School of Business, Economics, and Information Systems
Chair of Management Science/Operations
and Supply Chain Management

Master Seminar in Data Science and Optimization

“The Future of Multi-Variant Production: Car Sequencing”

Seminar Leader: Prof. Dr. Alena Otto

Lecturing tutors: Luis Rocha, Andrei Nevskii

Edited by:

Aleksandra Petrenko
108448
MBA
petren01@ads.uni-passau.de
5th Semester
+4915204877525

Thi Khanh Ninh Tran
111303
MIEB
thikha01@ads.uni-passau.de
3rd Semester
+4917685586472

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1. Introduction

1.1 Motivation

Aleksandra:

Modern manufacturing has witnessed a paradigm shift towards increased customization and a variety of models. This transformation is driven by consumer demands for customer-tailored features and unique design elements in the models. The manufacturers are aiming to satisfy all the demand niches and, as a result, are facing the challenge of efficiently managing the production process for a large array of various product models. Sequencing, a crucial aspect of production planning and the topic of this seminar paper, has become a key strategy to streamline the manufacturing process, reduce work overload and optimize resource utilization. In the era of Industry 4.0, where technology and data play essential roles in manufacturing, sophisticated algorithms are employed to optimize production sequences. Advanced data science methods help manufacturers anticipate demand, manage inventory, and dynamically adjust production sequences based on market trends and customer preferences. To sum up, sequencing will remain a relevant urgent topic in production in the next decades and requires attention from a management science perspective.

1.2 Introduction to sequencing theory

Ninh:

Sequencing and scheduling are one of the most crucial processes in production planning which direct factory's operations. On the one hand, scheduling tells when the tasks are processed on the machine, corresponding processing time and the resources required to complete the tasks. On the other hand, sequencing aims at determining optimal job orders when various products are being produced at a production line with multiple machines. Both processes target to explore production schedule and operation orders that could avoid workers' overload, improve throughput time, minimize materials' costs and fully utilize machines' capacity. The paper focuses on discussing sequencing theory, practical models and their role in preventing work overload at workstations. If planning fails and work overload is incurred, production line might stop due to technician not being able to finish earlier tasks to continue to the next and consequently, additional resources are required as a compensation, escalating production costs.

Sequencing theory has been developed under mathematical forms that turn manufacturing goals into objective functions, incorporating other constraints such as labor utility, capital capacity and technical limitations. The goals could be minimizing costs, work overload or reducing set up and throughput time. Solving the model helps manufacturers discover feasible job orders that satisfy objective functions while adhering to existing capacity and materials constraints. However, sequencing also involves various unpredictable events such as excursions or machine issues which are difficult to predict and therefore, it could not be explicitly depicted in the model. These days, manufacturers serve a variety of product portfolios and produce them in an intermixed sequence which intricates sequencing process. For example, in car production, cars with different roof materials with different setup times and logistic organization are assembled on the same line, multiple processing times would happen at one workstation. If products with long processing time are sequential without reasonable planning, workers might not be able to handle the work within the time defined and work overload will induce.

In practice, there are two common approaches for sequencing models which are car sequencing and mixed-model sequencing which both target to minimize work overload. While two models follow the same objective functions, they are different in approach. The mixed-model sequencing model explicitly examines operation time, logistic system, workstation design, labor handling time and derives workload measurement. Mixed-model sequencing directly determines workload from operation characteristics which results in tedious work for data collection, preciseness and computation time. A feasible sequence is the one that does not experience work overload. Car sequencing imposes an implicit rule $H_o : N_o$ which restricts the number of successive models with long processing time. Therefore, objective function is to minimize total number of violations in each subsequence and it is considered to be easy for companies to implement.

1.3 Research questions

Ninh:

Compared to mixed-model sequencing, car sequencing approach possesses certain advantages in simplicity of execution and data collection, however, the model heavily relies on sequencing rule to define solution space and therefore, the generation of adequate rules should become the top priority.

The paper focuses on the following questions which will be answered from literature review, including:

- What are the advantages of car sequencing compared to mixed-model sequencing?
- What are the sequencing rules that generate a result as good as mixed-model sequencing?

Aleksandra:

- What are the recent trends in literature about car sequencing?
- What are the suggestions for future research?
- In which industries could car sequencing be applied?

Ninh:

Section 2 would introduce key assumptions of car sequencing and the mathematical model. Then, a rundown of the past 10 years research work is summarized to answer research questions in the literature review which is followed by the future of car sequencing in session 3 and conclusion remarks will be provided at the end.

