BT6270: Computational Neuroscience

Assignment 3 - Report

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BE21B038

A Pair of Hopf Oscillators With Complex Coupling:

Hopf Oscillators with complex coupling follow the following equations:

$$\dot{z_1} = z_1 \left(\mu + i\omega - |z_1|^2\right) + W z_2$$

$$\dot{z_2} = z_2 \left(\mu + i\omega - |z_2|^2\right) + W^* z_1$$

where $z_1 = r_1 e^{i\varnothing_1}$, $z_2 = r_2 e^{i\varnothing_2}$ and $W = Ae^{i\theta}$, $W^* = Ae^{-i\theta}$

W and W* being the coupling coefficient (A and θ being the magnitude and the angle of complex coupling coefficient)

In polar coordinates :

$$\dot{r_1} = \left(\mu - r_1^2\right) r_1 + A r_2 \cos\left(\varnothing_2 - \varnothing_1 + \theta\right)$$

$$\dot{\varnothing_1} = \omega + A \frac{r_2}{r_1} \sin(\varnothing_2 - \varnothing_1 + \theta)$$

$$\dot{r_2} = \left(\mu - r_2^2\right) r_2 + A r_1 \cos\left(\varnothing_1 - \varnothing_2 - \theta\right)$$

$$\dot{\varnothing_2} = \omega + A \frac{r_1}{r_2} \sin(\varnothing_1 - \varnothing_2 - \theta)$$

Assumptions and Parameters:

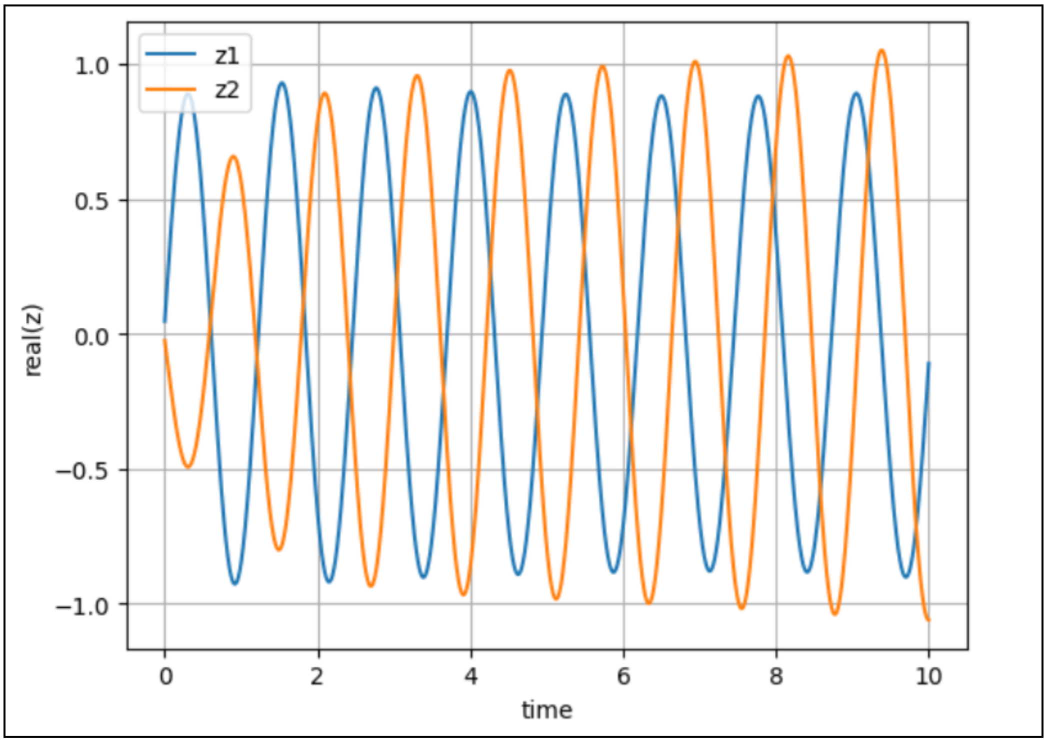
The following assumptions are employed while solving the oscillator equations:

1. ω is given as 5
2. We take $\mu = 1$
3. For Euler Integration, we have used time step as 1ms and total time as 10 secs, so the number of steps is 10,000 to analyse convergence

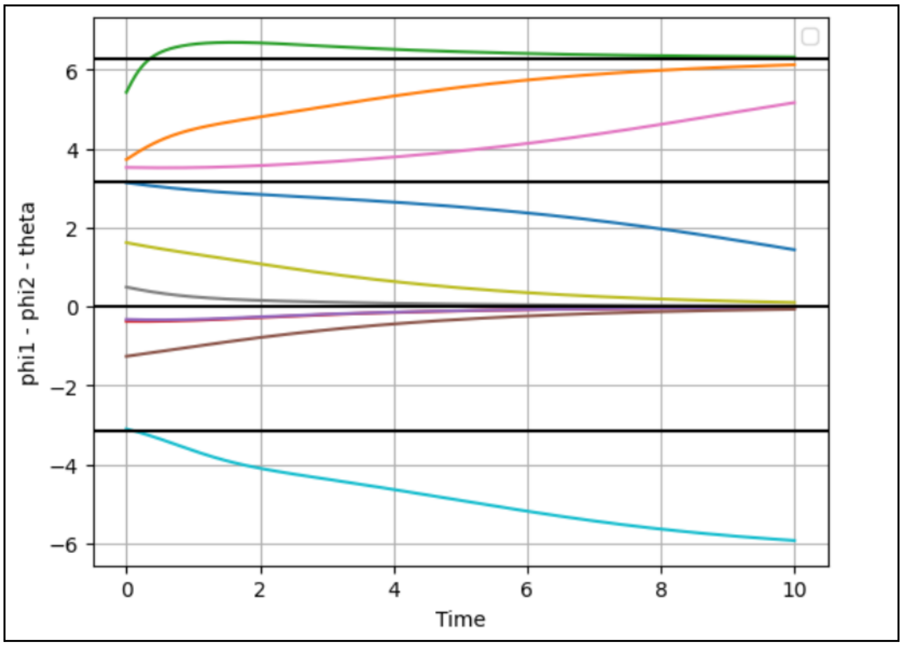
4. We plot the real(z) and phase change for different values of A (0.2, 0.5, 0.8) and θ(-47° and 98° as given) to find the effect of coupling coefficient on the trajectory.
5. 10 random trajectories are taken by the use of the Python library *random*
6. Initial values of r are taken from 0 to 1 and for phi are taken from 0 to 2Π at random
7. A seed is used for each so that it can be reproduced for every case
8. The *real(z) vs time* plots attached in the report are for seed 0.

Plots:

1. **Case 1:** $A = 0.2$ and $\theta = -47°$
   a. ***real(z) vs time (secs)***

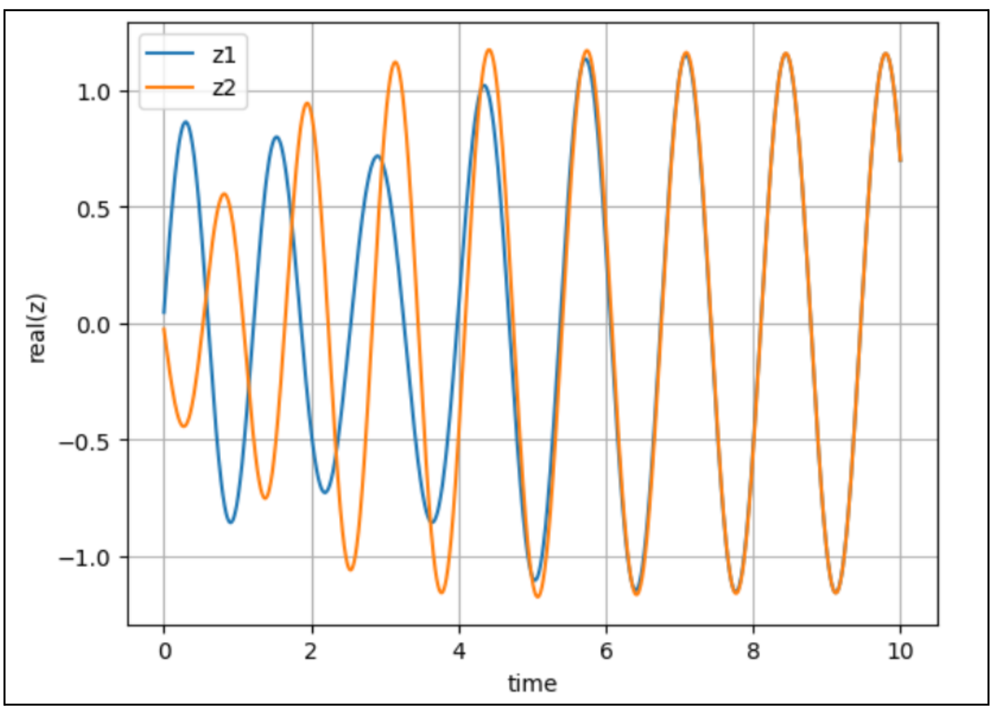


b. ***phase difference plot***

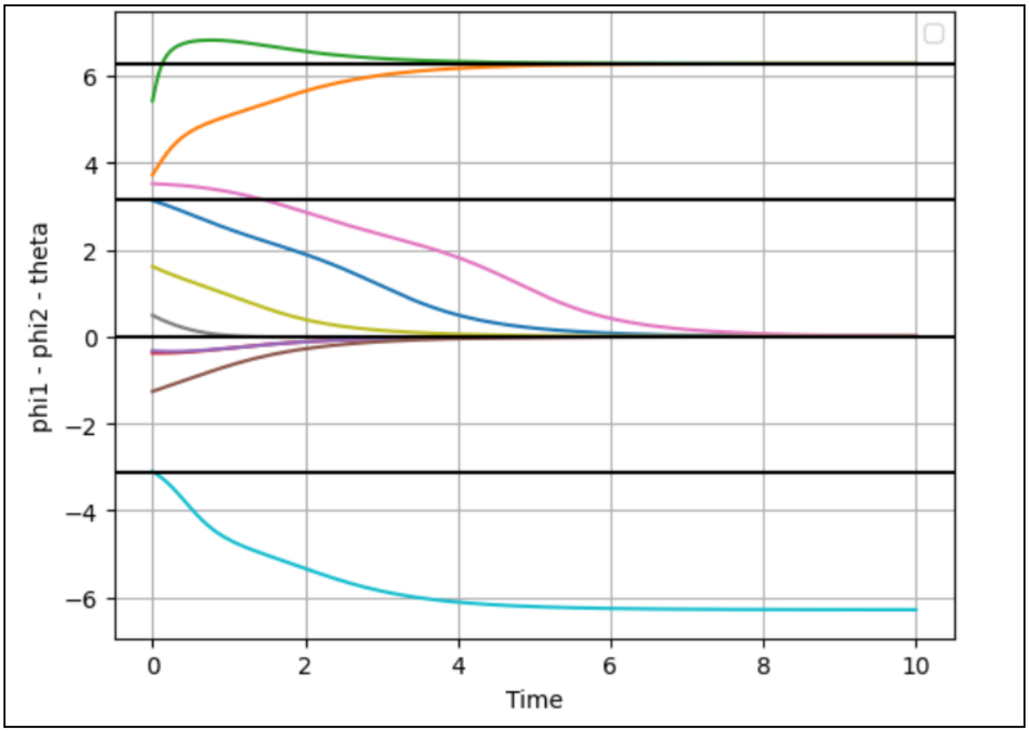


2. Case 2: $A = 0.2$ and $\theta = -47°$

a. ***real(z) vs time (secs)***

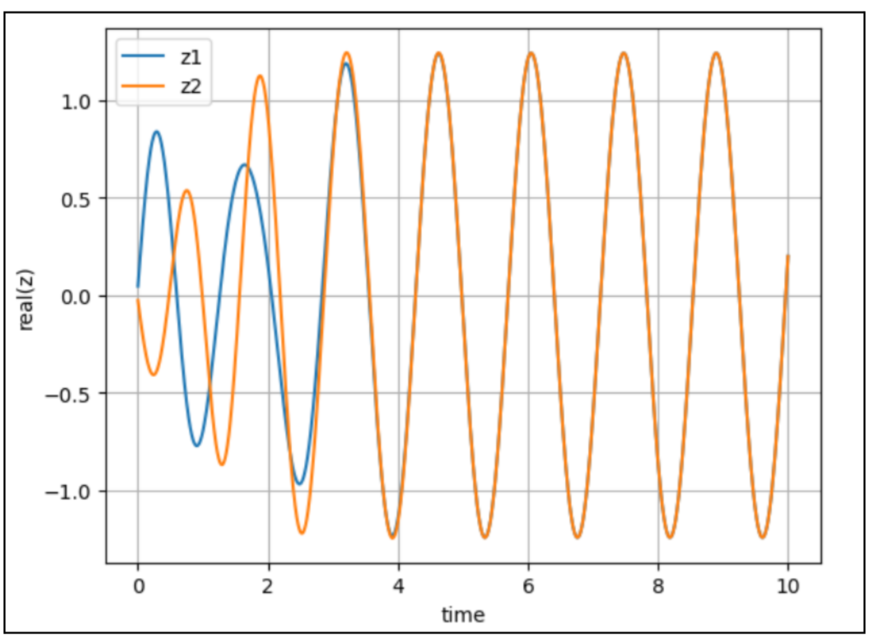


