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Advanced Data Science Methods for Fleet Dimensioning and Location of Drone Depots

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List of abbreviations and symbols

Abbreviation / symbol	Meaning
EMS	Emergency Medical Services
$\|A\|$	A cardinality of a set A
$\mathbb{Z}_0^+$	A set of non-negative integer numbers, including 0
$F(\cdot)$	A joint (cumulative) distribution function
$F_i(\cdot)$	A marginal (cumulative) distribution function of a random variable
$F^{-1}(\cdot)$	A quantile function

Abstract

This paper deals with a facility location and fleet dimensioning problem of a drone system for delivering emergent specimens to laboratories. Two probabilistic models with joint chance constraints, as well as their underlying deterministic models, were proposed. To solve the problem, deterministic equivalents of probabilistic constraints were defined based on p-efficient points. After that, a case study of Passau city was conducted, which showed that the models are applicable to real-life settings.

Introduction

The design of emergency medical services has always been an essential topic in healthcare operations. Saving human lives is a very important mission and requires sophisticated approaches. The invention of unmanned aerial vehicles (drones) has initiated a revolution in operations research in healthcare. With drones, one can quickly deliver essential medical supplies, equipment or medications to places that are hard to reach, such as remote areas. UAVs offer rapid response capabilities, avoiding traffic congestion and geographical barriers, thereby ensuring timely intervention during emergencies and disaster relief operations.

Therefore, the Federal Ministry of Germany has decided to run a pilot project for a drone delivery system. The aim of the project is to enhance the availability and quality of medical services during emergency situations in rural areas. One of the tasks is to design an infrastructure for delivering urgent human specimens from doctor’s offices to laboratories with drones. The purpose of this paper is to define, where to place drone bases and how many drones should be reserved for each location to ensure the service level.

Apart from a mathematical aspect of the problem, there are other challenges for this complicated task. Regulatory restrictions, privacy and safety concerns and ethical aspects should be considered to ensure a reliable use of the technology.

Problem description

The **basic setting** of the problem is described as follows. There is a set of doctor's offices, set of potential locations for drone bases and a set of medical laboratories. An emergency, where a human specimen needs to be collected and analysed, occurs at a doctor's office according to a Poisson distribution. A drone flies from the drone base to the doctor's office, picks up the specimen, delivers it to the laboratory and comes back to the same drone base. The trip from a base to a doctor's office should not exceed a predefined service time (reaction time). The whole trip must not exceed a drone's battery capacity. Capacities of drone bases are limited. Capacities of laboratories are unlimited. A drone starts with a 100% charged battery. After coming back to the drone base, a battery swap with a fully charged battery is conducted, while the used battery is charged to 100%. The number of spare loaded batteries is unlimited. Recharging stations are not used because of strict time limitations.

As soon as the paths are triangle, it makes sense to include medical laboratories and doctor's offices to the set of potential locations as soon as the costs of establishing these locations are lower and the overall distance is minimized.

Two **research questions** must be answered:

1) How many drone bases should be opened?
2) How many drones should be placed in these drone bases to ensure the demand coverage to a prescribed level of probability?

However, the assumption of returning to the same drone base is quite conservative since it reduces the number of doctor's offices which can be potentially served by drones because of enlarging a flying distance. Let us assume that we open drone bases in laboratories and a drone's battery can be swapped at the laboratory. After swapping the battery, a drone comes to the start base to avoid relocation costs and the battery is swapped again to ensure technical reliability. In a general case, a battery can be swapped at any location, however, swapping it in the laboratory automatically minimizes the overall distance. This setting will be our **problem extension**.

Both research questions will be addressed from a strategic perspective. We will not include routing and track the status of each drone. On the contrary, our model will address general designing of EMS.

The assumption of delivering a specimen to the closest laboratory is too restrictive for the basic setting where the drone returns to its base without changing the battery. Let us take a following example below (Figure 1). In this scheme, I denotes a doctor's office, $K1$ and $K2$ are laboratories, where $K1$ is the closest laboratory and J is a potential drone location. Let us assume that the drone's battery capacity equals to 50. Then the length of path $J - I - K1 - J = 13.5 + 13 + 25 = 51.5$ is below the battery limit and does not allow J to cover the doctor's office I. On the contrary, the path $J - I - K2 - J = 13.5 + 21.5 + 12 = 47$ is shorter than the battery capacity, which is makes J a feasible location to cover I.

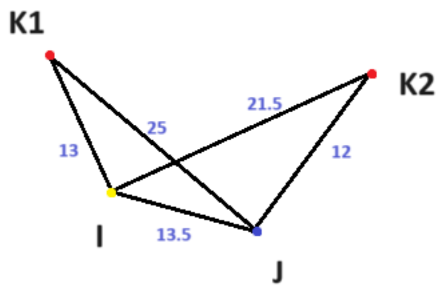

Figure 1. Numerical example with two laboratories.

This controversy raises a question about the cost of human lives. In both settings, travelling to the furthest laboratory minimizes total travelling costs per kilometre and allows to open less locations since more doctor's offices can be covered by one location. However, the most important service time from the request arrival to the specimen delivery is not minimized. The planner should consider these points when designing an acceptable solution.

Literature review

Methodology

To the best of our knowledge, our problem in exactly this setting has not been discussed in the literature before. However, there are many papers which elaborate on certain aspects of our problem. To transparently select literature and to shorten the pool of search, we define the main criteria of our drone specimen delivery problem, which are crucial for our setting. They are:

- Facility location;
- Fleet dimensioning;
- Focus on uncertainty in demand realization;
- Separate "pick-up and delivery" setting;
- Drones with limited battery consumption;
- One type of vehicle.

After that, we select papers, which cover at least four of six aspects, where at least one of the first two criteria is present. We pose no restrictions on journals' ratings and quality of papers because of a limited number of articles closely related to our problem.

Nevertheless, to get a general understanding of the topic, we will also include a short overview of articles in designing EMS relying on existing literature surveys of other authors.

Literature analysis

Designing of EMS has already been widely discussed in the literature. We refer the reader to the comprehensive surveys of, for example, Aringhieri et al. (2017), Brotcorne et al. (2003) or Bélanger et al. (2019), devoted to operations research problems appearing in EMS systems with standard ambulances. However, recent studies (for instance, Kim et al., 2017; Shavarani et al., 2019; Wang et al., 2023; Gentili et al., 2022) utilize unmanned aerial vehicles (or, simply, drones) as a promising technology in emergency situations, which is not susceptible to traffic congestions and other obstacles. A review on recent facility location papers for drone delivery is presented in Dukkanci et al. (2023). Our literature review will focus on strategic questions in designing EMS, namely, EMS facility location and fleet dimensioning, with possible applications of drones. Tactical and operational relocation models will not be covered.

Designing EMS requires sophisticated approaches due to uncertainty and dynamism of EMS as well as their socially oriented objectives like survival or equity (Bélanger et al., 2019). Early works were deterministic approaches: Single and multiple coverage models, which have not directly addressed these real-life aspects of the systems (see, for instance, Church and ReVelle, 1974; Gendreau et al., 1997). A detailed overview of these models is presented in Aringhieri et al. (2017), Brotcorne et al. (2003) and Bélanger et al. (2019). These deterministic problem formulations played an important role in our field since they have built a starting basis for future models.

There are also deterministic approaches in drone delivery systems that are close to our setting. For instance, Kim et al. (2017) model drone-aided delivery and pickup of medication and test kits for patients in rural areas. The demand is assumed to be known. Apart from a routing problem, the paper formulates a program to find the optimal number of drone base locations using the set covering approach. A preprocessing algorithm, a Partition method, and a Lagrangian relaxation method were developed as solution approaches to the problems. Furthermore, similar to the setting in this paper, a pick-up and delivery problem for biomedical material transportation with drones was introduced in Dhote and Limbourg (2020). Four location models (one basic model and three extensions) were developed and applied to the city of Brussels and its suburbs and solved by a commercial solver. The main difference from our model is that the demand is also assumed to be deterministic.

With the evolution of EMS planning, two main paradigms were determined: Incorporating social objectives and embodying uncertainty. Social objectives inclusion is a promising research direction as it directly addresses the main mission of EMS – to save lives. However, since there is still no consensus on measuring equity and other social criteria (Aringhieri et al., 2017), in this paper we will focus on approaches tackling uncertainty.

There are four main paradigms to reflect uncertain behaviour in the literature, where some of the paradigms can be combined:

- Queuing theory;
- Stochastic programming;
- Robust optimization;

- Fuzzy framework.

Stochastic programming directly addresses the uncertainty in demand, assuming that it follows a certain probability distribution. Ball and Lin (1993) were among the first to apply stochastic programming to designing EMS systems with ambulances. They pose, for each demand point, a reliability level on the probability of system failure, a situation where there are no feasible vehicles available for its service when a demand call occurs. A binary integer program is solved via branch-and-bound, implying valid inequalities as a preprocessing technique. Similar chance constraints are introduced in Zhang and Li (2015), which were approximated as second-order cone constraints. The model was transformed into a conic quadratic mixed-integer program by utilizing a conic transformation and solved with CPLEX. Beraldi et al. (2004) proposed joint chance constraints ensuring a service level for the entire area, formulate a deterministic equivalent of the integer linear program using p-efficient points, and solve it with a commercial solver. Later, Beraldi and Bruni (2009) propose the same joint constraints in a two-stage model with recourse and derive its deterministic counterpart with a big-M method. The problem is solved via exact branch-and-bound and several heuristics.

Similarly, Boujemaa et al. (2017) develop a two-stage stochastic programming location-allocation model and solve it using the sampling average approximation algorithm. A sampling approach with the assumption of a stochastic demand was used by Nickel et al. (2016): They draw several samples of scenarios and solve the model associated with each of them. Noyan (2010) introduces integrated chance constraints (risk constraints) on the violation of demand coverage and uses a scenario approach to derive a deterministic equivalent. However, the scenario approach has its drawbacks, namely, the difficulty to identify scenarios and computational complexity.

The queuing paradigm typically employs hypercube models, enabling the modeling of congestion (when all servers are busy) and addressing uncertainty in vehicle availability. Wang et al. (2022) designed an EMS system using drones with an $M/G/k$ queue and considered three aspects: a non-priority, static priority, and dynamic priority model. To tackle the problem, they approximated it as a mixed-integer second-order conic program and solved it with Gurobi.

The queuing paradigm is often combined with other methods, such as a fuzzy framework, which models uncertain and vague information. Shavarani et al. (2019) described a drone delivery system with refuel stations, warehouses, drone procurement, and transportation. They provided a fuzzy mathematical formulation where the waiting times are restricted based on an $M/G/k$ queuing system. A genetic algorithm was applied to the case of the Amazon Prime Air project. Wang et al. (2023) created a drone-based M/G/k-queuing location model, employing fuzzy theory to tackle vague drone endurance and demand arrival rate under a priority queuing system. They used weighted goal programming to simultaneously optimize the total cost, system efficiency, and equitable response time. Moreover, stochastic and second-order conic programming were applied to obtain a crisp mixed-integer second-order conic program, solved with Gurobi.

