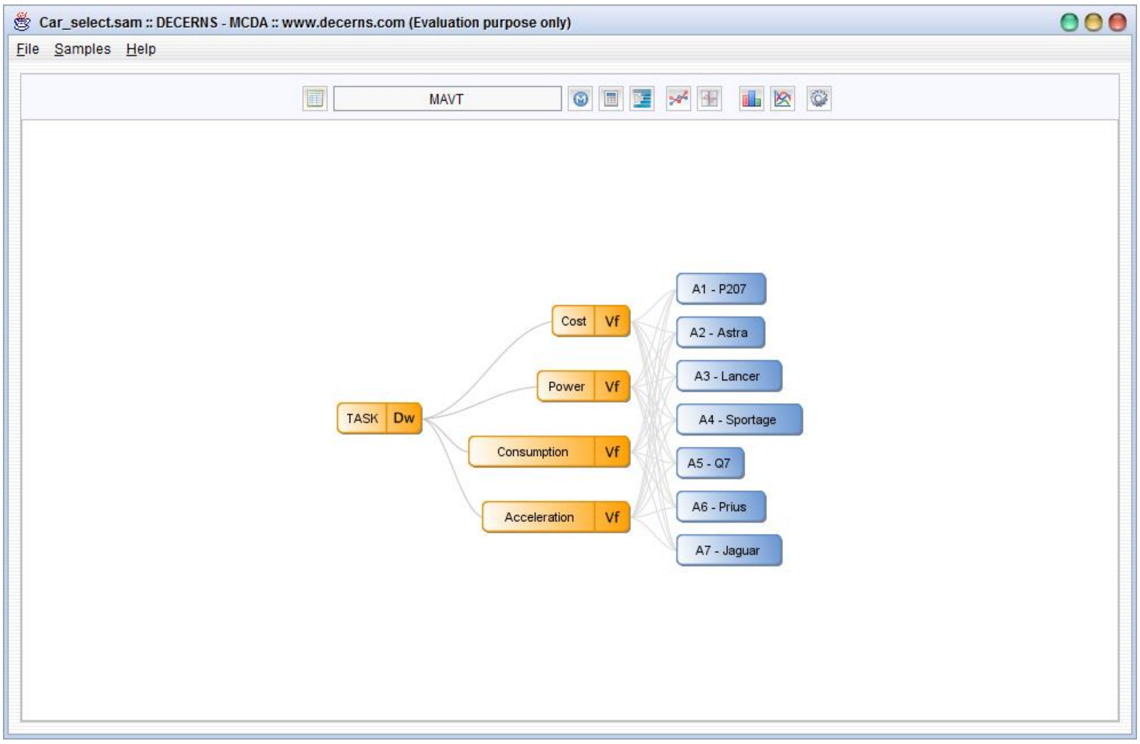


Fig.6 *DecernsMCDA*: Value tree

Then you can build a VT according to your problem. To do this: specify criteria with criterion pop-up menu (right click on criterion right area), specify alternatives with tree pop-up menu (right click on empty tree space).

The next step is assigning weights and alternatives scores for criteria (use double-click on right part of criterion). After that you can calculate model results and represent them in the output forms (calculate button in toolbar).

Adding criterion

- Click right mouse button on the parent criterion for new one.
- Click “Add criterion” in the pop-up menu.
- Specify criterion name.

Adding alternative

- Click right mouse button on the free tree space.
- Click “Add alternative” in the pop-up menu.
- Specify alternative name.

Arrange tree

- Click right mouse button on the free tree space.
- Click “Arrange tree” in the pop-up menu.

After these actions VT will be arranged with smooth animation.

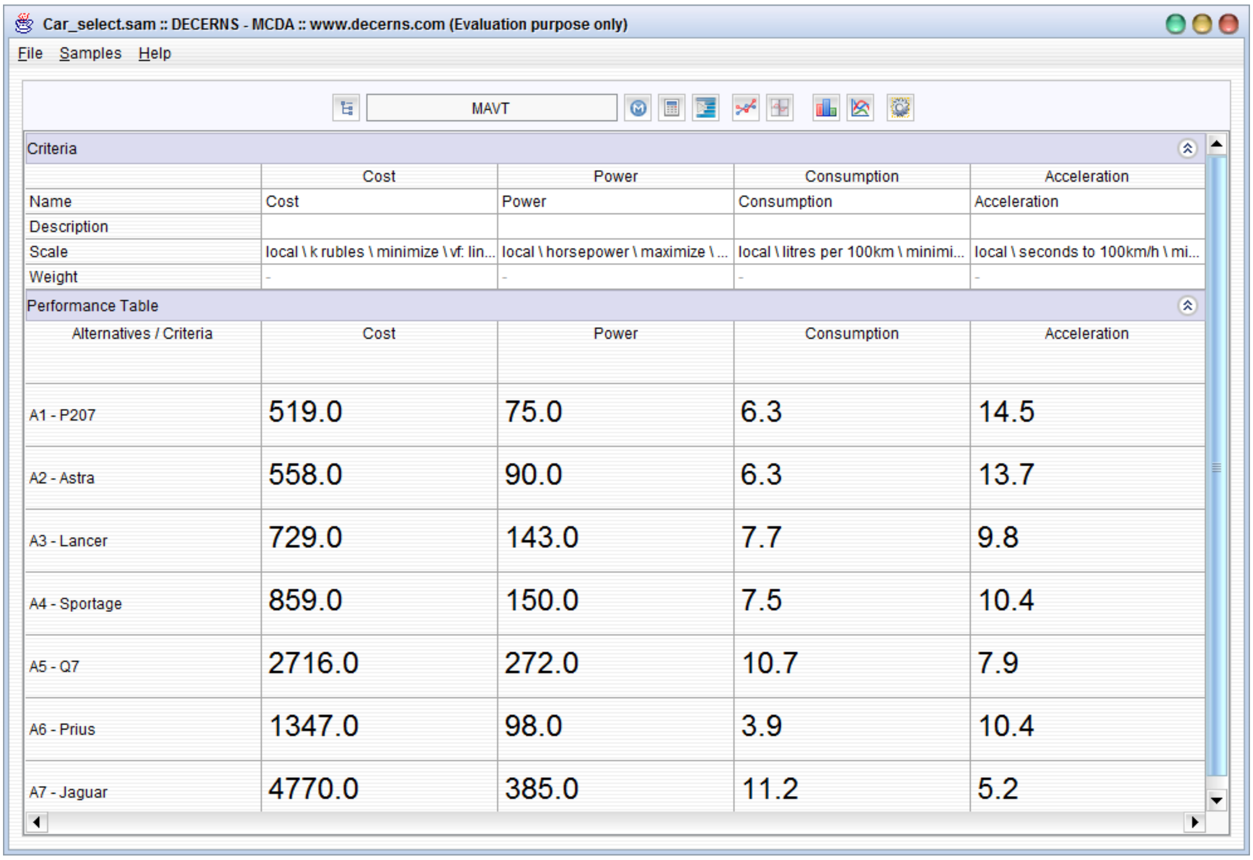
Performance table

Performance Table (PT) displays the values of alternatives for all leaf criteria, criterion weights, and additional descriptions. The interface of PT allows adding or deleting alternatives, editing parameters of alternatives and criteria.

Selection

Performance Table supports three types of selection:

- Column selection is performed by single clicking left mouse button on top cell in the column.
- Row selection is performed by single clicking left mouse button on first cell in the row; and
- Cell selection is produced by single clicking left mouse button on any other cell in the table.



	Cost	Power	Consumption	Acceleration
Name	Cost	Power	Consumption	Acceleration
Description				
Scale	local \ k rubles \ minimize \ vf: lin...	local \ horsepower \ maximize \ ...	local \ litres per 100km \ minimi...	local \ seconds to 100km/h \ mi...
Weight	-	-	-	-

Alternatives / Criteria	Cost	Power	Consumption	Acceleration
A1 - P207	519.0	75.0	6.3	14.5
A2 - Astra	558.0	90.0	6.3	13.7
A3 - Lancer	729.0	143.0	7.7	9.8
A4 - Sportage	859.0	150.0	7.5	10.4
A5 - Q7	2716.0	272.0	10.7	7.9
A6 - Prius	1347.0	98.0	3.9	10.4
A7 - Jaguar	4770.0	385.0	11.2	5.2

Fig.7 *DecernsMCDA*: Performance table

Structure

Performance Table consists of two parts:

- top part represents various information about leaf criteria,
- bottom part displays criterion values against each alternative.

Note: Part of table can be collapsed or expanded by clicking left mouse button on the header of the part with a title.

Top part and criteria attributes

Each column in the table represents information about single leaf criterion, including:

1. criterion name,
2. criterion description,
3. scale attributes,
4. criterion weight.

Criterion name and description can be changed just by clicking left mouse button on the cell. After that editing cursor will appear in the cell. Finishing editing is performed by pressing Enter button or selecting any other cell of the table.

Scale attributes cell displays row of scale properties separated by “/” symbol:

1. is scale local or global (local, global),
2. measure units (or “none” if undefined),
3. is scale minimized or maximized (minimize, maximize),
4. value function type (linear, exp-exponential, PW – piece-wise).

Double-click on scale attribute cell evokes Scale Properties Dialog.

Criterion weight cell represents criterion weight in the current model by means of cardinal/ordinal number(*eg:”0,25”*), fuzzy number(*eg:“Fuzzy Weight”*), random number (*eg:”Rand”*) or “not set” string if not calculated. Double-click on Criterion weight cell with random number or fuzzy number evokes Probability Tools or Fuzzy Tools dialog.

Criterion properties dialog can also be evoked by selecting whole column, clicking right mouse button and selecting Properties item in context menu.

Bottom part and alternative scales

In this part columns represent leaf criteria and rows represent alternatives. Headers of columns display criterion names and first cells of rows show the names of alternatives. Other cells display values of alternatives against the criteria.

In any value/score cell left part is represented by a given basic/deterministic value of alternative for this criterion and right part contains (if exists) information about (given by user(s)) fuzzy or random value implemented within the selected multicriteria method.

If fuzzy number is used: Right part contains type of fuzzy number (Singleton, Triangular, Trapezoidal, and Piece-Wise).

If random number is used: Right part contains:

- type if distribution (uniform, normal, delta, log-normal),
- expected value (*eg.:”E:0.9”*),
- standard deviation value (*eg.:”s:0.19”*),
- left bound of consideration interval (*eg.:“L:1.2”*),
- right bound of consideration interval (*eg.:“R:4.5”*).

Double-click on score cell will evokes Scores/Performance Dialog for editing scores information.

Alternative can be added by clicking right mouse button on headers or on first cells in rows and selecting “Add alternative” item in the context menu.

Alternative can be deleted by selecting whole row, clicking right mouse button on first cell of the row and selecting “Delete alternative” item in shown context menu.

Criterion properties dialog can be evoked by selecting whole column, clicking right mouse button and selecting Properties item in shown context menu.

Alternative properties dialog can be evoked by selecting whole row, clicking right mouse button and selecting Properties item in the context menu.

Results

The button "Calculate" invokes the dialog which presents the results of calculation of the current model. It is possible to scale the dialog using Mouse. To close the dialog press the cross button.