1.4 Recent trends of sequencing

Ninh:

Due to a dynamic and quickly changing environment, several heuristic methods have been developed to cope mixed model sequencing problems within a short computation time, increasing the efficiency of the process and enable quick decision-making process for managers. This allows real-time data and other relevant information from production to be timely considered, generating an optimized and efficient production system. Brammer et al., 2021 used reinforcement learning and metaheuristics algorithm to solve mixed model sequencing problem which was considered an improvement compared to initial greedy solutions (Boysen et al., 2011). The paper tried to minimize the number of overload scenarios in which initial sequence was generated from a simulated environment with predefined production line and demand orders. Any work overload situation would be penalized with a negative reward and a quick initial solution could be produced within a short period. In addition, the evolution and modifications of machines and production systems might force manufacturers to alter methodological approach which requires high level of flexibility of mathematical models so that sequencing model would quickly adapt to the transformation of production systems. Ebrahimi et al., 2023 proposed a model using parallel workstations and walking walkers to increase flexibility of a shop floor and reduce production costs. Walking workers were believed to help increase flexibility in dynamic sequencing and resource allocation in case of resequencing. The paper proved that adding additional workers did not actually help but in contrast, it increased blocking time in the assembly line while processing time would reduce by just adding a new parallel posts or buffers. The studied used a hyper-heuristic method to develop a mathematical programming and simulation model for sequencing problem that could help to tackle the problems in regard to the number of workers and configuration of the assembly line. In addition, the author also compared the effectiveness of two models against each other. A mathematical model appeared to be more complex in modelling stage which could be easily done with the help of simulation software. However, simulation model might induce additional effort if there is a change in parameter settings which could be adjusted effortlessly in mathematical model. According to the paper, the situation occurs infrequently in reality and there are just some minor changes. Despite that, simulation software brought huge advantages concerning data visualization and data analytics which could help provide insightful information for managers to make timely decisions. Last but not least, a simulation tool would normally generate feasible solutions faster which might come as a trade off with optimality and uncertainty.

In contrast to mix-model sequencing, car sequencing does not require detailed planning and data processing, its principle is based on an implicit rule which define the maximum number of product models containing additional option o that could be processed on the line and a sequence that is evaluated as feasible if they do not violate the defined rule. The idea is to balance the product model that has significant processing time exceeding the cycle time. There are three car sequencing rules that have been developed so far. Firstly, under the circumstance of having only two processing time per station, a single ratio had been introduced by Bolat and Yano (1992). The authors suggested a sequencing rule of H/N where H was the maximum numbers of option models allowed in N consecutive models in a subsequence. Golle et. al (2010) had developed a model that deployed multiple sequencing rules allowing to have multiple processing time per station and Lesert, Alpan, and Frein (2011) proposed a unique ratio per sequencing rule for the whole assembly line. The development of these models aims to boost performance of car sequencing in comparison to that of mixed-model sequencing.

1.5 Car sequencing versus mixed model sequencing

Ninh:

Because car sequencing approach minimizes work overload indirectly, there have been some concerns regarding the quality of classifications. Golle et al. (2014) discovered that the reliability of car sequencing models heavily depends on sequencing rules and the selected algorithms. If the rules are not prudently selected, sequences might be wrongly classified and therefore generate unreliable results.

There are two possible misclassifications that could appear mentioned in Golle et. al (2014). Firstly, the sequence is categorized as feasible under car sequencing rules, but it appears to be infeasible under mixed-model sequencing. This problem arises possibly because sequencing rules are not strict enough to eliminate circumstances where work overload would happen. It might lead to additional costs to deal with unforeseen work overload if it is incurred in reality. The other misclassification scenario is where mixed-model sequence defines one as feasible while it is excluded from car sequencing. Even though there is no unpredicted work overload, the elimination of feasible solutions would potentially lead to the removal of optimal solution or in the worst case, no feasible sequences could be found. It is due to the reason that car sequencing is unnecessarily strict that no sequence could fulfill. To measure the quality of car sequencing, the percentage of misclassifications would be considered and the lower the better.

Based on Golle et. el., 2014 and Louis et. al., 2023 car sequencing is considered to be easy to implement and the approach requires less effort in constructing the mathematical model and data collection process. As mix-model sequencing model is constructed from detailed measurements of processing time, logistic setups, there are thorough investigation and cooperation with production experts to build mathematical models. In addition, when companies expose to upgrades or adjustments in production systems, car sequencing can be updated and adapted quickly. However, all previous studies proved that mixed-model sequencing always generated less overload for assembly workers in case of real-case industry data and random instances. Louis et. al., 2023 and Golle et al., 2014 examined quality of car sequencing models with different sequencing rules and reached conclusions that two methods are as equivalently good when the number of cars on one sequence are limited otherwise, mixed model sequencing helps achieve better results and amongst three sequencing rules one sequencing rule, multiple sequencing rules, unique sequencing ratio per station (USR), USR delivered the best performances.

2. Problem statement

2.1 Notation

Ninh, reviewed and corrected by Aleksandra:

T	Number of production cycles (index t)
M	Number of models (index m)
K	Number of stations (index k)
O	Number of options (index o)
Q	Total number of sequencing rules (index q)
c	Cycle time
d_m	Demand for model m
l_k	Length of station k
a_{om}	Binary variables: 1, if model m contains option o, 0 otherwise
x_{mt}	Binary variables: 1, if model m is produced in slot t, 0 otherwise
$H_o : N_o$	Sequencing rule for option o: at most H_o out of N_o successively sequenced models require option o
p_o^+	Processing time of models with option o
p_o^-	Processing time of models without option o
O	Number of sequencing rules for option o (index q)
$H_o^q : N_o^q$	q-th sequencing rule for option o
q_o^{min}	Maximum number of successive models containing option o without inducing work overload, equals H_o
q_o^{max}	Maximum number of models with option o that can occur in a sequence of length T without work overload
λ_o	Weight of violations of rules

