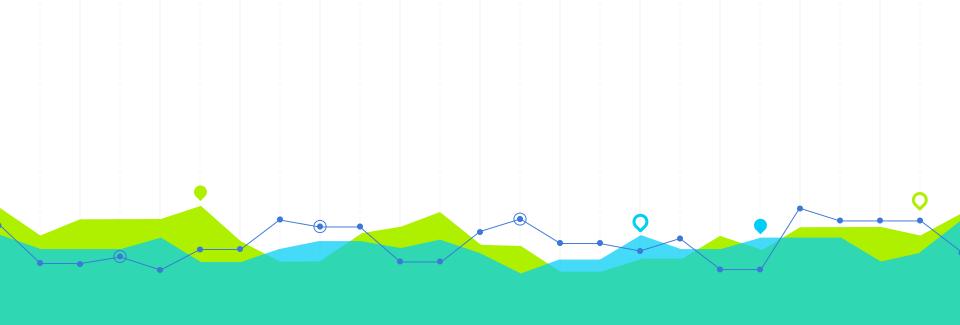
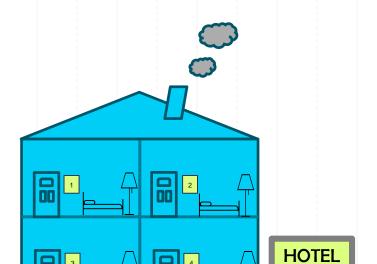


PARTIAL INFORMATION

Aleksandra Petrenko, Maya Marten

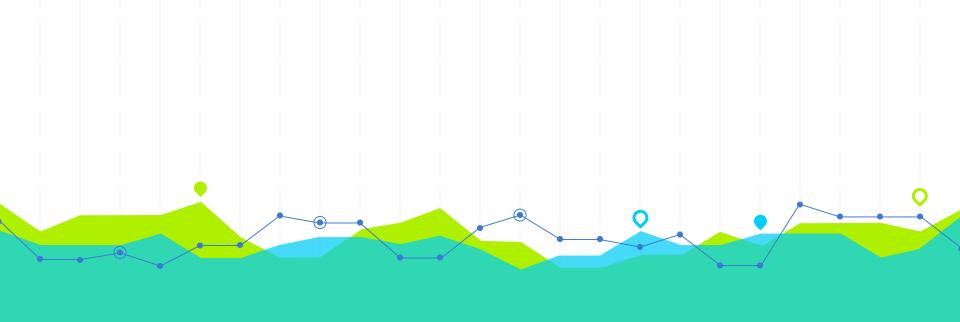


Motivation



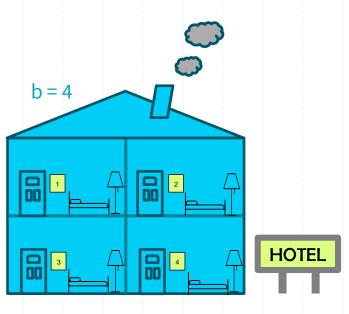
Your hotel

- Imagine you own this hotel
- 4 rooms to rent out
- You get requests from different types of customers
- → Which customers do you accept?



Partially predictable demand arrival model

Notations (1) - Resources



- Single resource
 - Hotel rooms
- Identical units
 - Same equipment
- → **b**: units of the product to sell

Notations (2) - Periods

- n: number of periods the product is sold
 - $-n \geq b$
- \bigcirc λ : time steps of the time horizon, normalized to 1:

$$\lambda = \frac{1}{n}, \frac{2}{n}, \dots, 1$$

Notations (3) - Types of Customers



Type-1 Customer

Pays \$1 for the room



Type-2 Customer

Pays \$a for the room (0<a<1)</p>



No Customer





Sequence of arrival (1)

- The demand realizes sequentially
- At each time $\lambda = \frac{1}{n}, \frac{2}{n}, \dots, 1$ we either face
 - a Type-1 Customer
 - a Type-2 Customer
 - no Customer
- At this moment we have to decide whether to accept or to reject the customer

Sequence of arrival (2)

- The sequence of arrival is denoted by $\vec{v} = (v_1, v_2, ..., v_n)$, with $v_i \in \{0, a, 1\}$
 - The customers are represented by the revenue they generate if accepted

Goal

Goal: Maximize Revenue

How to predict demand? (1)

Adversarial (unpredictable) demand

- Demand is unpredictable
- Worst-case approach
- Demand is controlled by an imaginary adversary

Stochastic (predictable) demand

- Demand is predictable
- Demand follows a known distribution

How to predict demand? (2) – Assessment

Adversarial (unpredictable) demand

- Demand can't be perfectly predicted
- Worst-case approach often results in **too conservative** online policies

Stochastic (predictable) demand

- If demand can be predicted, making online decisions incurs little loss
- Demand can't be perfectly predicted

→ Both approaches have their downsides!