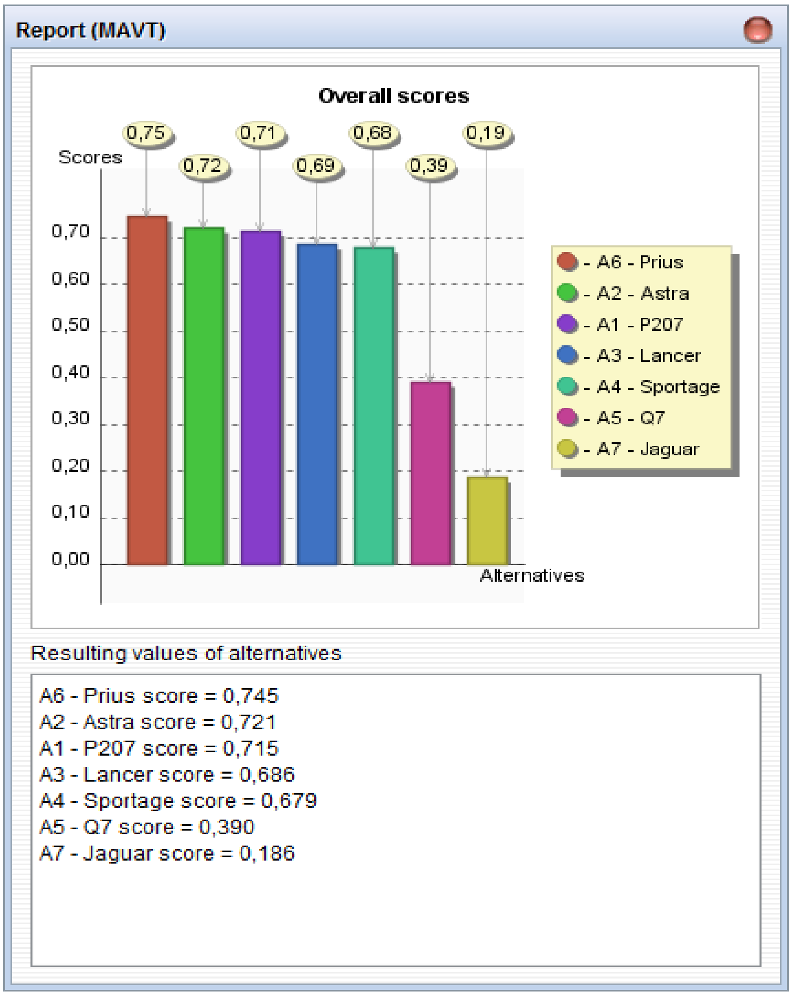


Fig.8 *DecernsMCDA*: Results – MAVT method

The dialog will be shown depends on selected scenario`s method. Sample dialog for MAVT/MAUT, AHP methods shown in Fig.8, other possible dialogs are represented in “Methods” chapter.

Weight Coefficients

Interface of *DecernsMCDA* allows users choosing several methods for setting weight coefficients depending on the MCDA model under use and experts’ experience, Fig.9.0; for selecting a weighting methods right mouse click in the right part of a parent criterion/goal/task is used; five weighting methods are suggested: Swing, Direct, Ranking, Rating, and Pairwise (comparison).

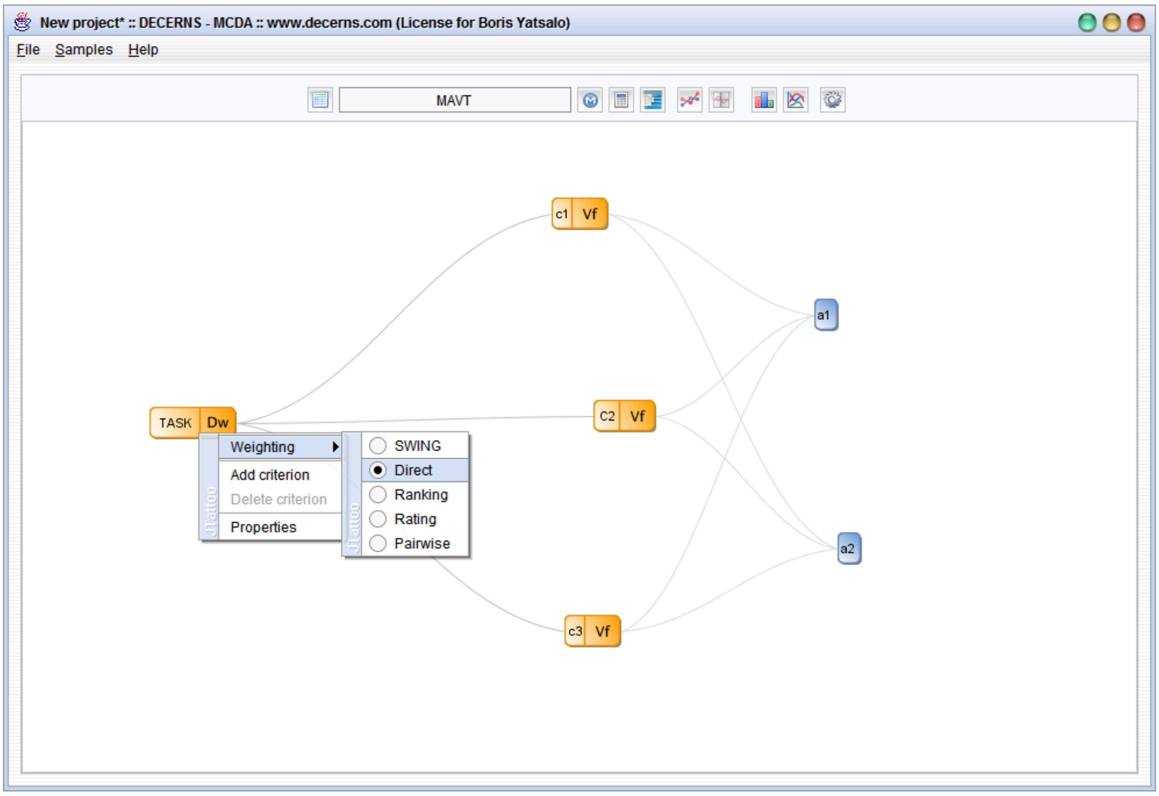


Fig.9.0 *DecernsMCDA*: the choice of weighting method

Direct method of weighting

In direct weighting user must specify the weight coefficients for all child criteria. Just use slider in second column or specify the number in third column, Fig.9. Weights sum must. After you click “Ok” button or “Normalize” button, the weights get normalized with the sum =1.

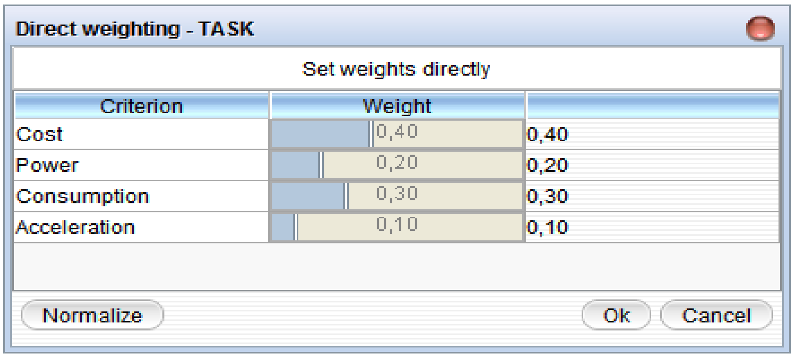


Fig.9 *DecernsMCDA*: Direct weighting

Ranking

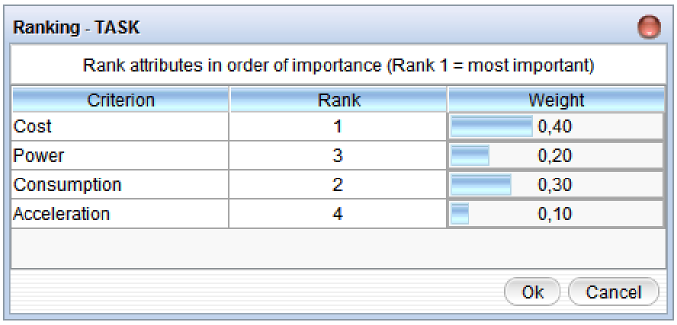


Fig.10 *DecernsMCDA*: Weighting: Ranking method

In ranking weighting you must specify the ranks for criteria. Weight coefficients will be calculated automatically. The most important criterion must have rank 1.

Rating

In rating weighting you must define rating points for every criterion. Most important criterion must have 100 points rating and all other importance points related to the most important one. Weight coefficients will be calculated automatically.

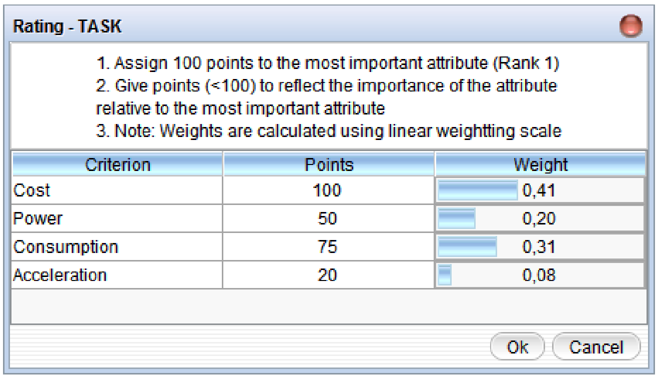


Fig.11 *DecernsMCDA*: Weighting: Rating method

Pairwise Comparison

Pairwise comparison used as weighting and scoring mechanism in AHP method. You need to fill matrix of relative scores for every pair of elements. Just select the cell in matrix and use vertical slider to specify the score for given pair of elements in Saaty scale:

- 9 – Extremely preference
- 7 – Very strong preference
- 5 – Strong preference
- 3 – Moderate preference
- 1 – Equal

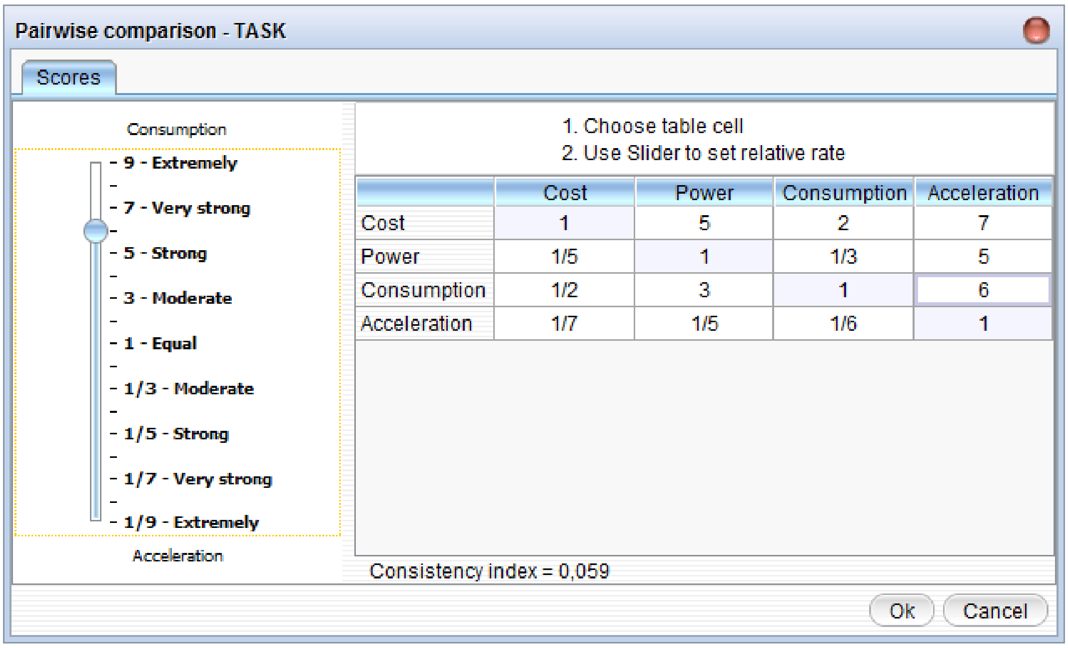


Fig.12 *DecernsMCDA*: Weighting: Pairwise comparison method

Swing

Swing weighting method allows you to take into account swings of criteria scales along with corresponding relative importance to assess *scaling factors*. On the first step criteria must be ranked from the most important one to least important (this could be done by table rows drag). On the second step every criterion is considered to evaluate its relative importance/scaling factor concerning the most important criterion, see Fig.13, (Belton V, Stewart T., 2002.

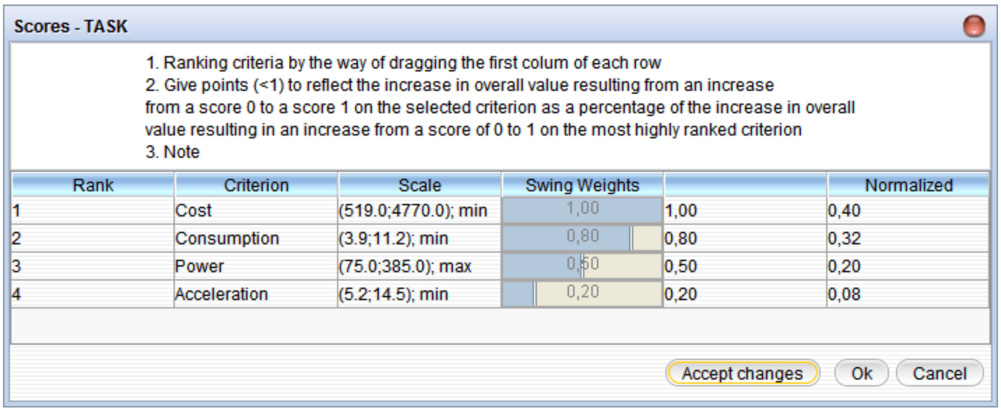


Fig.13 *DecernsMCDA*: Weighting: Swing method

Scoring

Value function

Value function, $V_j(x)$, translates the value (performance) from criterion scale to the range [0; 1].Then, these scores may be used in further calculations, (Belton V, Stewart T., 2002; Keeney RL, Raiffa H., 1976).