2.2 Assumptions

Ninh, reviewed and corrected by Aleksandra:

We consider the following assumptions for a car sequencing model (see Golle et. al., 2014):

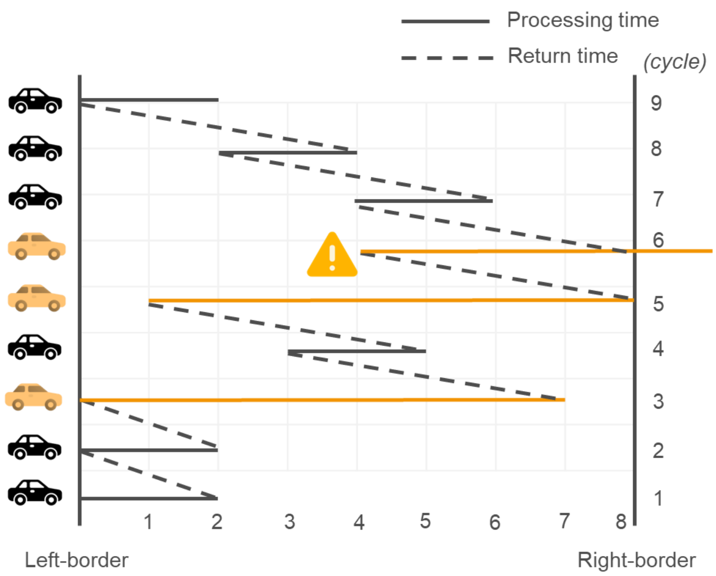
- The assembly line flows from the left to the right and stations are closed (so operators are not allowed to work beyond the station borders)

- Product models move at a constant speed across production line and all workstations are set up successively along the line.
- The products are started to be processed as soon as they meet the operators within the stations.
- When the operator has finished one model, he moves with infinite speed to the next model.
- Product models are placed on the conveyer at the same interval equal to a cycle time.
- Demand and all processing times are known in advance and are static and deterministic during the planning period.
- $p_{mk} \leq l_k \ \forall m \in M, k \in K$ holds, otherwise sequence independent work overload will occur.
- No line stoppage incurs as work overload will be compensated with additional workers within the border of the station.

2.3 Numerical example 1

Ninh:

Assume that there is a workstation is set up as with 9 cycles and border length l_k= 8 and the conveyor used to pick up the products move from the left station to the right station border. Black cars and orange cars are two models processed intermixed in the production line in which the black model is a car without feature "o", possessing shorter processing time p_o^-= 2 and the orange car is the model with additional feature "o" and therefore it has longer processing time p_o^+= 7. Additional workers will be used to compensate any work overload that occurs. Nine cars enter the production line with the order as describe below



The operator will proceed the model when she meets the car on the station. It can be seen that she cannot finish the 6th car because the remaining time on the workstation is not sufficient to complete the task and therefore, work overload incurs. From that observation, it is infeasible to process two orange cars consecutively. Car sequencing would help to define which sequence is appropriate for the set up without generating any work overload by imposing the rule H_0 : N_0 for each line in which H_o tells the maximum number of models with option o that could be processed consecutively and N_0 is the summation of H_0 and the number of models without option o to get the operators back to left-border station.

2.4 Problem statement

Ninh, reviewed and corrected by Aleksandra:

Multiple sequencing rules approach (Golle et al., 2014) introduces multiple sequencing rules $H_o^q : N_o^q$, with $q = 1, \dots, Q_o$ and Q_o being the total number of sequencing rules for option o. Q_o amounts to $q_o^{max} - q_o^{min} + 1$ with

$$q_o^{min} = \left\lfloor \frac{l_o - c}{p_o^+ - c} \right\rfloor \forall o \in O \tag{1}$$

$$q_o^{max} = \left\lfloor \frac{T(c - p_o^-) + (l_o - c)}{p_o^+ - p_o^-} \right\rfloor \forall o \in O \tag{2}$$

Then, $\forall q_o \epsilon \left[q_o^{min}, q_o^{max}\right]$, multiple sequencing rule generates a rule with:

$H_o^{q_o - q_o^{min} + 1} : N_o^{q_o - q_o^{min} + 1}$ with

$$H_o^{q_o - q_o^{min}+1} = q_o \forall o \in O \tag{3}$$

$$N_o^{q_o - q_o^{min}+1} = H_o^{q_o - q_o^{min}+1} + \left\lceil \frac{q_o .(p_o^+ - c) + (l_o - p_o^+)}{c - p_o^-} \right\rceil \forall o \in O \tag{4}$$

The equation (4) computes the minimum number of models without option o that are required after processing q_o models with o to process another model containing o without inducing work overload.

The model assumes that exactly one option $o \in O$ is processed at each station $k \in K$. Each option has merely two processing times. A model requiring option o has processing time p_o^+ larger than the cycle time c, while a model without this option needs processing time p_o^-, which is lower than c. Thus, $p_o^- < \text{c} < p_o^+ < l_o$.

