# Project 1

## Random variables

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# Outline

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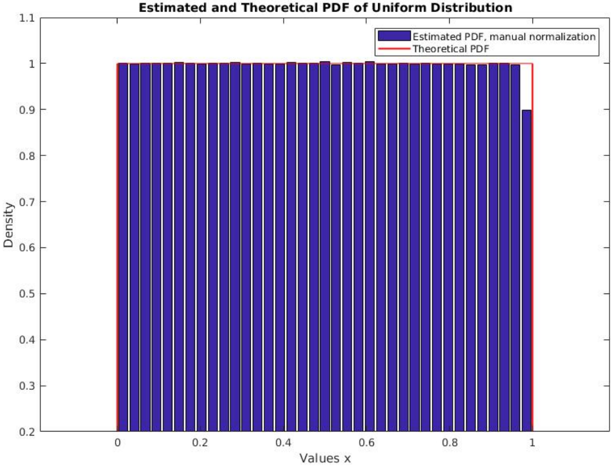
# Task 1.1

## Uniform PDF - Estimated probability density function

- Estimated PDF compared to the theoretical PDF



# Task 1.1

## Uniform PDF - Estimated values

- Normalization of the probability density function, dividing the PDF on the total area of the PDF. The quadratic integration method for n bins:

$$\frac{f_x(\alpha)}{\sum_n BinWidth_n * BinHeight_n}$$

- Estimated mean and variance

$$E\{x\} = m_x \approx 0.5$$

$$\sigma_x^2 \approx 0.083 = \frac{1}{12}$$

# Task 1.1

## Uniform PDF - Theoretical values

- Theoretical mean and variance eq. 3.11 & 3.15:

$$E\{x\} = \int_{-\infty}^{\infty} \alpha * f_x(\alpha) d\alpha = \int_{0}^{1} \alpha * 1 d\alpha = (\frac{1}{2} * 1^2) - (\frac{1}{2} * 0^2) = \frac{1}{2}$$

$$\sigma_x^2 = E\{[x - E\{x\}]^2\} = \int_{0}^{1} [\alpha - \frac{1}{2}]^2 * 1 d\alpha = \frac{1}{12}$$

# Task 1.1

## Uniform PDF - Repeated experiments

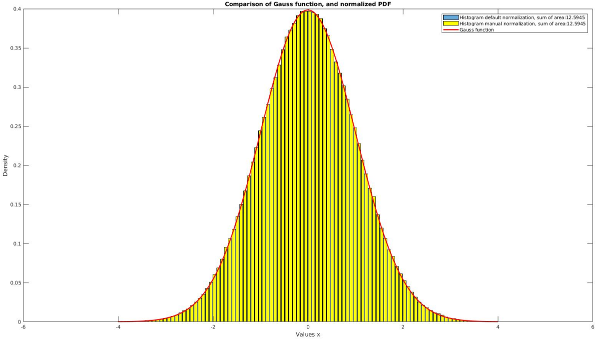
- The mean and variance varies more for lower values of N and greater number of bins in the plot
- We see that the estimated values stables out on the theoretical values when $N \to \infty$

# Outline

# Task 1.2

## Gaussian PDF - Estimated probability density function

- Estimated PDF compared to the theoretical PDF



# Task 1.2

## Gaussian PDF - Estimated values

- Same method for normalization of the probability density function
- Estimated mean and variance

$$E\{x\} = m_x \approx 0$$

$$\sigma_x^2 \approx 1$$

# Task 1.2

## Gaussian PDF - Theoretical values

- Theoretical mean and variance eq. 3.11 & 3.15:

$$E\{x\} = \int_{-\infty}^{\infty} \alpha f_x(\alpha) d\alpha = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \alpha e^{-\frac{(\alpha-\mu)^2}{2\sigma^2}} d\alpha = \mu = 0$$

$$Assuming: \int_{-\infty}^{\infty} e^{-\alpha^2} d\alpha = \sqrt{\pi}$$

$$\sigma_x^2 = E\{[x - E\{x\}]^2\} = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} [\alpha - \mu]^2 e^{-\frac{(\alpha-\mu)^2}{2\sigma^2}} d\alpha = 1$$

# Outline

# Task 1.3

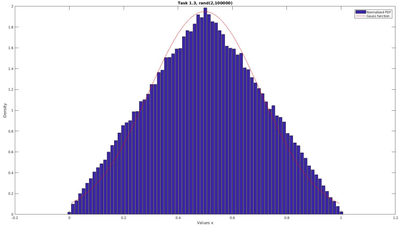
## The Central Limit Theorem

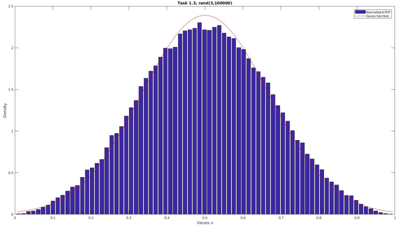
- The mean value of the process equals to the convolution of the individual realizations. Comparing 2 and 3 realizations with 12. We can see that the estimated PDF approaches a normal distribution.