The following types of value functions are supported:

- Linear
- Exponential
- Piecewise linear

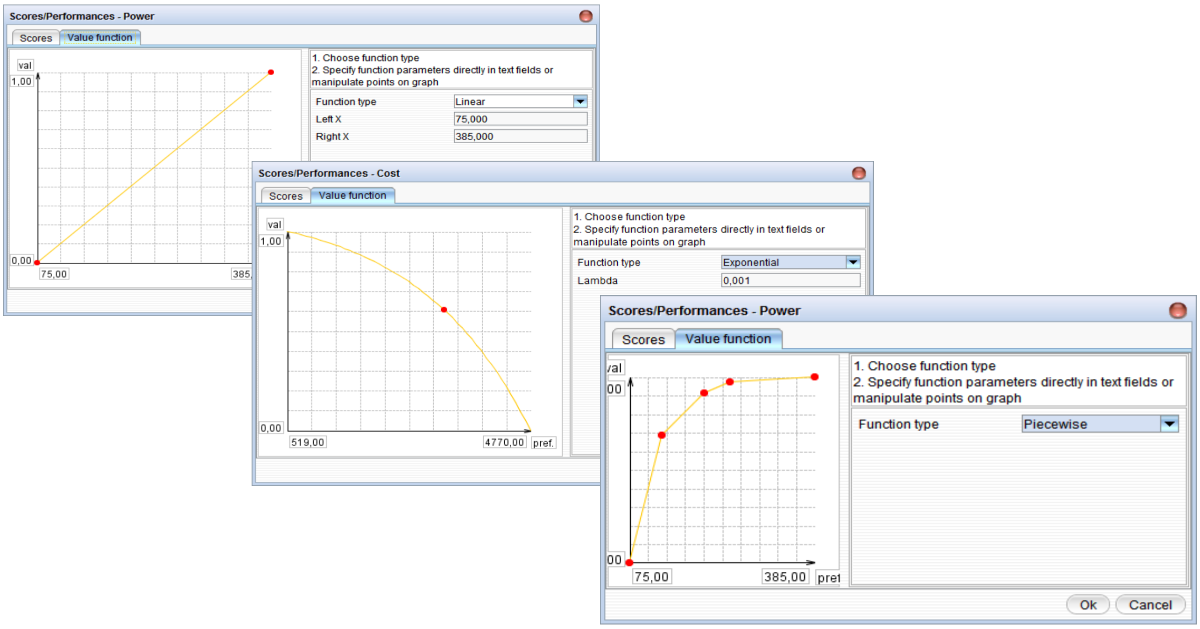


Fig.14 *DecernsMCDA*: Value functions

It should be pointed out, in MAUT model corresponding 'value' functions are called as *utility* functions, $U_j(x)$; utility functions can take into account relation to the risks and, in principle, may differ from value functions (Keeney RL, Raiffa H., 1976; Figueira J, Greco S, Ehrgott,M (Eds), 2005). The significance of such differences can be analyzed through value/utility function sensitivity analysis.

Preference function

Preference function translates the difference between criterion values/performance for every pair of alternatives to the range [0;1]; these scores are used in further flows calculations within the models PROMETHEE and FlowSort (Brans JP, Vincke P., 1985; Figueira J, Greco S, Ehrgott,M (Eds), 2005).

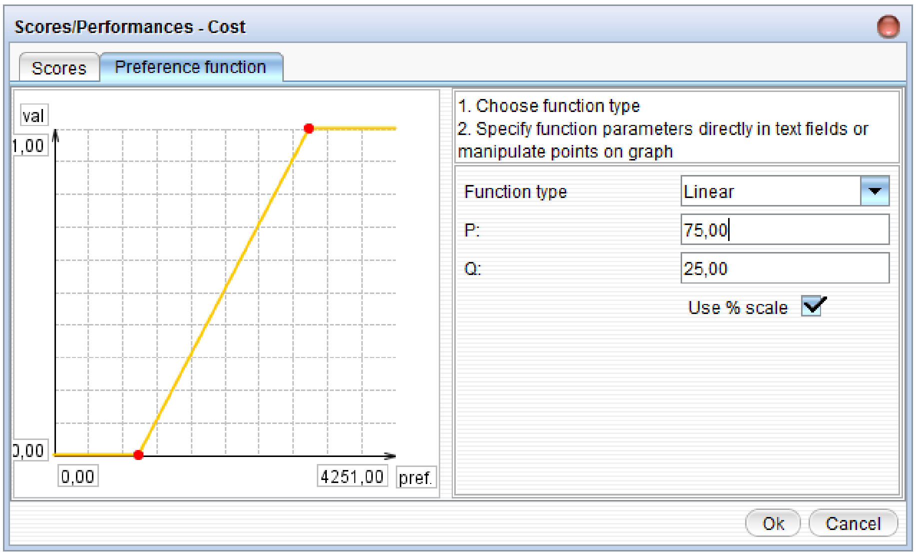


Fig.15 *DecernsMCDA*: Preference function - linear

The following types of preference functions are supported (see Fig.19):

- Usual
- U-Shape
- V-Shape
- Level
- Linear

Random values

Several methods in *DecernsMCDA* (MAUT, ProMAA) use random values for description of criterion performances/values against alternatives (and weights in ProMAA). Users can specify these values using special tool (see below).

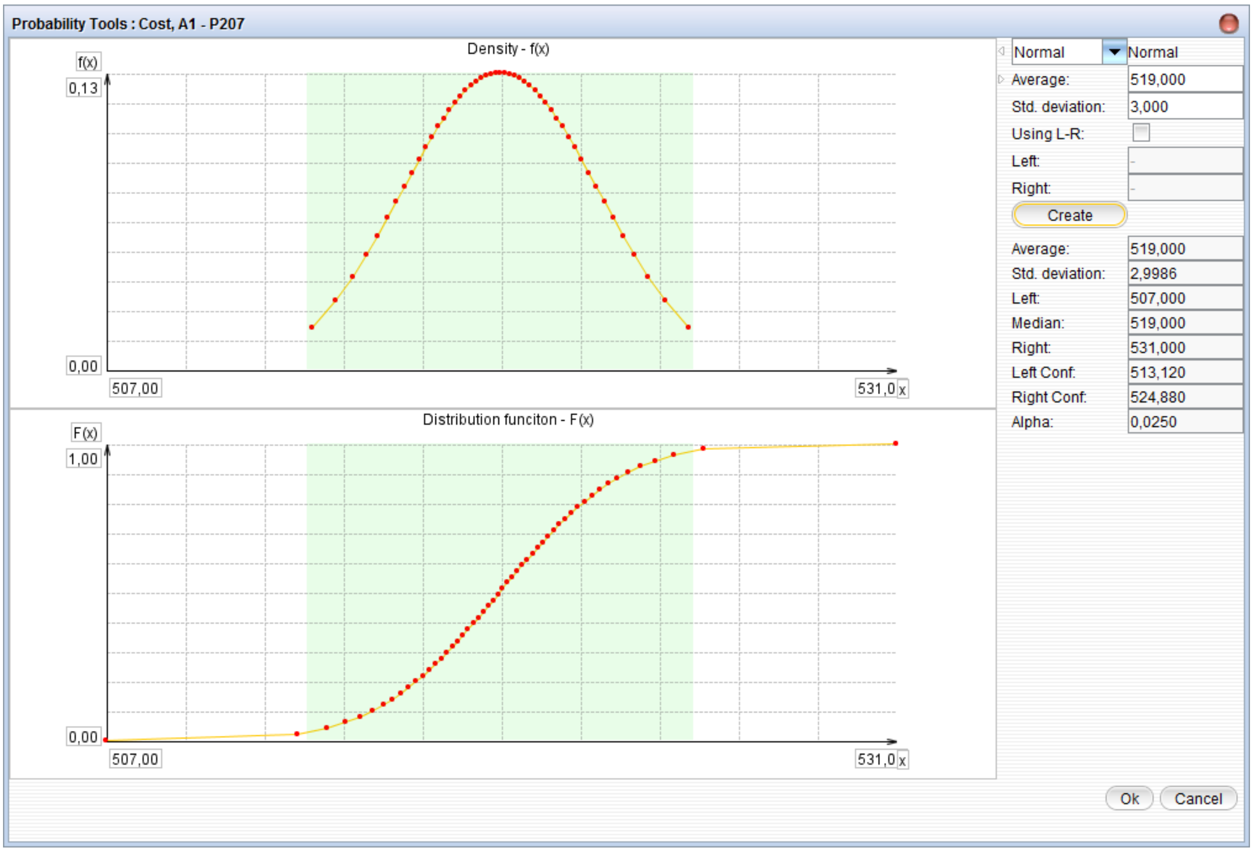


Fig.16 *DecernsMCDA*: Setting a probability distribution

The following types of probability distributions are supported:

- Delta distribution, $\delta(x\text{-}a)$, - uses only average value *a;*
- Uniform distribution, *U(a,b)*, - uses left, *a*, and right, *b*, borders, [x_{min},x_{max}], for changing such a truncated 'Normal' random value;
- Normal distribution, $N(a,s^2)$, where *a* is the average/math expectation, and *s* is the standard deviation value; user can set also the left, and right borders, [x_{min},x_{max}], if you need to truncate a chosen distribution (truncated 'Normal' distribution);
- Log-Normal distribution, *logN*, uses average, *a*, and standard deviation, *s*, values (of *lognormal* (!) distribution), and left, and right borders if you need to truncate a chosen distribution.

Fuzzy numbers

Several implemented *DecernsMCDA* methods, FMAVT/=Fuzzy MAVT, and FMAA, use fuzzy numbers for description of criterion performance and weight coefficients. Users can specify these values using special tool (see below).

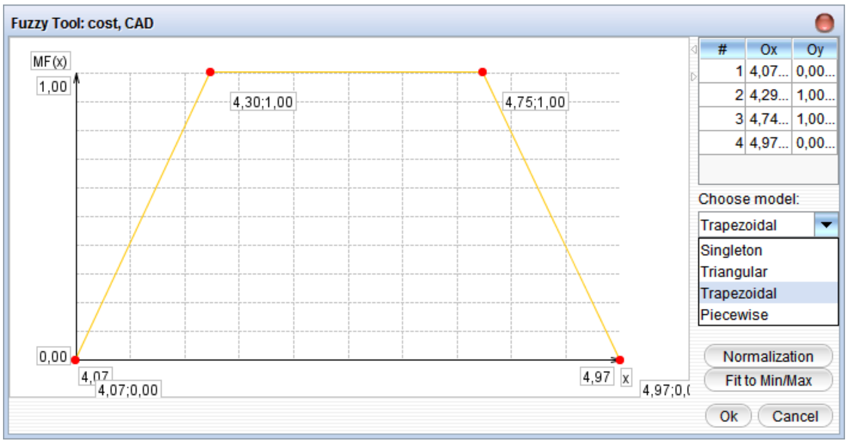


Fig.17 *DecernsMCDA*: setting a fuzzy number

The following types of membership functions for fuzzy numbers are supported:
- Singleton – uses only one point (crisp number);
- Triangular fuzzy number – uses three points in shape of triangle;
- Trapezoidal fuzzy number – uses four points in shape of trapezium/rectangle;
- Piecewise fuzzy number – user could define shape of membership function by himself using points and piecewise linear interpolations.

MCDA Models/Methods

In this section the MCDA models/methods, included in ***DecernsMCDA DE***, are briefly described. The details of these methods and their extensive discussions can be found in references at the end of this manual.

MAVT

Approaches, that use value functions, form so-called MAVT methods (MultiAttribute *Value* Theory) [Keeney and Raiffa, 1976; von Winterfeldt and Edwards, 1986; Belton and Stewart, 2002; Figueira, Greco and Ehrgott, 2005]. A value function describes a person's (expert's, decision maker's) preference regarding different levels of an attribute under *certainty* (see below description of MAUT method).

