Ralph & Coop: patchy selection

Let's start with the more obvious:

- eq. 56 is wrong, as easily seen with $\Theta = 0$

  in this case $\sin\Theta = 0$ and $\cos\Theta = 1$

  $$\Rightarrow \left(\left(1 - \frac{r_0}{r}\cos\Theta\right)^2 + \frac{r_0^2}{r^2}\sin^2\Theta\right)^{1/2} - 1 = 1 - \frac{r_0}{r} - 1 = -\frac{r_0}{r}$$

  eq 56 has an extra 2, which results from a wrong expansion of

  $$\sqrt{1+x} \simeq 1 + \frac{x}{2} \cdots \quad \Rightarrow \text{eq } 57-61 \text{ are also wrong}$$

- time in transit

  One can obtain these results by a simple saddle point approximation

  $$P(\text{getting to } x \text{ in time } t) = P(\text{diffusion to } x \text{ in time } t) \times P(\text{surviving})$$

  $$= \frac{1}{(2\pi\sigma^2)^{1/2}} e^{-\frac{x^2}{2\sigma^2 t} - st}$$

  the exponent is minimal when $\frac{x^2}{2\sigma^2 t^2} = s \Rightarrow t^* = \frac{x}{\sigma\sqrt{2s}}$

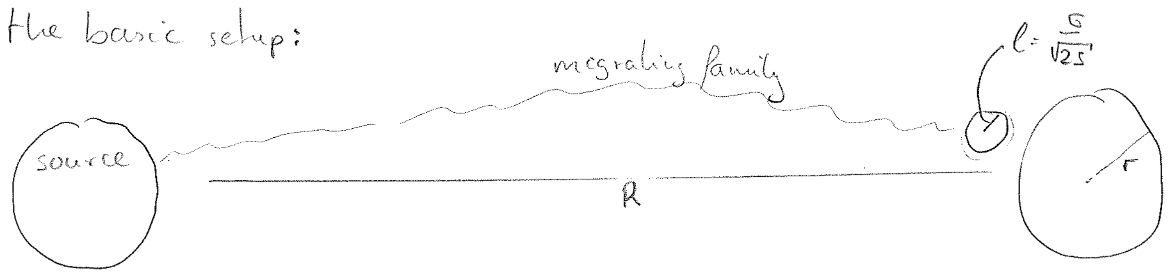
  this gives your eq 17

  the variance is simply the inverse of the second derivative of the exponent

  $$\frac{1}{\mathrm{var}[t]} = \frac{x^2}{\sigma^2 t^{*3}} = \frac{\sigma}{x(2s)^{3/2}}$$

- now to the thornier issues of dimension, geometry & fluxes

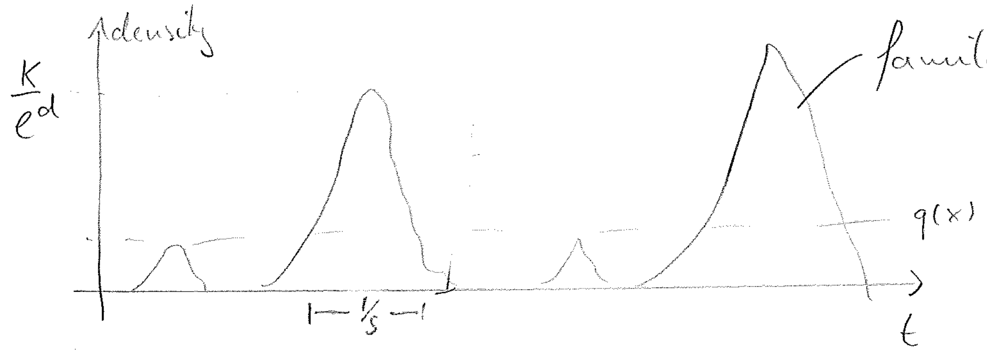
  the basic setup:



$$l \ll r \ll R$$

the average migrant density is $q(x) \sim e^{-x\frac{\sqrt{2s}}{\sigma}} = e^{-x/\ell}$

but fluctuates far from the source as



local density of a family: $\frac{K}{\ell^d}$

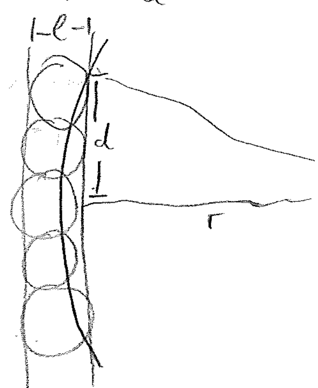
note that the distribution of family sizes $\sim \frac{e^{-sK}}{K}$
similar to that of deleterious alleles.

Families of size K live for K generation, such that they contribute K^2 to the time averaged density. q(x) is hence dominated by the largest families $K \sim s^{-1}$, but families reach this size only with prob. s

$$q(x) = \frac{\lambda_{arr}}{\ell^d} s K^2 \simeq \frac{\lambda_{arr}}{\ell^d} s^{-1}$$ where λ_{arr} is the rate for families to overlap with x

$\lambda_{arr} \approx q(x)\, s\, \ell^d$ note that the dimensions work out ?

To calculate λ_{mig}, we need the approximate number of equivalent locations for seeding of successful migrants.
since $q(x) \sim e^{-x/\ell}$, they need to be within the same distance from the source to approx $\sim \ell$
the number of ℓ^d ball scales as $\sqrt{\frac{r}{\ell}}$ in d = 2



$$\Rightarrow \lambda_{mig} \sim \rho\, q(R)\, s\, \ell^d \sqrt{\frac{r}{\ell}} \left(1 - e^{-P/s}\right)$$

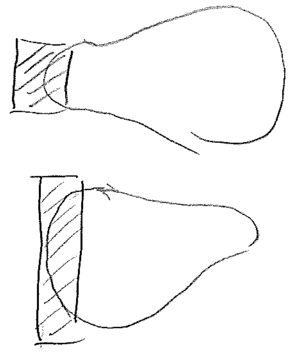
it is worth noting that this reduces correctly to

$\lambda_{mig} = p_e \int_A dx\, \rho q(x)$ in the limit $p_e \ll s$

(the integral is dominated by a stripe of width ℓ and height $\sqrt{\epsilon r}$)

more importantly, λ_{mig} depends on the geometry of the sink patch:

λ_{mig} for the lower is larger since it can absorb migrants from a much larger crossection.



- fluxes.

It might be worth noting that the flux of individuals is implicit in the equation for the average density:

$$\partial_t q(r) = 0 = \frac{\sigma^2}{2} \frac{\partial^2}{\partial r^2} q(r) + \frac{\sigma^2(d-1)}{2r} \frac{\partial}{\partial r} q(r) - s q(r)$$

$$= \underbrace{\frac{1}{r^{d-1}} \frac{\partial}{\partial r} \underbrace{\left[\frac{\sigma^2 r^{d-1}}{2} \frac{\partial}{\partial r} q(r) \right]}_{\text{flux}}}_{\text{divergence of flux}} - s q(r)$$

divergence of flux $=$ local death.