# BT6270: Computational Neuroscience

Assignment 2 - Report

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## The Fitzhugh-Nagumo model:

The Fizhugh-Nagumo model is a two variable neuron model (*v, w*) developed by simplifying the four variable Hodgkin-Huxley model (*v, m, h, n*), by the use of one gating variable instead of three.

After applying suitable assumptions and approximations, the FN model is formulated as

$$\frac{dv}{dt} = f(v) - w + I_m$$

where $$f(v) = v(a - v)(v - 1)$$ and

$$\frac{dw}{dt} = bv - rw$$

where *a, r;* and *b* are the parameters.

## Assumptions:

The following assumptions are employed to the HH model parameters to convert to FN model:

1. We assume that the gating variable *m* relaxes faster than *h* and *n* since the time scale for the *m* is much smaller than that of *h* and *n.*
2. We assume the gating variable *h* to be a constant (=$h_0$) as *h* varies slowly

## Plots:

For cases 1, 2, and 3, the values of parameters taken are $a = 0.5, r = 0.1, b = 0.1$

1. **Case 1:** $I_{ext} = 0$ ($I_{ext} <$ I1)
    a. **Phase Plot**

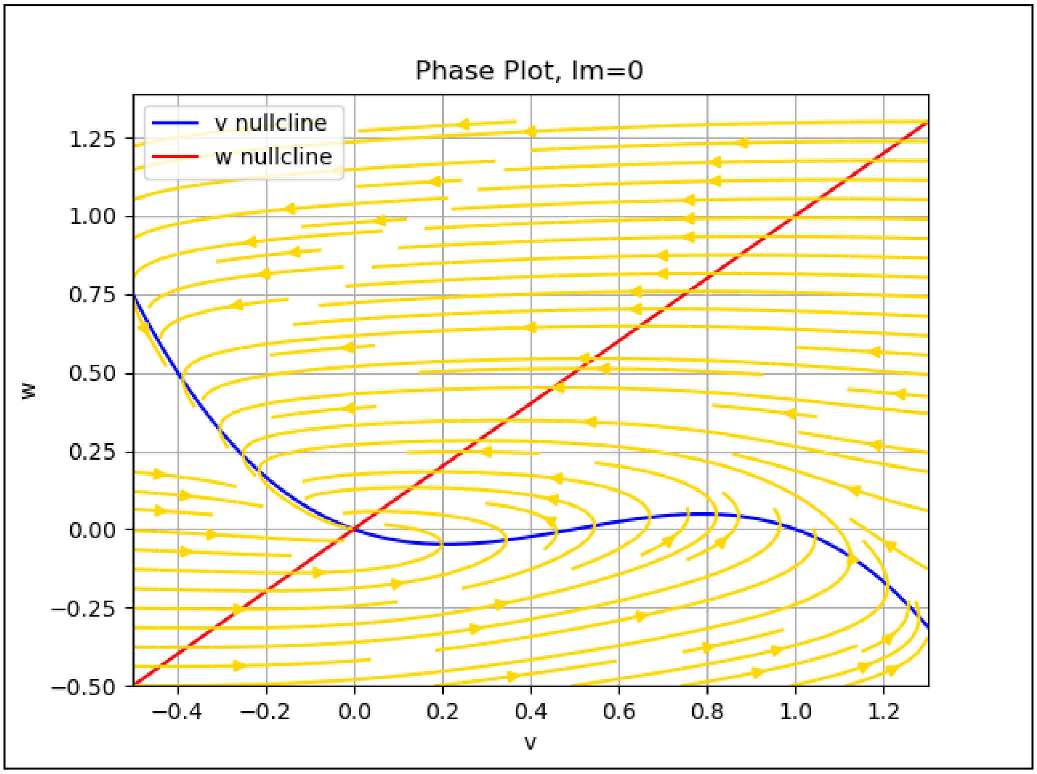


**Fig 1**: Phase plot of the system when $I_{ext} = 0$.

Analysing the phase plot, it is observed that the curve always approaches the stationary point at (0,0). Hence the intersection of the two nullclines is a stable point and excitability is observed.

