Ralph & Coop: patchy selection

Let's start with the more obvious:

- eq. 56 is wrong, as easily seen with $\Theta = 0$

  in this case $\sin\Theta = 0$ and $\cos\Theta = 1$

  $$\Rightarrow \left(\left(1-\frac{r_0}{r}\cos\Theta\right)^2+\frac{r_0^2}{r^2}\sin^2\Theta\right)^{1/2}-1 = 1-\frac{r_0}{r}-1 = -\frac{r_0}{r}$$

  eq 56 has an extra 2, which results from a wrong expansion of

  $$\sqrt{1+x} \simeq 1+\frac{x}{2}\ \cdots \quad \Rightarrow \text{eq } 57-61 \text{ are also wrong}$$

- time in transit

  One can obtain these results by a simple saddle point approximation

  $$P(\text{getting to } x \text{ in time } t) = P(\text{diffusion to } x \text{ in time } t) \times P(\text{surviving})$$
  $$= \frac{1}{(2\pi\sigma^2)^{1/2}}\, e^{-\frac{x^2}{2\sigma^2 t} - st}$$

  the exponent is minimal when $\frac{x^2}{2\sigma^2 t^2} = s \Rightarrow t^* = \frac{x}{\sigma\sqrt{2s}}$

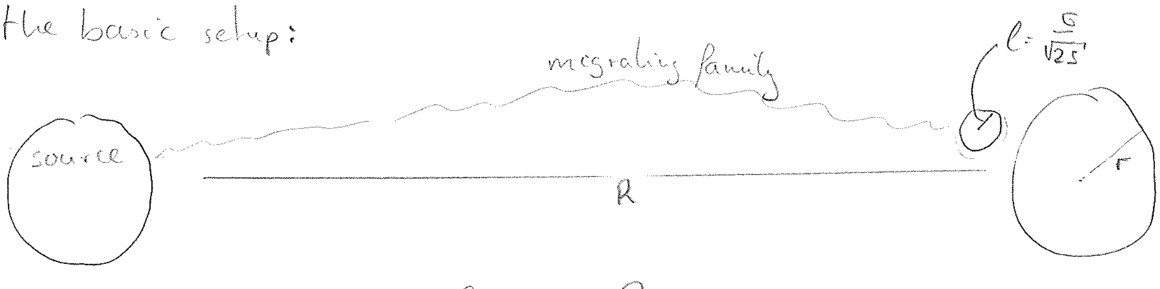
  this gives your eq 17

  the variance is simply the inverse of the second derivative of the exponent

  $$\frac{1}{\mathrm{Var}[t]} = \frac{x^2}{\sigma^2 t^{*3}} = \frac{\sigma}{x(2s)^{3/2}}$$

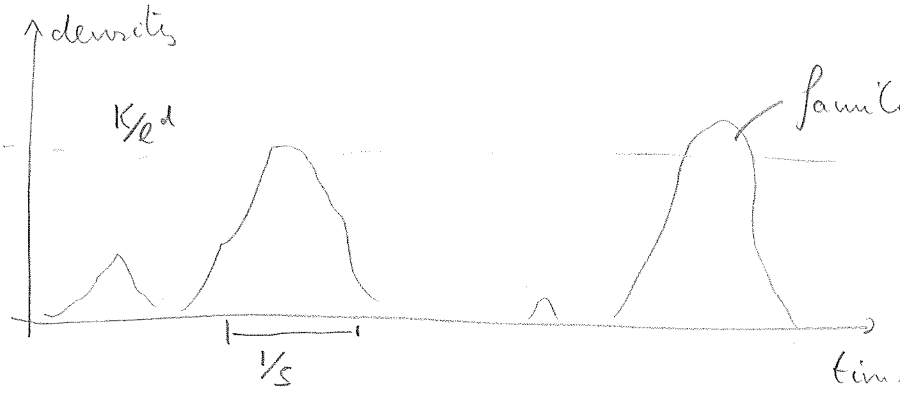
- now to the thornier issues of dimension, geometry & fluxes

  the basic setup:



$$l \ll r \ll R$$

the average density is $q(x) \sim e^{-x\sqrt{2\zeta}} = e^{-x/\ell}$ but fluctuates far from the source as



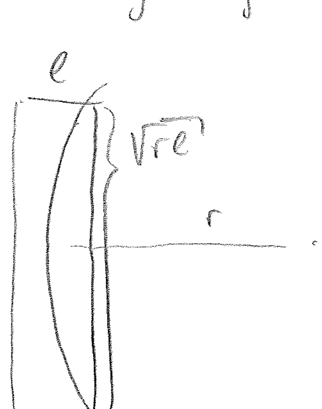
→ local density of a family $\sim K/\ell^d \sim \frac{1}{s\ell^d}$

→ Families of size $\frac{1}{s}$ live for $\frac{1}{s}$ generation and hence contribute $\frac{1}{s^2}$ to the time averaged density $q(x)$

$q(x) \approx \lambda_{arr}(x) \frac{1}{s^2 \ell^d}$ where λ_{arr} is the rate at which families arrive that overlap with x

$\lambda_{arr}(x) = q(x)\, s^2 \ell^d$ and $\Lambda_{arr}(x) = \lambda(x)\, \ell^{-d} = q(x)\, s^2$ would be an "arrival density"

to get at λ_{mig}, arrival needs to be integrated over the relevant area. In $d=1$, this is simply a stretch of length ℓ, in $d=2$ it should be $\sqrt{\ell^3 r}$



hence one should get

$\lambda_{mig} \sim \rho\, q(x)\, s^2 \ell^d \sqrt{\frac{r}{\ell}} \left(1 - e^{-\rho_0/s}\right)$

→ result does depend on target geometry

→ formulation in terms of fluxes (suggested by ref. 2) might be easier