Finally, another research direction is robust optimization. For instance, Zhang and Jiang (2014) introduce a bi-objective robust program balancing costs and responsiveness and conduct a case study using the data of China. The underlying deterministic and the robust model are solved using CPLEX.

Literature analysis summary is presented in the table below.

Table 1. Literature summary.

Paper	Ball and Lin (1993)	Beraldi and Bruni (2009)	Boujemaa et al. (2017)	Dhote and Limbourg (2020)
Problem:				
Facility location	✓	✓	✓	✓
Fleet dimensioning	✓	✓	✓	✗
Uncertainty:				
Vehicle availability	✗	✗	✗	✗
Demand realization	✓	✓	✓	✗
Types of vehicles	Ambulances	Ambulances	Two types of ambulances	Drones
Setting	One-to one trips (and back)	One-to one trips (and back)	One-to one trips (and back)	Separate pick-up and delivery (and back)
Objective	Min costs	Min costs	Min costs	Min costs
Approach	Probabilistic (chance-) constraints	Two-stage location-allocation model with probabilistic constraints	Two-stage stochastic location-allocation program with recourse	Deterministic
Demand covering	Probabilistic constraints: Each demand point with reliability p	Probabilistic constraints: Each demand zone or whole demand with reliability p	Penalty cost on uncovered demand in the objective	1) Whole demand should be satisfied 2) r% of demand should be satisfied
Crossing coverage zones	✓	✓	✓	✗
Congestion	✗	✓	✓	✗
Solution method	Heuristic: Branch-and-bound	Exact: Branch-and-bound; heuristics: Mincard, Approx, Comb	Exact: Sampling Average Approximation Algorithm	Exact: CPLEX

Table 1 (continued). Literature summary.

Paper	Kim et al. (2017)	Nickel et al. (2016)	Noyan (2010)	Shavarani et al. (2019)
Problem:				
Facility location	✓	✓	✓	✓
Fleet dimensioning	✓	✓	✓	✓
Uncertainty:				
Vehicle availability	✗	✗	✗	✓
Demand realization	✗	✓	✓	✓
Types of vehicles	Drones	Ambulances	Ambulances	Drones
Setting	Delivery and pick-up and back	One-to one trips (and back)	One-to one trips (and back)	One-to one trips (and back) with refuel stations
Objective	Min costs	Min costs	Min costs	Min costs
Approach	Deterministic	Drawing samples of scenarios and obtaining a sample of optimal solutions	Singe- and two-stage model with risk constraints on violation of demand coverage constraints	Fuzzy programming, waiting times restricted based on queuing theory
Demand covering	Each demand point is covered by at least one facility	Whole demand for all scenarios is covered with a service level p	Risk constraints: Expected uncovered demand for each demand point is larger than the one with a risk aversion parameter	Whole demand should be satisfied
Crossing coverage zones	✓	✗	✓	✗
Congestion	✗	✗	✓	✓
Solution method	Exact: Preprocessing algorithm, Partition method, Lagrangian Relaxation	Exact: CPLEX	Heuristic: HICXP, CPLEX	Heuristic: Genetic algorithm

Table 1 (continued). Literature summary.

Paper	**Wang et al. (2022)**	**Wang et al. (2023)**	**Zhang and Jiang (2014)**
Problem:			
Facility location	✓	✓	✓
Fleet dimensioning	✓	✓	✓
Uncertainty:			
Vehicle availability	✓	✓	✗
Demand realization	✗	✓	✓
Types of vehicles	Drones	Drones	Ambulances
Setting	One-to one trips (and back)	One-to one trips (and back)	One-to one trips (and back)
Objective	Min the longest expected response time	Min costs, min average/total service time, min max response time	Min costs, min costs with weighted penalty on unfulfilled demand
Approach	Queuing theory, approximation of the model as a mixed-integer second-order conic program	Multi-objective model based on stochastic, second-order conic, fuzzy and weighted goal programming, waiting times restricted based on queuing theory	Bi-objective robust optimization
Demand covering	Stability of the queue	Stability of the queue, for high priority: a new request can receive an immediate response with a reliability at least *p*	Penalty term on uncovered demands in the objective
Crossing coverage zones	✗	✗	✗
Congestion	✓	✓	✗
Solution method	Exact: Gurobi; SimPy Python simulation	Exact: Gurobi	Heuristic: CPLEX

Table 1 (continued). Literature summary.

Paper	Zhang and Li (2015)	Beraldi et al. (2004)	This paper
Problem:			
Facility location	✓	✓	✓
Fleet dimensioning	✓	✓	✓
Uncertainty:			
Vehicle availability	✗	✗	✗
Demand realization	✓	✓	✓
Types of vehicles	Ambulances	Ambulances	Drones
Setting	One-to one trips (and back)	One-to one trips (and back)	Separate pick-up and delivery and back
Objective	Min costs	Min costs	Min costs
Approach	Probabilistic constraints	Joint probabilistic constraints	Joint probabilistic constraints
Demand covering	Probabilistic constraints: Each demand point with reliability p	Probabilistic constraints: Each demand zone or whole demand with reliability p	Probabilistic constraints: Each demand zone or whole demand with reliability p
Crossing coverage zones	✓	✓	✓
Congestion	✓	✓	✓
Solution method	Heuristic: CPLEX	Exact: CPLEX	Exact: Gurobi

Model formulation

The main challenge is how to incorporate uncertain nature of demand (emergency situations). We assume that demand has a stochastic nature and follows a known probability distribution – in our case it is a Poisson distribution. For this paper, we will choose a stochastic programming paradigm – an approach with joint probabilistic (chance) constraints. As in other papers (Zhang and Li, 2015; Noyan, 2015; Beraldi et al., 2004), we assume that the average service time is 1,5 hours, which seems to be a reasonable time for one service request. Therefore, we count the number of emergency calls come in the subsequent one-and-a-half-hour period and due to that exclude time from the model.

Deterministic model without relaxation

Let us firstly formulate a deterministic model, which will serve as a basic model for our further probabilistic models. We will refer to this model as a deterministic model without relaxation. Let us assume that the demand is known. To formulate the problem, we introduce the following notation below:

Table 2. Notation for the deterministic model without relaxation

Notation	Type	Description
$I;\ i = 1, \dots, \lvert I \rvert$	Set	A finite set of doctor's offices
$K; k = 1, \dots, \lvert K \rvert, K \neq I,$ $\lvert K \rvert < \lvert I \rvert$	Set	A finite set of medical laboratories
$J;\ j = 1, \dots, \lvert J \rvert$ $I, K \subseteq J$	Set	A finite set of potential locations for drone bases
$d_{ji}, d_{ji} \geq 0$	Parameter	Travel time (Euclidean distance) from a potential drone base j to a doctor's office i
$d_{ik}, d_{ik} \geq 0$	Parameter	Travel time (Euclidean distance) from a doctor's office i to a laboratory k
$d_{kj}, d_{kj} \geq 0$	Parameter	Travel time (Euclidean distance) from a laboratory k to a potential drone base j
$B, B > 0$	Parameter	Maximal allowed overall travel time of the drone due to a battery capacity
$S, S > 0$	Parameter	Maximal allowed service time from accepting a request to arriving to doctor's office

$L = \{(k,j): d_{kj} \leq 0.5B\}$, $L \subseteq K \times J$	Set	A set of paths from a laboratory k to a drone base location j that last not longer than a half of the battery capacity
$P = \{(i,k): d_{ik} \leq 0.5B\}$, $P \subseteq I \times K$	Set	A set of paths from a doctor's office i to a laboratory k that last not longer than a half of the battery capacity
$M_{(k,j)} = \{i \in I,$ $(k,j) = 1, \dots, \lvert L\rvert :$ $\begin{cases} d_{ji} \leq S \\ d_{ji} + d_{ik} + d_{kj} \leq B \end{cases}\}$	Set	A set of doctor's offices i that can be covered by a path (k,j). A doctor's office i is covered by a path (k,j) if 1) the distance "start base – doctor's office" is not larger than a predefined service time and 2) the distance "start base – doctor's office – laboratory – start base" is not larger than the drone's battery capacity.
$N_i = \{(k,j) \in L,$ $i = 1, \dots, \lvert I\rvert :$ $\begin{cases} d_{ji} \leq S \\ d_{ji} + d_{ik} + d_{kj} \leq B \end{cases}\}$	Set	A set of all paths (k,j) that can cover a doctor's office i. A path (k,j) can cover a doctor's office i if 1) the distance "start base – doctor's office" is not larger than a predefined service time and 2) the distance "start base – doctor's office – laboratory – start base" is not longer than the drone's battery capacity.
$O_j = \{(i,k) \in P,$ $j = 1, \dots, \lvert J\rvert :$ $\begin{cases} d_{ji} \leq S \\ d_{ji} + d_{ik} + d_{kj} \leq B \end{cases}\}$	Set	A set of all paths (i,k) that can be covered by a location j. A path (i,k) can be covered by a location j if 1) the distance "start base – doctor's office" is not larger than a predefined service time and 2) the distance "start base – doctor's office – laboratory – start base" is not longer than the drone's battery capacity
$KJ_{feas} = \{(k,j) : M_{(k,j)} \neq \emptyset\}$	Set	Set of paths (k,j), which cover at least one doctor's office i
$I_{feas} = \{i : N_i \neq \emptyset\}$, $i_{feas} = 1, \dots, \lvert I_{feas}\rvert$	Set	Set of doctor's offices i, which are covered at least by one path (k,j)
$J_{feas} = \{j : O_j \neq \emptyset\}$, $j_{feas} = 1, \dots, \lvert J_{feas}\rvert$	Set	Set of locations j, which cover at least one path (i,k)
$x_{ikj}, x_{ikj} \in \mathbb{Z}_0^+, i \in I, (k,j) \in L$	Decision variable	A number of drones located at j which are used to cover the service requests at the doctor's office i using a laboratory k
$y_j = \begin{cases} 1 \\ 0 \end{cases}, j = 1, \dots, \lvert J\rvert$	Decision variable	1, if the location j is open, 0 otherwise
α	Parameter	Costs for buying and maintaining one drone
β_j	Parameter	Fixed costs for opening and maintaining a drone base location j
γ	Parameter	Variable costs per kilometre
$q_j, q_j \in \mathbb{Z}_0^+, j = 1, \dots, \lvert J\rvert$	Parameter	A number of drones which can be placed at the location j (a drone base capacity)

$\theta_i, \theta_i \in \mathbb{Z}_0^+, i = 1, \dots, \lvert I \rvert$	Parameter	A service request (emergency situations) generated at the doctor's office i
$p, p \in [0; 1]$	Parameter	A reliability level, i. e. a probability with which random service requests are covered

Thus, the deterministic problem without relaxation can be formulated as follows:

$minimize$	$\sum_{i \in I_{feas}} \sum_{(k,j) \in KJ_{feas}} (a + \gamma \cdot (d_{ji} + d_{ik} + d_{kj})) \cdot x_{ikj} + \beta_j \sum_{j \in J_{feas}} y_j$	Minimize total costs: 1) Costs for buying and maintaining of drones, 2) variable costs per kilometre and 3) fixed costs for opening and maintaining a base location *j*	(1)
$s.t.$	$\sum_{(k,j) \in N_i \neq \emptyset} x_{ikj} \geq \theta_i, \quad i = 1, \dots, \lvert I_{feas} \rvert$	Covering constraints ensuring that the whole demand is covered	(2)
	$\sum_{(i,k) \in O_j \neq \emptyset} x_{ikj} \leq q_j y_j, \quad j = 1, \dots, \lvert J_{feas} \rvert$	Capacity constraints limiting the number of drones which can be located at each candidate station *j*	(3)
	$(d_{ji} + d_{ik} + d_{kj}) \cdot x_{ikj} \leq B \cdot x_{ikj}, \quad (k,j) \in KJ_{feas}, i \in I_{feas}$	Distance constraints limiting the overall distance according to drones' battery capacity	(4)
	$d_{ji} x_{ikj} \leq S \cdot x_{ikj}, \quad (k,j) \in KJ_{feas}, i \in I_{feas}$	Distance constraints limiting the service time according to a threshold	(5)
	$x_{ikj} \in \mathbb{Z}_0^+$, $y_j \in \{0, 1\}, \forall i = 1, \dots, \lvert I_{feas} \rvert, j = 1, \dots, \lvert J_{feas} \rvert, k = 1, \dots, \lvert K_{feas} \rvert$	Non-negative integer and binary variables	(6)

As you have noticed, there is a data preprocessing procedure in the problem formulation to reduce the feasible solution space. Firstly, sets L and P are defined since the length of a one-way trip should be less or equal to a half of the battery (in case that a drone base is located in a doctor's office or in a laboratory). After that, sets $M_{(k,j)}$, N_i and O_j are determined to fit the battery and service time constraints and non-empty sets are used in the model. Moreover, KJ_{feas}, J_{feas} and I_{feas} are defined to miss infeasible points and paths – that is the way to avoid infeasible instances.

Probabilistic model without relaxation

Now let us assume that θ_i is a discrete random variable following the Poisson distribution. We introduce joint chance constraints posing a reliability level – the minimum probability that the demand for the whole area will be covered. Then the model can be reformulated as follows:

$minimize$	$\sum_{i\in I_{feas}} \sum_{(k,j)\in KJ_{feas}} (a + \gamma \cdot (d_{ji} + d_{ik} + d_{kj})) \cdot x_{ikj} + \beta_j \sum_{j\in J_{feas}} y_j$	Minimize total costs: 1) Costs for buying and maintaining of drones, 2) variable costs per kilometre and 3) fixed costs for opening and maintaining a base location *j*	(7)
$s.t.$	$P(\sum_{(k,j)\in N_i \neq \emptyset} x_{ikj} \geq \theta_i,\ i = 1, \ldots, \lvert I_{feas}\rvert) \geq p$	Probabilistic constraints Reliability - the ability of the emergency service to guarantee a high level of service by covering the random requests with a prescribed value of probability p	(8)

$$\sum_{(i,k)\in O_j\neq\emptyset} x_{ikj} \leq q_j y_j,$$ $$j = 1, \dots, \lvert J_{feas} \rvert$$	Capacity constraints limiting the number of drones which can be located at each candidate station j	(9)
$$\left(d_{ji} + d_{ik} + d_{kj}\right) \cdot x_{ikj} \leq B \cdot x_{ikj},$$ $$(k,j) \in KJ_{feas}, i \in I_{feas}$$	Distance constraints limiting the overall distance according to drones' battery capacity	(10)
$$d_{ji} x_{ikj} \leq S \cdot x_{ikj},$$ $$(k,j) \in KJ_{feas}, i \in I_{feas}$$	Distance constraints limiting the service time according to a threshold	(11)
$$x_{ikj} \in \mathbb{Z}_0^+$$ $$y_j \in \{0,1\}, \forall i = 1, \dots, \lvert I_{feas} \rvert,$$ $$j = 1, \dots, \lvert J_{feas} \rvert, k = 1, \dots, \lvert K_{feas} \rvert$$	Non-negative integer and binary variables	(12)

As soon as we assume that θ_i are independent random variables (under the assumption that we exclude catastrophes), the constraints (8) can be rewritten as follows:

$$\prod_{i=1}^{\lvert I_{feas} \rvert} F_i\left(\sum_{(k,j)\in N_i\neq\emptyset} x_{ikj}\right) \geq p \tag{13}$$

The problem cannot be solved in such formulation because of probabilistic constraints, that is why we will try to formulate their deterministic equivalents. Similar joint constraints were introduced and reformulated by Beraldi et al. (2004) based on the reformulation proposed by Dentcheva et al. (2000) using so-called p-efficient points.

Let us introduce the following replacement:

Table 3. Notation for the probabilistic model without relaxation

Notation	Type	Description
$z_i = \sum_{(k,j)\in N_i\neq\emptyset} x_{ikj}$, $i = 1, \dots, \lvert I_{feas} \rvert$	Decision variable	The sum of numbers of drones located at j which are used to cover the service requests at the doctor's office i using a laboratory k, $(k,j) \in N_i \neq \emptyset$

Then the constraints (13) can be rewritten as:

$$\prod_{i=1}^{|I_{feas}|} F_i(z_i) \geq p \tag{14}$$

We can take a logarithm:

$$\sum_{i=1}^{|I_{feas}|} \ln\left(F_i(z_i)\right) \geq \ln p \tag{15}$$

As $F_i(z_i)$ is monotonically increasing, let us introduce the terminology of a p-efficient point (Dentscheva et al., 2000).

Table 4. Notation for the probabilistic model without relaxation

Notation	Type	Description
$l_i = F_i^{-1}(p), i = 1, \ldots, \lvert I_{feas} \rvert$	Parameter	A p-efficient point of the marginal distribution F_i (in other words, a p-quantile of the marginal distribution F_i, that is the smallest integer value such that $F_i(z_i) \geq p$)

As soon as values $z_i = 1, \ldots, |I_{feas}|$have to satisfy $\prod_{i=1}^{I} F_i(z_i) \geq p$ and $0 \leq F_i(z_i) \leq 1$, it is evident that $F_i(z_i) \geq p$. As soon as l_i is the smallest integer value of z_i such that $F_i(z_i) \geq p$, the inequality $z_i \geq l_i$ holds.

Let us introduce the concept of log-concavity for discrete integer probability distributions. A sequence of non-negative elements $\ldots, a_{-1}, a_0, a_1, \ldots$ is log-concave, when the following expression holds (Prékopa, 2013):

$$a_m^2 \geq a_{m-1} a_{m+1} \tag{16}$$

A discrete probability distribution, defined on the integers, is log-concave if the sequence of corresponding probabilities is log-concave. Discrete Poisson distribution is defined as:

$$P_k = \begin{cases} \frac{\lambda^n}{n!} e^{-\lambda}, & n \in \mathbb{Z}_0^+ \\ 0, & n < 0 \end{cases}, \tag{17}$$
$$\lambda > 0$$

It can be shown that the Poisson distribution is log-concave. Indeed, imagine, for non-negative n holds:

$$\left(\frac{\lambda^n}{n!}e^{-\lambda}\right)^2 \geq \left(\frac{\lambda^{n-1}}{(n-1)!}e^{-\lambda}\right)\left(\frac{\lambda^{n+1}}{(n+1)!}e^{-\lambda}\right) \tag{18}$$

As soon as $e^{-\lambda}$ is positive, we get

$$\left(\frac{\lambda^n}{n!}\right)^2 \geq \left(\frac{\lambda^{n-1}}{(n-1)!}\right)\left(\frac{\lambda^{n+1}}{(n+1)!}\right) \tag{19}$$

$$\frac{\lambda^{2n}}{(n!)^2} \geq \frac{\lambda^{n-1+n+1}}{\frac{n!}{n}\cdot n!\cdot(n+1)} \tag{20}$$

$$\frac{\lambda^{2n}}{(n!)^2} \geq \frac{\lambda^{2n}}{(n!)^2\cdot\frac{(n+1)}{n}} \tag{21}$$

$\frac{\lambda^{2n}}{(n!)^2}$ is positive, that is why the following holds true for all $n \in \mathbb{Z}_0^+$:

$$1 \geq \frac{n}{n+1} \tag{22}$$

For $n < 0$, the proof is trivial. ■

As soon as the Poisson distribution is log-concave, it is possible to rewrite z_i in a $0-1$ formulation (Prékopa, 2013). If $l_i + g_i$ is a known upper bound, z_i can be replaced with:

$$z_i = l_i + \sum_{g=1}^{g_i} z_{ig},\ \ z_{ig} \in \{0,1\} \tag{23}$$

Let us show that the constraint (15) can be rewritten as:

$$\sum_{i=1}^{|I_{feas}|}\sum_{g=1}^{g_i} h_{ig}z_{ig} \geq \varphi, \tag{24}$$

where

$$h_{ig} = \ln\big(F_i(l_i+g)\big) - \ln\big(F_i\ (l_i+g-1)\big) \tag{25}$$

$$\varphi = \ln p - \ln F(l) = \ln p - \sum_{i=1}^{|I_{feas}|} \ln(F_i(l_i)) \tag{26}$$

Setting z_i in a binary reformulation, we get:

$$\sum_{i=1}^{|I_{feas}|} \ln\,(F_i(l_i + \sum_{g=1}^{g_i} z_{ig}) \geq \ln p \tag{27}$$

We rewrite the constraint (24) and open the brackets:

$$\begin{aligned}
&\sum_{i=1}^{|I_{feas}|} \sum_{g=1}^{g_i} \left[\ln\big(F_i(l_i + g)\big) - \ln\big(F_i\,(l_i + g - 1)\big)\right] z_{ig} \geq \ln p - \sum_{i=1}^{|I_{feas}|} \ln\big(F_i(l_i)\big) \Leftrightarrow \\
&\sum_{i=1}^{|I_{feas}|} \left[\ln\big(F_i(l_i)\big) + \sum_{g=1}^{g_i} \left[\ln\big(F_i(l_i + g)\big) z_{ig} - \ln\big(F_i\,(l_i + g - 1)\big)\, z_{ig}\right]\right] \geq \ln p \Leftrightarrow
\end{aligned} \tag{28}$$

$$\begin{aligned}
\sum_{i=1}^{|I_{feas}|} &\left[\ln\big(F_i(l_i)\big) + \ln\big(F_i(l_i + 1)\big)\, z_{i1} - \ln\big(F_i(l_i)\big)\, z_{i1} + \ln\big(F_i(l_i + 2)\big)\, z_{i2}\right. \\
&- \ln\big(F_i(l_i + 1)\big)\, z_{i2} + \cdots + \ln\big(F_i(l_i + g_i)\big) z_{ig_i} \\
&\left. - \ln\big(F_i\,(l_i + g_i - 1)\big)\, z_{ig_i}\right] \geq \ln p
\end{aligned} \tag{29}$$

By log-concavity of $F_i(\cdot)$ we get (see (16)):

$$\begin{aligned}
\big(F_i(l_i + g)\big)^2 &\geq F_i(l_i + g - 1) F_i(l_i + g + 1) \Leftrightarrow \\
2\ln\big(F_i(l_i + g)\big) &\geq \ln\big(F_i(l_i + g - 1)\big) + \ln\,(F_i(l_i + g + 1)) \Leftrightarrow \\
\ln\big(F_i(l_i + g)\big) - \ln\big(F_i(l_i + g - 1)\big) &\geq \ln\big(F_i(l_i + g + 1)\big) - \ln\,(F_i(l_i + g))
\end{aligned} \tag{30}$$

If we can notice, (29) with two negative sides is equivalent to a knapsack constraint. Implying (30), binary z_{ig} must be non-increasing to get a solution. We can assume that $z_{i1}, z_{i2}, \dots, z_{ir} = 1$ and $z_{i(r+1)}, \dots, z_{ig} = 0$ for $r \in \mathbb{Z}_0^+$ and replace them in (29). Curtailing zero terms from (29), we get:

$$\sum_{i=1}^{I} \ln\,(F_i(l_i + r) \geq \ln p \tag{31}$$

As we can notice, $r = \sum_{g=1}^{g_i} z_{ig}$, so with (23) we get the original constraint (15).