**b.** ***v(t), w(t)*** **plots**

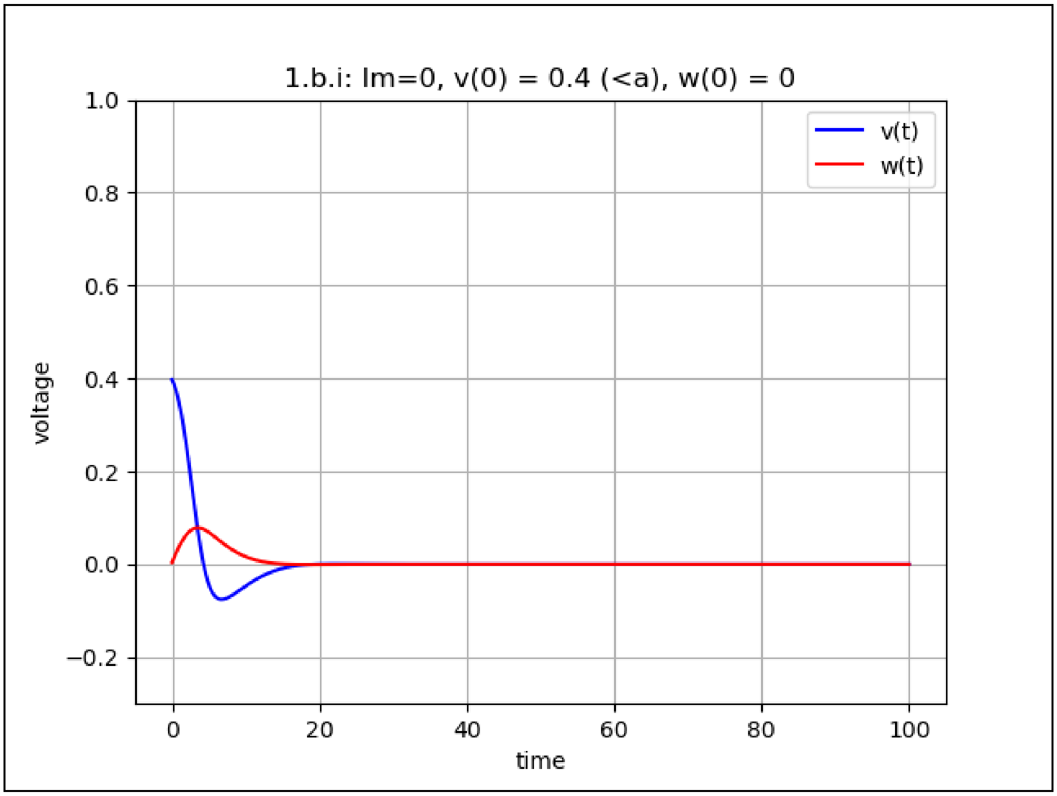


**Fig 2**: *v* and *w* as functions of time for *v(0)<a*

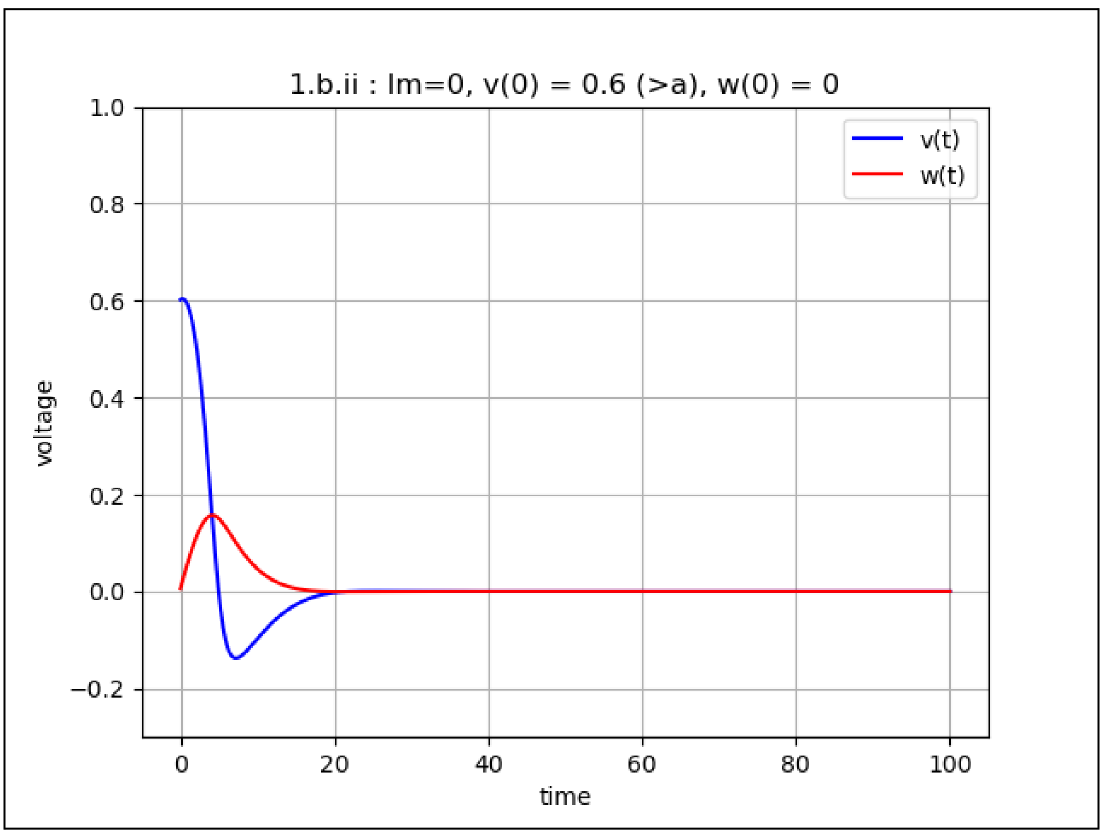


**Fig 3**: *v* and *w* as functions of time for *v(0)>a*

It is observed that for both cases (v(0)>a and v(0)<a), no action potentials are observed for sub-threshold pulse injections ($I_{ext} = 0$).