b. ***phase difference plot***

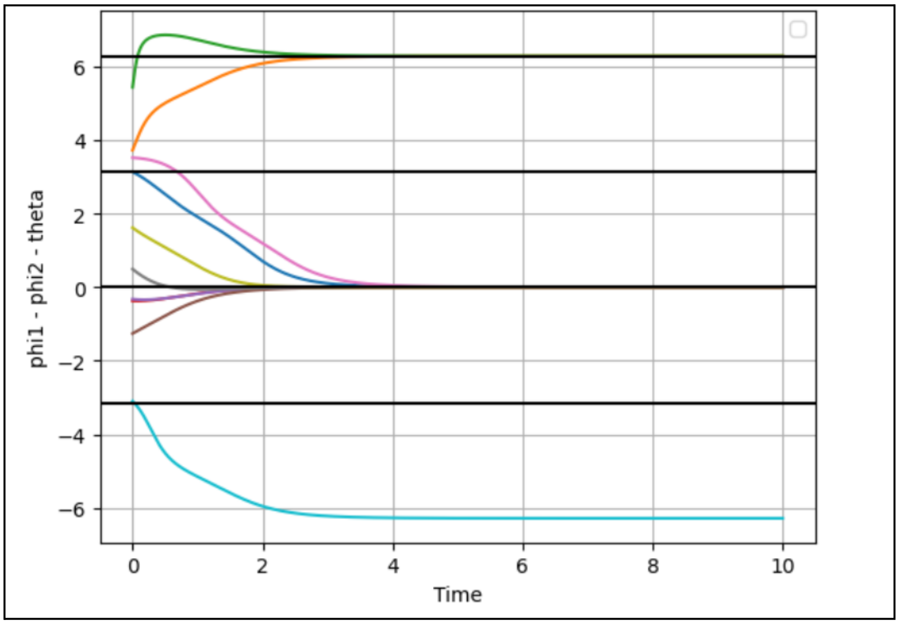


3. Case 3: $A = 0.8$ and $\theta = -47°$

a. ***real(z) vs time (secs)***

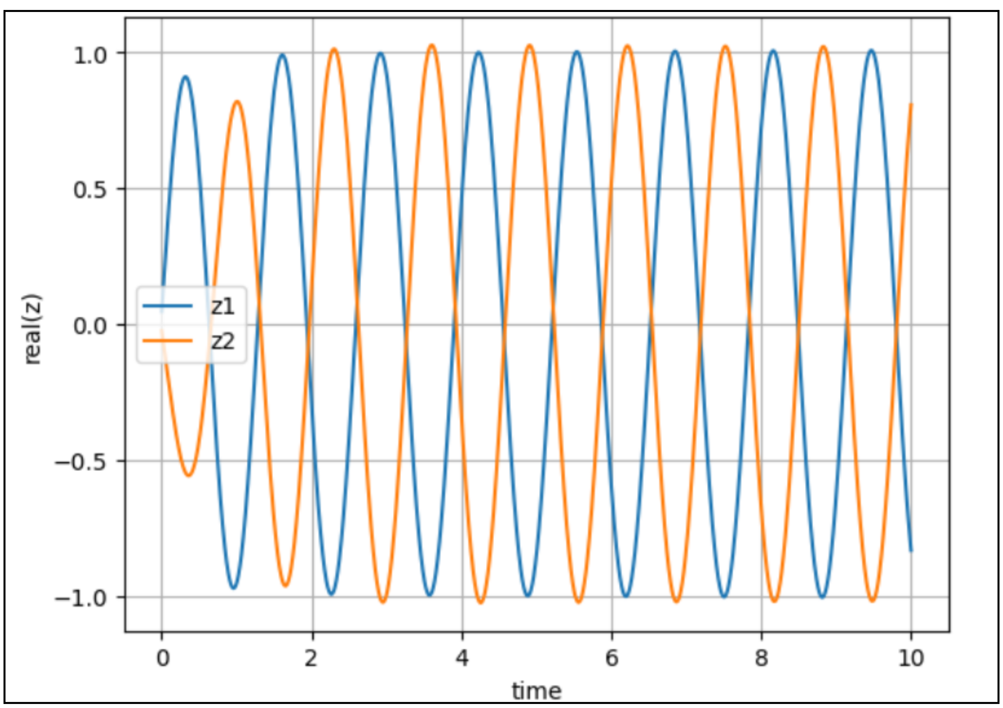


b. ***phase difference plot***

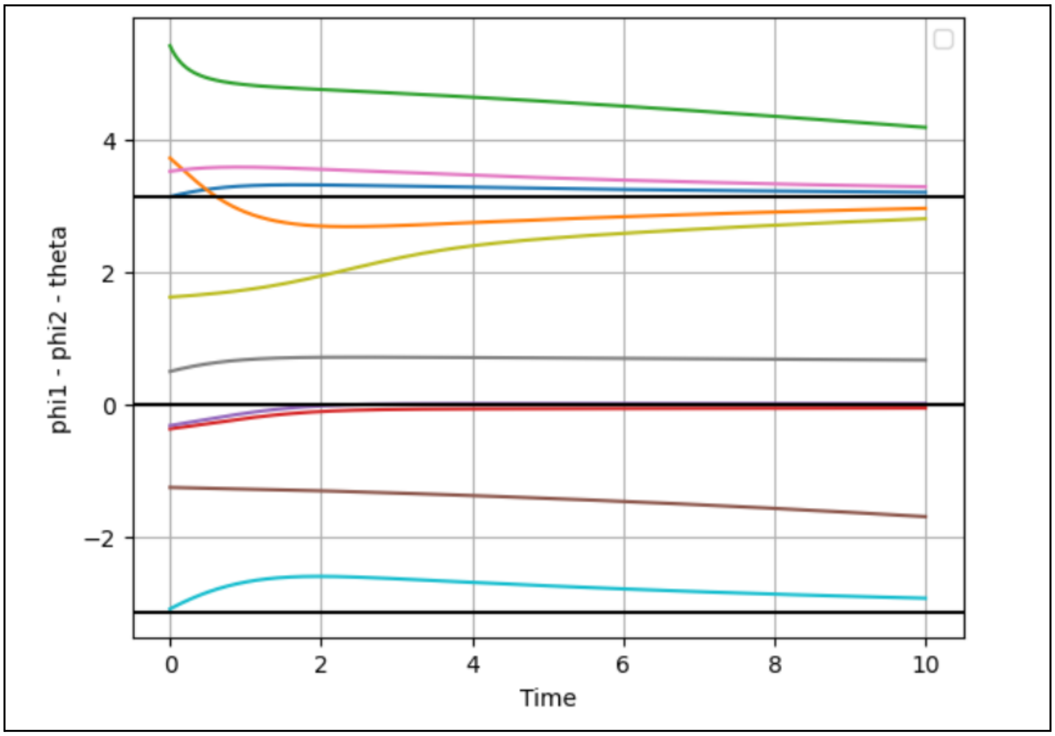


4. Case 4: $A = 0.2$ and $\theta = 98°$

a. real(z) vs time (secs)