Therefore, the probabilistic model without relaxation can be rewritten as follows:

$minimize$	$\sum_{i \in I_{feas}} \sum_{(k,j) \in KJ_{feas}} (a + \gamma \cdot (d_{ji} + d_{ik} + d_{kj})) \cdot x_{ikj} + \beta_j \sum_{j \in J_{feas}} y_j$	Minimize total costs: 1) Costs for buying and maintaining of drones, 2) variable costs per kilometre and 3) fixed costs for opening and maintaining a base location *j*	(32)
$s.t.$	$\sum_{i=1}^{\lvert I_{feas} \rvert} \sum_{g=1}^{g_i} h_{ig} z_{ig} \geq \varphi$	Deterministic constraints replacing probabilistic ones	(33)
	$\sum_{(k,j) \in N_i \neq \emptyset} x_{ijk} = l_i + \sum_{g=1}^{g_i} z_{ig}$	Sum of drones in a binary reformulation	(34)
	$\sum_{(i,k) \in O_j \neq \emptyset} x_{ikj} \leq q_j y_j, \quad j = 1, \dots, \lvert J_{feas} \rvert$	Capacity constraints limiting the number of drones which can be located at each candidate station *j*	(35)
	$(d_{ji} + d_{ik} + d_{kj}) \cdot x_{ikj} \leq B \cdot x_{ikj}, \quad (k,j) \in KJ_{feas}, i \in I_{feas}$	Distance constraints limiting the overall distance according to drones' battery capacity	(36)
	$d_{ji} x_{ikj} \leq S \cdot x_{ikj}, \quad (k,j) \in KJ_{feas}, i \in I_{feas}$	Distance constraints limiting the service time according to a threshold	(37)
	$x_{ikj} \in \mathbb{Z}_0^+$, $y_j \in \{0,1\}, \forall i = 1, \dots, \lvert I_{feas} \rvert, j = 1, \dots, \lvert J_{feas} \rvert, k = 1, \dots, \lvert K_{feas} \rvert$	Non-negative integer and binary variables	(38)

Deterministic model with relaxation

The notation below will be used to model the problem extension. From now on drones' batteries are swapped directly in the laboratory. We redefine the sets of data preprocessing and enlarge them, relaxing the constraints concerning battery. Moreover, we add constraints creating a drone base in laboratories. As before, for this basic model we assume that the service requests are already known.

Table 5. Notation for the deterministic model with relaxation

Notation	Type	Description
$\theta_i, \theta_i \in \mathbb{Z}_0^+, i = 1, \dots, \lvert I \rvert$	Parameter	A service request (emergency situations) generated at the doctor's office i
$L = \{(k,j): d_{kj} \leq B\}$, $L \subseteq K \times J$	Set	A set of paths from a drone base location j to a laboratory k that are not larger than a battery capacity
$P = \{(i,k): d_{ik} \leq B\}$, $P \subseteq I \times K$	Set	A set of paths from a doctor's office i to a laboratory k that last not longer than the battery capacity
$M_{(k,j)} = \{i \in I, (k,j) = 1, \dots, \lvert L \rvert : \begin{cases} d_{ji} \leq S \\ d_{ji} + d_{ik} \leq B \end{cases}\}$	Set	A set of doctor's offices that can be covered by a path (k,j). A doctor's office i is covered by a path (k,j) if 1) the distance "start base – doctor's office" is not larger than a predefined service time, 2) the distance "start base – doctor's office – laboratory" is not larger than a drone's battery capacity and 3) the distance "laboratory – start base" is not larger than a drone's battery capacity
$N_i = \{(k,j) \in L, i = 1, \dots, \lvert I \rvert : \begin{cases} d_{ji} \leq S \\ d_{ji} + d_{ik} \leq B \end{cases}\}$	Set	A set of all paths that can cover a doctor's office i. A path (k,j) can cover a doctor's office i if 1) the distance "start base – doctor's office" is not larger than a predefined service time, 2) the distance "start base – doctor's office – laboratory" is not larger than the drone's battery capacity and 3) the distance "laboratory – start base" is not larger than the drone's battery capacity
$O_j = \{(i,k) \in P, j = 1, \dots, \lvert J \rvert : \begin{cases} d_{ji} \leq S \\ d_{ji} + d_{ik} \leq B \end{cases}\}$	Set	A set of all paths (i,k) that can be covered by a location j. A path (i,k) can be covered by a location j if 1) the distance "start base – doctor's office" is not larger than a predefined service time and 2) the distance "start base – doctor's office – laboratory" is not longer than the drone's battery capacity

The deterministic model with relaxation can be formulated as follows:

$minimize$	$\sum_{i \in I_{feas}} \sum_{(k,j) \in KJ_{feas}} (a + \gamma \cdot (d_{ji} + d_{ik} + d_{kj})) \cdot x_{ikj} + \beta_j \sum_{j \in J_{feas}} y_j$	Minimize total costs: 1) Costs for buying and maintaining of drones, 2) variable costs per kilometre and 3) fixed costs for opening and maintaining a base location *j*	(39)
$s.t.$	$\sum_{(k,j) \in N_i \neq \emptyset} x_{ikj} \geq \theta_i,$ $i = 1, \dots, \lvert I_{feas} \rvert$	Covering constraints ensuring that the whole demand is covered	(40)
	$\sum_{(i,k) \in O_j \neq \emptyset} x_{ikj} \leq q_j y_j,$ $j = 1, \dots, \lvert J_{feas} \rvert$	Capacity constraints limiting the number of drones which can be located at each candidate station *j*	(41)
	$d_{ji} x_{ikj} \leq B \cdot x_{ikj},$ $(k,j) \in KJ_{feas}, i \in I_{feas}$	Distance constraints limiting the overall distance according to drones' battery capacity	(42)
	$d_{ji} x_{ikj} \leq S \cdot x_{ikj},$ $(k,j) \in KJ_{feas}, i \in I_{feas}$	Distance constraints limiting the service time according to a threshold	(43)
	$y_{j \in K} = 1$	Creating a drone base in laboratories	(44)
	$x_{ikj} \in \mathbb{Z}_0^+$ $y_j \in \{0,1\}, \forall i = 1, \dots, \lvert I_{feas} \rvert,$ $j = 1, \dots, \lvert J_{feas} \rvert, k = 1, \dots, \lvert K_{feas} \rvert$	Non-negative integer and binary variables	(45)

Battery constraints (4) are relaxed to (42). The constraints $d_{kj} y_j \leq B,\ (j,k) \in N_i$ are redundant and can be omitted because of the triangulation inequality and, therefore, the way "drone base – doctor's office – laboratory" is always larger or equal to the way "laboratory – drone base".

Probabilistic model with relaxation

For the probabilistic model with relaxation, we assume that the demand is a random variable, replace the demand covering constraints by joint chance constraints in the deterministic model and formulate the problem's deterministic counterpart according to the p-efficient points method discussed above.

Table 6. Notation for the deterministic model with relaxation

Notation	Type	Description
$\theta_i, \theta_i \in \mathbb{Z}_0^+, i = 1, \dots, \lvert I \rvert$	Parameter	A service request (emergency situations) generated at the doctor's office i

$minimize$	$$\sum_{i \in I_{feas}} \sum_{(k,j) \in KJ_{feas}} (a + \gamma \cdot (d_{ji} + d_{ik} + d_{kj})) \cdot x_{ikj} + \beta_j \sum_{j \in J_{feas}} y_j$$	Minimize total costs: 1) Costs for buying and maintaining of drones, 2) variable costs per kilometre and 3) fixed costs for opening and maintaining a base location j	(46)
$s.t.$	$$\sum_{i=1}^{\lvert I_{feas} \rvert} \sum_{g=1}^{g_i} h_{ig} z_{ig} \geq \varphi$$	Deterministic constraints replacing probabilistic ones	(47)
	$$\sum_{(k,j) \in N_i \neq \emptyset} x_{ijk} = l_i + \sum_{g=1}^{g_i} z_{ig}$$	Sum of drones in a binary reformulation	(48)

$\sum_{(i,k)\in O_j\neq\emptyset} x_{ikj} \leq q_j y_j,$ $j = 1, \dots, \lvert J_{feas} \rvert$	Capacity constraints limiting the number of drones which can be located at each candidate station j	(49)
$(d_{ji} + d_{ik}) \cdot x_{ikj} \leq B \cdot x_{ikj},$ $(k,j) \in KJ_{feas}, i \in I_{feas}$	Distance constraints limiting the overall distance according to drones' battery capacity	(50)
$d_{ji} x_{ikj} \leq S \cdot x_{ikj},$ $(k,j) \in KJ_{feas}, i \in I_{feas}$	Distance constraints limiting the service time according to a threshold	(51)
$y_{j\in K} = 1$	Creating a drone base in laboratories	(52)
$x_{ikj} \in \mathbb{Z}_0^+$ $y_j \in \{0,1\}, \forall i = 1, \dots, \lvert I_{feas} \rvert,$ $j = 1, \dots, \lvert J_{feas} \rvert, k = 1, \dots, \lvert K_{feas} \rvert$	Non-negative integer and binary variables	(53)

Computational experiments

Methodology

The Passau region was chosen by the Federal Ministry of Germany as a pilot region for testing emergency specimen delivery because it is located in a rural area where EMS face transportation difficulties in providing reliable service to city and suburbs residents.

For the case study in Passau, we identified 77 doctor's offices (along with their latitudes and longitudes) available on Google Maps. We placed no restrictions on the specialization of doctors, as detailed information regarding which doctors may require urgent specimen analysis was not available. For simplicity, we assume that all identified doctor's offices can utilize an emergency delivery service.

Additionally, we identified two medical laboratories in Passau using Google Maps. The first laboratory is situated in the city hospital and caters exclusively to the requests of this institution. Therefore, only one laboratory, "Medical Laboratories Passau," was selected for this case study.

To conduct calculations, coordinates were converted to meters, considering that one degree equals approximately 111,111 meters.