The model is based on Golle et al. (2014) which delivered a minimum resulting gap in the amount of work overload when compared to MMS.

Minimize $$obj^w = \sum_{o \in O} \lambda_o \frac{1}{Q_o} \sum_{q \in Q_o} v_{oq} \tag{5}$$

$$\text{with } v_{oq} = \sum_{t=1}^{T - N_o^q + 1} \min\left\{1; \max\left\{\sum_{t'=t}^{t+N_o^q-1} \sum_{m \in M} a_{om} . x_{mt'} - H_o^q ; 0\right\}\right\} \tag{6}$$

$$\forall o \in O; q \in Q_o$$

$$\lambda_o = p_o^+ - c \; \forall o \in O \tag{7}$$

subject to

$$\sum_{t=1}^{T} x_{mt} = d_m \tag{8}$$

$$\sum_{m \in M} x_{mt} = 1 \; \forall t = 1, \dots, T \tag{9}$$

Objective function (5) minimizes total violation of rules where each of the violation rule has different weight λ_o, defined in (7). (6) counts the number of excessive occurrences of option o in each subsequence. (8) and (9) are to ensure that the demand of each model is met and each slot in the sequence contains exactly one model.

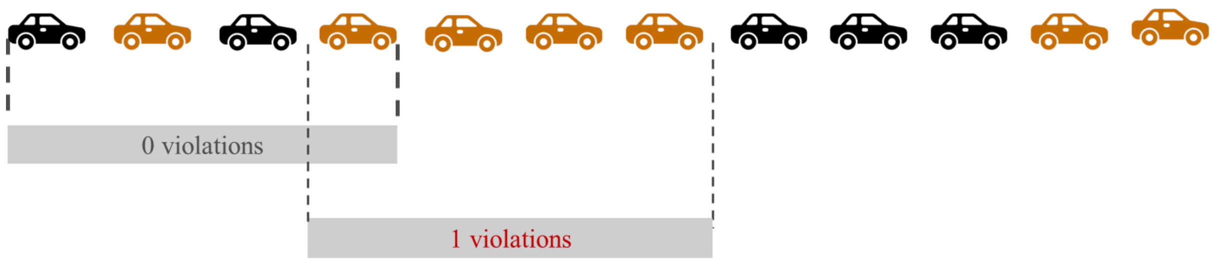
2.5 Numerical example 2

Ninh, reviewed by Aleksandra:

Assume that there is a work station in a car manufacturer with length $l_o = 20$ and there are two car models with two processing time $p_o^+ = 15$ and $p_o^- = 5$ respectively. The cycle time is 10 and we assume that the length of the sequence T = 12, then from (6) and (7) we will have $q_o^{min} = 2$ and $q_o^{max} = 7$, which indicates that we would have 6 sequencing rules in overall.

From (8) and (9), we get $H_o^1 : N_o^1 = 2:3$, $H_o^2 : N_o^2 = 3:4$, $H_o^3 : N_o^3 = 4:5$, $H_o^4 : N_o^4 = 5:6$, $H_o^5 : N_o^5 = 6:7$ and $H_o^6 : N_o^6 = 7:8$

Let's examine the second sequencing rule $H_o^2 : N_o^2 = 3:4$ which indicates that under a subsequence of 4 cars, there are maximum of 3 orange cars that can be processed consecutively to avoid work overload. Each subsequence will be assessed as followed:



3. Literature analysis

3.1 Literature review methodology

Aleksandra:

To define the current status of the literature in car sequencing, the literature analysis was conducted. For this purpose, SCOPUS database was used. The search was restricted to the journals with ranking from A+ to B according to VHB-JOURQUAL[1] rating for operations research field. The search keyword was "car sequencing". Moreover, we restricted our search to papers published from years 2004 to 2023 inclusively. The following search query was used for the papers:

```
ALL("car sequencing") AND ISSN(1047-7047 OR 0030-364X OR 0276-7783 OR 0377-2217 OR 0041-1655 OR 0025-5610 OR 0740-817X OR 0165-1889 OR 0166-218X OR 0171-6468 OR 0364-765X OR 1523-4614 OR 1094-6136 OR 0272-6963 OR 1091-9856 OR 0097-5397 OR 0895-5646 OR 0160-5682 OR 0191-2615 OR 0925-5273 OR 0254-5330 OR 0020-7543 OR 1381-1231 OR 0167-6377 OR 0894-069X OR 0167-9236 OR
```

[1] https://vhbonline.org/vhb4you/vhb-jourqual/vhb-jourqual-3/tabellen-zum-download