Partially predictable demand model (1)

- Combine the 2 approaches:
 - 1. Adversary component
 - 2. Stochastic component
- lacktriangledown Parameter $m{p}$, with 0 , specifies predictability of demand in the model
 - Extreme case $p = 0 \rightarrow pure Adversarial model$
 - Extreme case $p = 1 \rightarrow pure Stochastic model$

Partially predictable demand model (2)

An imaginary adversary first sets an initial sequence

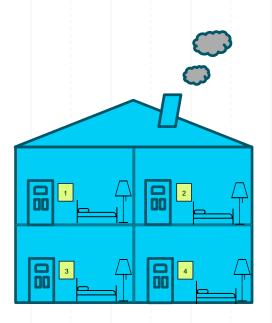
$$\overrightarrow{v_I} = (v_{I,1}, v_{I,2}, \dots v_{I,n})$$
 with $v_{I,j} \in \{0, a, 1\}$, for $1 \leq j \leq n$

- A subset S of customers (p%) represents the stochastic group
 - won't follow the order determined by the adversary
- A subset A of customers ((1-p)%) represents the adversarial group
 - will follow the order determined by the adversary

Partially predictable demand model (3)

- Customers in S
 - join the subset independently
 - with the same propability *p*
 - Are permuted uniformly at random among themselves: $\sigma_S: S \to S$

Example 1 a)



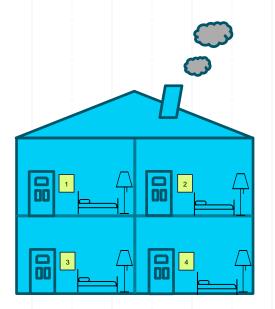
- \odot 4 rooms to allocate: b = 4
- We start allocating our hotel rooms 10 days in advance, we expect at most 1 customer per day: n = 10
- We have historical data and assume, that 50% of our guests are predictable and permuted uniformly: p = 0.5
- Type-2 Customers are willing to pay 0.6\$ for the room: a = 0.6



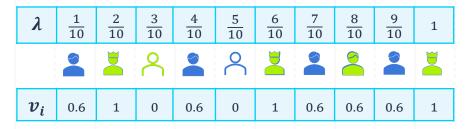




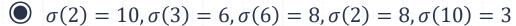
Example 1 a)

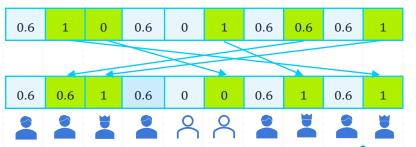


Customers arrive in the following order:



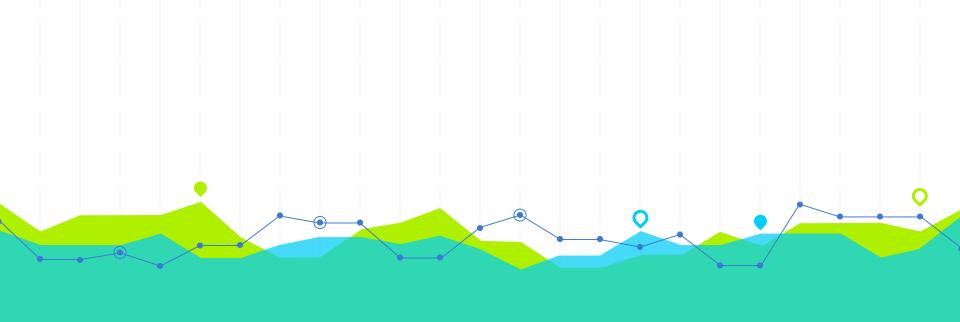
We assume:





Further Proceeding





Competitive Ratio

How to assess the performance of online algorithms?