For determining potential drone base locations, we decided to establish a rectangular grid. The size of the rectangle was estimated using the minimal and maximal longitudes and latitudes of all doctor's offices and the laboratory. We assumed that every 200 meters, there exists a potential drone base location. Since Passau is divided by three rivers, we manually excluded locations that were on the rivers' surface. Moreover, we dropped the locations which were situated at the Austrian territory. To sum up, we ended up with 1048 potential drone bases: 77 doctor's offices, 1 laboratory and 970 other locations.

After that, two-dimensional Euclidean distances between points of three data sets were calculated.

The drone infrastructure maps are presented below.

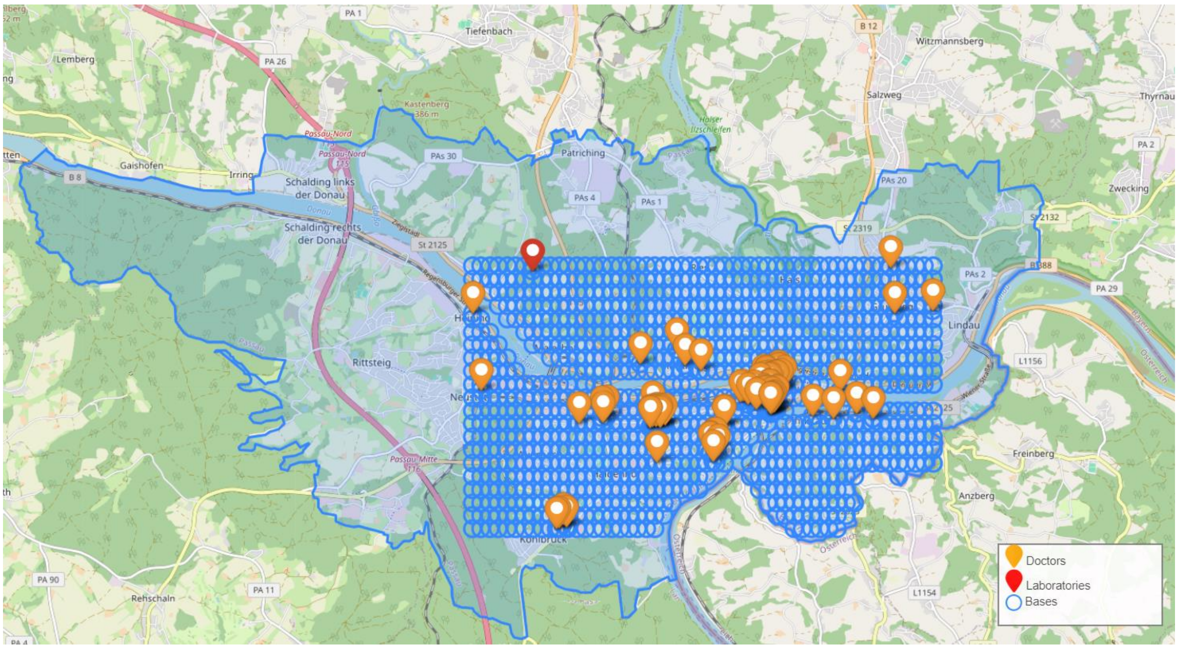

Figure 2. The map of Passau with an initial infrastructure plan

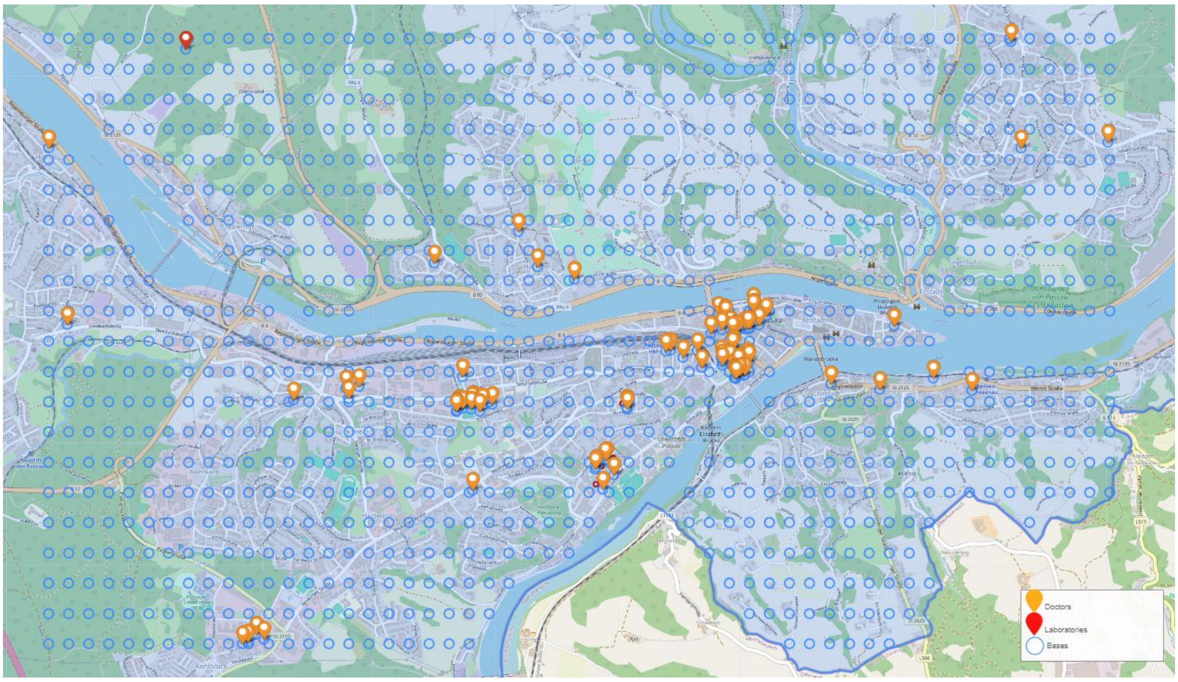

Figure 3. The map of Passau with an initial infrastructure plan

The parameters of the drones which were selected for the project as well as costs of drone bases are calculated in the table below. All the measures were calculated in metres and euros. Due to privacy concerns, we have no access to historical data to estimate the Poisson λ_i. Therefore, we randomly assign lambdas in the interval [0;10] to doctor's offices, which seems to be a reasonable estimation of demand for emergency specimen analysis.

Table 7. Calculation of parameters of drones and drone bases for the case study in Passau

Parameter	Calculation
λ_i	Randomly (uniformly) from [1; 10], fixed for all instances for a fair comparison
B	Speed of a drone 17 m/s × (90 × 60 s of travel) = 91 800 m
S	Speed of a drone 17 m/s × 60 s × {1; 5; 10 min} = {1 020; 5 100; 10 200 m}
α	Purchasing cost of a drone 14 760 € + transport box 500 € + spare battery 640 € = 15 900 €
$\beta_{j\in J\setminus\{I\cup K\}}$	Rent with extra costs 15 €/m^2 × free space 100 m^2 × 12 months + salary of 3 drone technicians 55 000 € × 3 + equipment 20 000 € = 203 000 €
$\beta_{j\in\{I\cup K\}}$	Rent with extra costs 8 €/m^2 × free space 20 m^2 × 12 months + salary of a drone technician 55 000 € + equipment 20 000 € = 76 920 €
γ	0.0045 €/km / 1000
$\beta_{j\in J\setminus\{I\cup K\}}$	3 drones/m^2 (using shelves) × shelf space (100 – 15) m^2 = 255 drones
$\beta_{j\in\{I\cup K\}}$	3 drones/m^2 (using shelves) × shelf space (20 – 5) m^2 = 45 drones
p	{0.97; 0.98; 0.999}
Computational time limit	600 seconds

All the following calculations are performed using Gurobi 10.0.3 – Python API.

Results

Unfortunately, only the models with $S = 1020$ could be solved to optimality during the time limit of 600 seconds. However, the largest gap is only 0.5664% (Figure 4). The most computationally expensive model is a probabilistic model without relaxation followed by a probabilistic model with relaxation (Figure 5). It took maximum 200 seconds for the models to reach the corresponding gap.

Detailed computational results for the case of Passau are presented in Appendix 1.

