# The Title of Your Project

## The Sub-Title of Your Project

Author1 (ID, email), Author2 (ID, email), Author3 (ID, email)

Department of Computer Science,
Shanghai Jiao Tong University, Shanghai, China

**Abstract.** The abstract should briefly summarize the contents of the paper in 15–250 words.
**Keywords:** keywords 1, keywords2.

## 1 Preliminaries

In this section, we describe the variables and denotations we use and the assumptions we propose.

### 1.1 Denotations

$\mathbf{L} = \{1, 2, 3...\}$: The cities set.

$N = |\mathbf{L}|$: The total number of cities.

$\mathbf{H} = \{1, 2, 3...\}$: The hubs set.

$D_{ij}$: Direct distance between city $i$ and $j$.

$T_{ij}$: Direct travel time between city $i$ and $j$.

$F_{ij}$: Flow demands from city $i$ to city $j$.

$U_i$: Update cost for city $i$.

$C_{ijkm}$: The transportation cost per unit flow from city $i$ to city $j$ routed from hub $k$ and $m$.

$K_i$: The capacity of city $i$.

$X_{ijkm}$ : The fraction of flow from origin $i$ to destination $j$ routed from hubs $k$ and $m$, in other words, the fraction of $F_{ij}$ through the path $i \to k \to m \to j$.

### 1.2 Assumptions

We assume that the transportation cost per unit flow directly from city $p$ and city $q$ is is proportional to the direct distance between $p$ and $q$ and we denote the coefficient as $s$. Then we have:

$$C_{ijkm} = s(D_{ik} + D_{km} + D_{mj}) \tag{1}$$

### 1.3 Decision variables

$$X_{ijkm} \in [0, 1] \tag{2}$$

$X_{ijkm}$ is the fraction of flow from origin $i$ to destination $j$ routed from hubs $k$ and $m$, in other words, the fraction of $F_{ij}$ through the path $i \to k \to m \to j$.

$$Y_i = \begin{cases} 1 \text{ where city } i \text{ is selected as a hub} \\ 0 \text{ where city } i \text{ is not selected as a hub} \end{cases} \tag{3}$$

$$A_{ik} = \begin{cases} 1 \text{ where city } i \text{ is allocated to hub } k \\ 0 \text{ where city } i \text{ is not allocated to hub } k \end{cases} \tag{4}$$

### 1.4 Other variables

Then the **transportation distance** from $i$ to $j$ through network is:

$$\sum_k \sum_m (D_{ik} + D_{km} + D_{mj}) A_{ik} A_{jm} \tag{5}$$

The **transportation time** from $i$ to $j$ through network is:

$$\sum_k \sum_m (T_{ik} + T_{km} + T_{mj}) A_{ik} A_{jm} \tag{6}$$

The **flow** transferred from city $i$ to city $j$ through network is:

$$\sum_k \sum_m F_{ij} X_{ijkm} A_{ik} A_{jm} \tag{7}$$

## 2 Uncapacitated single allocation formulation

In this section, we formulate the uncapacitade single allocation logistic network design as a ILP problem to minimize 2 objects:

1. the cost including transportation cost and update cost .
2. the total travel time between all O-D pairs.

$$\begin{aligned}
\min \quad & \sum_i \sum_j \sum_k \sum_m F_{ij} X_{ijkm} C_{ijkm} + \sum_k U_k Y_k \\
& \sum_i \sum_j \sum_k \sum_m (T_{ik} + T_{km} + T_{mj}) A_{ik} A_{jm} && \text{(LP1)} \\
\text{s.t.} \quad & Y_k \in \{0,1\} & \forall k & \quad (8) \\
& A_{ik} \in \{0,1\} & \forall i,k & \quad (9) \\
& A_{ik} \le Y_k & \forall i,k & \quad (10) \\
& \sum_k A_{ik} = 1 & \forall i & \quad (11) \\
& \sum_k \sum_m X_{ijkm} = 1, & \forall i,j & \quad (12) \\
& X_{ijkm} \in \{0,1\} & \forall i,j,k,m & \quad (13) \\
& X_{ijkm} = A_{ik} * A_{jm} & \forall i,j,k,m & \quad (14)
\end{aligned}$$

As Eq 1 shows, we have $C_{ijkm} = s(D_{ik} + D_{km} + D_{mj})$.

Eq 8,9 assure that $Y_k$ and $A_{ik}$ are binary variables.

Eq 10 enforces that when city $i$ is allocated to city $k$, city $k$ must be selected as a hub.

Eq 11 enforces that one city $i$ must be allocated to a single hub $k$.

Eq 12 enforces that all required flow between city $i$ and $j$ must be routed through proper hub $k$ and $m$, as the sum of the fractions of flow distributed to different paths is 1.