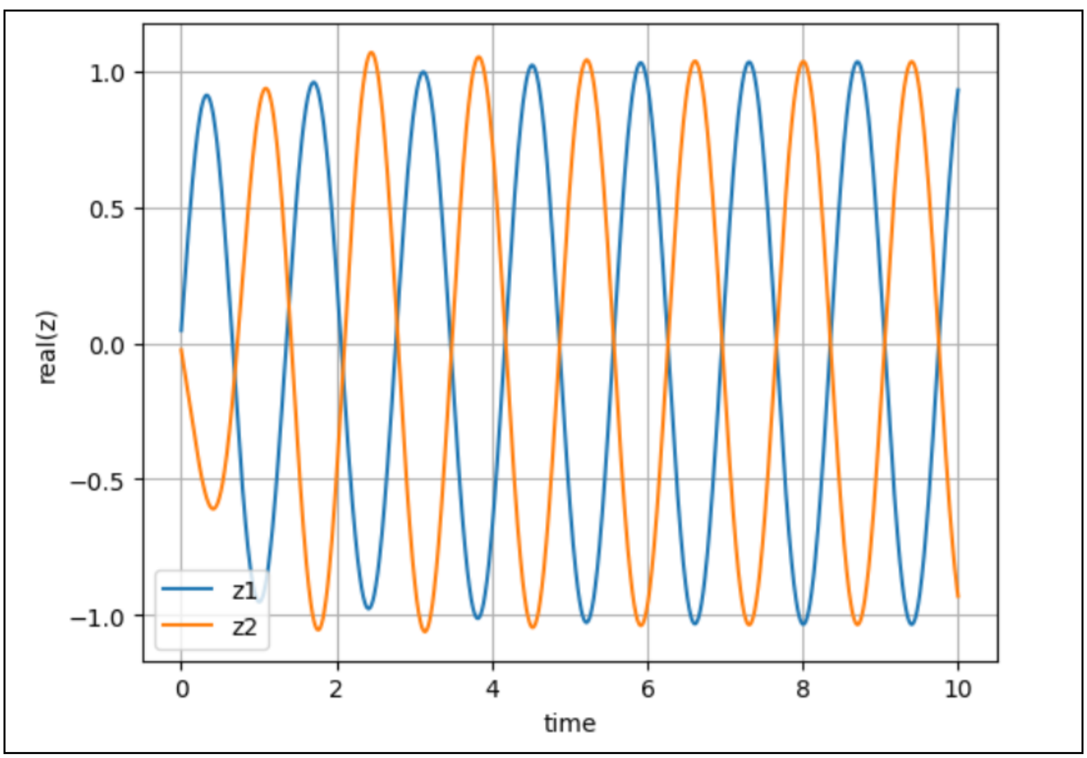


b. ***phase difference plot***

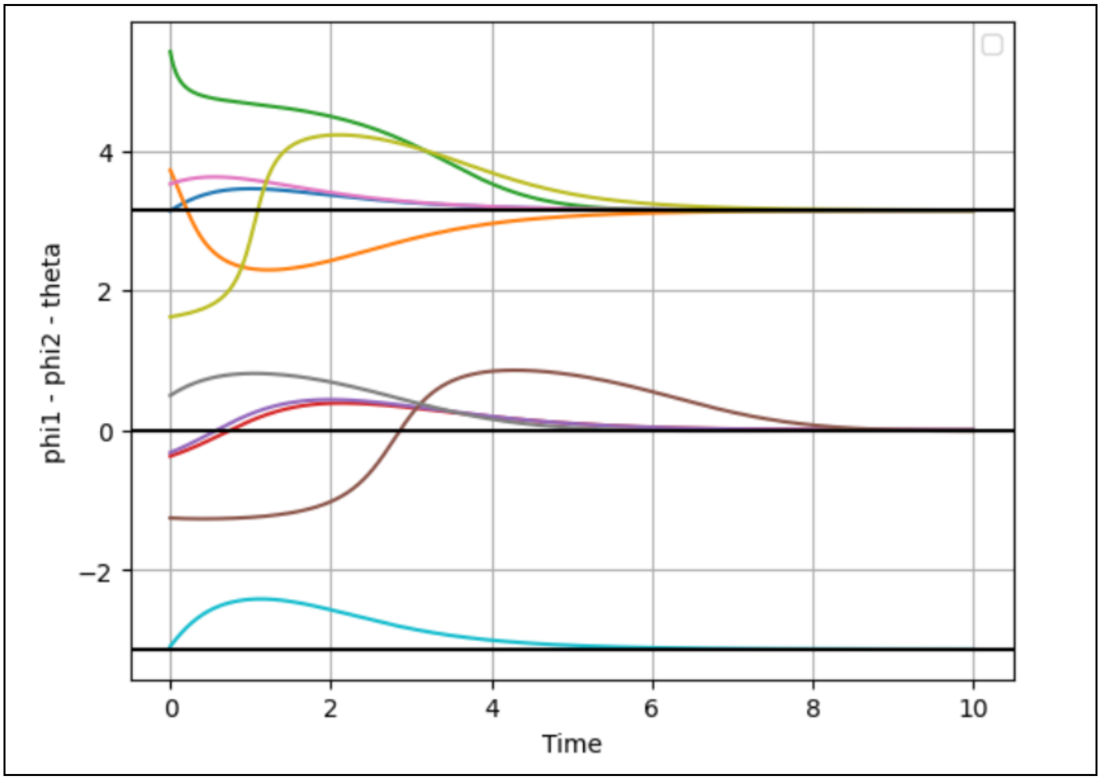


5. Case 5: $A = 0.5$ and $\theta = 98°$

a. ***real(z) vs time (secs)***

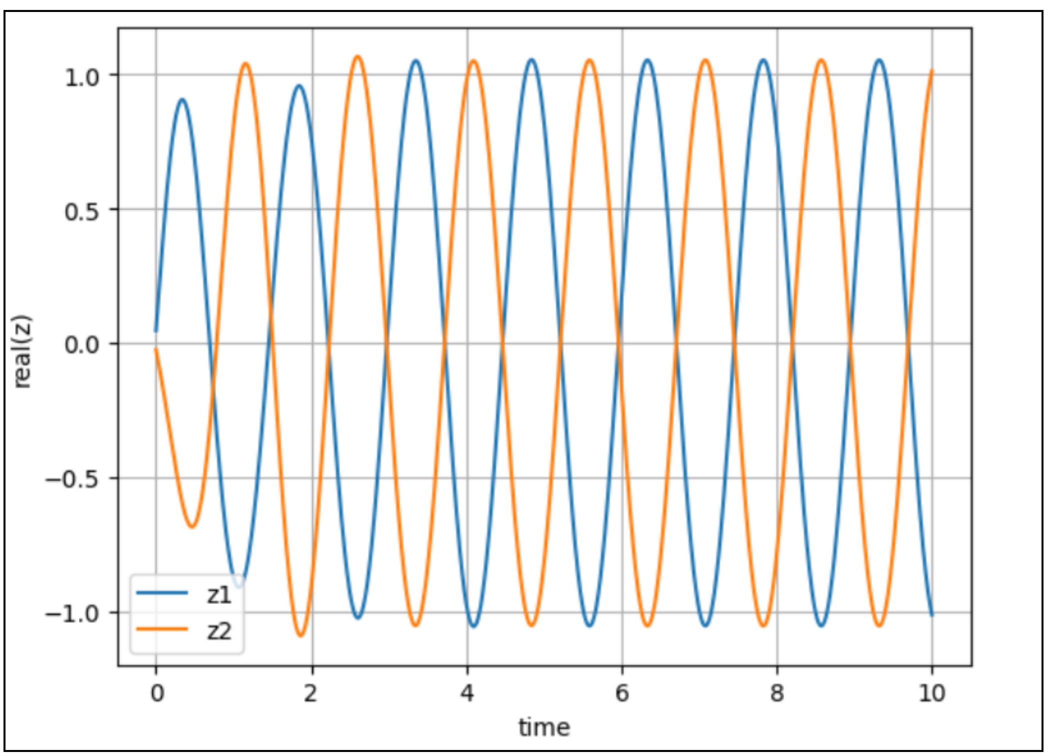


b. phase difference plot

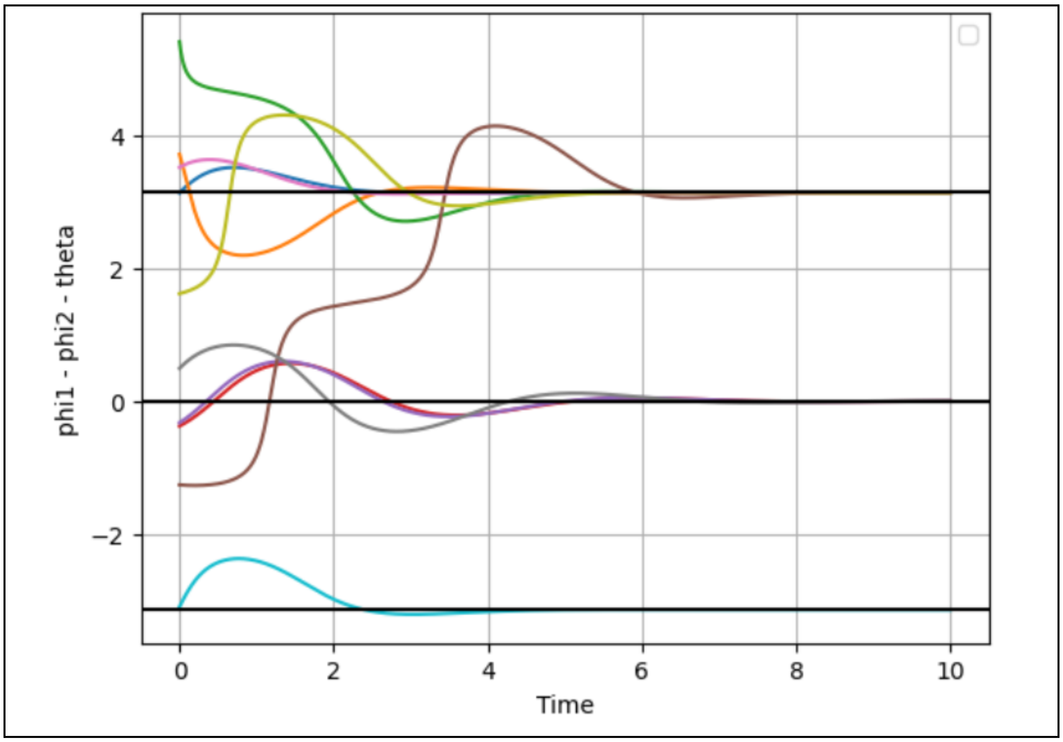


6. Case 6: $A = 0.8$ and $\theta = 98°$

a. ***real(z) vs time (secs)***



b. phase difference plot



Observations:

The real(z) vs t plots confirm oscillatory behaviour.

From the above 6 cases, from phase difference plots, we see that it always converges to 0 or multiples of Π, irrespective of the choice of A and θ, which are the magnitude and phase of the coupling coefficients.

Cases 1,2,3 have common θ and increasing A. We infer that with increasing A, the plots converge faster.

Cases 1 & 4, 2 & 5, 3& 6 have common A and different θ. We infer that for higher θ, there is more variation in the plots before converging.

A Pair of Hopf Oscillators With Power Coupling:

Hopf Oscillators with complex coupling follow the following equations:

$$\dot{z_1}=z_1\left(\mu+i\omega_1-|z_1|^2\right)+A_{12}e^{i\frac{\theta_{12}}{\omega_2}}z_2^{\frac{\omega_1}{\omega_2}}$$

$$\dot{z_2}=z_2\left(\mu+i\omega_2-|z_2|^2\right)+A_{21}e^{i\frac{\theta_{21}}{\omega_1}}z_1^{\frac{\omega_2}{\omega_1}}$$

where $z_1 = r_1e^{i\emptyset_1}$, $z_2 = r_2e^{i\emptyset_2}$ and $W = Ae^{i\theta}$, $W^*=Ae^{-i\theta}$

W and W* being the coupling coefficient (A and θ being the magnitude and the angle of complex coupling coefficient)

In polar coordinates :

$$\dot{r_1} = \left(\mu - {r_1}^2\right) r_1 + A_{12}r_2^{\frac{\omega_1}{\omega_2}} \cos\omega_1\left(\frac{\emptyset_2}{\omega_2} - \frac{\emptyset_1}{\omega_1} + \frac{\theta_{12}}{\omega_1\omega_2}\right)$$

$$\dot{\emptyset_1} = \omega_1 + A_{12}\frac{r_2^{\frac{\omega_1}{\omega_2}}}{r_1}\sin\omega_1\left(\frac{\emptyset_2}{\omega_2} - \frac{\emptyset_1}{\omega_1} + \frac{\theta_{12}}{\omega_1\omega_2}\right)$$

$$\dot{r_2} = \left(\mu - {r_2}^2\right) r_2 + A_{21}r_1^{\frac{\omega_2}{\omega_1}} \cos\omega_2\left(\frac{\emptyset_1}{\omega_1} - \frac{\emptyset_2}{\omega_2} + \frac{\theta_{21}}{\omega_1\omega_2}\right)$$

$$\dot{\emptyset_2} = \omega_2 + A_{21}\frac{r_1^{\frac{\omega_2}{\omega_1}}}{r_2}\sin\omega_2\left(\frac{\emptyset_1}{\omega_1} - \frac{\emptyset_2}{\omega_2} + \frac{\theta_{21}}{\omega_1\omega_2}\right)$$

where $A_{12}e^{i\frac{\theta_{12}}{\omega_2}}=W_{12}$ and $A_{21}e^{i\frac{\theta_{21}}{\omega_1}}=W_{21}$ are the weights of power coupling