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0011-7315 OR 0305-0548 OR 0277-6693 OR 2192-4376 OR 0004-3702 OR 1432-2994
OR 0926-2644 OR 0092-2102 OR 0144-3577 OR 0883-7066 OR 1246-0125 OR 0143-
6570 OR 1936-6582) AND PUBYEAR > 2003 AND PUBYEAR < 2024
```

All in all, 33 relevant papers devoted to car sequencing were found. The distribution of the papers can be seen on the graph below:

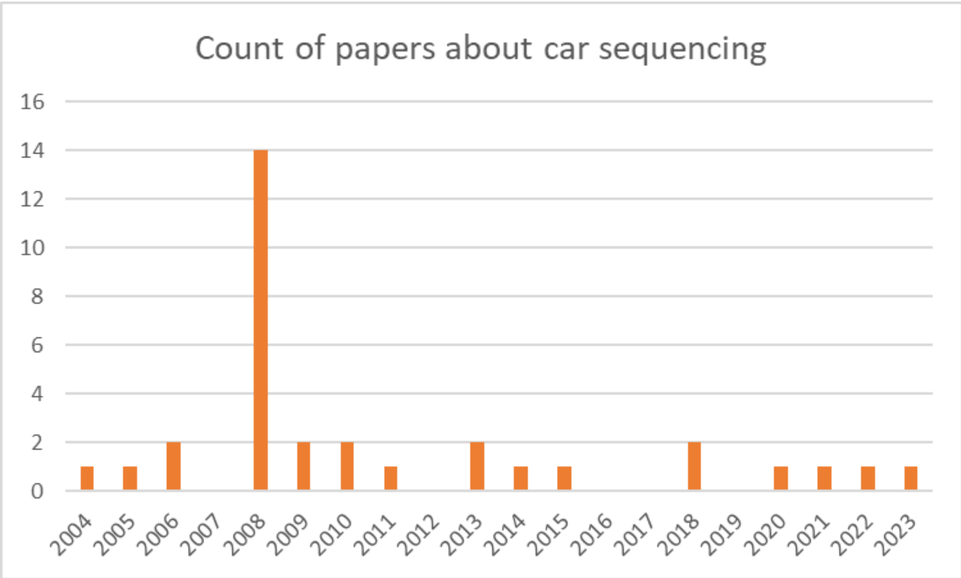

Figure 1. The distribution of car sequencing papers in the period from 2004 to 2023

As it can be seen from the graph, there is a spike of published papers in 2008. This is explained by a ROADEF'05 challenge in 2005 initiated by Renault company: It was a scientific competition in constructing an algorithm for car sequencing using the data set provided by the company. Papers around 2008 mostly propose solution algorithms for a car sequencing problem, conduct computational experiments using Renault instances and analyse the quality of solutions.

However, in the latest 8 years the interest to the topic has decreased and there were years with no publications to the topic. As soon as the scope of this seminar paper should not exceed a certain level, we decided to cut off the articles to the years from 2010 to 2023 and therefore exclude the majority of ROADEF'05 contributions. We also excluded an Estellon and Gardi (2013) paper, which is a short proof that a car sequencing problem is NP-hard. To sum up, we ended up with 12 high-quality papers that will be analysed further.

3.2 Literature review

Aleksandra:

With the time, papers about car sequencing are more often devoted to different aspects of the problem, rather than for pure development of algorithms for solving it. For instance, the topic of sequencing rules creation, which is essential for a successful solution of the problem, was raised by some authors in the early 2010-s. Golle et al. (2010), as it was previously mentioned, proposed a multiple sequencing rules (MSR) approach, which allows to tackle multiple variants of options (e. g. a car with no sunroof, with a sunroof or a with a convertible roof) instead of standard binary yes/no options. It correctly classifies sequences to be either feasible or infeasible, compared to a result of a mixed-model sequencing approach. Almost at the same time, Lesert et al. (2011) proposed a unique sequencing rule method, which aims to find a ratio that is compatible with all the other ratios and that allows the production of the highest possible number of cars. Their method is also able to consider multiple variants of options and it was successfully applied in the production process of PSA Peugeot Citroën.

Many papers are devoted to the modifications of a car sequencing problem. For example, Giard and Jeunet (2010) formulated a problem where the number of temporary utility workers and the number of sequence-dependent setups are to be minimized simultaneously using a cost function and solved it with a standard solver ILOG-Cplex 9.0. Yavuz (2013) formulated a combined car sequencing and level scheduling problem with the objective to minimize the sum of absolute differences between actual and ideal positions of variant copies. Moreover, a parametric iterative beam search algorithm was developed. Later, Yavuz and Ergin (2018) proposed an advanced constraint propagation algorithm for this problem. Jahren and Achá (2018) considered an industrial car sequencing problem from the ROADEF'05 challenge – a problem that also takes into account colours of cars. They formulated a new integer linear program, derived new lower bounds for some instances from the competition and developed an exact column-generation-based algorithm to solve the problem. Hottenrott et al. (2021) incorporated uncertainty and formulated a robust car sequencing problem, considering vehicles' failure probabilities. Short-term disruptions such as missing parts or quality problems often occur in the real world, which affects predefined sequences and leads to work overload. Robust car sequences can be operated without changes when the cars fail. The authors developed an exact branch-and-bound algorithm for small instances and a sampling-based adaptive large neighbourhood search heuristic for larger instances, which were proven to perform better compared to benchmarks.

A newly published paper of Sun et. al. (2022) provides new insights in car sequencing and a new paper direction. They conduct an instance space analysis by deriving different features of instances, applying a principal component analysis to reduce the dimensions to two and visualizing the results. This helps them to answer the question: How instances, which are hard to solve, are characterized and how these characteristics influence the behaviour of a solution algorithm. Accordingly, they construct new benchmark instances, which are shown to be more diverse and harder than the existing ones. After that, the authors introduce two solution approaches – an adaptive large neighbourhood search algorithm and a mixed-integer program method with lazy constraints solved with a standard solver Gurobi, however, the solution approaches could not outperform the others over all instances. That is why the authors constructed an algorithm selection model based on machine learning classification methods to select an appropriate algorithm for solving this or that instance with certain features, which significantly improved the solution quality. To summarize, the paper sheds a light on the car sequencing problem hardness and can serve as a useful basis for further research.

There are also several papers that compare sequencing approaches – car sequencing (CS) and mixed-model sequencing (MMS), and which tend to conclude that MMS produces less work overload. For instance, Golle et al. (2014) compared CS with different rules' generation procedures and objective functions to MMS and found out that a sequence found by CS leads to at least 15% more work overload, for one method of sequencing rules' creation and one objective function type the solution gap is higher than 75%. The study was conducted with different randomly generated test instances. A recent paper of Louis et al. (2023) supports the findings of Golle et al. (2014). The authors develop two dynamic programming procedures for counting the numbers of distinct CS- and MMS-feasible sequences. Computational experiments with Renault Group data show that MMS generates more feasible sequences than CS for all sequencing rules' calculation methods. The authors also advise the practitioners to reconsider the sequencing approach and to try implementing MMS.