Eq 13 sets $X_{ijkm}$ to binary variable, which means the flow between city $i$ and $j$ could only be routed through a singe path.

Eq 14 enforces the route from city $i$ and $j$ through hub $k$ and $m$ is valid, in other words, city $k$ and $m$ must all be selected as hubs.

The decision variable $X_{ijkm}$ makes the model easier to extend to capacitated or multiple allocation problems, but in fact it is redundant here, as it would be either 0 or 1 and could be replaced with

$X_{ijkm} = A_{ik} * A_{jm}$. Thus we could simplify the model as:

$$
\begin{aligned}
\min \quad & \sum_i \sum_j \sum_k \sum_m F_{ij} A_{ik} A_{jm} C_{ijkm} + \sum_k U_k Y_k \\
& \sum_i \sum_j \sum_k \sum_m (T_{ik} + T_{km} + T_{mj}) A_{ik} A_{jm} && \text{(LP2)} \\
\text{s.t.} \quad & Y_k \in \{0,1\} \quad \forall k && (15) \\
& A_{ik} \in \{0,1\} \quad \forall i,k && (16) \\
& A_{ik} \le Y_k \quad \forall i,k && (17) \\
& \sum_k A_{ik} = 1 \quad \forall i && (18)
\end{aligned}
$$

### 2.1 Solution

This is a multi-objective programming problem, there are 2 common approaches to solve this kind of problems:

1. Focus on the first object function and solve the problem, get the optimal value *opt*. Then set the value as a up bound and find the optimal value for for the second object function.
2. Simply add the 2 object function with proper weights $w_1$ and $w_2$, and to optimize the new object function $w_1 * OBJ1 + w_2 * OBJ2$.

We prefer the first approach.

## 3 Uncapacitated multiple allocation formulation

Similarly to the single allocation problem, we just modify the constrains of $A_{ik}$ and $X_{ijkm}$ to model the multiple allocation problem. Obviously, the flow demands from city $i$ and $j$ could be distributed to multiple different paths here.

$$
\begin{aligned}
\min \quad & \sum_i \sum_j \sum_k \sum_m F_{ij} X_{ijkm} C_{ijkm} + \sum_k U_k Y_k \\
& \sum_i \sum_j \sum_k \sum_m (T_{ik} + T_{km} + T_{mj}) A_{ik} A_{jm} && \text{(LP3)} \\
\text{s.t.} \quad & Y_k \in \{0,1\} \quad \forall k && (19) \\
& A_{ik} \in \{0,1\} \quad \forall i,k && (20) \\
& A_{ik} \le Y_k \quad \forall i,k && (21) \\
& \sum_k \sum_m X_{ijkm} = 1, \quad \forall i,j && (22) \\
& 0 \le X_{ijkm} \le 1 \quad \forall i,j,k,m && (23) \\
& X_{ijkm} \le A_{ik} * A_{jm} \quad \forall i,j,k,m && (24)
\end{aligned}
$$

Comparing the single allocation formulation, we make the following changes:

1. Remove the constraint $\sum_k A_{ik} = 1, \forall i$ to assure one city $i$ could be allocated to multiple hubs.
2. $X_{ijkm}$ represents the flow distributed fractions of city $i$ to city $j$, we modify it from a binary variable 0 or 1 to a decimal number in range of $[0,1]$.
3. Eq 24 means that, if there are some flows routed through the path $i \to k \to m \to j$, then city $i$ must be allocated to hub $k$ and city $j$ must be allocated hub $m$.

## 4 Capacitated single allocation formulation

We simply add a capacity constraint for the sum of all valid paths routed through hub $m$.

$$
\begin{aligned}
\min \quad & \sum_i \sum_j \sum_k \sum_m F_{ij} X_{ijkm} C_{ijkm} + \sum_k U_k Y_k \\
& \sum_i \sum_j \sum_k \sum_m (T_{ik} + T_{km} + T_{mj}) A_{ik} A_{jm} && \text{(LP4)} \\
\text{s.t.} \quad & Y_k \in \{0,1\} && \forall k \quad (25) \\
& A_{ik} \in \{0,1\} && \forall i,k \quad (26) \\
& A_{ik} \le Y_k && \forall i,k \quad (27) \\
& \sum_k A_{ik} = 1 && \forall i \quad (28) \\
& \sum_k \sum_m X_{ijkm} = 1, && \forall i,j \quad (29) \\
& X_{ijkm} \in \{0,1\} && \forall i,j,k,m \quad (30) \\
& X_{ijkm} = A_{ik} * A_{jm} && \forall i,j,k,m \quad (31) \\
& \sum_i \sum_j \sum_k F_{ij} X_{ijkm} \le K_m && \forall m \quad (32)
\end{aligned}
$$