The objective of MAVT is to model and represent the decision maker's preferential system into an 'integrated' value function $V(\boldsymbol{a})$,

$$V(\boldsymbol{a}) = F(V_1(a_1),\ldots,V_m(a_m)); \tag{1}$$

where alternative $\boldsymbol{a}$ is presented as a vector of the evaluation criteria $\boldsymbol{a}=(a_1,\ldots,a_m)$; a_j is an estimate of this alternative against a criterion C_j, $j=1,\ldots,m$; and $V_j(a_j)$ is the value score of the alternative reflecting its performance on criterion j via use of a value function $V_j(x)$ $(0 \le V_j(x) \le 1)$. The goal of decision-makers in this process is to identify the alternative $\boldsymbol{a}$ which maximizes the overall value of $V(\boldsymbol{a})$. The most widely used form of function $F(\ )$ is an additive model (this model is used in ***DecernsMCDA DE***):

$$V(\boldsymbol{a}) = w_1 V_1(a_1) + \ldots + w_m V_m(a_m), \tag{2}$$

$$w_j > 0\ , \sum w_i = 1, \tag{3}$$

where w_j, j=1,...,m, are the criterion weights reflecting the *scaling factors* (relative importance of criteria) [Keeney and Raiffa, 1976; von Winterfeldt and Edwards, 1986], and for their assessment *swing* weighting method is recommended.

It should be stressed, however, that for a justified implementation of the additive model (2) some requirements of MAVT concerning the problem under investigation should be held, especially the *preferential independence* requirements [Keeney and Raiffa, 1976; von Winterfeldt and Edwards, 1986]. MAVT relies on the assumption that the decision-maker is rational, preferring more value to less value, for example, Fig.18, and that the decision-maker has perfect knowledge, and is consistent in his judgments.

Because poor scores on some criteria can be compensated by high scores on other criteria, MAVT is part of MCDA techniques known as "*compensatory*" methods.

Various methods for defining partial value functions $V_j(x)$ and assessing weights/ *scaling factors* w_j have been developed both for quantitative and qualitative criteria [Keeney and Raiffa, 1976; von Winterfeldt and Edwards, 1986; Belton and Stewart, 2002].

Other functions *F(.)* in (1) may also be used, e.g., multiplicative or multilinear forms of MAVT [Keeney and Raiffa, 1976; von Winterfeldt and Edwards, 1986].

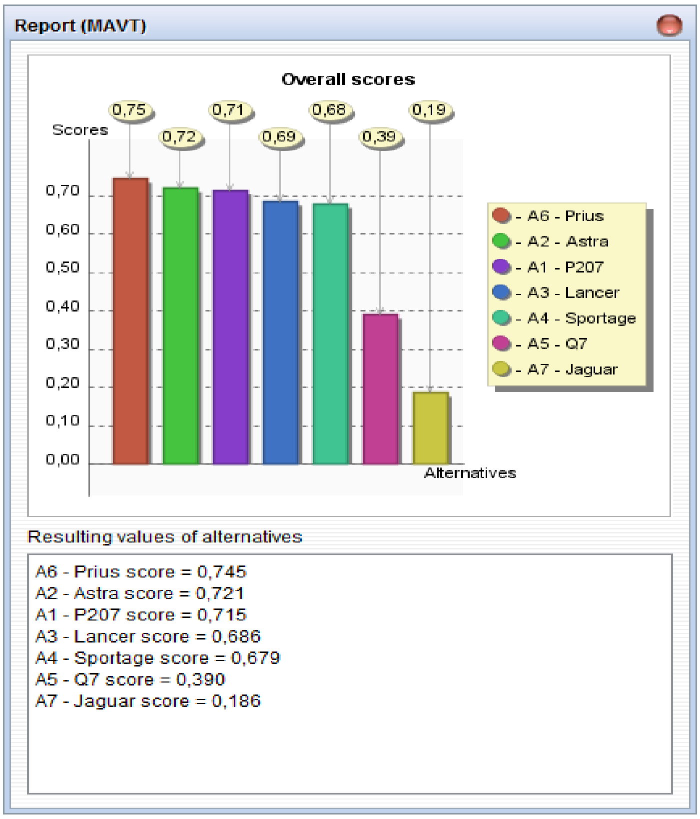


Fig.18 *DecernsMCDA*: Results of ranking alternatives with MAVT method

MAUT

MAUT methods (Multi Attribute Utility Theory) are also often used within multicriteria decision analysis. While MAVT and MAUT methods are not always seen as fundamentally different [von Winterfeldt and Edwards, 1986], they are typically differentiated (according to an agreement in terms) on the basis of certainty. A value function describes a person's preference regarding different levels of an attribute under certainty, whereas utility theory extends the method to use probabilities and expectations to deal with uncertainty [Keeney and Raiffa, 1976; von Winterfeldt and Edwards, 1986].

Within MAUT methodology ranking alternatives is based on using the overall utility $U(\boldsymbol{a})$:

$$U(\boldsymbol{a}) = F(U_1(a_1),\dots,U_m(a_m)); \quad (4)$$

where alternative $\boldsymbol{a}$ is presented by a vector $\boldsymbol{a}=(a_1,\dots,a_m)$; here a_j - is an estimate of this alternative against a criterion C_j, $j=1,\dots,m$; $U_j(a_j)$ is an assessment of alternative $\boldsymbol{a}$ in a utility scale with the use of a partial utility function $U_j(x)$ for criterion/attribute C_j, $(0 \leq U_j(x) \leq 1)$.

Strictly speaking, the type of MAUT model (function $F(.)$) depends on the requirements (preferential independence, utility independence, and additive independence), which provide implementation of the appropriate function $F(.)$ in (4) [Keeney and Raiffa, 1976; von Winterfeldt and Edwards, 1986].

For practical MAUT based applications the additive model is most widely used [Keeney and Raiffa, 1976; von Winterfeldt and Edwards, 1986]:

$$U(\boldsymbol{a}) = w_1\, U_1(a_1) + \dots + w_m\, U_m(a_m), \quad (5)$$

$$w_i > 0\,,\ \sum w_i = 1, \quad (6)$$

weight coefficients w_j are interpreted in (5) as scaling factors.

At that, multiplicative and multilinear MAUT models are also used [Keeney and Raiffa, 1976; von Winterfeldt and Edwards, 1986].

Uncertainty of the criterion value a_j are presented in MAUT by a random variable $X_j = X_j(\boldsymbol{a})$ with density of distribution $\varphi_j(x)$, $j=1,\dots,m$. The overall utility for the alternative $\boldsymbol{a}$ can be considered in this case as a random variable

$$U(\boldsymbol{a}) = w_1\, U_1(X_1) + \dots + w_m\, U_m(X_m), \quad (7)$$

where weight coefficients w_j satisfy the normalization condition (6). Ranking of alternatives within MAUT is based on the comparison of expected utilities: the alternative $\boldsymbol{a}_1$ exceeds the alternative $\boldsymbol{a}_2$, $\boldsymbol{a}_1 > \boldsymbol{a}_2$, if and only if

$$E(U(a_1)) > E(U(a_2)) \quad (8)$$

where $E(X)$ is the mathematical expectation of random variable X. According to (5),

$$E(U(\boldsymbol{a})) = w_1\, E(U_1(X_1)) + \dots + w_m\, E(U_m(X_m)), \quad (9)$$

Despite extensive use of the expected utility concept, it's use is not universally accepted as the only approach within decision analysis, and other approaches which do not use expected utility methods are implemented [Brans and Vincke, 1985; Belton and Stewart, 2002; Figueira, Greco and Ehrgott, 2005].

AHP

The AHP method (Analytic Hierarchy Process), developed by T.Saaty [Saaty, 1980], is based on 3 principles:

- *Decomposition*: AHP hierarchy development (with the use of Value Tree);
- *Comparative judgments*: pairwise comparisons of criteria, and pairwise comparisons of alternatives against each criterion (of the lowest level);
- *Synthesis of priorities*: determination of weights based on pairwise comparison of criteria (including comparison through hierarchy/Value Tree), and determination of scores (assessment of eigenvectors for the maximum eigenvalue); determination of the overall score using linear additive model.

AHP presents an integration of the additive model (2) with a distinctive determination of the decision matrix, $V_{i,a}$, and criteria weights, w_i, $i=1,...,m$. Within AHP a systematic pairwise comparison of alternatives with respect to each criterion is used based on a special ratio scale: for a given criterion, alternative i is preferred to alternative j with the strength of preference given by $a_{ij}=s$, $1 \leq s \leq 9$, correspondingly, $a_{ji}=1/s$. Then, the same procedure is implemented for $m(m-1)/2$ pairwise comparisons in the same scale for m criteria. The obtained matrices are processed (by extracting the eigenvector corresponding to the maximum eigenvalue of the pairwise comparison matrix), and yield the values $V_{i,a}$ and weights w_i for subsequent use with the model, when preferences are aggregated across different criteria according to (2).

AHP may thus be considered as an MAVT approach with a specific elicited value function (scoring) and criteria weights (weighting). However, taking into account different assumptions and approaches, proponents of AHP insist that it is not a value function method [Belton and Stewart, 2002]. Additionally, AHP relies on the supposition that humans are more capable of making relative judgments than absolute judgments. Consequently, the rationality assumption in AHP is more relaxed than in MAVT.

AHP popularity is due to its flexibility and ease of use, and availability of software packages.

AHP method has not been without criticism:

- ambiguity in the meaning of the relative importance of one element of the decision hierarchy when it is compared to another element;
- the number of comparisons for large problems;
- the use of 1-9 scale.

Some researches argue that the type of questions asked during the process of pairwise comparisons are meaningless; another criticism is related to the *rank reversal problem* [Belton and Stewart, 2002; Figueira, Greco and Ehrgott, 2005].

Decerns authors recommend using AHP method (if ratio scale within the pairwise comparison is considered by experts as suitable for the problem under investigation) as a preliminary step, and in the cases when implementation of other methods seems for stakeholders more complicated.

PROMETHEE

The PROMETHEE method, developed by Brans and Vincke, belongs to so called *outranking* (ORT, Outranking Relation Theory) methods [Brans and Vincke, 1985; Belton and Stewart, 2002; Figueira, Greco and Ehrgott, 2005].

ORT approaches imply forming an ordered relation of a given set of alternatives. Outranking methods are based on a pairwise comparison of alternatives for each criterion under consideration with subsequent integration of obtained preferences according to a chosen algorithm. Among outranking approaches, the ELECTRE family of methods, developed by Roy [Belton and Stewart, 2002; Figueira, Greco and Ehrgott, 2005], and the PROMETHEE method are most used [Brans and Vincke, 1985; Figueira, Greco and Ehrgott, 2005].

PROMETHEE is based on utilization of a performance matrix $\{z_i(\boldsymbol{a})\}$ (where $z_i(\boldsymbol{a})$ is an evaluation of alternative $\boldsymbol{a}$ against criterion i) and a chosen preference function $p_j(d)$, $0 \le p_j(d) \le 1$, with specified indifference (q_j) and preference (p_j) thresholds.

The main types of preference functions are presented in Fig.19,

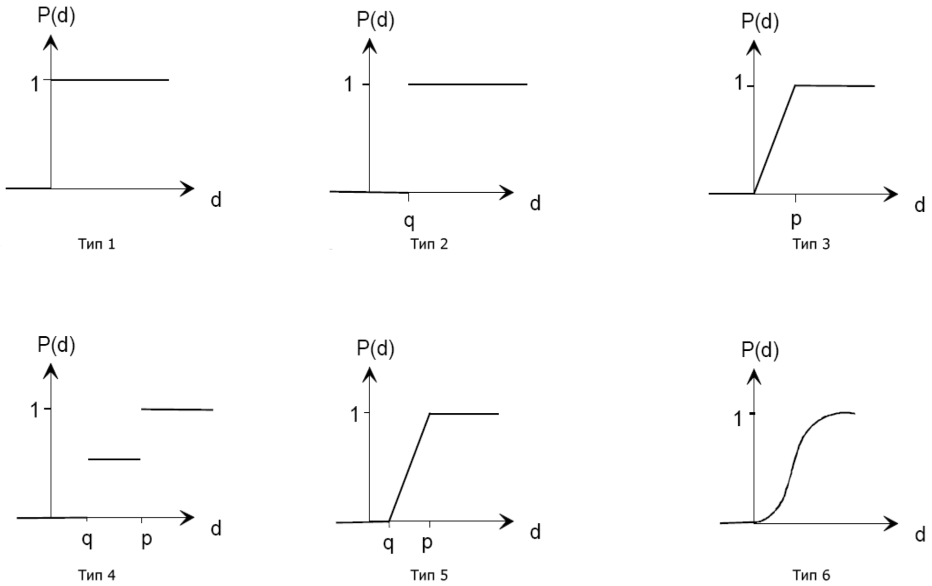


Fig.19 Preference functions

here $d = z_j(\boldsymbol{a}) - z_j(\boldsymbol{b})$ for criterion j under consideration, q and p are indifference and preference thresholds, correspondingly, chosen for criterion j.