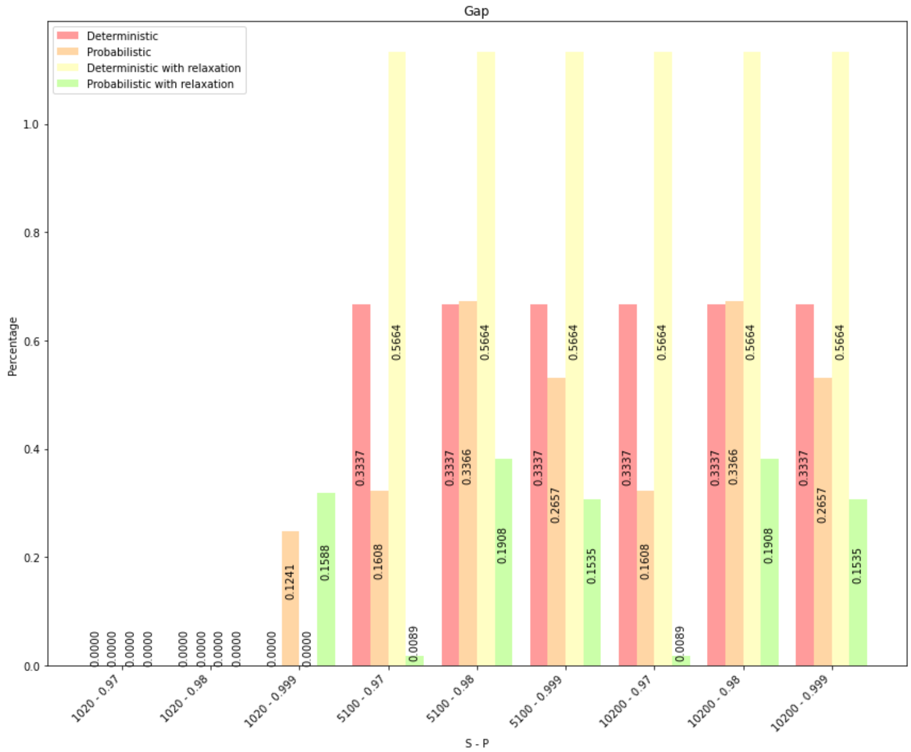

Figure 4. Optimality gap

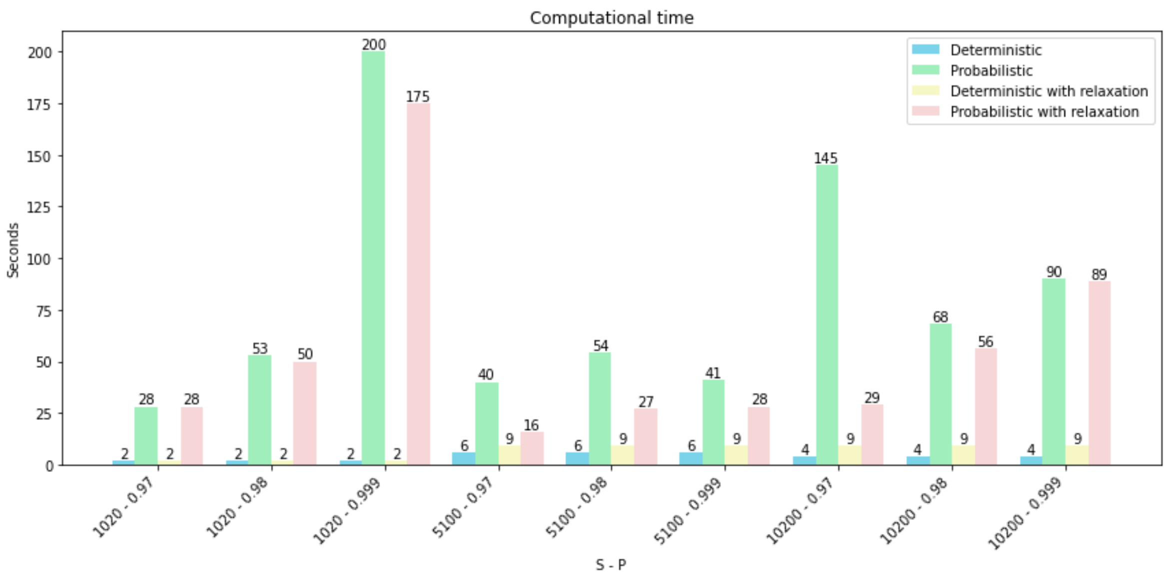

Figure 5. Computational time

As it was expected, the number of drones is rapidly increasing with a higher reliability level (Figure 6). Below is the solution for the case with $S = 5100, p = 0.97$ and $p = 0.999$ (Figure 7, Figure 8). As we can see, the increasing number of drones requires additional base locations.

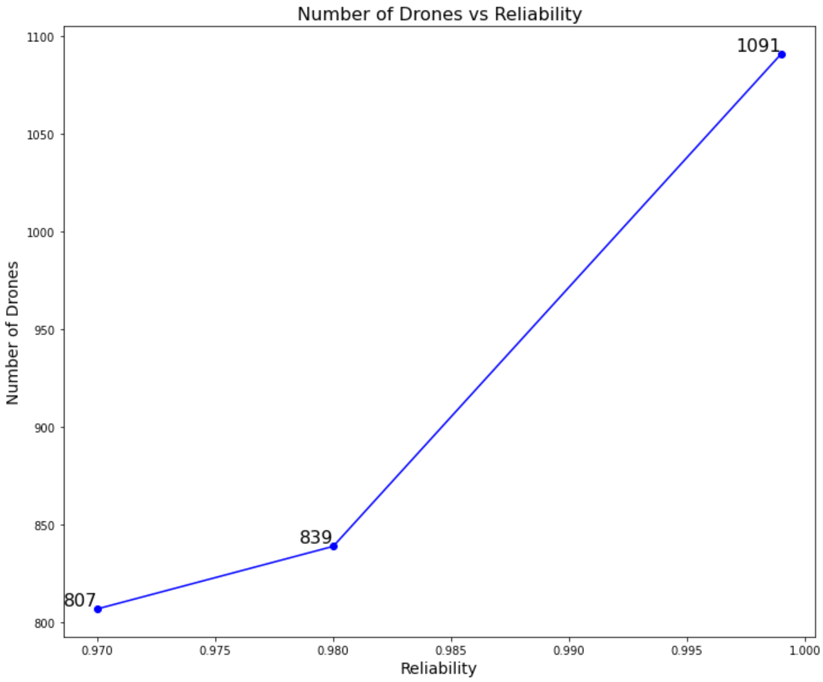

Figure 6. Number of drones – Reliability trade-off

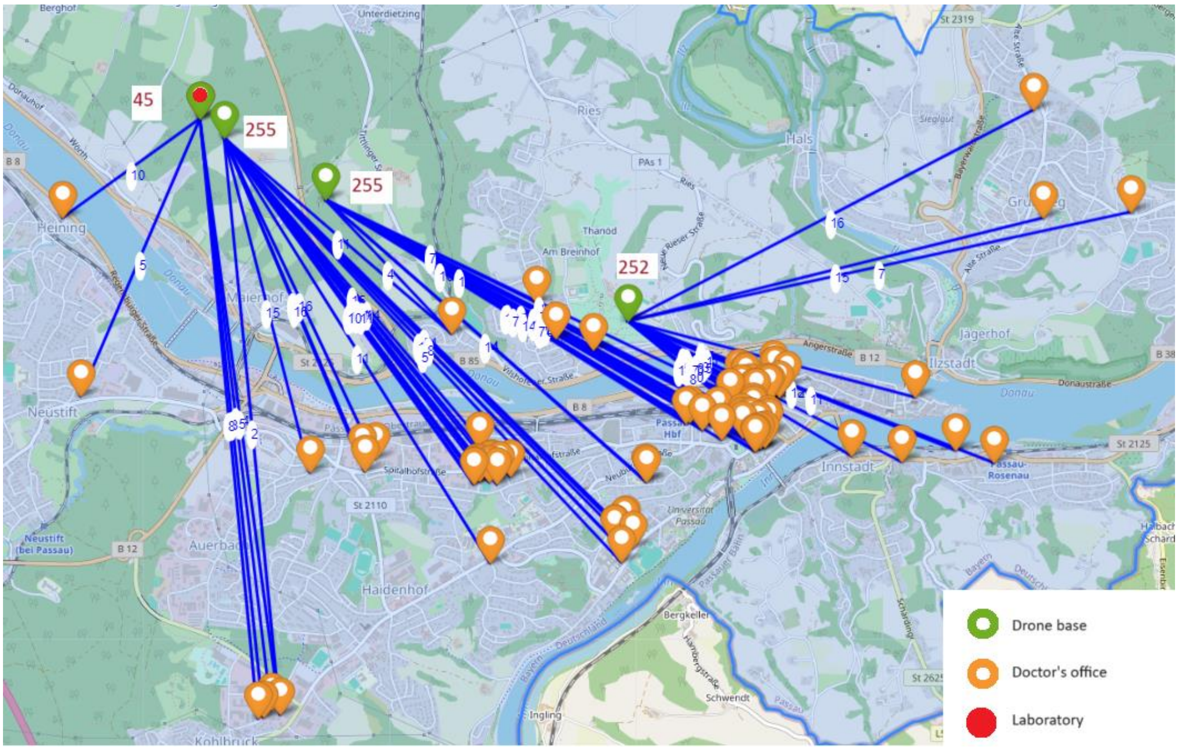

Figure 7. Probabilistic model without relaxation with $S = 5100, p = 0.97$

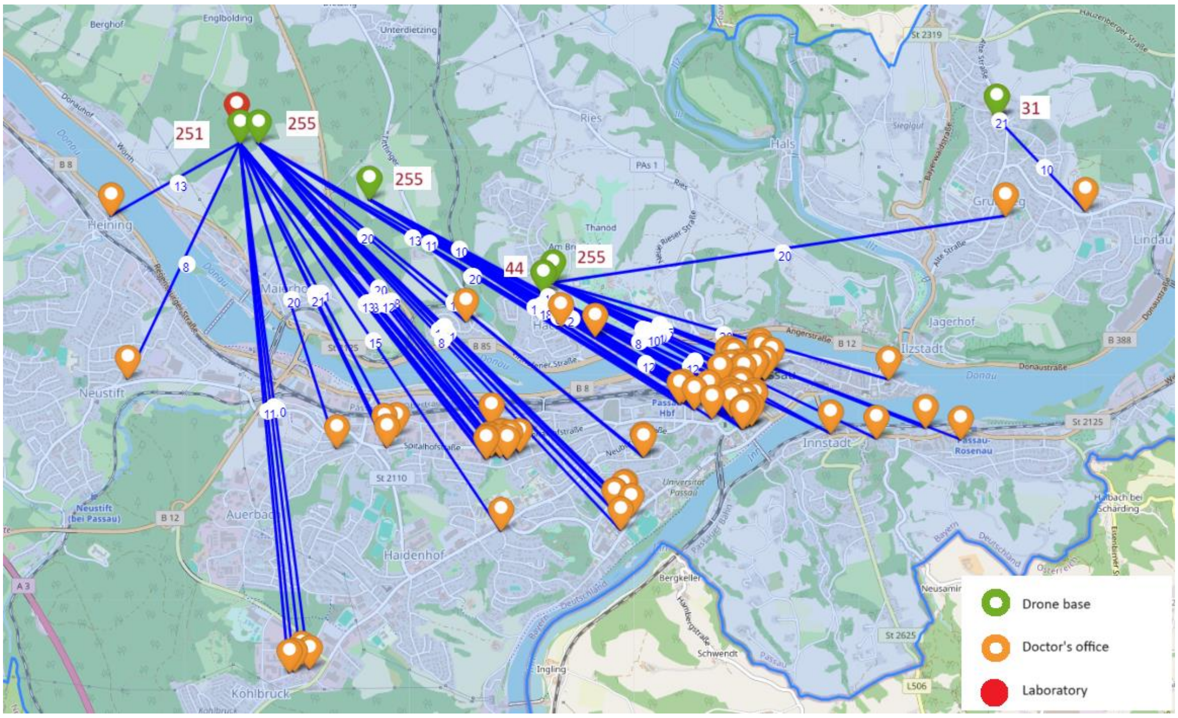

Figure 8. Probabilistic model without relaxation with $S = 5100, p = 0.999$

The number of locations is increasing with the reliability level and is declining with growing S (Figure 9). The distances in Passau are not so large, that is why the solutions and objective values are similar both for $S = 5100$ and $S = 10200$. However, if we calculate the model for Passau district, the service time will matter. This similarity indicates that the coverage zone for this type of drones allows us to conduct the project for the whole district.

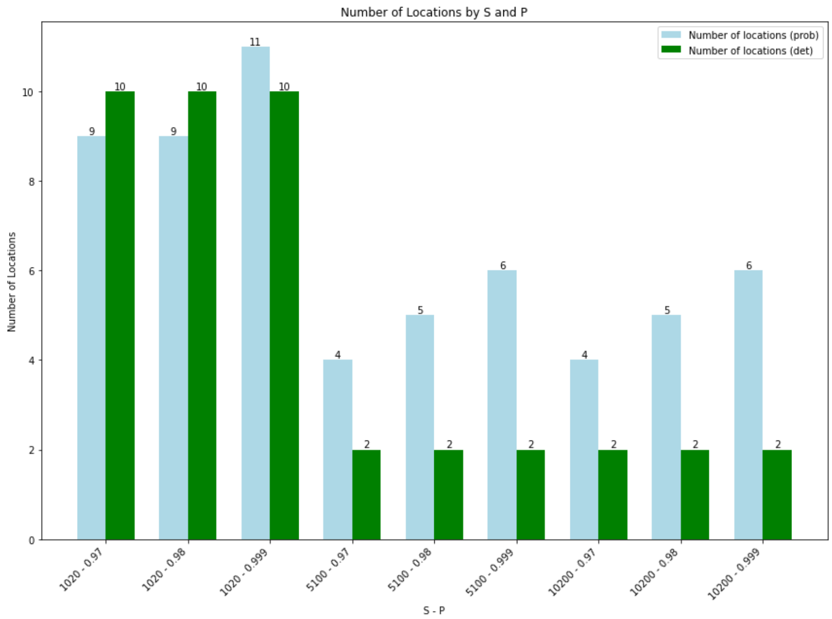

Figure 9. Number of locations

The price of uncertainty is calculated as a difference between objective values for deterministic and corresponding probabilistic models. Obviously, the higher the reliability, the higher is the price of uncertainty. For $p = 0.999$, the difference is about 10,7 million euros (Figure 10).