Nevertheless, despite recent trends, there are still some papers of the last decade that are focusing purely on the development of solution algorithms for classical car sequencing. For instance, Golle et al. (2015) constructed a graph representation of the problem and proposed an iterative beam search algorithm, which outperformed developed to that time algorithms. The solution algorithm can be used as an exact approach or a heuristic. Thiruvady et al. (2020) developed a large neighbourhood search algorithm based on mixed integer programming and ant colony optimisation, which was shown to find high quality solutions in lower run-times compared to existing algorithms in the literature.

To finalize the review, the literature is summarized in the table below. Papers are sorted by year and classified according to the problem formulation: Type of the problem (pure car sequencing or its modification), objective function, options per station (whether non-binary options and colours are considered). After that, papers are also classified by contribution: Solution method, algorithm type (exact approach or a heuristic), improvement of bounds. Moreover, such papers' contributions as comparison of CS and MMS, development of sequencing rules creation approaches, and instance space analysis are marked.

Aleksandra:

Title		Problem formulation			Contribution					
Paper	**Year**	**Problem type**	**Objective function**	**Options per station**	**Solution method**	**Algorithm type**	**Bounds improved**	**Comparison of CS and MMS**	**Sequencing rules creation**	**Instance space analysis**
Giard and Jeunet	2010	industrial CS with sequence-dependent setups and utility workers	minimize the cost function involving costs associated with additional utility workers and setup costs	2, colours	MIP solved with ILOG-Cplex 9.0	exact algorithm	-	-	-	-
Golle et al.	2010	CS	-	multiple	-	-	-	-	**X**	-
Lesert et al.	2011	CS	minimize the number of violations	multiple	sequencing tool of an automotive company	-	-	-	**X**	-
Yavuz	2013	combined CS and LS	minimize the sum of absolute differences between actual and ideal positions of variant copies	2	iterative beam search	both heuristic or exact algorithm	tight lower bounds, upper bounds	-	-	-
Golle et al.	2014	CS	minimize the number of (weighted) violations with 3 objective functions: 1) sliding window (Gravel et al., 2005), 2) Fliedner and Boysen (2008) function, 3) Bolat and Yano (1992) function	2	MIP solved with CPLEX 12.2	exact algorithm	-	**X**	-	-
Golle et al.	2015	CS	minimize the number of (weighted) violations with 3 objective functions: 1) sliding window (Gravel et al., 2005), 2) Fliedner and Boysen (2008) function	2	iterative beam search	both heuristic or exact algorithm	lower bounds improved	-	-	-
Jahren and Achá	2018	industrial CS	minimize the weighted sum of 2 objectives: ratio costs and paint changes	2, colours	Column-generation-based algorithm	exact algorithm	lower, upper bounds improved	-	-	-

Yavuz and Ergin	2018	combined CS and LS	minimize sum of absolute differences between actual and ideal positions of variant copies	2	constraint propagation, branch-and-bound	heuristic	-	-	-	-
Thiruvady et al.	2020	CS	min the sum of upper over- and under-assignments (see Bautista et al., 2008)	2	large neighbourhood search	heuristic	-	-	-	-
Hottenrott et al.	2021	Industrial CS, robust against short-term sequence alterations	minimize the expected number of violations	2, colours	1) branch-and-bound for small instances, 2) a sampling-based adaptive large neighbourhood search heuristic for large instances	1) exact algorithm 2) heuristic	lower bounds improved	-	-	-
Sun et al.	2022	CS	minimize the sum of upper over- and under-assignments (see Bautista et al., 2008)	2	adaptive large neighbourhood search, MIP with lazy constraints solved with Gurobi	1) heuristic 2) exact algorithm	-	-	-	**X**
Louis et al.	2023	CS, industrial CS	minimize the number of violations	2 or multiple	-	-	-	**X**	-	-

Aleksandra:

We can observe some common trends in the literature as well as literature gaps. Accordingly, we formulate possible future research directions. The conclusions are listed below.

- Authors often approach not only pure car sequencing, but also its modifications, which reflect better real industrial problems. For example, robust car sequencing (Hottenrott et al., 2021) or other approaches incorporating uncertainty could be a promising future research direction.
- Car sequencing with multiple options gained attention in the early 2010-s, however, the interest to it came back only in 2023. Modern production is not limited to the presence or absence of one option, but rather to multiple variants of it. Therefore, multiple options should be more considered in the future research.
- There is a lack of research in the application of car sequencing in other industries than an automotive one in high quality journals.
- Many well-performing algorithms were already created for car sequencing. The ROADEF'05 challenge has also contributed to it. Future research should be focused on other aspects of this problem.
- There is a tendency in convincing practitioners to switch to mixed-model sequencing as it was proven to perform better and leads to less work overload. Future case studies should focus on mixed-model sequencing and how it can be applied in real companies.
- Sequencing rules generation topic was popular in the early 2010-s, however the attention to it decreased in the past ten years. It could be a possible research direction to revisit this topic.
- Instance space analysis is a relatively new topic in car sequencing, despite its wide practical application. Future research can be devoted to developing new and improving existing algorithms according to new benchmark instances and hardness characteristics.

3.3 Industries' analysis

Aleksandra:

To identify future classes and markets, where car sequencing could play an important role, an additional literature analysis should be conducted, because all case studies found in our research are devoted to an automotive industry. An automotive industry includes such subindustries as car, truck, or motorbike manufacturing. Our idea was to search for papers about industries with mixed-model assembly lines and sequencing, which could be potential industries where sequencing can be applied.

We used Google Scholar and SCOPUS databases to search for papers and posed no limitations on journals' ranking. The following search queries were used:

```
Google Scholar: mixed model assembly sequencing case study -car -automotive -truck -motorbike
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SCOPUS: mixed AND model AND assembly AND "case stud*" AND sequencing AND NOT ("car*" OR "automotive" OR "truck*" OR "motorbike*")
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The industries that were found are listed below. In some papers, the authors provide details about the manufacturing process. These details are also described below.

- Boiler manufacturing (Hwang and Katayama, 2010): E. g. various fan seals, fan motors, burners, air conditioners, etc.
- Food processing (Savsar et al., 2017): E. g. a salad line with different core and additional ingredients
- GPU cards (Wang et al., 2023)
- Hydraulic pumps (Rauf et al., 2020): E. g. different valves, levers
- Industrial air-dryers (Faccio et al., 2016)
- Industrial machines manufacturing (Rabbani et al., 2015)
- Off-Highways systems manufacturing (production of steering axles, three-point linkages, hooks, quick couplers for three-point linkages, trailer units, front and rear Power Take Off) (Tiacci and Mimmi, 2018): E. g., two main types of front linkages: the first type has no variants, while the second one has 4 variants, corresponding to the presence/absence of the Power Take Off (PTO) and/or quick couplers
- Plastic bag manufacturing (Mamun et al., 2012): E. g. presence/absence of different prints, various U-panels, etc.
- Turbocharger assembly (Yadav et al., 2020)
- Wooden furniture (Nouri and Abdul-Nour, 2019): E. g. presence/absence of reclining sofa backs, cabinet handles, different colours, etc.

As we see, there is a high demand for sequencing in durable goods' manufacturing, especially for models with more than two variants of one option, which is in line with defined literature trends. Despite certain companies like Tesla tend to reduce the number of models, sequencing remains an important process in modern production.

4. Conclusion

Ninh, reviewed and corrected by Aleksandra:

The research paper introduces car sequencing, the problem is formulated in detail and is followed by a mathematical model and a numerical example. It also explains fundamental differences between car sequencing and mixed-model sequencing, including their advantages over the other. We also present recent trends in the development of two approaches in recent papers. We also illustrate how quality of car sequencing is compared to mixed-model sequencing in previous research work and how different car sequencing rules could generate different performance results.

For our contribution, we summarize high-quality papers from the past 13 years to define the recent trends in literature. The literature was classified by problem formulation and contribution in a table. After that, the main trends were derived, followed by corresponding future research directions. We also summarize further papers without quality restrictions to discover which industries will still apply car sequencing. Our findings show that despite some companies preferring to reduce the number of models, car sequencing will remain an important process in durable goods' industries.

5. References

Aleksandra:

Bautista, J., Pereira, J., & Adenso-Díaz, B. (2008). A beam search approach for the optimization version of the car sequencing problem. *Annals of Operations Research*, *159*, 233-244.

Bolat, A., & Yano, C. A. (1992). A surrogate objective for utility work in paced assembly lines. *Production Planning & Control*, *3*(4), 406-412.