Then the intensity of preference for alternative $\boldsymbol{a}$ over alternative $\boldsymbol{b}$, $P_j(\boldsymbol{a},\boldsymbol{b}) = p_j(z_j(\boldsymbol{a}) - z_j(\boldsymbol{b}))$, and the preference index, $P(\boldsymbol{a},\boldsymbol{b})$, are assessed:

$$P(\boldsymbol{a},\boldsymbol{b}) = \Sigma w_j P_j(\boldsymbol{a},\boldsymbol{b}), \tag{10}$$

where weights w_j reflect the relative importance of the criteria. According to the features of preference functions $p_j(x)$, if $P_j(\boldsymbol{a},\boldsymbol{b})>0$, then $P_j(\boldsymbol{b},\boldsymbol{a})=0$. Preference indices are used for determination of positive outranking flow $Q^+(\boldsymbol{a})$:

$$Q^+(\boldsymbol{a}) = \Sigma_b P(\boldsymbol{a},\boldsymbol{b})/(n-1) \tag{11}$$

and negative outranking flow $Q^-(\boldsymbol{a})$:

$$Q^-(\boldsymbol{a}) = \Sigma_b P(\boldsymbol{b},\boldsymbol{a})/(n-1), \tag{12}$$

summed over all alternatives $\boldsymbol{b} \neq \boldsymbol{a}$, n is the number of alternatives under consideration.

According to the PROMETHEE 1 method, $\boldsymbol{a}$ outranks $\boldsymbol{b}$ if $Q^+(\boldsymbol{a}) \ge Q^+(\boldsymbol{b})$ and $Q^-(\boldsymbol{a}) \le Q^-(\boldsymbol{b})$;

a is indifferent to ***b*** if $Q^{+}(\boldsymbol{a}) = Q^{+}(\boldsymbol{b})$ and $Q^{-}(\boldsymbol{a})=Q^{-}(\boldsymbol{b})$;
a and ***b*** are incomparable if $Q^{+}(\boldsymbol{a})>Q^{+}(\boldsymbol{b})$ and $Q^{-}(\boldsymbol{b})<Q^{-}(\boldsymbol{a})$, or $Q^{+}(\boldsymbol{b})>Q^{+}(\boldsymbol{a})$ and $Q^{-}(\boldsymbol{a})<Q^{-}(\boldsymbol{b})$. Thus, PROMETHEE 1, like some other outranking methods, does not presuppose that a single best alternative can be identified, since some alternatives may be incomparable.

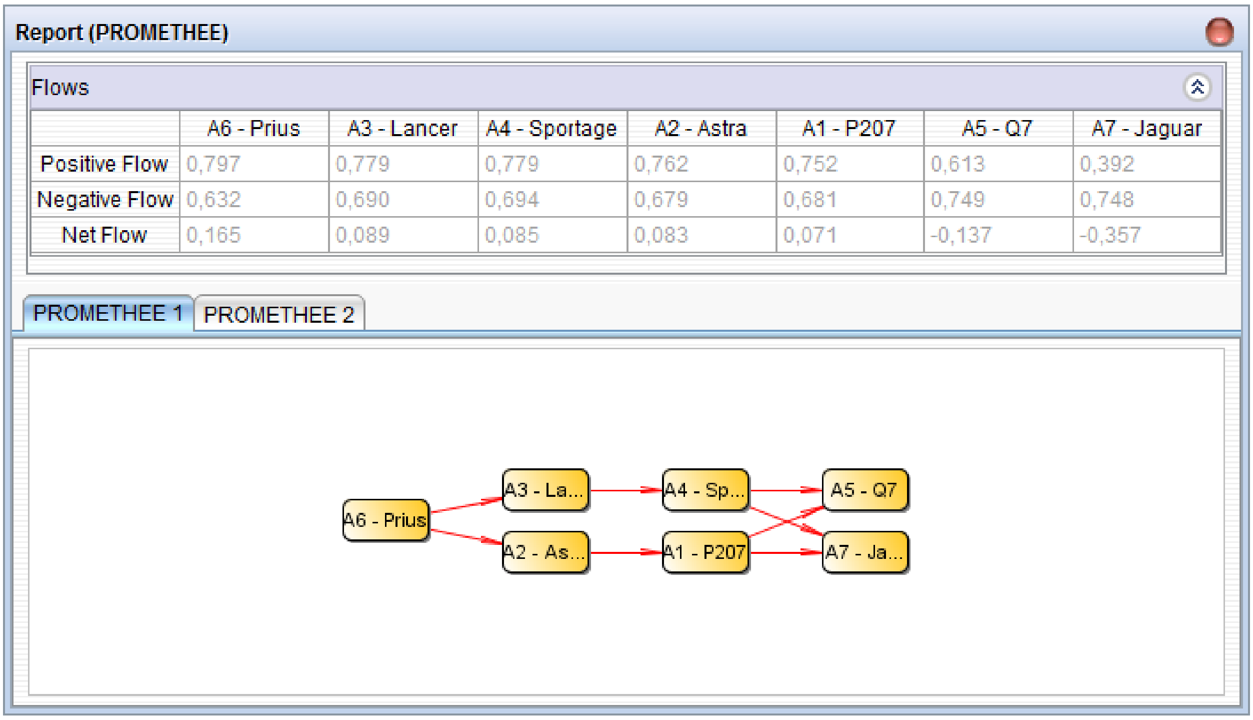
The PROMETHEE-2 method is based on the "*net flow*" $Q(\boldsymbol{a})$ for alternative ***a***:

$$Q(\boldsymbol{a}) = Q^{+}(\boldsymbol{a}) - Q^{-}(\boldsymbol{a})\ , \tag{13}$$

and it may be used for a complete ranking of alternatives: alternative ***a*** outranks ***b*** if $Q(\boldsymbol{a})>Q(\boldsymbol{b})$.

PROMETHEE, like other outranking methods, is considered an attractive and transparent method, although both positive and negative flows depend on the complete set of alternatives under consideration. However, a 'drawback' of outranking is that "indifference" and "preference" thresholds – though often based on expert knowledge – are essentially arbitrary, and the relationship representing which alternatives outrank depends on selection of those thresholds [Belton V, Stewart T 2002]. One way to analyze the robustness and check consistency between thresholds is to manipulate by the thresholds.

Outranking techniques allow inferior performance on some criteria to be compensated for by superior performance on others. They do not necessarily, however, take into account the magnitude of relative underperformance in a criterion versus the magnitude of over-performance in another criterion. Therefore, outranking models are known as "*partially compensatory*".


Report (PROMETHEE)

Flows

	A6 - Prius	A3 - Lancer	A4 - Sportage	A2 - Astra	A1 - P207	A5 - Q7	A7 - Jaguar
Positive Flow	0,797	0,779	0,779	0,762	0,752	0,613	0,392
Negative Flow	0,632	0,690	0,694	0,679	0,681	0,749	0,748
Net Flow	0,165	0,089	0,085	0,083	0,071	-0,137	-0,357

Fig.20 *DecernsMCDA*: Results – PROMETHEE I method

TOPSIS

TOPSIS orders a set of alternatives on the basis of their distances to the ideal and anti-ideal points [Hwang and Yoon, 1981; Malczewski, 1999]. These points represent hypothetical alternatives that consist of the most desirable (ideal) and the less desirable (anti-ideal) levels of each criterion across the alternatives under consideration.

Within TOPSIS method the following distance to the '*ideal point*' is used:

$$s_{i+} = (\sum_j w_j^p (x_{ij} - x_{+j})^p)^{1/p} \tag{14}$$

where w_j is a weight assigned to the *j*-th criterion, x_{ij} is the *standardized* criterion value of the *i*-th alternative, x_{+j} is the ideal value for the *j*-th criterion, *p* is a parameter (p=1,2, ∞ is the most often used); in *DecernsMCDA* parameter ***p*=2** is implementeed.

There are several approaches to standardization of criterion values. One of them: $C_{ij}=C_j(\boldsymbol{a}_i)\rightarrow x_{ij}$ (C_{ij} is estimation of alternative $\boldsymbol{a}_i$ for criterion *j*; x_{ij} – corresponding standardized value), and

$$x_{ij} = C_{ij} / (\sum_{j=1}^{m} C_{ij}^2)^{1/2})$$

The negative (anti) ideal point and distances s_i- are defined similarly:

$$s_{i-} = (\sum_j w_j^p (x_{ij} - x_{-j})^p)^{1/p} \tag{15}$$

There are several decision rules which are implemented within TOPSIS. The following rule is most often used:

$$c_{i+} = s_{i-} / (s_{i+} + s_{i-}) \tag{16}$$

(the case *p*=2 in (14-15) and the formula (16) are implemented in *DecernsMCDA*).

The Alternative(s) with the highest c_{i+} is considered as the "best" one.

TOPSIS is very attractive method to decision problems when the dependency among criteria is difficult to test or verify. That is especially true in case of spatial decision problems, which typically involve complex interdependencies among attributes.

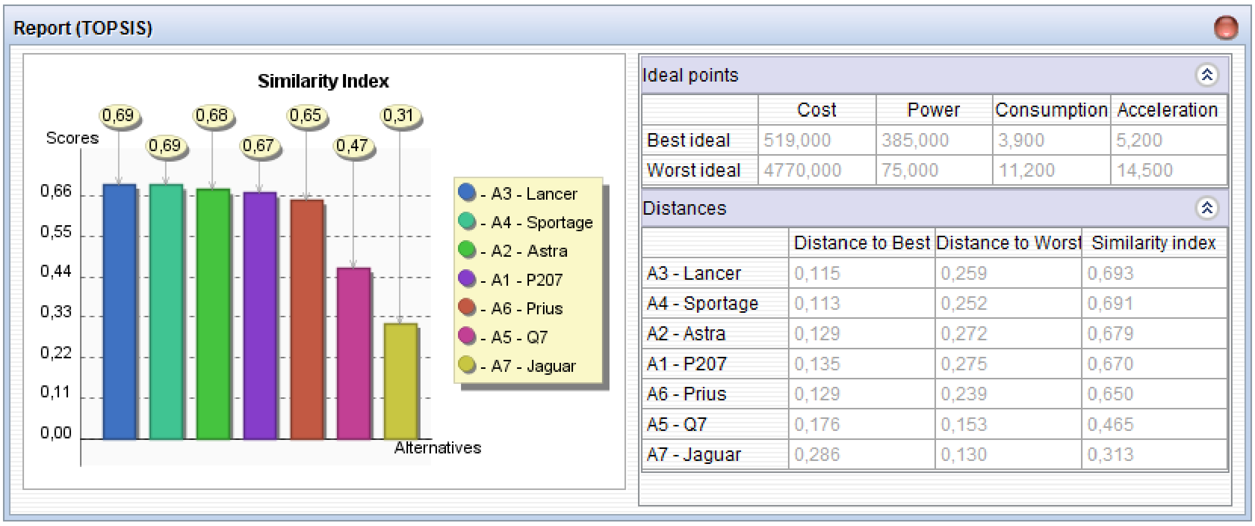


Fig.21 *DecernsMCDA*: Results – TOPSIS method

FuzzyMAVT /FMAVT

FMAVT model is intended for uncertainty treatment when solving multicriteria problems with the use of ‘value function concept’.

Within FMAVT, implemented in *DecernsMCDA*, the expression/model (2) is used, where criterion values a_{ij}, scores $V_j(a_{ij})$, and weights w_j are considered as fuzzy numbers, i=1,…,n, j=1,…,m; partial value functions $V_j(x)$ are considered as given by experts usual/crisp functions. The approach for assigning fuzzy weights w_j in FMAVT is similar to weighting process in ProMAA method, described below.

Ranking alternatives within FMAVT is based on comparison of overall fuzzy values $V(\boldsymbol{a}_i)$ with the use of visual analysis (see the left part of Fig.22) and several methods for ranking fuzzy numbers (using 4 defuzzification methods and 3 methods for comparison of fuzzy numbers).