Assumptions and Parameters:

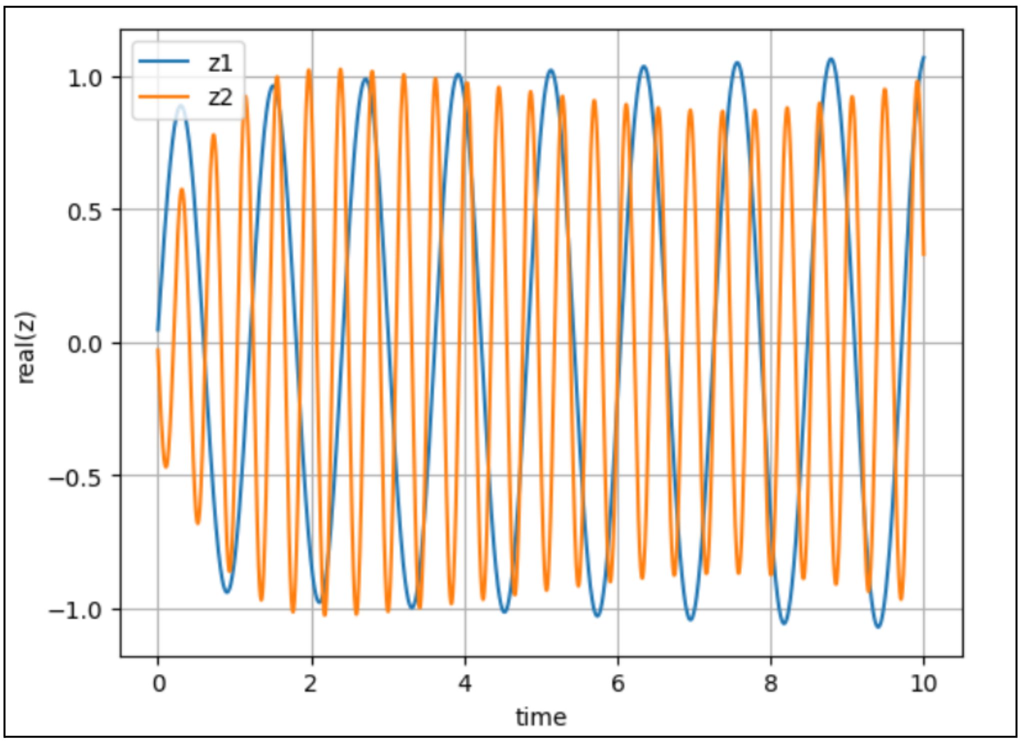
The following assumptions are employed while solving the oscillator equations:

1. We assume the A_{21} and A_{12} are equal, i.e., the coupling coefficients have equal magnitude
2. We take $\theta_{12} = -\theta_{21}$
3. $\omega_1 = 5$ and $\omega_2 = 15$
4. We take $\mu = 1$

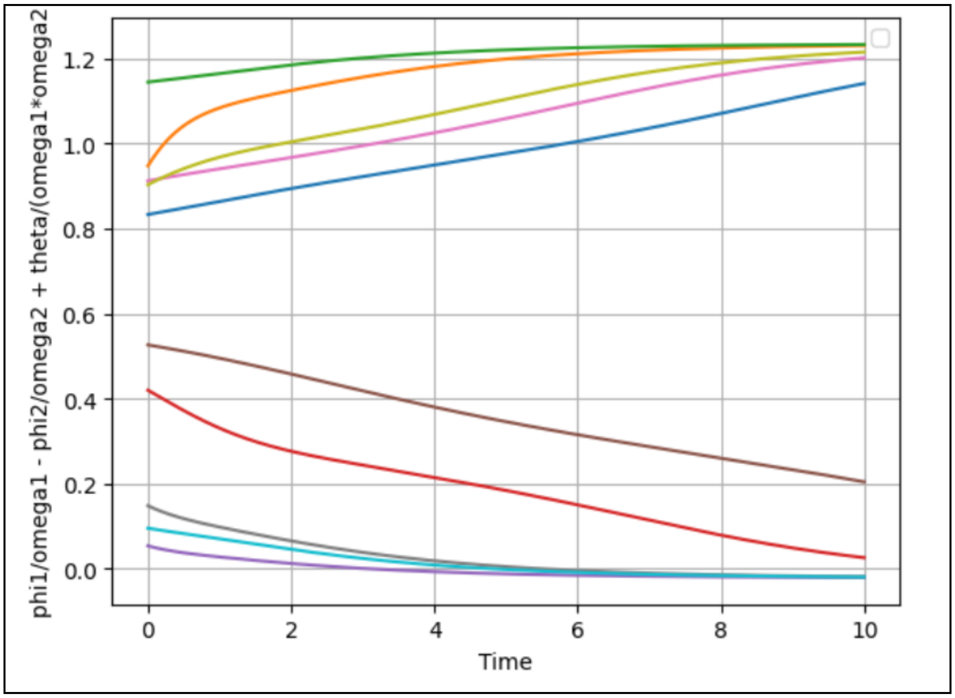
5. For Euler Integration, we have used time step as 1ms and total time as 10 secs, so the number of steps is 10,000 to analyse convergence
6. We plot the real(z) and phase change for different values of A (0.2, 0.5, 0.8) and θ(-47° and 98° as given) to find the effect of coupling coefficient on the trajectory.
7. 10 random trajectories are taken by the use of the Python library *random*
8. Initial values of r are taken from 0 to 1 and for phi are taken from 0 to 2Π at random
9. A seed is used for each so that it can be reproduced for every case
10. The *real(z) vs time* plots attached in the report are for seed 0.

Plots:

7. **Case 1:** $A = 0.2$ and $\theta = -47°$
   a. ***real(z) vs time (secs)***

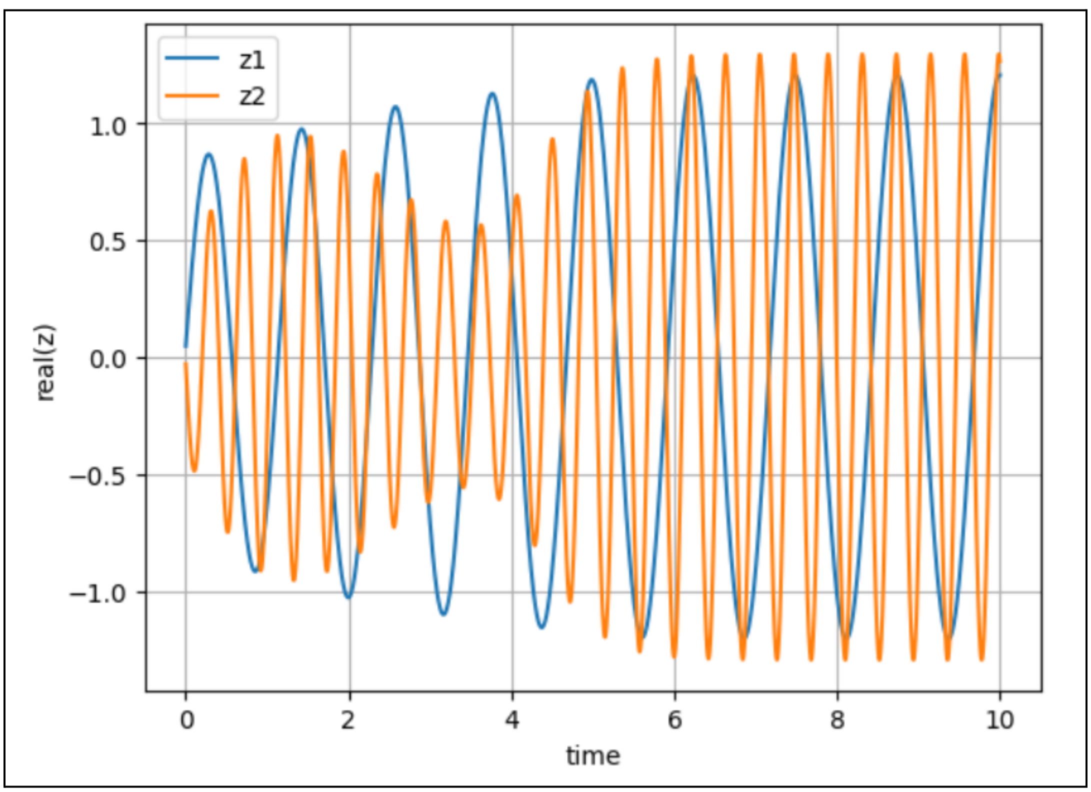


b. ***phase difference plot***

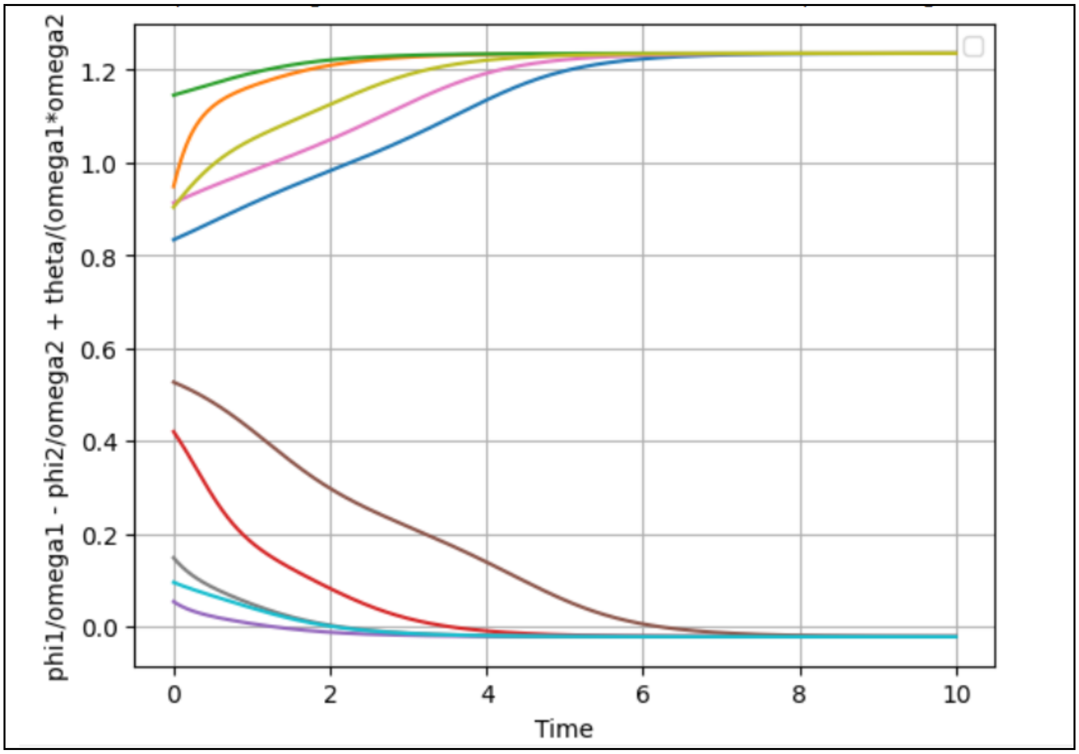


8. Case 2: $A = 0.2$ and $\theta = -47°$

a. ***real(z) vs time (secs)***

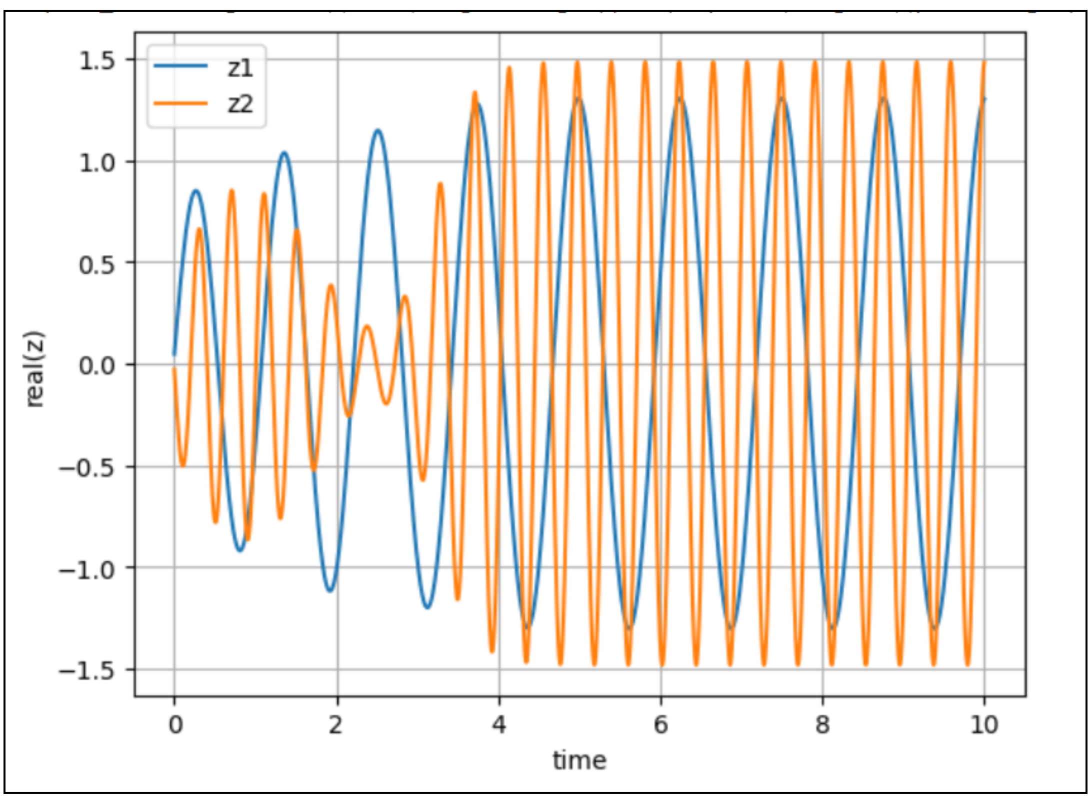