## 5 Capacitated multiple allocation formulation

Similarly to the capacitated single allocation formulation, we add a capacity constraint for the sum of all valid paths routed through hub $m$.

$$
\begin{aligned}
\min \quad & \sum_i \sum_j \sum_k \sum_m F_{ij} X_{ijkm} C_{ijkm} + \sum_k U_k Y_k \\
& \sum_i \sum_j \sum_k \sum_m (T_{ik} + T_{km} + T_{mj}) A_{ik} A_{jm} && \text{(LP5)} \\
\text{s.t.} \quad & Y_k \in \{0,1\} && \forall k \quad (33) \\
& A_{ik} \in \{0,1\} && \forall i,k \quad (34) \\
& A_{ik} \le Y_k && \forall i,k \quad (35) \\
& \sum_k \sum_m X_{ijkm} = 1, && \forall i,j \quad (36) \\
& 0 \le X_{ijkm} \le 1 && \forall i,j,k,m \quad (37) \\
& X_{ijkm} \le A_{ik} * A_{jm} && \forall i,j,k,m \quad (38) \\
& \sum_i \sum_j \sum_k F_{ij} X_{ijkm} \le K_m && \forall m \quad (39)
\end{aligned}
$$

## 6 Evalution

In this section we evalute our LP2 model on the given dataset.

## 7 First Section

### 7.1 A Subsection Sample

Please note that the first paragraph of a section or subsection is not indented. The first paragraph that follows a table, figure, equation etc. does not need an indent, either.

Subsequent paragraphs, however, are indented.

**Sample Heading (Third Level)** Only two levels of headings should be numbered. Lower level headings remain unnumbered; they are formatted as run-in headings.

*Sample Heading (Fourth Level)* The contribution should contain no more than four levels of headings. Table 1 gives a summary of all heading levels.

**Table 1.** Table captions should be placed above the tables.

| Heading level | Example | Font size and style |
|---|---|---|
| Title (centered) | **Lecture Notes** | 14 point, bold |
| 1st-level heading | **1 Introduction** | 12 point, bold |
| 2nd-level heading | **2.1 Printing Area** | 10 point, bold |
| 3rd-level heading | **Run-in Heading in Bold.** Text follows | 10 point, bold |
| 4th-level heading | *Lowest Level Heading.* Text follows | 10 point, italic |

Displayed equations are centered and set on a separate line.

$$x + y = z \tag{40}$$

Please try to avoid rasterized images for line-art diagrams and schemas. Whenever possible, use vector graphics instead (see Fig. 1).

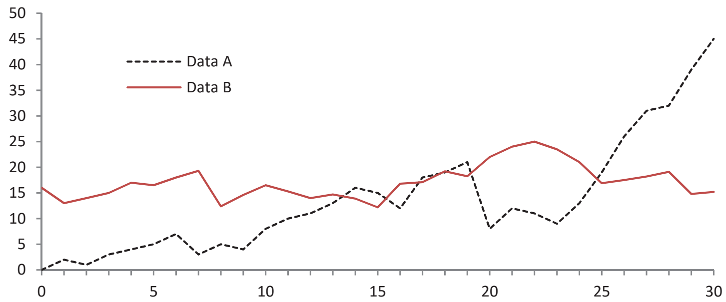


**Fig. 1.** A figure caption is always placed below the illustration. Please note that short captions are centered, while long ones are justified by the macro package automatically.

**Theorem 1.** *This is a sample theorem. The run-in heading is set in bold, while the following text appears in italics. Definitions, lemmas, propositions, and corollaries are styled the same way.*

*Proof.* Proofs, examples, and remarks have the initial word in italics, while the following text appears in normal font.

For citations of references, we prefer the use of square brackets and consecutive numbers. Citations using labels or the author/year convention are also acceptable. The following bibliography provides a sample reference list with entries for journal articles [1], an LNCS chapter [2], a book [3], proceedings without editors [4], and a homepage [5]. Multiple citations are grouped [1,2,3], [1,3,4,5].


## Acknowledgements

Here is your acknowledgements. You may also include your feelings, suggestion, and comments in the acknowledgement section.

## References

1. Author, F.: Article title. Journal **2**(5), 99–110 (2016)
2. Author, F., Author, S.: Title of a proceedings paper. In: Editor, F., Editor, S. (eds.) CONFERENCE 2016, LNCS, vol. 9999, pp. 1–13. Springer, Heidelberg (2016).
3. Author, F., Author, S., Author, T.: Book title. 2nd edn. Publisher, Location (1999)
4. Author, A.-B.: Contribution title. In: 9th International Proceedings on Proceedings, pp. 1–2. Publisher, Location (2010)
5. LNCS Homepage, http://www.springer.com/lncs. Last accessed 4 Oct 2017