- Competitive ratio
 - Worst-case ratio between the revenue of the online scheme to that of a clairvoyant solution
- An online algorithm is c-competitive in the proposed partially predictable model if for any adversarial instance $\overrightarrow{v_I}$, $\mathbb{E}[ALG(\overrightarrow{V})] \geq c \cdot OPT(\overrightarrow{v_I})$

 n_1 :number of Type-1 Customers in the sequence n_2 :number of Type-2 Customers in the sequence b: units of the product to sell

Optimal offline solution $OPT(\vec{v})$: Strategy

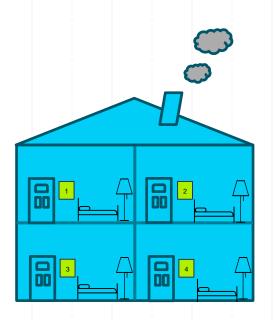
- Accept as many Type-1 Customers as possible $\min\{n_1, b\}$
- If $n_1 < b$: Allocate the remaining units to as many Type-2 Customers as possible $\min\{n_2, b n_1\}$

$$\rightarrow$$
 OPT(\vec{v}) = min{n₁, b} + a · min{n₂, (b - n₁)⁺}

 $(b-n_1)^+ \triangleq \max(b-n_1,0)$

 n_1 :number of Type-1 Customers in the sequence n_2 :number of Type-2 Customers in the sequence b: units of the product to sell

Example OPT(\vec{v})



$$\lambda$$
 $\frac{1}{10}$ $\frac{2}{10}$ $\frac{3}{10}$ $\frac{4}{10}$ $\frac{5}{10}$ $\frac{6}{10}$ $\frac{7}{10}$ $\frac{8}{10}$ $\frac{9}{10}$ 1

 v_i 0.6 0.6 1 0.6 0 0 0 0.6 1 0.6 1

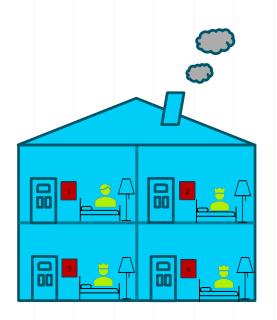
$$OPT(\vec{v}) = min\{n_1, b\} + a \cdot min\{n_2, (b - n_1)^+\}$$

$$a = 0.6$$
 $b = 4$ $n_1 = 3$ $n_2 = 5$

OPT(
$$\vec{v}$$
) = min{3,4} + 0.6 · min{5,(4-3)⁺}
= 3 + 0.6 · min{5,1}
= 3 + 0.6 · 1
= 3 + 0.6
= 3.6

 n_1 :number of Type-1 Customers in the sequence n_2 :number of Type-2 Customers in the sequence b: units of the product to sell

Example OPT(\vec{v})



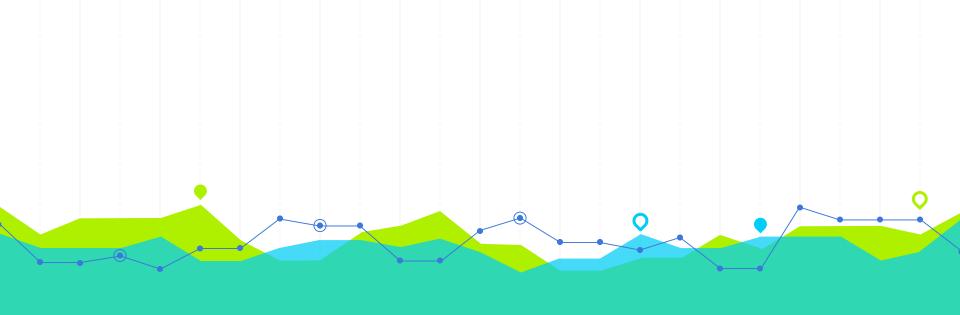
$$\lambda$$
 $\frac{1}{10}$ $\frac{2}{10}$ $\frac{3}{10}$ $\frac{4}{10}$ $\frac{5}{10}$ $\frac{6}{10}$ $\frac{7}{10}$ $\frac{8}{10}$ $\frac{9}{10}$ 1

 v_i 0.6 0.6 1 0.6 0 0 0 0.6 1 0.6 1

$$OPT(\vec{v}) = min\{n_1, b\} + a \cdot min\{n_2, (b - n_1)^+\}$$

$$a = 0.6$$
 $b = 4$ $n_1 = 3$ $n_2 = 5$

OPT(
$$\vec{v}$$
) = min{3,4} + 0.6 · min{5,(4-3)⁺}
= 3 + 0.6 · min{5,1}
= 3 + 0.6 · 1
= 3 + 0.6
= 3.6