b. ***phase difference plot***

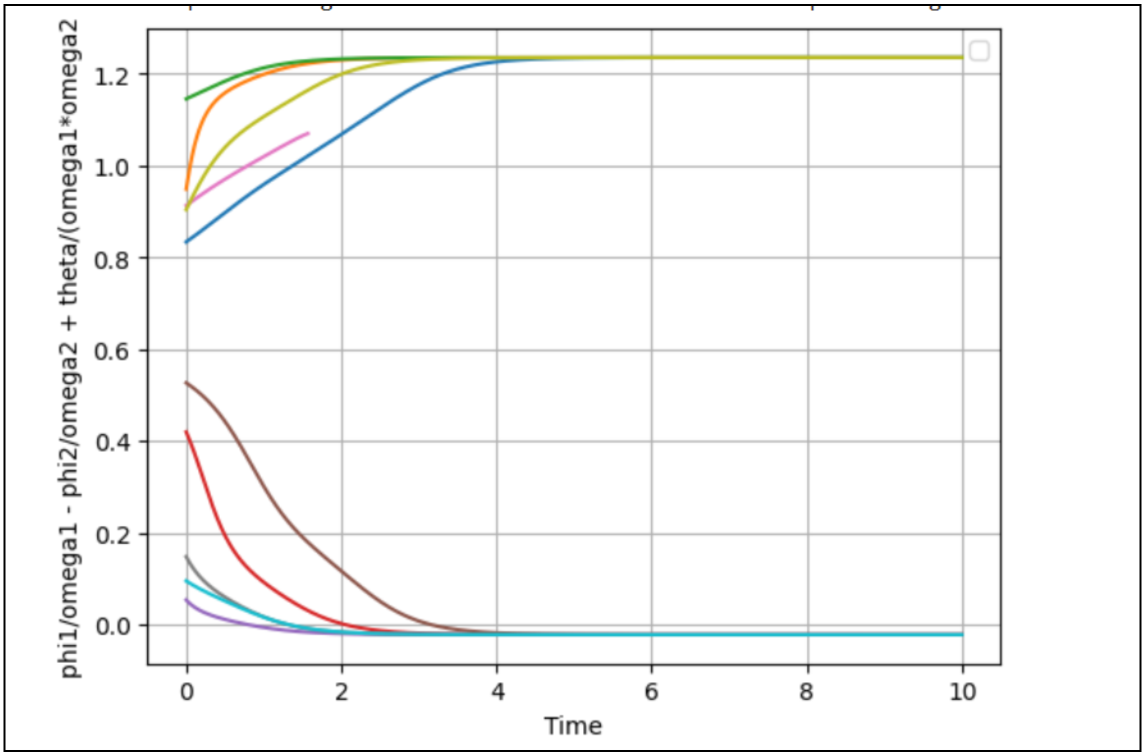


9. Case 3: $A = 0.8$ and $\theta = -47°$

a. ***real(z) vs time (secs)***

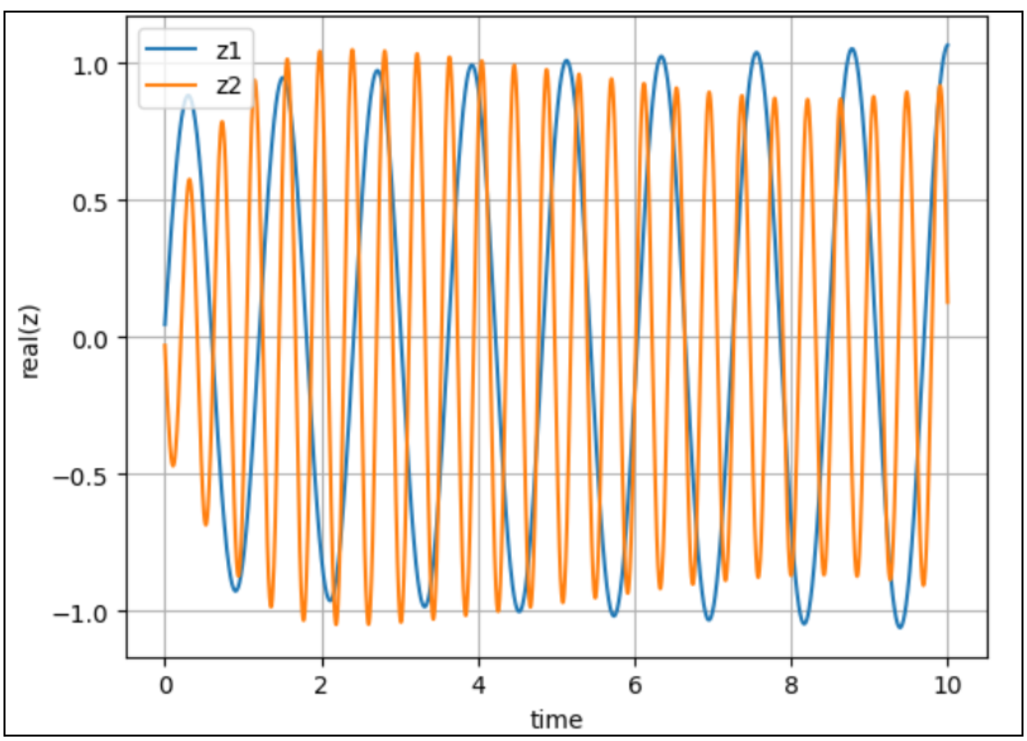


b. ***phase difference plot***

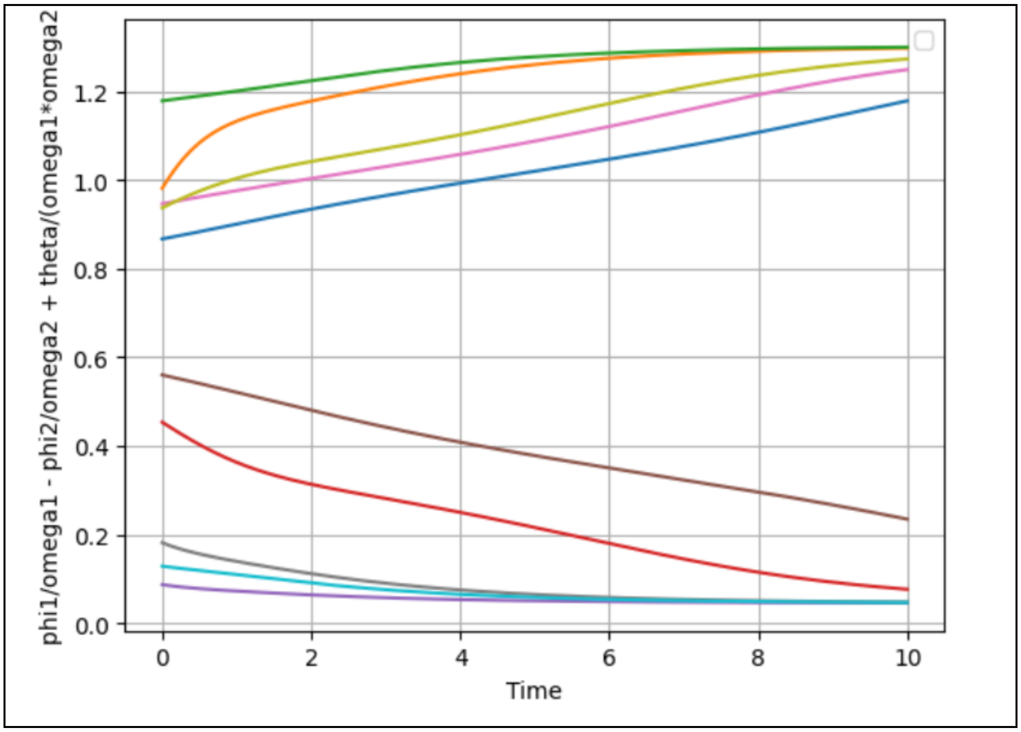


10. Case 4: $A = 0.2$ and $\theta = 98°$

a. real(z) vs time (secs)