## 2. Case 2: $I_{ext} = 0.5$ ($I1 < I_{ext} < I2$)

I1 and I2 have been obtained by solving manually. The minimum and maximum of the v nullcline are calculated (act as boundary conditions). These values have been back substituted into the v nullcline equation to obtain I1 and I2 respectively. The final solution is as follows:

I1 = 0.26

I2 = 0.74

### a. Phase Plot

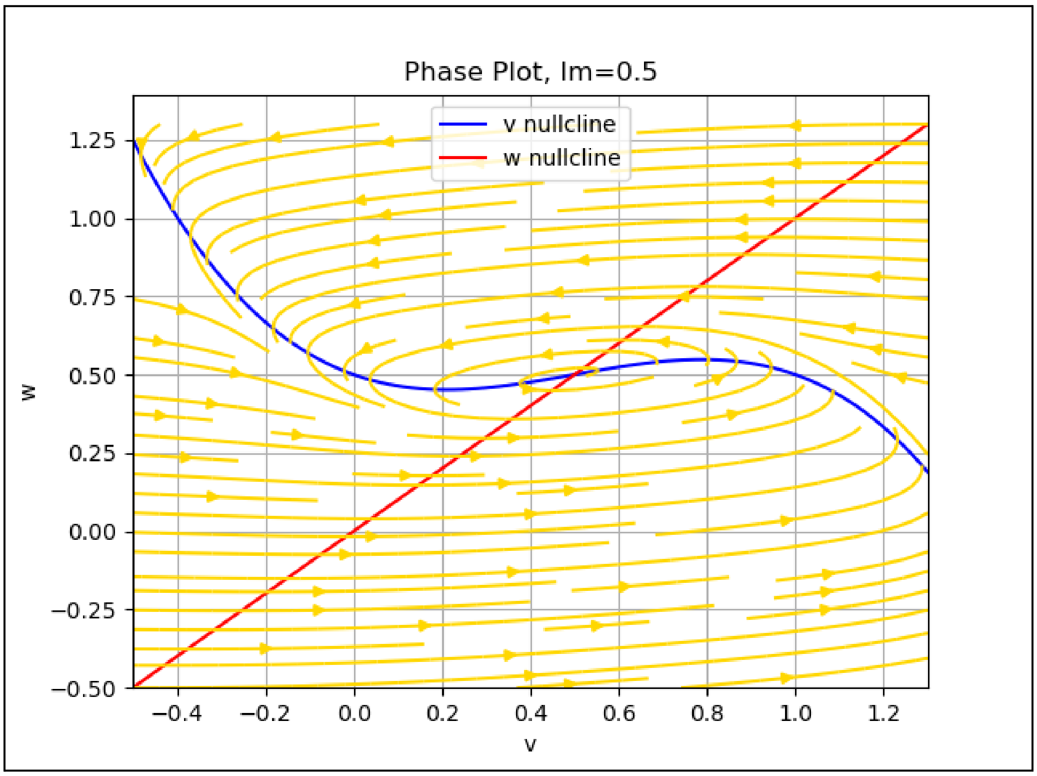


**Fig 4**: Phase plot of the system when $I_{ext} = 0.5$

## b. Stability analysis

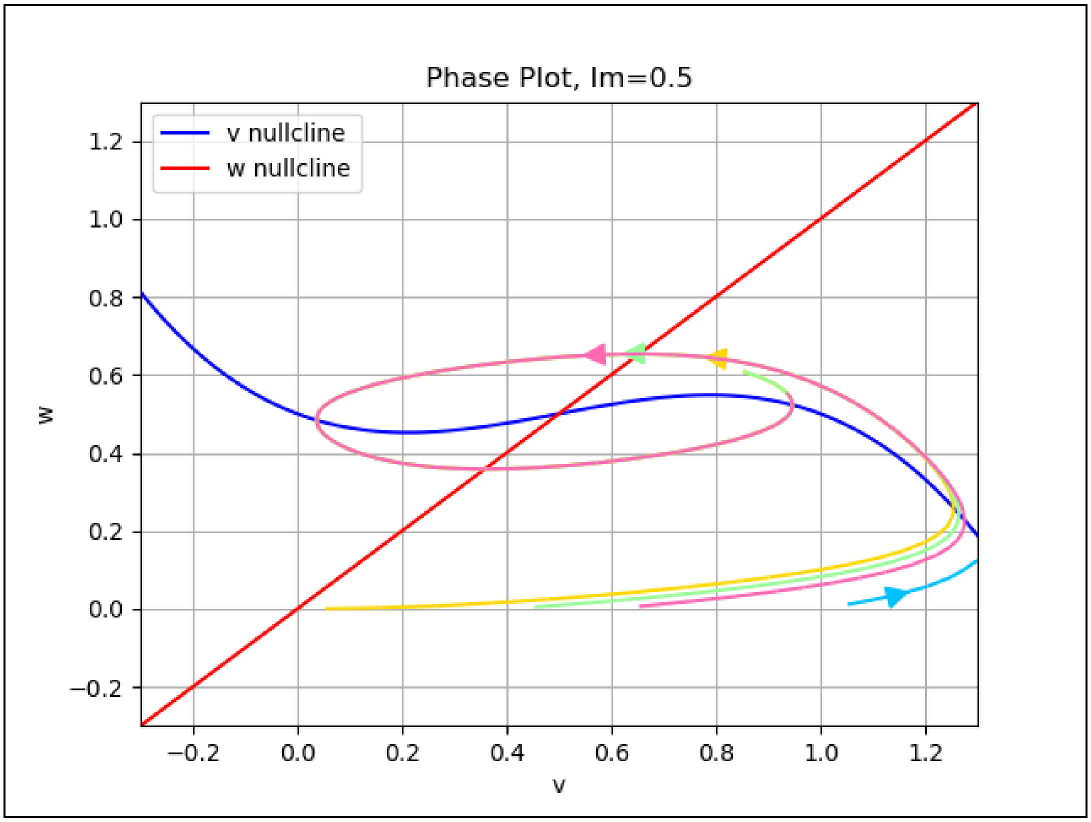