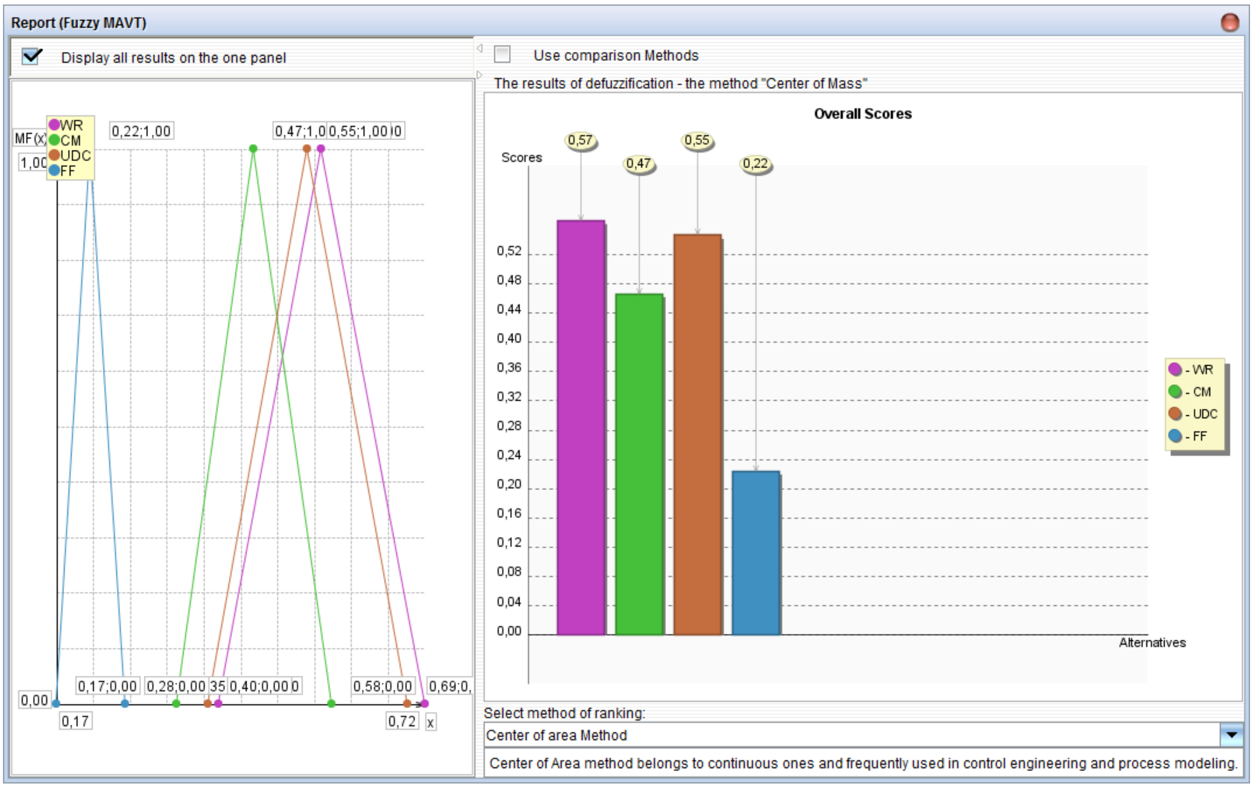


Fig.22 *DecernsMCDA*: Results – FMAVT method

ProMAA

The ProMAA method (Probabilistic Multicriteria Acceptability Analysis) [Yatsalo, Gritsyuk, Mirzeabasov, and Vasilevskaya, 2011], developed within the ***DECERNS*** project, assimilates uncertainties of objective values and subjective judgments within the discrete multicriteria decision analysis. ProMAA algorithm utilizes probability distributions of both criteria values and weight coefficients based on pairwise comparison of alternatives in an integrated scale.

Within the ProMAA, probabilities $P_{ik} = P\{S_{ik}\}$ of ‘*likely rank events*’ S_{ik} are determined, where event S_{ik} is defined as follows:

$$S_{ik}=\{\text{Alternative } \boldsymbol{a}_i \text{ has the rank } k\},\ i,k=1,\ldots,n,$$

(i.e., k-1 alternatives are better $\boldsymbol{a}_i$ in a chosen scale).

For values/probabilities P_{ik} $=P\{S_{ik}\}$ the term '*rank acceptability indices*' are often used [Lahdelma, Hokkanen and Salminen, 1998; Tervonen and Figueira, 2008]. For aggregation of the indicated probabilities a weighted sum may also be used:

$$R_i = \sum_{k=1}^{n} w_k^{ac} P_{ik} \quad , \tag{17}$$

where w_k^{ac} are weights of relative importance of ranks.

Thus, based on analysis of the matrix $\{P_{ik}\}$, $i,k=1,\dots,n$, choosing "best" alternatives among $\{\boldsymbol{a}_i$, $i=1,\dots n\}$, screening alternatives or, in some cases, ranking alternatives within ProMAA can be implemented; ranking alternatives can be realized based on the 'holistic acceptability indices' R_i; $i=1,\dots,n$, however, the recommendations concerning implementation of such a secondary ranking (17) are restricted.

Utility based ProMAA method, ProMAA-U, has been implemented within the ***DecernsMCDA***. It is based on (probabilistic) extension of the classical MAUT additive model (5) with implementation of acceptability analysis instead of expected utilities. Within ProMAA-U both values/utilities $U_j(a_j)$ and weights w_j may be considered as random variables with the given probability distributions, $j=1,\dots,m$; the following distributions may be chosen by users in ***DecernsMCDA***: delta function, uniform, and (truncated) normal, and lognormal distributions.

Realization of ProMAA is based on numerical approximation of functions of random variables and numerical assessment of integrals (for approximate determination of probabilities $P_{ik}=P\{S_{ik}\}$). Algorithm of ProMAA is presented in detail in [Yatsalo, Gritsyuk, Mirzeabasov and Vasilevskaya, 2011].

The user interface of ProMAA-U module and corresponding functions allow the user(s) to:

- specify the probability distribution of $C_j(\boldsymbol{a}_i)$ for criterion C_j, $j=1,\dots$, m, and the set of alternatives $\{\boldsymbol{a}_i$, $i=1,\dots n\}$;
- specify the utility functions $U_j(x)$ (from the class of linear, exponential, and piecewise-linear functions);
- specify the probability distribution for weight coefficient w_j, $j=1,\dots,m$, see also below a recommended approach for setting weights in ProMAA; then:
- the distributions of random variables $\eta_i=U(\boldsymbol{a}_i)$, $i=1,\dots,n$, and rank acceptability indices P_{ik} , $i,k=1,\dots,n$, are calculated by the system as a numerical implementation of the corresponding math expressions;
- the users analyze graphical and tabular representation of the output results for subsequent decision making; and
- users have the possibility to implement *utility functions sensitivity analysis* of the output results (through changing one or several selected partial utility functions $U_j(x)$).

Setting weights within ProMAA

Within MAVT/MAUT and within other *classical* multicriteria methods, weight coefficients are considered as constant/non-random positive numbers. In this case, for uncertainty analysis, as a rule, one-parameter sensitivity analysis to changing the chosen weight coefficient is used. However, extended uncertainty treatment/analysis, when weights are not single-valued and are considered as distributed in the intervals given by experts, is justified for most practical multicriteria problems.

Weight coefficients can be assessed with the use of different weighting methods, including swing method(s) for determination of *scaling factors* in MAVT/MAUT, voting approach for outranking methods and some others [Keeney and Raiffa, 1976; von Winterfeldt and Edwards, 1986; von Winterfeldt and Edwards, 1986; Belton and Stewart, 2002; Figueira, Greco and Ehrgott, 2005]. Weights of relative importance have uncertainties which are the result of both experts'/stakeholders' judgments and the weighting method chosen.

In most cases experts can more easily set a range for a weight/scaling factor as opposed to a precise value. For example, to state "the relative value of a swing from worst to best on the second ranked criterion is between 30-60% from a swing from worst to best on the most highly weighted criterion", is easier to do than state this value is equal exactly 45%. The uncertainties of weight coefficients can be a result of both individual and group implementation of a weighting process.

Within ProMAA, the distributions of weights w_j in the given variation intervals $[w_j^{min}, w_j^{max}]$ may be used. Therefore, the approach to setting distributed weight coefficients in ProMAA needs a special discussion.

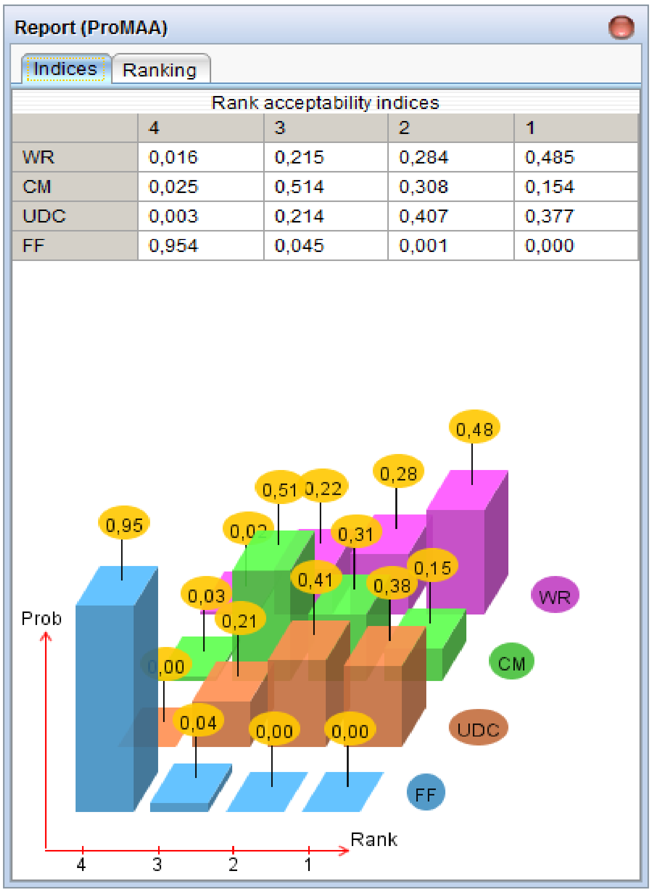
The recommended approach to setting weight coefficients in ProMAA-U is a natural one and corresponds to the steps for assignment of scaling factors as in *swing weighting method*, adapted for setting distributed weights:

- weight coefficient w_1=1 is assigned for the most highly weighted criterion (let us denote this criterion as C_1), taking into account, according to the method, evaluation of increase in overall value as a result of swing from worst to best for each criterion;
- the *variation interval* $[w_2^{min}, w_2^{max}]$, $0 < w_2^{min} \le w_2^{max} \le 1$, is assigned for the weight coefficient w_2 of the second ranked criterion (we denote it as C_2) based on evaluation of a range for relative value of a swing from worst to best for this criterion in comparison with the corresponding value of swing for the most highly weighted criterion;
- the previous step is repeated for the third, fourth, and subsequent criteria;
- the probability distributions (as subjective probabilities or as a result of statistical analysis of expert judgments) for (independent) weight coefficients w_j in the given interval $[w_j^{min}, w_j^{max}]$, j=2,…,m, is assigned by experts.

Within the classical MAVT/MAUT methods, the weights, assigned through the swing procedures, are usually normalized according to (6). This seems often to be useful for several reasons, including an interpretation of the importance of weights in percent, or presenting an overall value/utility function, etc. [Keeney and Raiffa, 1976; Belton and Stewart, 2002; von Winterfeldt and Edwards, 1986]. However, in specific cases experts may find it more intuitive to specify a reference criterion whose units is weighted at 1 and against which all other criteria are compared [Belton and Stewart, 2002].

It is evident that a (forced) proportional change of all weights w_j, j=1,...,m, ($w_j \rightarrow dw_j$, where d is any real positive number) does not change *ranking of alternatives* in MAVT/MAUT methods and in ProMAA-U (rank acceptability matrix $\{P_{ik}\}$ remains the same for distributed or standard/non-distributed type of weights).

In ProMAA-U, according to the current realization within the ***DecernsMCDA***, the original swing weight coefficients are then automatically normalized to the sum of their mathematical expectations; thus, the *sum of mean values* for (distributed) weights equals 1. Although, this is not necessary step for ranking alternatives within ProMAA-U, but this is useful for some comparison of ProMAA weights with weights used for other multicriteria methods, where weight normalization is traditionally implemented.



	4	3	2	1
WR	0,016	0,215	0,284	0,485
CM	0,025	0,514	0,308	0,154
UDC	0,003	0,214	0,407	0,377
FF	0,954	0,045	0,001	0,000

Fig.23 *DecernsMCDA*: Results – ProMAA method

FMAA

In many cases, when we use vague/uncertain values within a multicriteria problem, the application of *fuzzy numbers* may be considered as justified and more natural then utilization of (subjective) probability distributions. The use of fuzzy sets in such cases can assist with uncertainty assimilation both for criterion values and weight coefficients [Kahraman, 2008].