Non-adaptive algorithm

Non-adaptive algorithm: Basics

For each time period

- Always accept a T1 customer
- **Evolving threshold rule:** Accept a T2 customer, if the number of accepted T1 and T2 customers by the evolving threshold does not exceed $\lfloor \lambda pb \rfloor$
- Otherwise, **fixed threshold rule**: accept a T2 customer, if the number of T2 customers accepted by the fixed threshold does not exceed $\lfloor \theta b \rfloor$, where $\theta = \frac{1-p}{2-a}$
- Otherwise, reject T2 customer
- Repeat until there is no more inventory or no more customers

Non-adaptive algorithm: Pseudocode

1: Initialize:

$$q_{1} \leftarrow 0$$

$$q_{2,e} \leftarrow 0$$

$$q_{2,f} \leftarrow 0$$

$$rem.inv \leftarrow b$$

$$\theta \triangleq \frac{1-p}{2-a}$$
2: for $\lambda = \frac{1}{n}$ to 1 do
3: $i = \lambda \cdot n$
4: if $rem.inv > 0$ then
5: if $v_{i} = 1$ then
6: $q_{1} \leftarrow q_{1} + 1$
7: $rem.inv \leftarrow rem.inv - 1$

Non-adaptive algorithm: Pseudocode

```
else if v_i = a and q_1 + q_{2,e} < \lfloor \lambda pb \rfloor then
 8:
                    q_{2,e} \leftarrow q_{2,e} + 1
 9:
                     rem.inv \leftarrow rem.inv - 1
10:
                else if v_i = a and q_{2,f} < \lfloor \theta b \rfloor then
11:
                    q_{2,f} \leftarrow q_{2,f} + 1
12:
                     rem.inv \leftarrow rem.inv - 1
13:
                end if
14:
          end if
15:
```

16: **end for**

Competitive ratio

$$p + \frac{1-p}{2-a} + O\left(\frac{1}{a(1-p)p}\sqrt{\frac{\log n}{b}}\right)$$

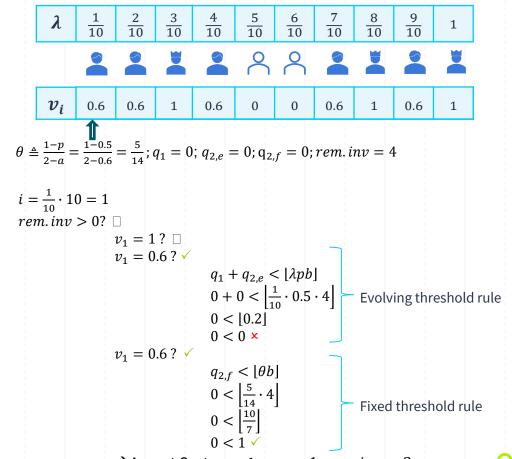


Algorithm 1 Non-adaptive Algorithm ALG_1

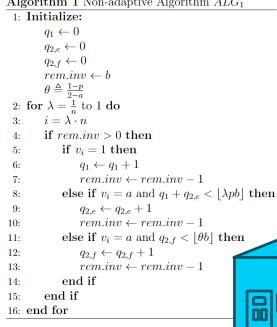
```
1: Initialize:
          q_1 \leftarrow 0
         q_{2,e} \leftarrow 0
          q_{2,f} \leftarrow 0
          rem.inv \leftarrow b
         \theta \triangleq \frac{1-p}{2-a}
 2: for \lambda = \frac{1}{n} to 1 do
          i = \lambda \cdot n
          if rem.inv > 0 then
               if v_i = 1 then
                    q_1 \leftarrow q_1 + 1
                    rem.inv \leftarrow rem.inv - 1
               else if v_i = a and q_1 + q_{2,e} < |\lambda pb| then
                    q_{2,e} \leftarrow q_{2,e} + 1
                    rem.inv \leftarrow rem.inv - 1
10:
               else if v_i = a and q_{2,f} < |\theta b| then
11:
                    q_{2,f} \leftarrow q_{2,f} + 1
12:
                    rem.inv \leftarrow rem.inv - 1
13:
              end if
14:
          end if
15:
16: end for
```



$$\lambda = \frac{1}{10}$$
:

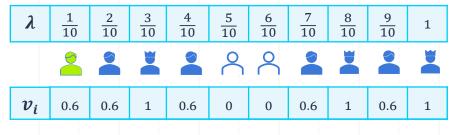


Algorithm 1 Non-adaptive Algorithm ALG_1



Initialize:

$$\lambda = \frac{1}{10}$$
:



$$\theta \triangleq \frac{1-p}{2-a} = \frac{1-0.5}{2-0.6} = \frac{5}{14}$$
; $q_1 = 0$; $q_{2,e} = 0$; $q_{2,f} = 0$; rem. $inv = 4$

$$i = \frac{1}{10} \cdot 10 = 1$$

rem. inv > 0?
$$\Box$$
 $v_1 = 1$? \Box

$$v_{1} = 0.6? \checkmark$$

$$q_{1} + q_{2,e} < \lfloor \lambda pb \rfloor$$

$$0 + 0 < \left\lfloor \frac{1}{10} \cdot 0.5 \cdot 4 \right\rfloor$$

$$0 < \lfloor 0.2 \rfloor$$

$$0 < 0 \times$$

$$v_{1} = 0.6? \checkmark$$

$$a_{2,\epsilon} < |A|$$

$$q_{2,f} < \lfloor \theta b \rfloor$$

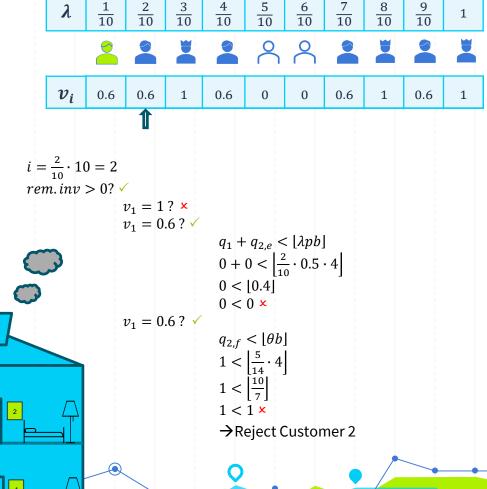
$$0 < \left\lfloor \frac{5}{14} \cdot 4 \right\rfloor$$

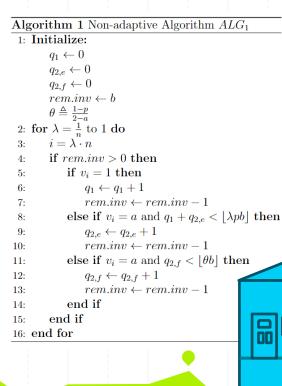
$$0 < \left\lfloor \frac{10}{7} \right\rfloor$$