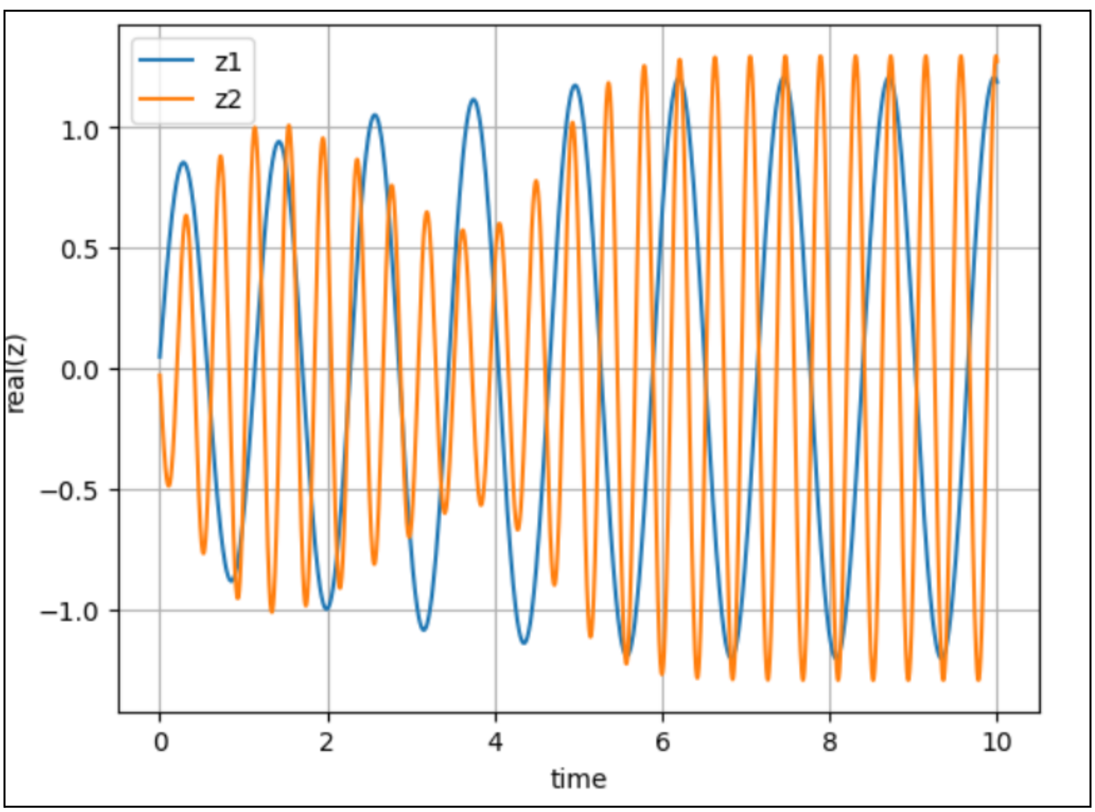


b. ***phase difference plot***

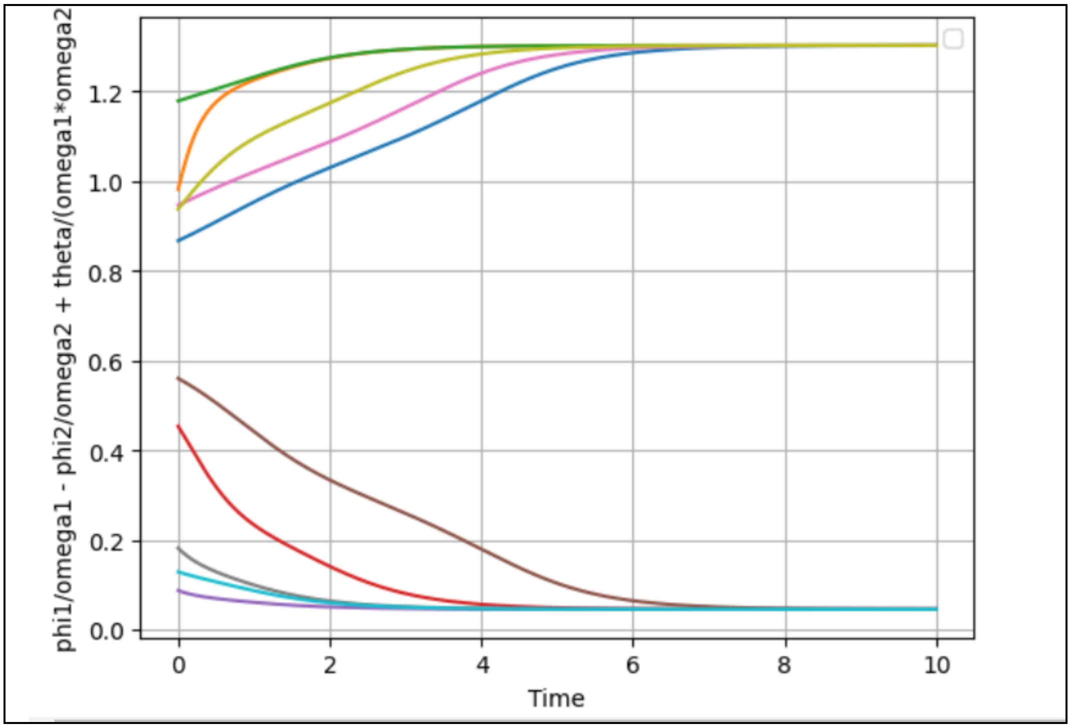


11. **Case 5:** $A = 0.5$ and $\theta = 98°$

a. ***real(z) vs time (secs)***

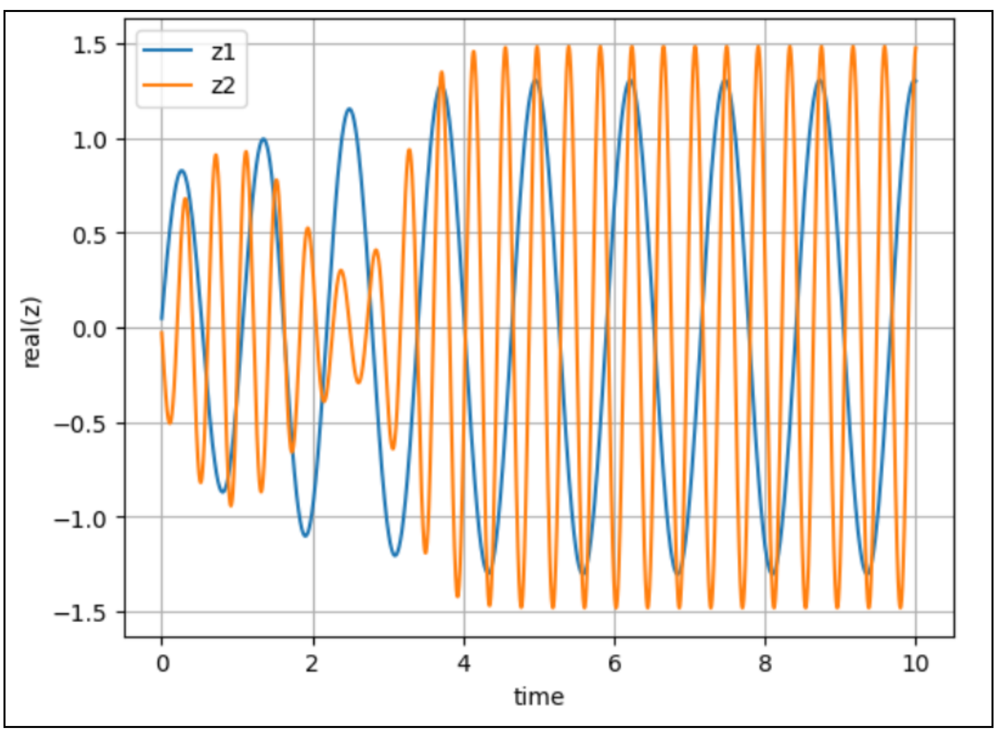


b. phase difference plot

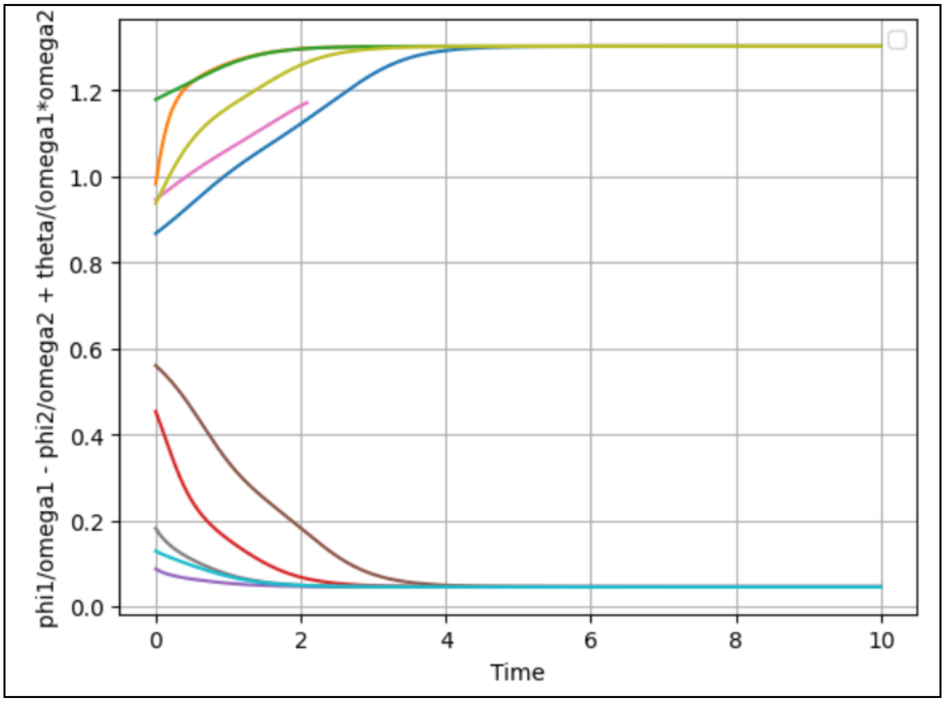


12. **Case 6:** $A = 0.8$ and $\theta = 98°$

a. ***real(z) vs time (secs)***



b. phase difference plot



Observations:

The real(z) vs t plots confirm oscillatory behaviour.

From the above 6 cases, from phase difference plots, we see that it always converges to 0 or a constant number, irrespective of the choice of A and θ, which are the magnitude and phase of the coupling coefficients.

Cases 1,2,3 have common θ and increasing A. We infer that with increasing A, the plots converge faster.

Cases 1 & 4, 2 & 5, 3& 6 have common A and different θ. Not much change is observed in the plot due to change in θ.

References:

1. A Complex-Valued Oscillatory Neural Network for Storage and Retrieval of Multidimensional Aperiodic Signals. *Dipayan Biswas, Sooryakiran Pallikkulath, and V. Srinivasa Chakravarthy*