FMAA is a fuzzy analog of ProMAA described above: instead of random utilities (criterion values) and random weights, correspondingly, fuzzy criterion values and fuzzy weights are used. Corresponding math algorithms are described in details in [Yatsalo, Gritsyuk, Mirzeabasov and Vasilevskaya, 2011].

Within the FMAA, criterion values $a_{ij}=X_j(\boldsymbol{a}_i)$, scores $V_j(a_{ij})$, and weights w_j are considered as fuzzy numbers, $i=1,\ldots,n$, $j=1,\ldots,m$, and overall (fuzzy) value $V(\boldsymbol{a}_i)$ is determined by the expression

$$V(\boldsymbol{a}_i)=\sum_{j=1}^{m} w_j V_j(\boldsymbol{a}_i). \tag{18}$$

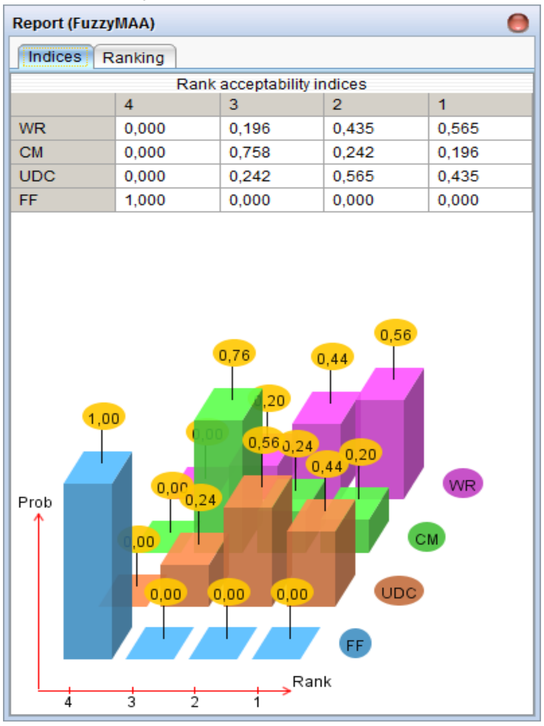
The partial value function $V_j(x)$ is considered here as the usual/crisp function, defined by experts on the variation interval of the criterion C_j, $j=1,\ldots,m$, for alternatives under consideration.

Within FMAA the measure $\mu(S_{ik})$ of the events S_{ik} as a degree of confidence that alternative i has rank k, is determined (as a fuzzy analogue of corresponding assessments in ProMAA) with the use of fuzzy logic and fuzzy calculations.

Within FMAA, using matrix $\{\mu_{ik} = \mu(S_{ik})\}$, experts/decision-makers can select the most acceptable alternative(s) (as in ProMAA method).

For aggregation of these measures a weighted sum (17) may also be used.

The approach, presented in ProMAA for assigning distributed/random weights, is similarly adjusted for assigning fuzzy weights w_j in FMAA.



	4	3	2	1
WR	0,000	0,196	0,435	0,565
CM	0,000	0,758	0,242	0,196
UDC	0,000	0,242	0,565	0,435
FF	1,000	0,000	0,000	0,000

Fig.24 *DecernsMCDA*: Results – FMAA method

FlowSort

(The text below was presented by Dr.Ph.Nemery)

The FlowSort method [Nemery and Lamboray, 2008] is a *multicriteria* sorting method which helps a decision maker (DM) to *assign* alternatives (e.g. geographical regions, projects, candidates, etc.) into *predefined* categories or groups. The DM defines thus in a first step the categories to which the alternatives will be assigned to. The particularity is that the DM can express a *transitive preference relation* on the categories: the categories are thus *ordered* from the best to the worst (e.g. high risk zones, medium risk zones and low risk zone).

In order to define the meaning of the categories, the DM needs to specify *limiting profiles* which characterize completely the categories. Each category is thus defined by an *upper* and a *lower* boundary: the category C_h (h=1, …, K) is thus defined by the upper limiting profile r_h and the lower profile r_{h+1} of the limiting profiles set $R = \{r_1, \dots, r_{K+1}\}$. Since the categories are completely ordered, each limiting profile *dominates* all the successive ones: $r_1 \succ^D r_2 \succ^D \dots \succ^D r_{K+1}$. Formally, the profiles respect thus the following condition if we suppose that the q criteria (noted f_l) have to be maximized:

$$\forall h = 1, \dots, K \,; l = 1, \dots, q \;:\; r_h \succ^D r_{h+1}\colon\; f_l(r_h) \geq f_l(r_{h+1}) \; and \; \exists\, j\colon f_j(r_h) > f_j(r_{h+1})$$

The FlowSort method is the outcome of the following main idea: an alternative $a_i \in A$ to be sorted is compared to *solely* the reference profiles by means of the PROMETHEE ranking method. The category, to which the alternative will be

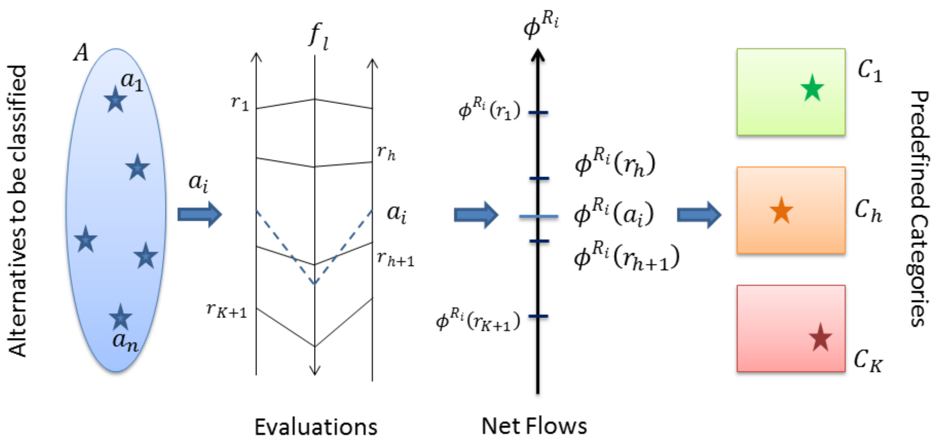


Fig.25 FlowSort method scheme

assigned to, is *deduced* from its *relative* position with respect to solely the reference profiles.

Let us note the set of reference profiles $\mathrm{R_i} = \{\mathrm{r_1}, \dots, \mathrm{r_{K+1}}\} \cup \{a_i\}$ and an alternative a_i to be classified. The alternative a_i is first pairwise compared to all the reference profiles and then a complete ranking is computed.

1. First, for each criterion a global uni-criterion net flow is computed for a_i by comparing the evaluation of a_i to the evaluations of the profiles:

$$\phi_l^{R_i}(a_i) = \frac{1}{|R_i| - 1} \sum_{\mathrm{j}=1}^{\mathrm{K}+1} [P_l(a_i, r_j) - P_l(r_j, a_i)]$$

Where

- $P_l(a_i, r_j)$ represents the preference degree of a_i on r_j for criterion f_l based on the preference functions of PROMETHEE [Brans and Vincke, 1985; Belton and Stewart, 2002; Figueira, Greco and Ehrgott, 2005].
- $|R_i|$ represents the numbers of elements belonging to the particular set R_i.

This uni-criterion net flow score (always between -1 and 1) represents the strength (if near 1) or the weakness (if near -1) of an alternative a_i in regards of solely the reference profiles of *R*.

2. In a second step, the PROMETHEE II ranking is computed for this set $\mathrm{R_i}$ by means of the net flows while taking the weights of the criteria w_l:

$$\phi^{R_i}(a_i) = \sum_{l=1}^{q} w_l * \phi_l^{R_i}(a_i)$$

3. Finally, the assignment of alternative a_i to the class C_h is based on its relative position with respect to the reference profiles r_h and r_{h+1} :

$$C(a_i) = C_h, \ \ if \quad \phi^{R_i}(r_h) > \phi^{R_i}(a_i) \geq \phi^{R_i}(r_{h+1})$$

Alongside the category to which an alternative is assigned, the net score of each alternative compared solely to reference profiles, gives the decision maker an idea of the strength or weakness of an alternative.

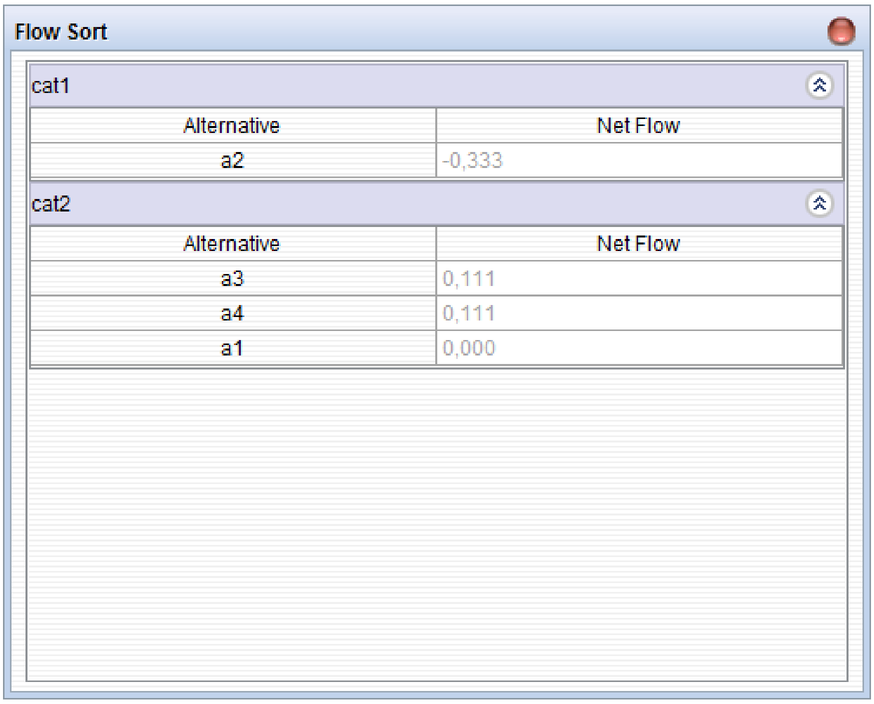


Fig.26 *DecernsMCDA*: Results – FlowSort method

Tools

Domination

Domination tool (see toolbar button) provides domination report for constructed decision support model (based on mean values for all the criteria/alternatives). It demonstrates whether an alternative is dominated by another one.

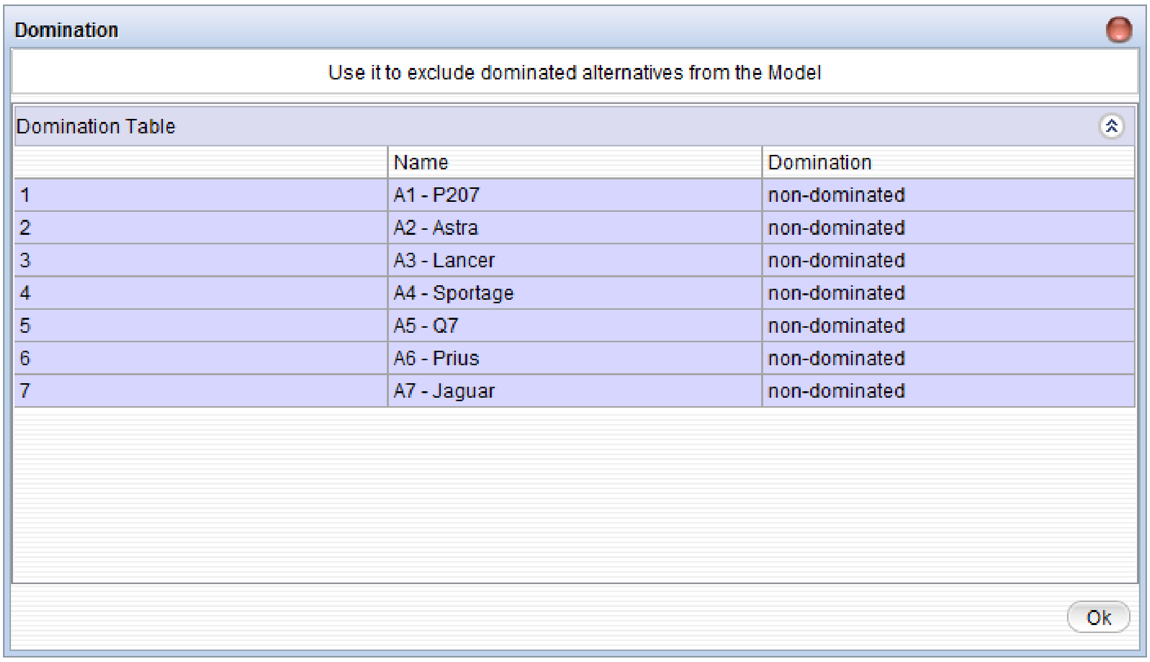


Fig.27 *DecernsMCDA*: Domination tool

Scatter plot

Scatter plot is graphical analysis tool which consists of 2D plots. The points represent values for alternatives in the chosen (C_i,C_j) plane for the two selected criteria C_i and C_j. Sometimes it is useful for understanding and comparison of alternatives.