**Fig 5**: Stability analysis of phase plot for $I_{ext} = 0.5$

Analysing the trajectory with initial points as (0, 0.4, 0.6, 1), it is observed there are circulating fields around the stationary point. Hence, it is an unstable point. This leads to Limit cycle behaviour in the region.

## c. *v(t), w(t)* plots

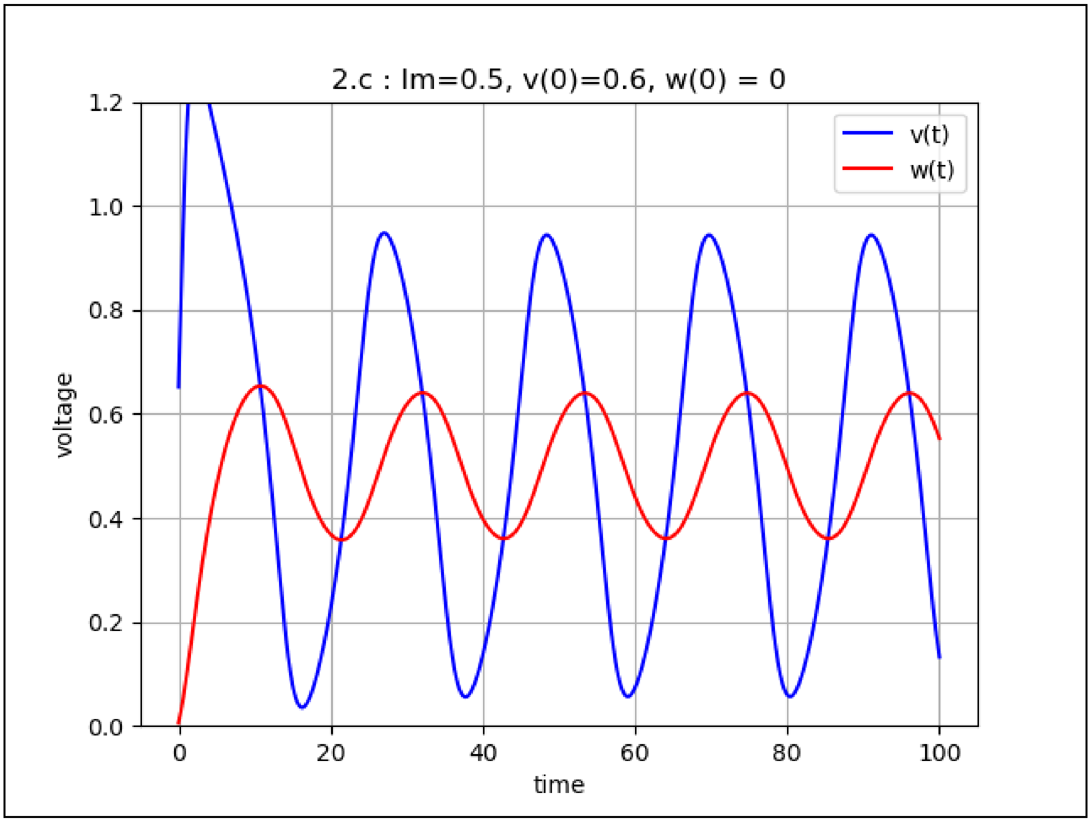


**Fig 6**: *v* and *w* as functions of time for $I_{ext} = 0.5$

Limit cycle behaviour is shown. Sustained oscillations can be observed

3. **Case 3:** $I_{ext} = 0.9$ ($I_{ext} >$ I2)