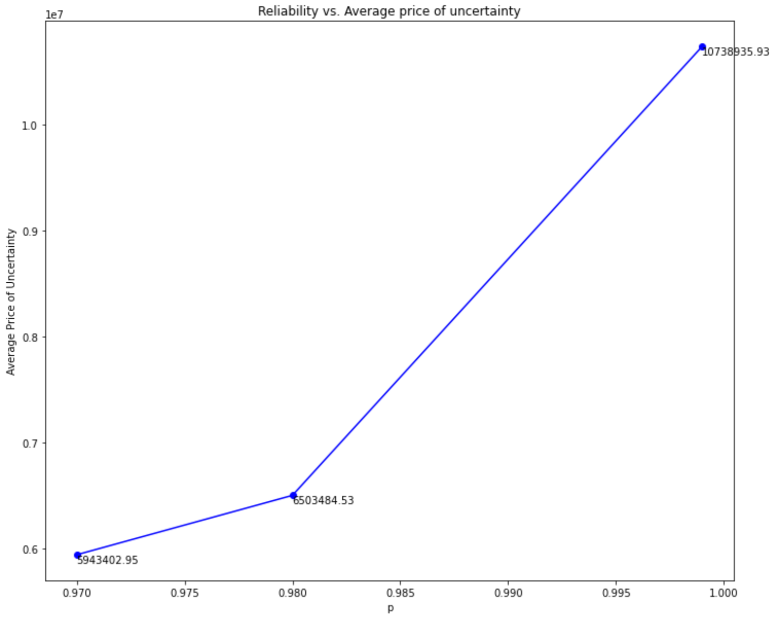

Figure 10. Reliability – Average price of uncertainty trade off

An interesting observation is that the models tend to put the locations near the laboratory with increasing service time (Figure 11, Figure 12). As it was previously mentioned, it indicates that using a cost objective solely can be inappropriate in designing an EMS system since it does not minimize the service time, which is crucial for saving lives. A multi-objective approach, introduction of a reasonable penalty on the distance from the base to doctor's office or a service time parameter S applied for the entire area or for each demand point or region separately could be a solution for this issue. However, the optimal trade-off between service time and costs should be determined by the experts.

Consequently, it seems to be a reasonable solution to choose the laboratory as a drone base location. The models do not always do it because of the assumption that the laboratory's capacity is equally limited as the capacity of a doctor's office (in our case, 45 drones). This assumption can be relaxed in practice, depending on the laboratory and free space in it, which can improve the cost function.

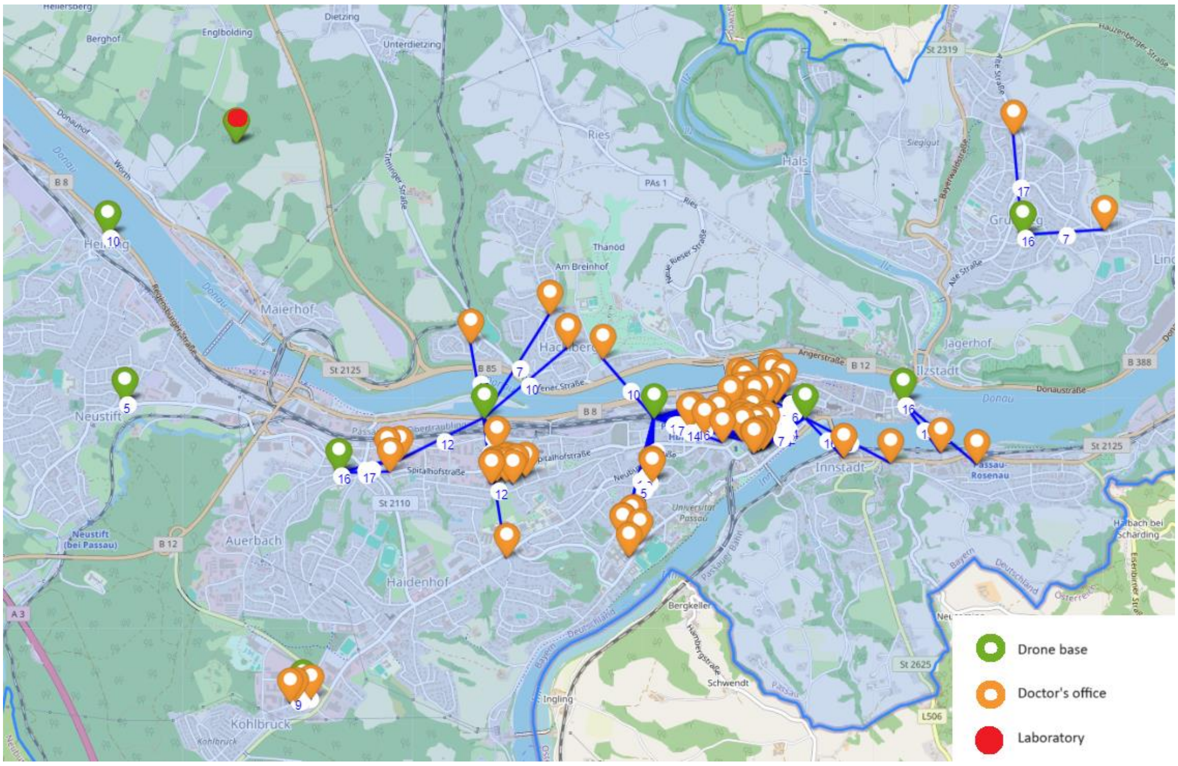

Figure 11. Probabilistic model with relaxation with $S = 1020, p = 0.98$

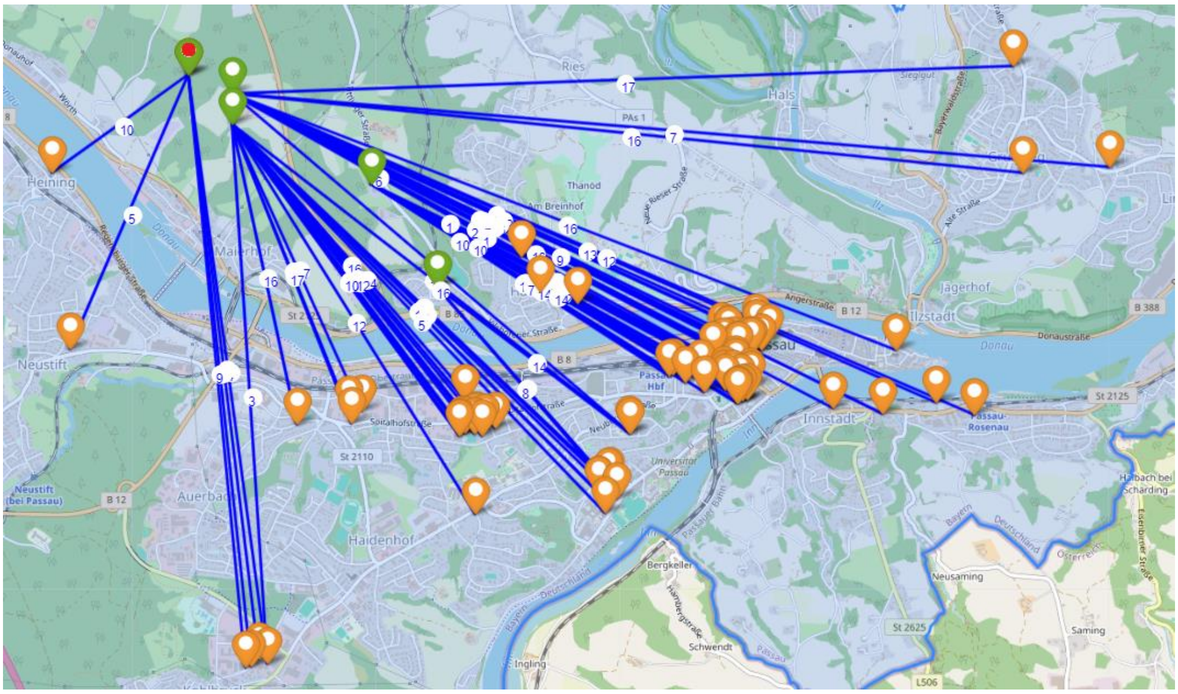

Figure 12. Probabilistic model with relaxation with $S = 10200, p = 0.98$

In general, the probabilistic models obtain similar solutions to deterministic cases so that the demand can be covered (for example, Figure 13, Figure 14). To prove the robustness of the model, we conduct a simulation where λ_i is chosen randomly and the deterministic case and random variables θ_i are generated according to it. The other parameters remain the same. After that, we compute the difference between the overall number of drones of the deterministic and corresponding probabilistic model. The results are presented in Figure 15.