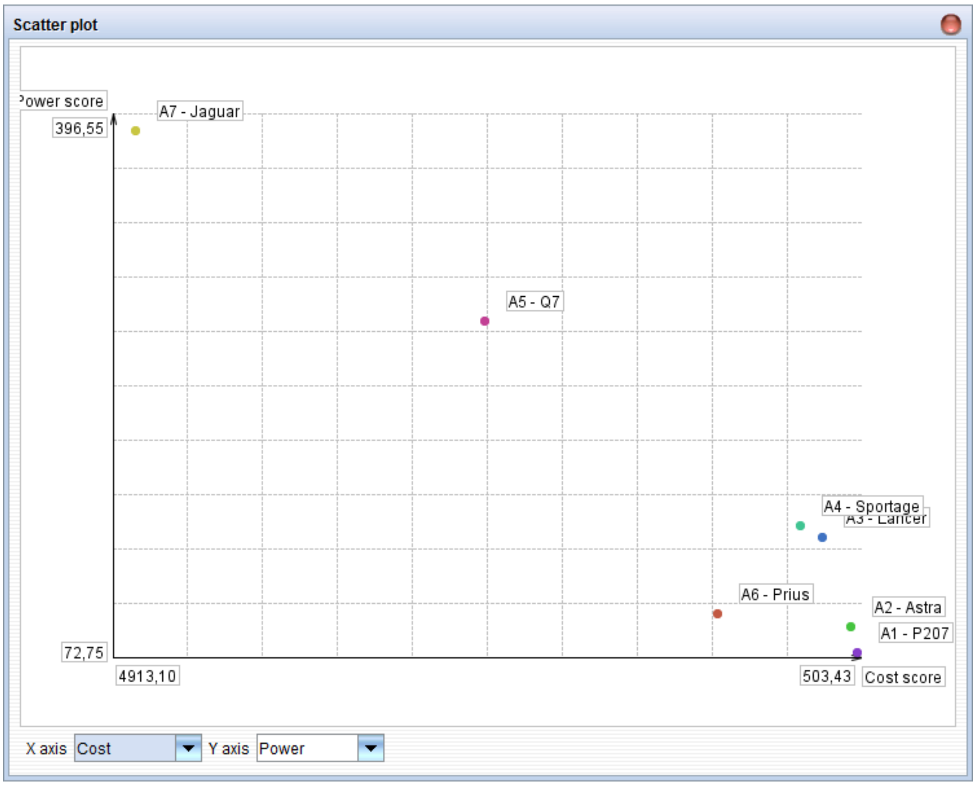


Fig.28 *DecernsMCDA*: Scatter plot tool

Value path

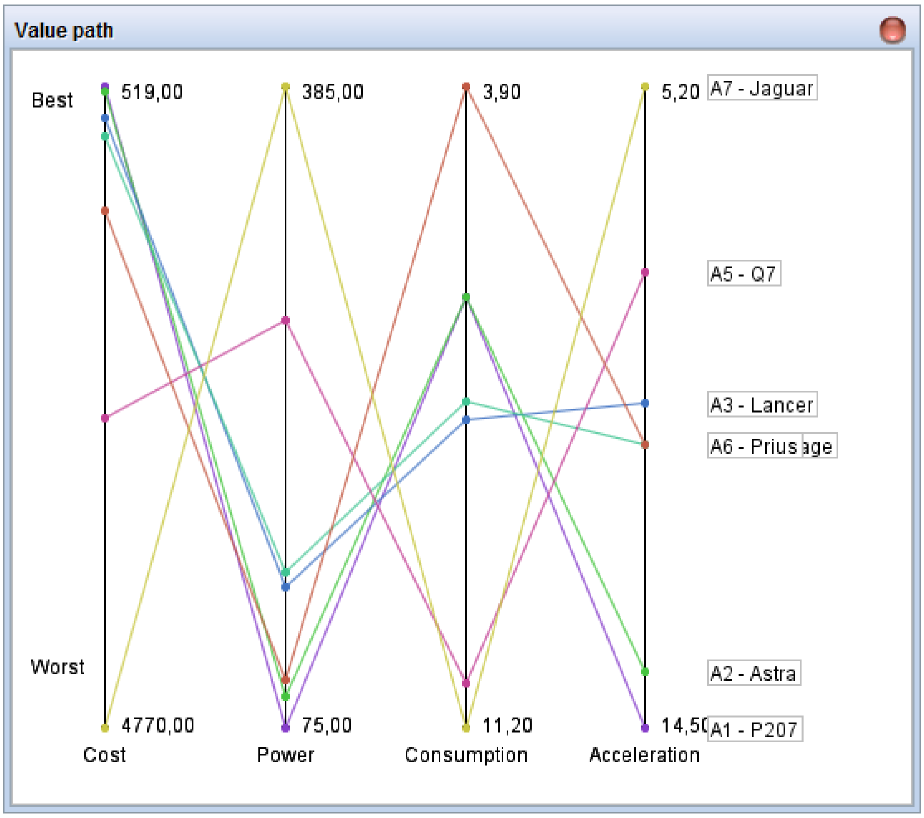


Fig.29 *DecernsMCDA*: Value path tool

Value path is another graphical analysis tool which represents values of alternatives against all criteria; this tool is useful for demonstration non-dominated/dominated alternatives.

Analysis

Weight sensitivity analysis

Weight sensitivity analysis is a powerful tool for understanding an influence of the assigned weights on the output results (e.g. ranking alternatives). It is used with the following methods: MAVT, MAUT, TOPSIS, PROMETHEE, and AHP. User can choose the criterion for weight sensitivity analysis (left-top part of the dialog); then with slider (bottom part of the dialog) user can change weight from 0 to 1 (other weights are automatically changed proportionally holding weight sum = 1) and observe possible changes of output results (ranking alternatives). There are the two forms for weight sensitivity analysis and representation of the results: Lines form and Bars form (walking weights). User can restore base weights values at any moment (see button at the top-right part of dialog).

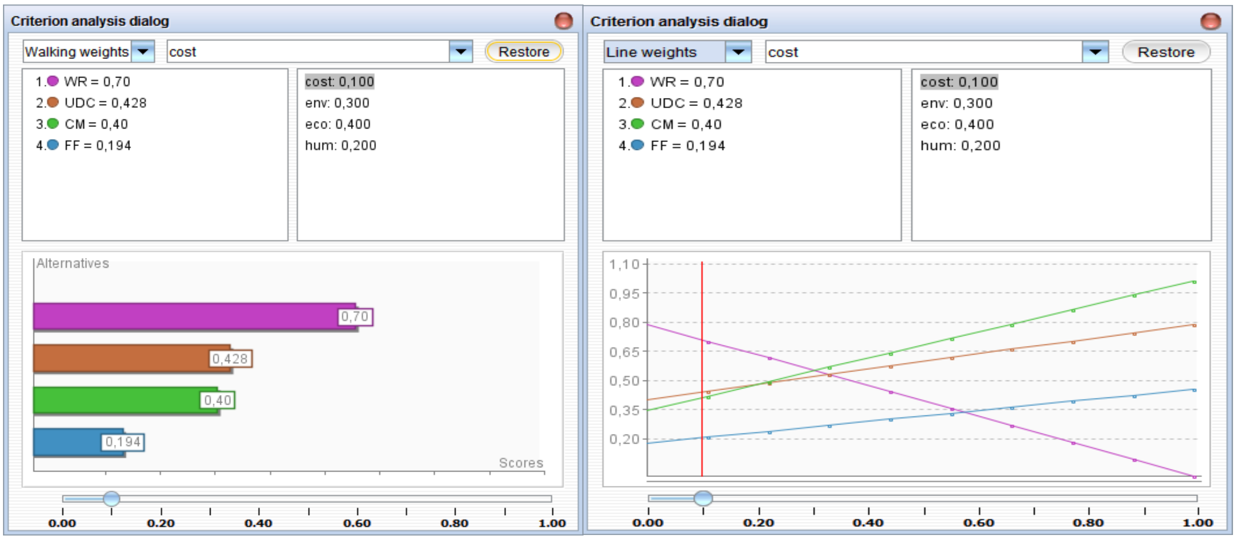


Fig.30 *DecernsMCDA*: Weight sensitivity analysis tools

a) walking weights; б) line weights

Value function sensitivity analysis

Value function sensitivity analysis is another powerful tool for assessing influence of the assigned value/utility functions on the output results (e.g. ranking alternatives). It is used with the following methods: MAVT, MAUT, FMAVT, ProMAA, and FMAA. After VF analysis started you will see Criteria choosing dialog. Here you can choose one or several criteria for VF sensitivity analysis. To choose the type of value function use right-click and popup dialog.

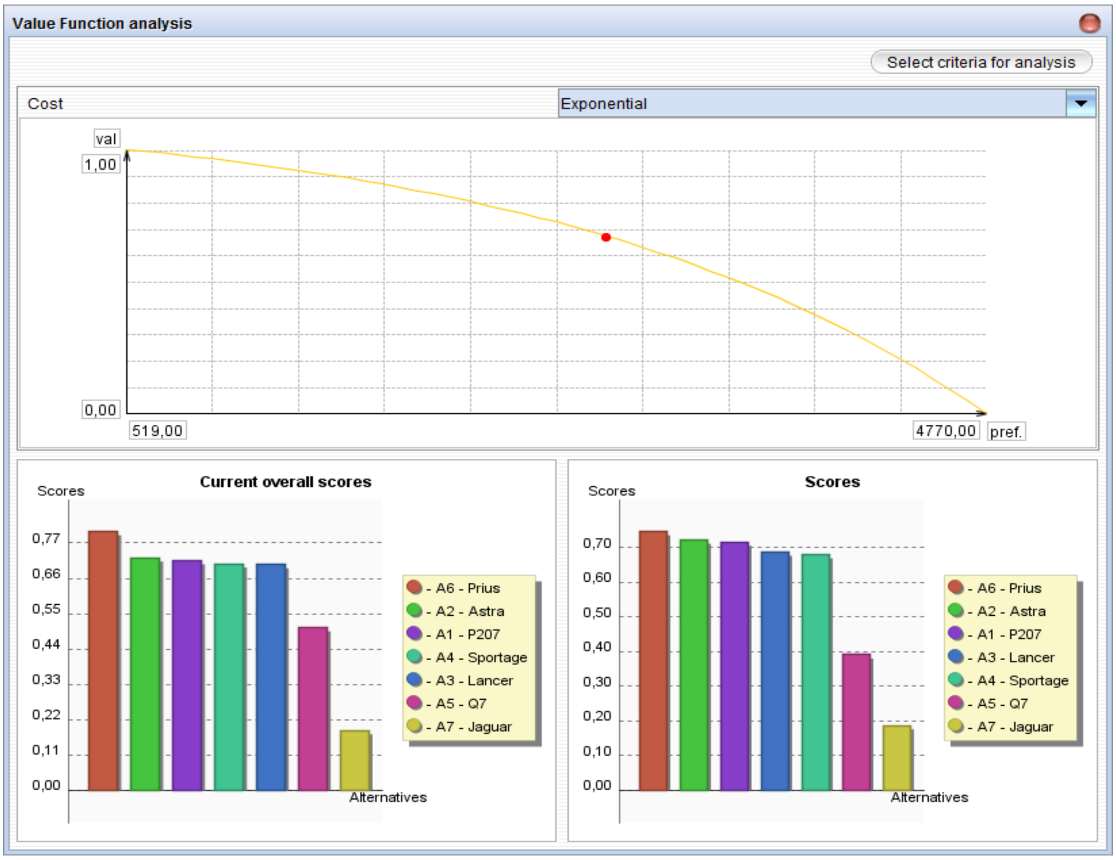


Fig.31 *DecernsMCDA*: Value function analysis tool

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