   a. **Phase Plot**

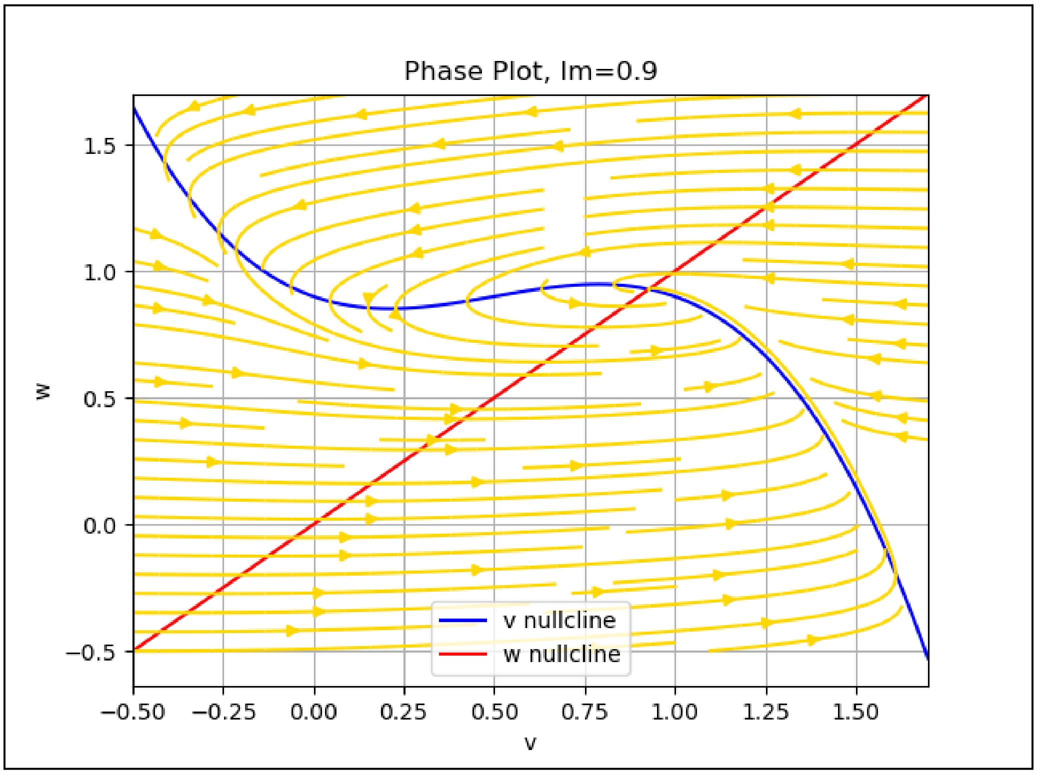


**Fig 7**: Phase plot of the system when $I_{ext} = 0$.

## b. Stability analysis

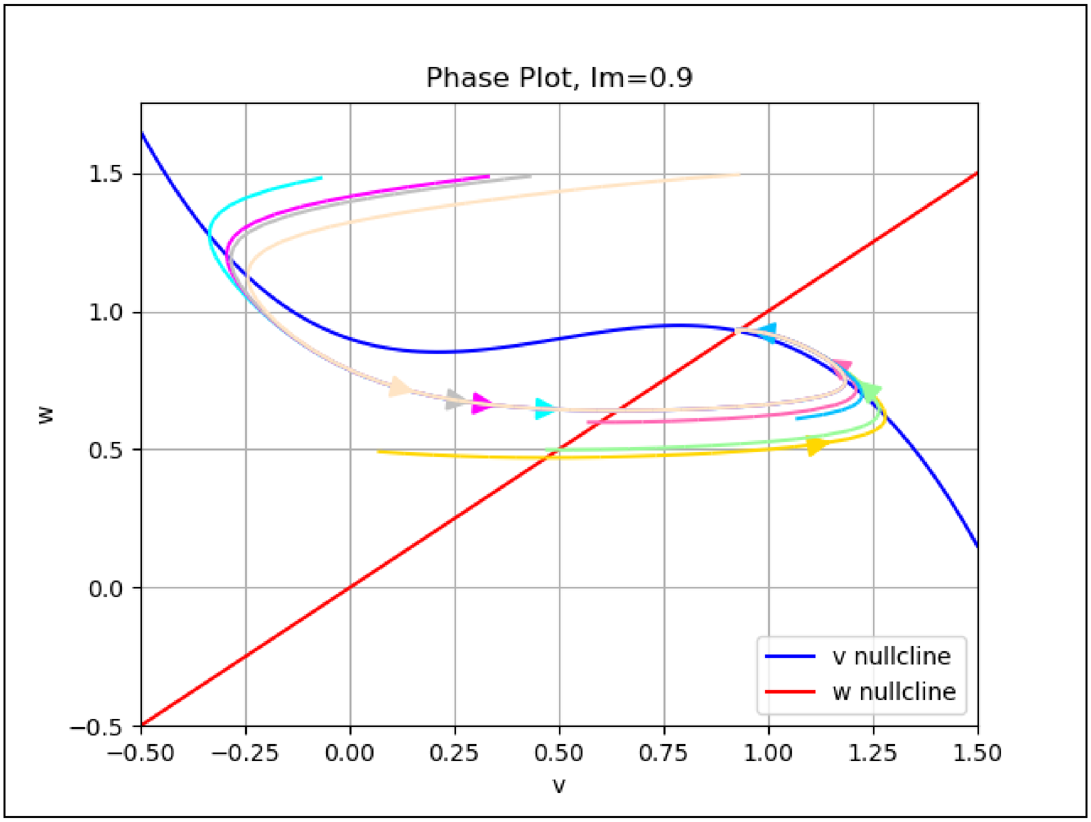


**Fig 8**: Stability analysis of phase plot for $I_{ext} = 0.9$

Analysing the phase plot with initial points of (v,w) as (0,0.5), (0.4,0.5), (0.5, 0.6), (1, 0.6), (0,1.5), (0.4,1.5), (0.5, 1.5), (1, 1.5) it is observed that even for big perturbations the phase curves approach the stationary point. Hence, it is a stable point.