Estellon, B., & Gardi, F. (2013). Car sequencing is NP-hard: a short proof. *Journal of the Operational Research Society, 64(*10), 1503-1504.

Faccio, M., Gamberi, M., & Bortolini, M. (2016). Hierarchical approach for paced mixed-model assembly line balancing and sequencing with jolly operators. *International journal of production research*, *54*(3), 761-777.

Fliedner, M., & Boysen, N. (2008). Solving the car sequencing problem via branch & bound. *European Journal of Operational Research*, *191*(3), 1023-1042.

Giard, V., & Jeunet, J. (2010). Optimal sequencing of mixed models with sequence-dependent setups and utility workers on an assembly line. *International Journal of Production Economics*, *123*(2), 290-300.

Golle, U., Boysen, N., & Rothlauf, F. (2010). Analysis and design of sequencing rules for car sequencing. *European Journal of Operational Research*, *206*(3), 579-585.

Golle, U., Rothlauf, F., & Boysen, N. (2014). Car sequencing versus mixed-model sequencing: A computational study. *European Journal of Operational Research*, *237*(1), 50-61.

Golle, U., Rothlauf, F., & Boysen, N. (2015). Iterative beam search for car sequencing. *Annals of Operations Research*, *226*, 239-254.

Gravel, M., Gagne, C., & Price, W. L. (2005). Review and comparison of three methods for the solution of the car sequencing problem. *Journal of the Operational Research Society*, *56*(11), 1287-1295.

Hottenrott, A., Waidner, L., & Grunow, M. (2021). Robust car sequencing for automotive assembly. *European Journal of Operational Research*, *291*(3), 983-994.

Hwang, R., & Katayama, H. (2010). Integrated procedure of balancing and sequencing for mixed-model assembly lines: a multi-objective evolutionary approach. *International Journal of Production Research*, *48*(21), 6417-6441.

Jahren, E., & Achá, R. A. (2018). A column generation approach and new bounds for the car sequencing problem. *Annals of Operations Research*, *264*, 193-211.

Lesert, A., Alpan, G., Frein, Y., & Noire, S. (2011). Definition of spacing constraints for the car sequencing problem. *International Journal of Production Research*, *49*(4), 963-994.

Louis, A., Alpan, G., Penz, B., & Benichou, A. (2023). Mixed-model sequencing versus car sequencing: comparison of feasible solution spaces. *International Journal of Production Research*, *61*(10), 3415-3434.

Mamun, A. A., Khaled, A. A., Ali, S. M., & Chowdhury, M. M. (2012). A heuristic approach for balancing mixed-model assembly line of type I using genetic algorithm. *International Journal of Production Research*, *50*(18), 5106-5116.

Nouri, K., & Abdul-Nour, G. (2019). Optimization via Computer Simulation of a Mixed Assembly Line of Wooden Furniture-A Case Study. *Procedia Manufacturing*, *39*, 956-963.

Rabbani, M., Sadri, S., Manavizadeh, N., & Rafiei, H. (2015). A novel bi-level hierarchy towards available-to-promise in mixed-model assembly line sequencing problems. *Engineering Optimization, 47*(7), 947-962.

Rauf, M., Guan, Z., Sarfraz, S., Mumtaz, J., Shehab, E., Jahanzaib, M., & Hanif, M. (2020). A smart algorithm for multi-criteria optimization of model sequencing problem in assembly lines. *Robotics and Computer-Integrated Manufacturing*, *61*, 101844.

Savsar, M., Elsaadany, A. K., Hassneiah, R., & Alajmi, A. (2017). Analysis of a manual mixed-model assembly line in food processing industry: a case study. In *International Conference on Industrial Engineering and Operations Management* (pp. 5689-5695).

Sun, Y., Esler, S., Thiruvady, D., Ernst, A. T., Li, X., & Morgan, K. (2022). Instance space analysis for the car sequencing problem. *Annals of Operations Research*, 1-29.

Thiruvady, D., Morgan, K., Amir, A., & Ernst, A. T. (2020). Large neighbourhood search based on mixed integer programming and ant colony optimisation for car sequencing. *International Journal of Production Research*, *58*(9), 2696-2711.

Tiacci, L., & Mimmi, M. (2018). Integrating ergonomic risks evaluation through OCRA index and balancing/sequencing decisions for mixed model stochastic asynchronous assembly lines. *Omega*, *78*, 112-138.

Wang, K. J., Eunike, A., Kurniawan, I., Ardi, R., & Chiu, J. M. (2023). Autonomous agent-based simulation modelling – A case study on a flexible GPU-card final assembly line. *Robotics and Autonomous Systems*, *169*, 104511.

Yadav, A., Verma, P., & Agrawal, S. (2020). Mixed model two-sided assembly line balancing problem: an exact solution approach. *International Journal of System Assurance Engineering and Management*, *11*(Suppl 2), 335-348.

Yavuz, M. (2013). Iterated beam search for the combined car sequencing and level scheduling problem. *International journal of production research*, *51*(12), 3698-3718.

Yavuz, M., & Ergin, H. (2018). Advanced constraint propagation for the combined car sequencing and level scheduling problem. *Computers & Operations Research*, *100*, 128-139.