# Task 1.3

## The Central Limit Theorem

- Estimated PDF compared to the theoretical PDF superimposed

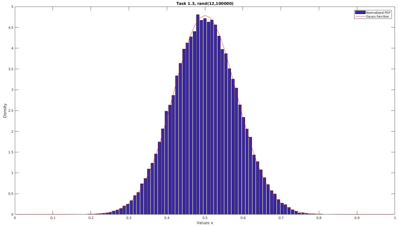


# Task 1.3

## The Central Limit Theorem

- Estimated PDF compared to the theoretical PDF superimposed, 12 realizations




- Estimated mean and variance:

$$E\{x\} \approx 0.5$$

$$E\{[x - E\{x\}]^2\} \approx 0.007$$

# Task 1.3

## The Central Limit Theorem - Theoretical values

- Theoretical mean and variance:

$$E\{\sum_i a_i x_i\} = \sum_i a_i E\{x_i\}$$

- Where $a_i$ is the number of realizations

$$E\{x\} = m_x = \sum_i \frac{1}{12}\frac{1}{2} = 12\frac{1}{12}\frac{1}{2} = \frac{1}{2}$$

$$\sigma_x^2 = Var\{\sum_i a_i x_i\} = \sum_i a_i^2 Var\{x_i\} = \sum_i \frac{1}{12^2}\int_0^1 [\alpha - \frac{1}{2}]^2 d\alpha$$

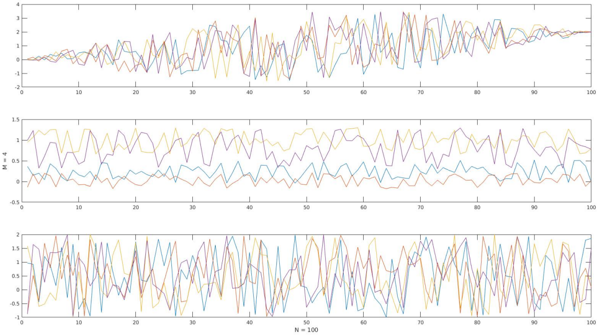
$$= 12\frac{1}{12^2}\frac{1}{12} = \frac{1}{144} \approx 0.007$$

# Outline

# Task 2.1

## Stationary or Ergodic?

- Plot of all 3 processes:



# Task 2.1

## Stationary or Ergodic? - Theory

- Stationary process if the distribution remains the same over the whole realization
- Ergodic process if the statistical properties of the whole process can be deduced by one random realization in the process, e.g the "Time Average" of one realization is equal to the "Ensembled Mean" over all realizations.

# Task 2.1

## Stationary or Ergodic? - Processes

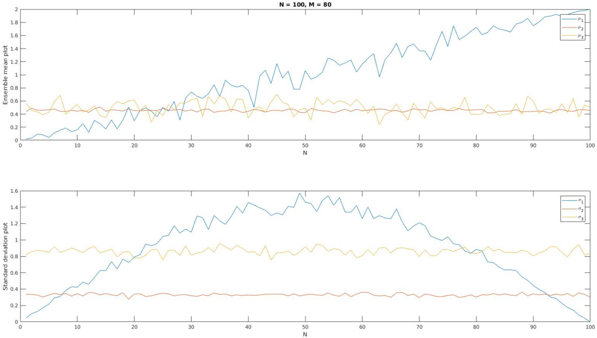
- 1. Process is not stationary, because the statistical properties changes over time. Since the process is not stationary, it can not be ergodic.
- 2. Process keeps its statistical properties over the whole process, and is a stationary process. Does not look ergodic, due to the ensembled mean does not look equal to the time average of any realization.
- 3. Process has the same statistical properties as the 2. process. I would say that this process is also ergodic because of the time average of each realization looks to be pretty similar.

# Outline

# Task 2.2

## Ensemble Averages

- Plot for the mean and standard deviation of all processes:



# Outline

# Task 2.3

## Time Averages

- Time averages for process 2:

  $A_2\{\cdot\}_1 = 0.941, A_2\{\cdot\}_2 = 0.595, A_2\{\cdot\}_3 = 0.266, A_2\{\cdot\}_4 = 0.849$

- Time averages for process 3:

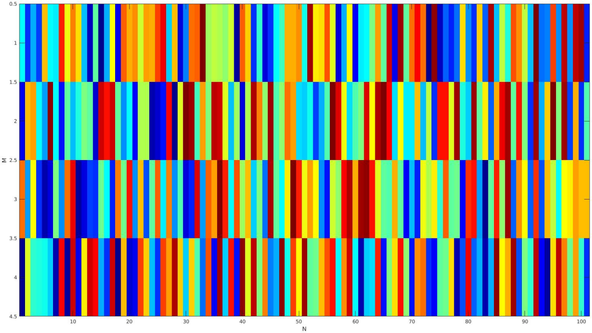
  $A_3\{\cdot\}_1 = 0.476, A_3\{\cdot\}_2 = 0.477, A_3\{\cdot\}_3 = 0.503, A_3\{\cdot\}_4 = 0.482$

- Can conclude with that process 3 is ergodic process, by looking at the time average values. And, the values from process 2 varies much more, which tells me that the process is at most very close to being ergodic.

# Task 2.3

## Time Averages

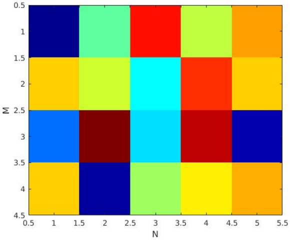
- By looking at the whole interval of the processes, it is hard to visually see any proof of ergodicity.



# Task 2.3

## Time Averages

- If we zoom in on 5 or 10 values of the realizations, we can use the image to see a relation between time average and ensemble mean of these.




- We could draw a conclusion that process 3 is ergodic with visual inspection on the 5 last values, if we say that:

$$E\{x(996)_3\} = \frac{1}{5} \sum_{996}^{1000} x(n)_3$$