### c. *v(t), w(t)* plots

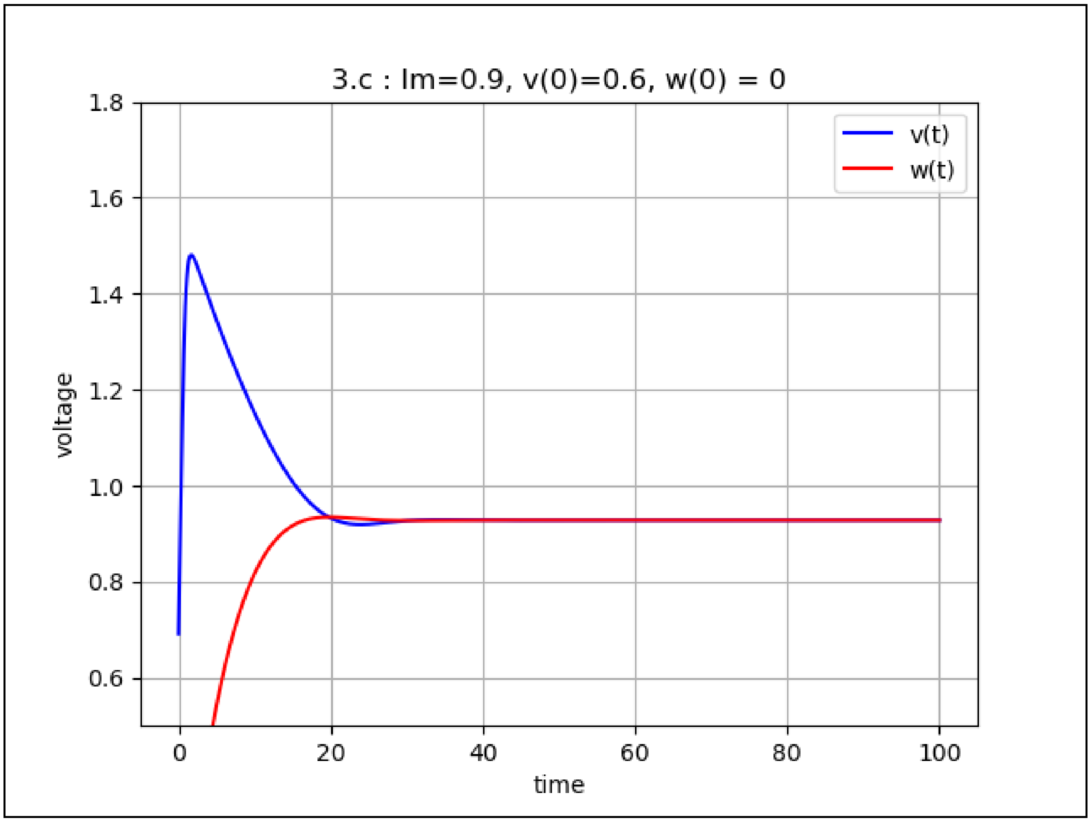


**Fig 9**: *v* and *w* as functions of time for $I_{ext} = 0.9$

At $I_{ext}$ > I2, depolarisation in action potential is observed

## 4. Case 4: *Choosing parameters*

A range of values of (b/r) and $I_{ext}$ can be obtained for which the given behaviour of the potential plot can be observed. One such situation can be obtained by taking the following values for these parameters:

$I_{ext} = 0.05$

$b = 0.1$

$r = 0.9$

$b/r = 0.11$

## a. Phase Plot

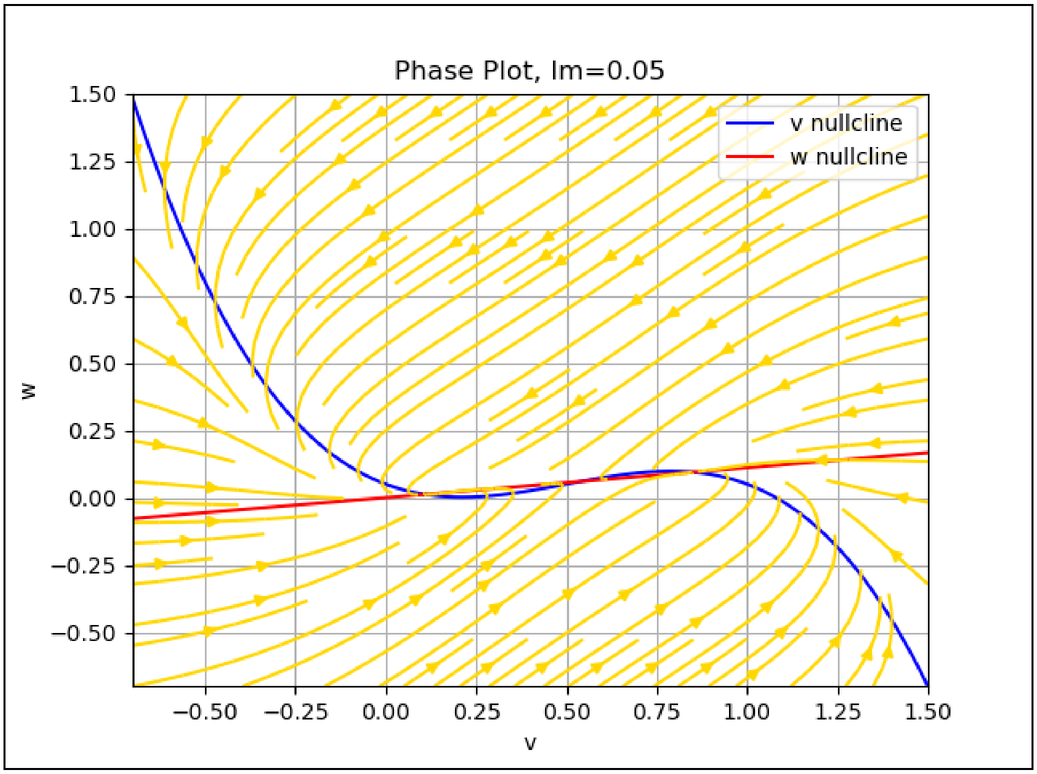


**Fig 10**: Phase plot of the system when $I_{ext} = 0.05$

Three stationary points are obtained at the intersections of the two nullclines. These are at v = 0.11, 0.54, 0.85.

## b. Stability analysis

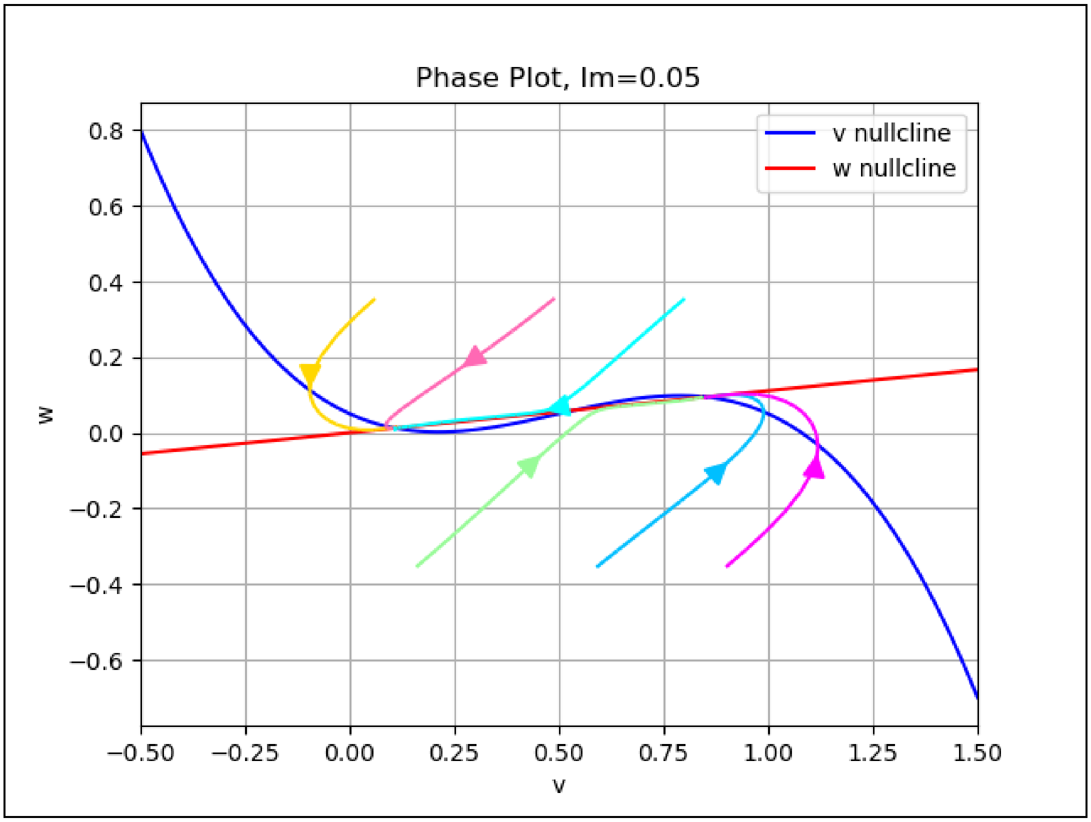


**Fig 11**: Stability analysis of phase plot when $I_{ext} = 0.05$

Analysing the phase plot with initial points of v at P1, P2, P3 (the stationary points). For P1 and P2 it is observed that perturbations lead back to the respective points. Hence P1 and P3 are stable points. For P2, perturbations in one axis lead to travelling along the nullcline. Hence P2 is a saddle point.