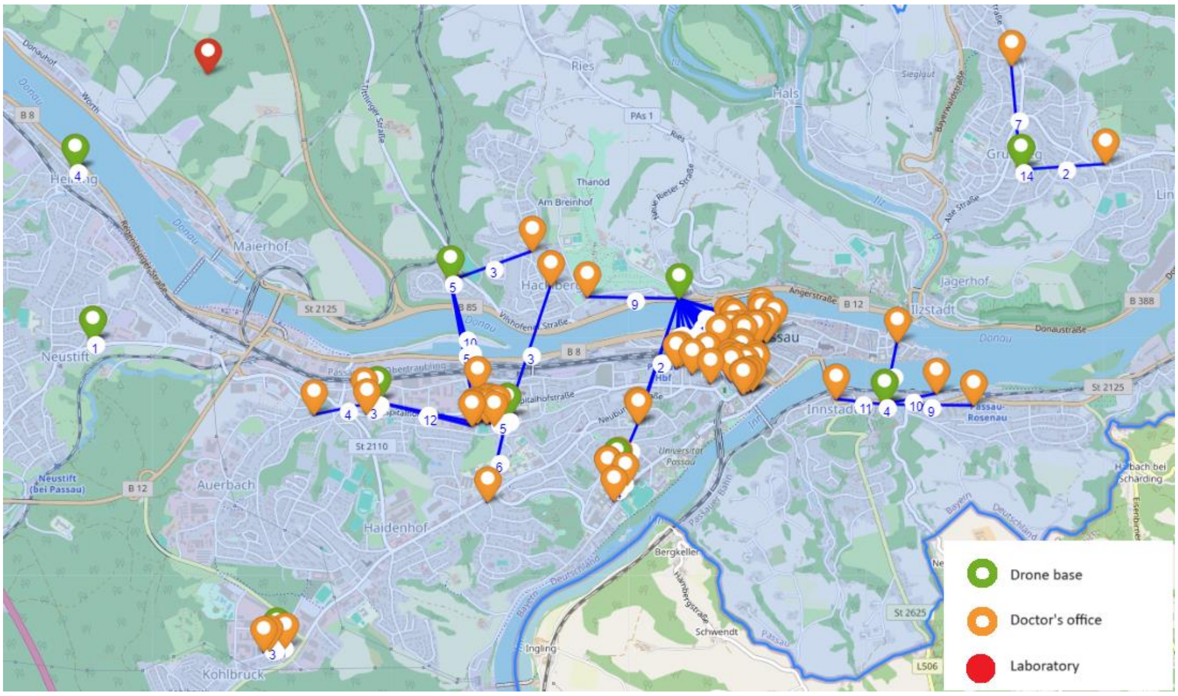

Figure 13. Deterministic model without relaxation with $S = 1020, p = 0.999$

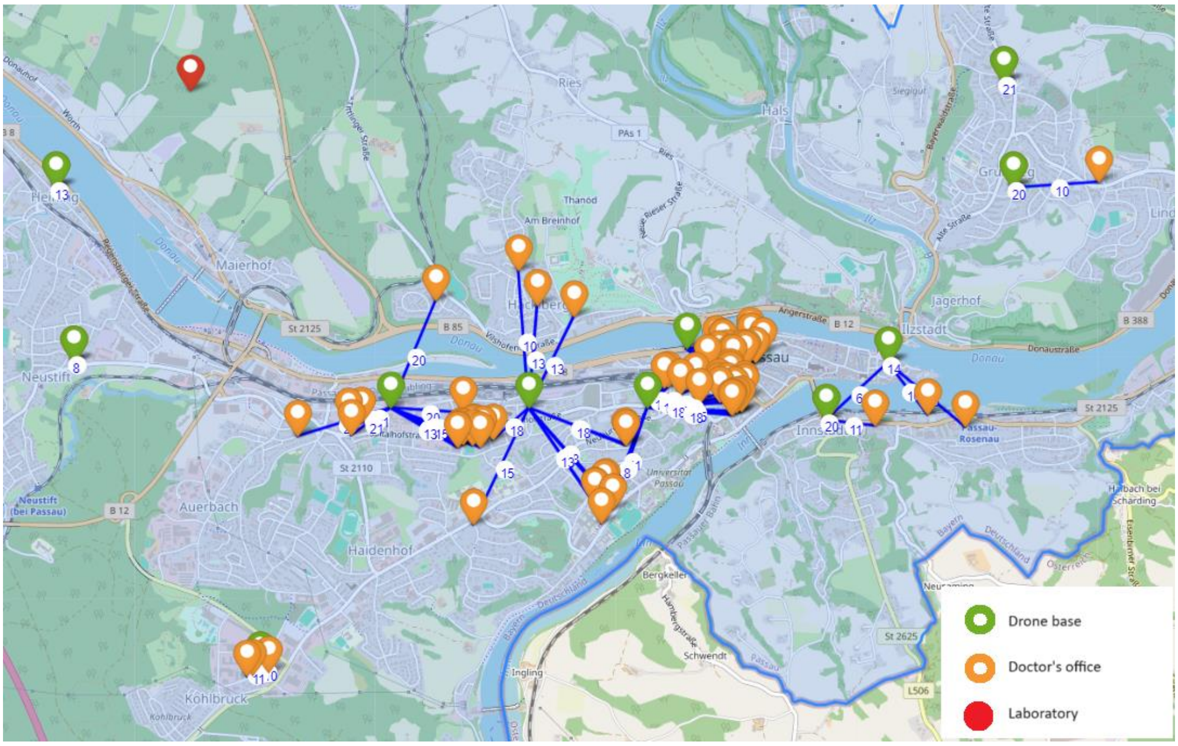

Figure 14. Probabilistic model without relaxation with $S = 1020, p = 0.999$

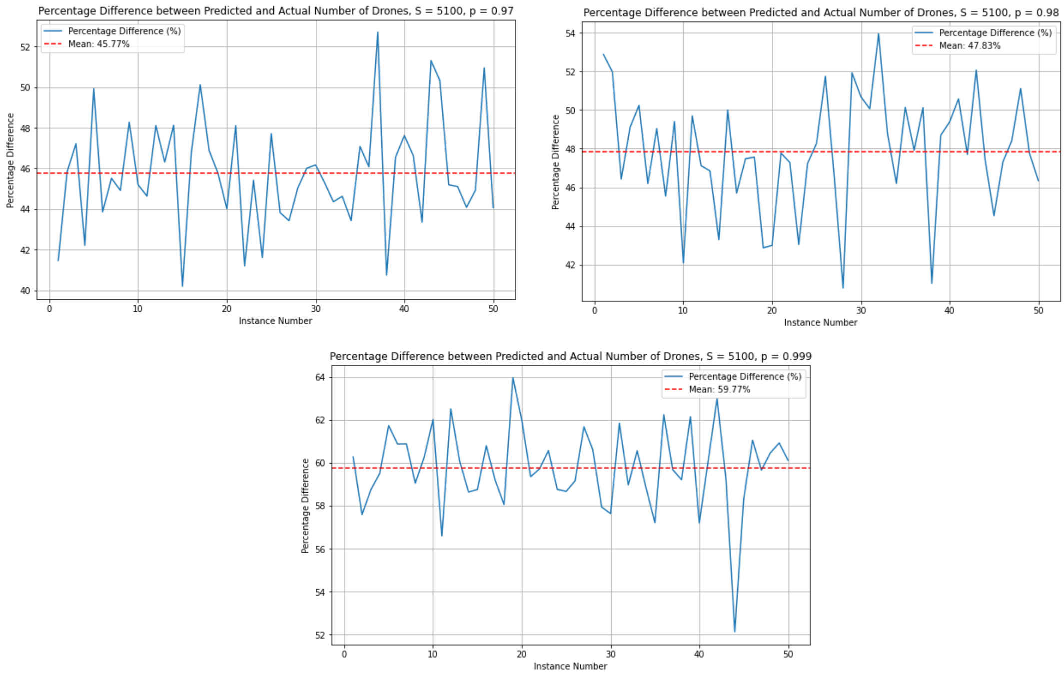

Figure 15. The percentage differences between probabilistic and deterministic models for 50 random instances (example of $S = 5100, p = \{0.97; 0.98; 0.999\}$, probabilistic model without relaxation).

The results indicate that for this particular example the probabilistic model for $p = 0.97$ produces 46% more drones than it was required, 48% - for $p = 0.98$ and 60% for $p = 0.99$. Although there should be some "safety fleet", the probabilistic models tend to overestimate the required number of drones, although they are less conservative that, for example, double coverage models. Since the drones are reserved for each doctor's office i, it is not correct to compare the overall number of drones. However, if we look at the original data, we see that in almost all cases the need for drones for each doctor's office is covered. Few cases where it is not the case can be compensated with other drones from other locations.

Conclusion

In this paper, a drone system for delivering emergent specimens from doctor's offices to laboratories was designed. The problem is formulated as a facility location and fleet dimensioning problem. To our best knowledge, this setting is unique in the literature. Four models were proposed: The probabilistic model without relaxation, the probabilistic model with relaxation, and their deterministic underlying models. Joint chance constraints, posing a probability level on the entire area with which the demand should be covered, were introduced, and their deterministic equivalents based on p-efficient points were derived, based on Beraldi et al. (2004) and Dentcheva (2000). Computational experiments were conducted on the case of Passau in Germany.

Higher reliability levels come at a cost, as the number of drones and locations increases. A higher threshold on service (reaction) time reduces the number of locations. The models serve as a strategic tool to design an EMS system, which has the consequence that the model overestimates the number of drones. Moreover, for the case of Passau, the model was not solved to optimality, despite preprocessing techniques. However, the gaps were small. A new solution algorithm is required to solve the model to optimality. Finally, the model tends to put the locations closer to the laboratories, which has the consequence that the service time becomes larger. It indicates that solely minimizing costs in the objective is not an appropriate measure for designing EMS.

Further research could include the development of a solution algorithm for the current problem, formulating alternative relocation models with drone tracking (for example, dynamic models), multi-objective optimization models, and models utilizing other ways of incorporating uncertainty (robust optimization, fuzzy programming, queuing paradigm).

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Appendix

Appendix 1

Table 1. Computational results for the case of Passau

Parameters		Det				
S	p	Objective	Gap	Time	Number of drones	Number of locations
1020	0,97	8.002.601,83	0,0000%	2	447	10
1020	0,98	8.002.601,83	0,0000%	2	447	10
1020	0,999	8.002.601,83	0,0000%	2	447	10
5100	0,97	7.513.321,53	0,6675%	6	447	2
5100	0,98	7.513.321,53	0,6675%	6	447	2
5100	0,999	7.513.321,53	0,6675%	6	447	2
10200	0,97	7.513.321,45	0,6675%	4	447	2
10200	0,98	7.513.321,45	0,6675%	4	447	2
10200	0,999	7.513.321,45	0,6675%	4	447	2
Parameters		**DetRel**				
S	p	Objective	Gap	Time	Number of drones	Number of locations
1020	0,97	8.002.601,83	0,0000%	2	447	10
1020	0,98	8.002.601,83	0,0000%	2	447	10
1020	0,999	8.002.601,83	0,0000%	2	447	10
5100	0,97	7.590.241,49	1,1327%	9	447	3
5100	0,98	7.590.241,49	1,1327%	9	447	3
5100	0,999	7.590.241,49	1,1327%	9	447	3
10200	0,97	7.590.241,43	1,1327%	9	447	3
10200	0,98	7.590.241,43	1,1327%	9	447	3
10200	0,999	7.590.241,43	1,1327%	9	447	3
Parameters		**Prob**				
S	p	Objective	Gap	Time	Number of drones	Number of locations
1020	0,97	13.901.858,81	0,0000%	28	807	9
1020	0,98	14.410.660,43	0,0000%	53	839	9
1020	0,999	18.697.391,41	0,2482%	200	1091	11
5100	0,97	13.517.257,44	0,3217%	40	807	4
5100	0,98	14.102.978,99	0,6731%	54	839	5
5100	0,999	18.312.790,61	0,5314%	41	1091	6
10200	0,97	13.517.257,38	0,3217%	145	807	4
10200	0,98	14.102.978,92	0,6731%	68	839	5
10200	0,999	18.312.790,58	0,5314%	90	1091	6

Parameters		ProbRel				
S	p	Objective	Gap	Time	Number of drones	Number of locations
1020	0,97	13.901.858,81	0,0000%	28	807	9
1020	0,98	14.410.660,43	0,0000%	50	839	9
1020	0,999	18.697.391,41	0,3177%	175	1.091	11
5100	0,97	13.517.257,45	0,0177%	16	807	4
5100	0,98	14.102.978,99	0,3817%	27	839	5
5100	0,999	18.312.790,62	0,3070%	28	1.091	6
10200	0,97	13.517.257,38	0,0177%	29	807	4
10200	0,98	14.102.978,94	0,3817%	56	839	5
10200	0,999	18.312.790,52	0,3070%	89	1.091	6