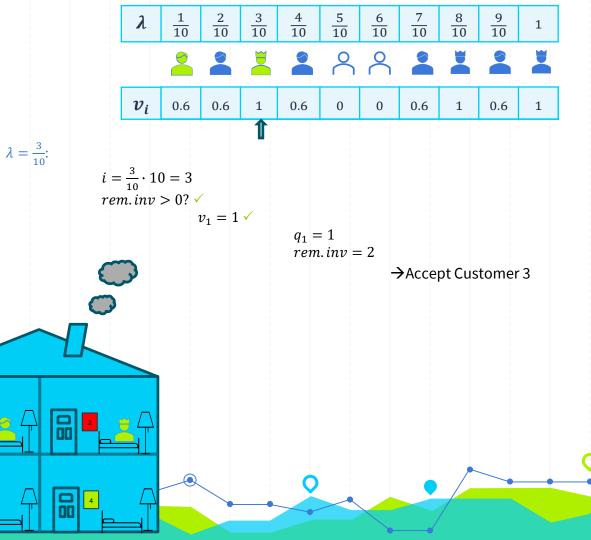
$$0 < 1 \checkmark$$

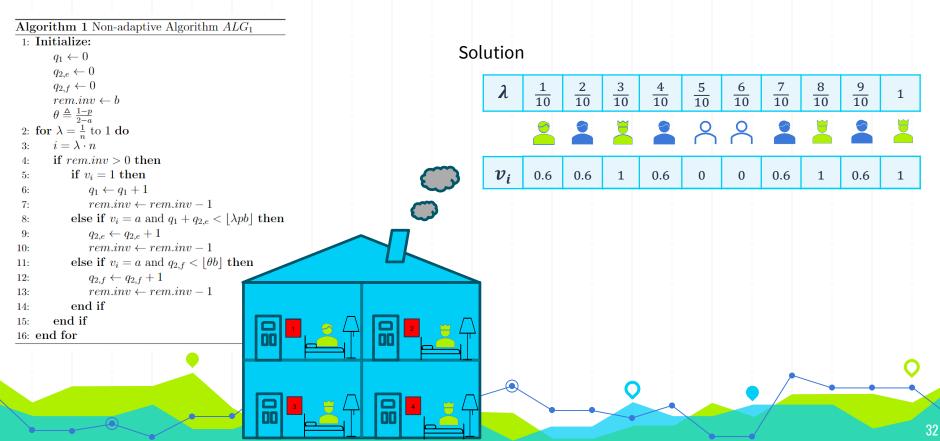
$$\rightarrow$$
Accept Customer 1: $q_{2,f} = 1$, rem. inv = 3

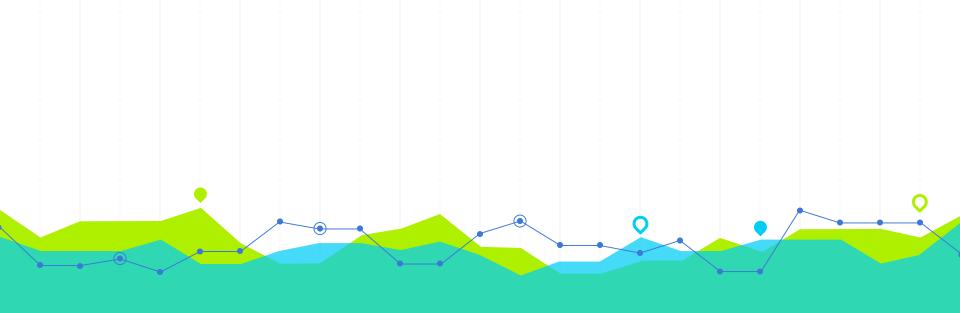
Algorithm 1 Non-adaptive Algorithm ALG_1 1: Initialize: $\lambda = \frac{2}{10}$: $q_1 \leftarrow 0$ $q_{2,e} \leftarrow 0$ $q_{2,f} \leftarrow 0$ $rem.inv \leftarrow b$ $\theta \triangleq \frac{1-p}{2-a}$ 2: for $\lambda = \frac{1}{n}$ to 1 do $i = \lambda \cdot n$ if rem.inv > 0 then if $v_i = 1$ then $q_1 \leftarrow q_1 + 1$ $rem.inv \leftarrow rem.inv - 1$ else if $v_i = a$ and $q_1 + q_{2,e} < \lfloor \lambda pb \rfloor$ then $q_{2,e} \leftarrow q_{2,e} + 1$ $rem.inv \leftarrow rem.inv - 1$ 10: 11: else if $v_i = a$ and $q_{2,f} < \lfloor \theta b \rfloor$ then 12: $q_{2,f} \leftarrow q_{2,f} + 1$ $rem.inv \leftarrow rem.inv - 1$ 13: end if 14: 15: end if 16: end for











Adaptive algorithm 1

Adaptive algorithm: Basics

For each time period

- Introduce $c \in [0,1]$
- Always accept a T1 customer
- **©** First condition: Accept a T2 customer, if $u_{1,2}(\lambda) < b$
- Otherwise, second condition: accept a T2 customer, if

$$q_2 > [\Phi b + c(b - u_1(\lambda))^+], \text{ where } \Phi = \frac{1-c}{1-a}$$

- Otherwise, reject a T2 customer
- Repeat until there is no more inventory or no more customers