### c. *v(t), w(t)* plots

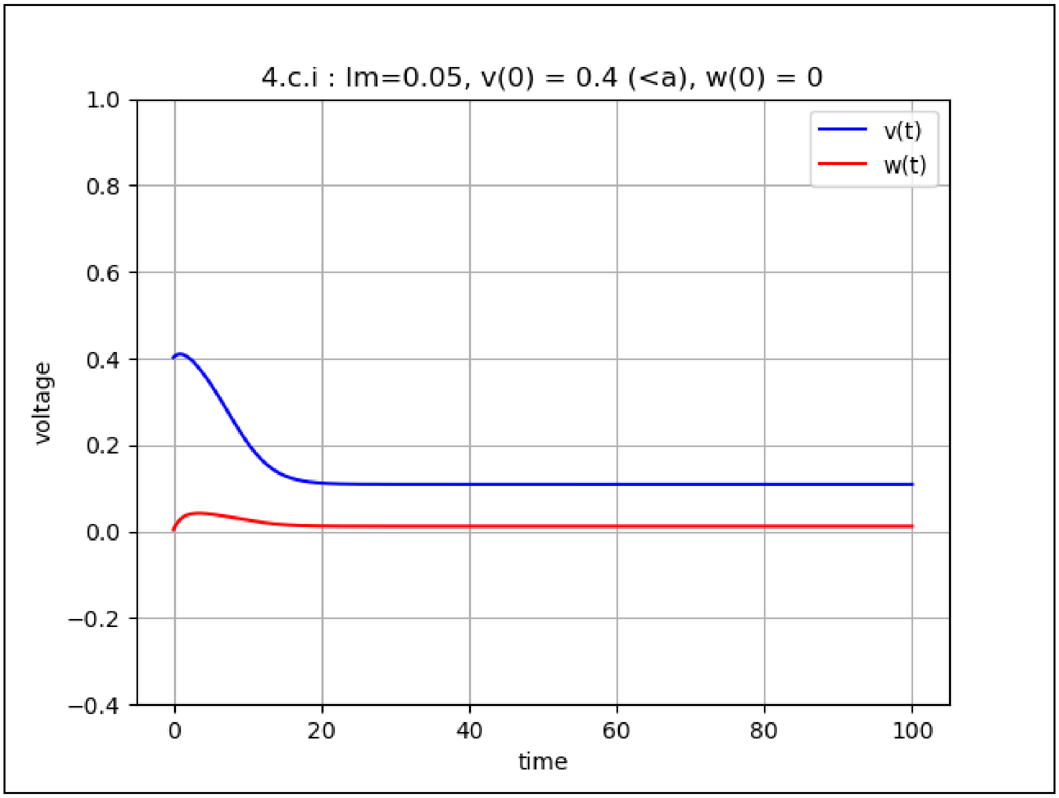


**Fig 12**: *v* and *w* as functions of time for *v(0)<a*

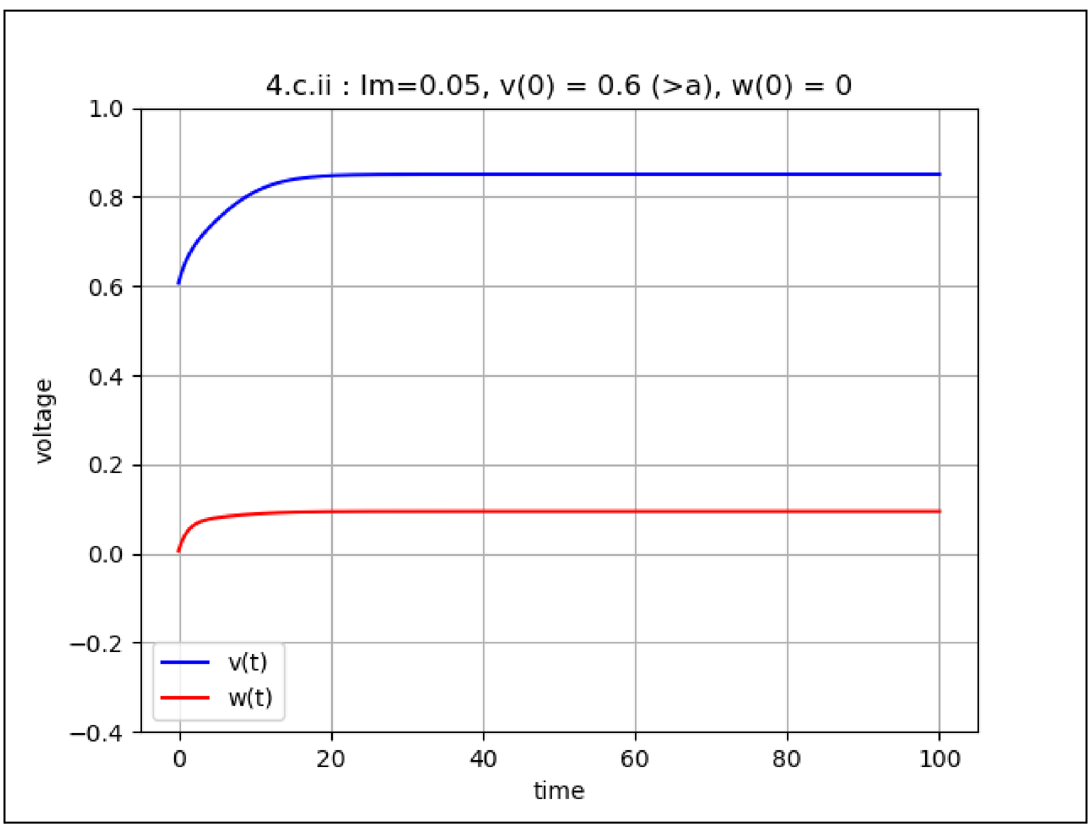


**Fig 13**: *v* and *w* as functions of time for *v(0)>a*

Bi-stability is observed at the values of parameters taken. For $v(0)>a$, a neuron exists in an up state (at P3), and for $v(0)<a$ (at P1), a neuron exists in a down state.