Adaptive algorithm: Pseudocode

```
1: Initialize:
                 q_1 \leftarrow 0
                 q_2 \leftarrow 0
                 \Phi \triangleq \frac{1-c}{1-a}
                \delta \triangleq \frac{\Phi b}{\pi}
rem.\overset{n}{inv} = b
2: for \lambda = \frac{1}{n} to 1 do
                 if \lambda < \delta then
                        u_1(\lambda) \triangleq b
                         u_{1,2}(\lambda) \triangleq b
                 else if \lambda \geq \delta then
                         u_1(\lambda) \stackrel{\triangle}{=} \min\{\frac{o_1(\lambda)}{\lambda p}, \frac{o_1(\lambda) + (1-\lambda)(1-p)n}{1-p+\lambda p}\}
                         u_{1,2}(\lambda) \triangleq \min\{\frac{o_1(\lambda) + o_2(\lambda)}{\lambda p}, \frac{o_1(\lambda) + o_2(\lambda) + (1 - \lambda)(1 - p)n}{1 - p + \lambda p}\}
                 end if
9:
```

Adaptive algorithm: Pseudocode

```
i = \lambda \cdot n
10:
         if rem.inv > 0 then
11:
             if v_i = 1 then
12:
13:
                  q_1 \leftarrow q_1 + 1
                  rem.inv \leftarrow rem.inv - 1
14:
             else if v_i = a and u_{1,2}(\lambda) < b then
15:
16:
                  q_2 \leftarrow q_2 + 1
                 rem.inv \leftarrow rem.inv - 1
17:
             else if v_i = a and q_2 \leq |\Phi b + c(b - u_1(\lambda))^+| then
18:
                  q_2 \leftarrow q_2 + 1
19:
                  rem.inv \leftarrow rem.inv - 1
20:
             end if
21:
         end if
22:
23: end for
```

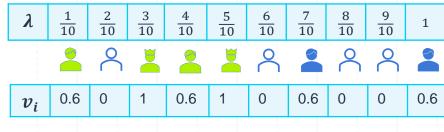
Competitive ratio

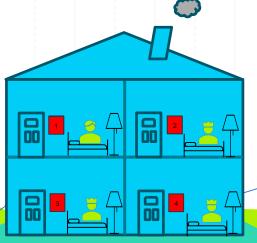
$$c - O\left(\frac{1}{(1-c)^2 a p^{\frac{3}{2}}} \sqrt{\frac{n^2 \log n}{b^3}}\right)$$

$$- competitive for \forall c \leq c^*, c < 1$$

Example ALG₁







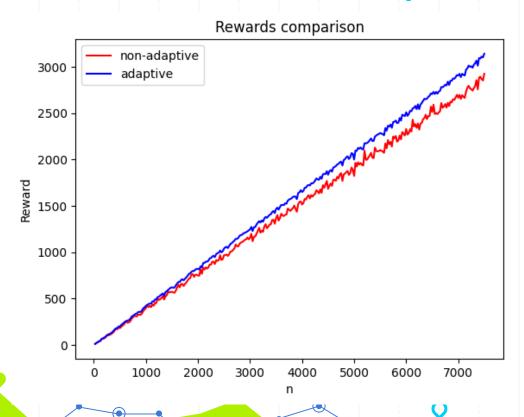


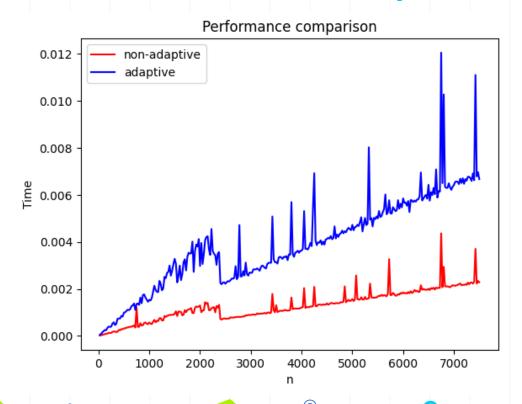
Comparison of both algorithms

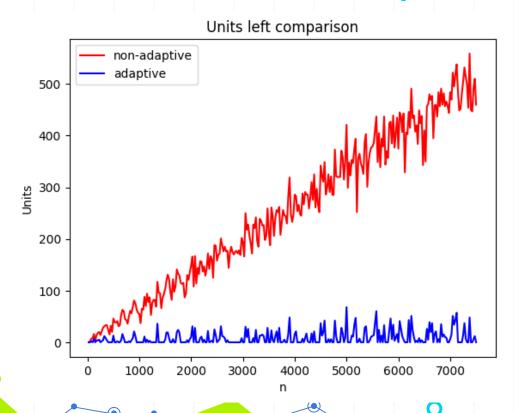
Methodology

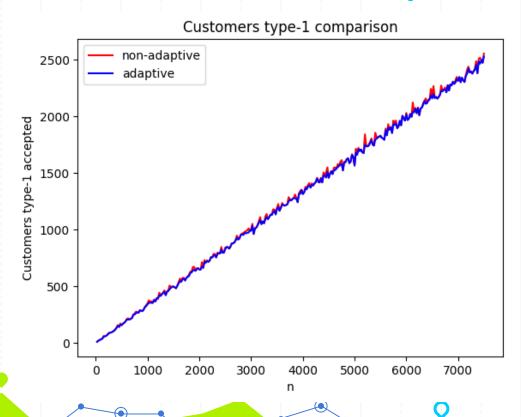
With the help of Python and Google Colab we create:

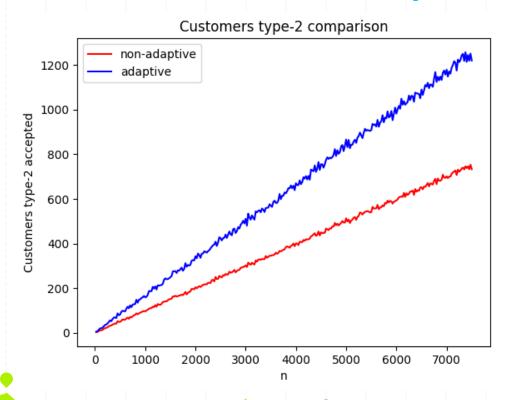
- Customer sequence generator according to demand arrival model
- Two functions for the non-adaptive algorithm (detailed and not detailed)
- Two functions for the adaptive algorithm (detailed and not detailed)
- Simulation:
- We begin with n=25 and increase it by 25
- For each n we create a customer sequence and run both algorithms and repeat it k=300 times
- We run the simulation for 25 different combinations of parameters $a, d = \frac{b}{n}, p, c$











Simulation table (fragment)

a	d	р	C	Larger	Less computational	Units left	Customers T1 and T2
				reward	time		
0,2	0,8	0,2	0,2	Α	NA	Units left	T1 - equally, A accepts more
						increase with n,	T2
						NA leaves more	
						units	
0,5	0,2	0,2	0,5	NA	NA	No units left	NA accepts more T1 and less
							T2
0,2	0,2	0,2	0,8	Α	NA	No units left	A accepts more T1 and less
			1			1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	T2
0,5	0,5	0,2	0,5	Α	NA	Units left	T1 - equally, NA a bit more
						increase with for	and less T2
						NA n, NA leaves	
						more units	

Observations

- With larger d, the number of units left increases with n. The non-adaptive algorithm leaves more units, accepts less type-2 customers and, consequently, gains less reward (on average). Type-1 customers are accepted equally.
- However, with small d, the non-adaptive algorithm gains more reward as it accepts more type-1 customers and less type-2. In this case, none of the algorithms leaves unsold units.
- For medium d, the share of units left for the adaptive algorithm becomes smaller with increasing n. The adaptive algorithm gets more reward by accepting a bit less customers of type-1 and much more customers of type-2.

Observations

- The non-adaptive algorithm is much faster than the adaptive algorithm.
- The adaptive algorithm gains larger reward more often than the non-adaptive one.
- The non-adaptive algorithm tends to leave more units unsold, so it rejects too many type-2 customers.
- The adaptive algorithm gets larger reward for a = 0.5.
- With growing c, the adaptive algorithm rejects more type-2 customers.

References

- Hwang, D., Jaillet, P., and Manshadi, V. (2021). Online resource allocation under partially predictable demand. Operations Research, 69(3):895–915.
- Link to the Python code: https://colab.research.google.com/drive/1VozvH_0Yh9msRQJQxYwDIafiGUH vC9jN?hl=de#scrollTo=McmQsgaF_s4H

THANKSI

Any questions?