Answers to Reviewer #1

This paper is a very high quality paper for the Journal of Computational Physics and should be accepted for publication without reservation. One of the main strengths of this paper is its very thorough review of spatial discretization technology for radiation diffusion. It is well written, and includes a comprehensive set of test problems applied to the new discretization proposed.

Thank you for your thorough review and appreciation of our work.

I have some suggestions for improvement, but as a reviewer, I do not require these additions to the paper for publication.

Thank you. Even though you do not require these additions for publication, I take it at heart to publish the possible papers and I will include as many suggestions for improvement as possible in the revised version of this manuscript.

On the list or prior works, a reference or references should be included for each item in the list. For the first two items, no references are included.

In this short enumeration of prior works, our intent was not to add new references but to recall the various techniques we have just reviewed in the above paragraphs. However, we have forgotten to discuss FV techniques for polygonal meshes. This is now corrected and a sentence has been added in the above literature review. Mimetic FD were already described and reviewed above so we do not think we are missing references for this type of discretization techniques.

The derivation of the discretization is complicated enough to warrant an appendix which goes through the details of the mathematics (e.g. how to get to Eq. 3 and Eq. 8), like one would find in a thesis or dissertation. I am a proponent of including these detailed types of explanations in papers because I think they enhance the read-ability of the paper; however, some disagree with this philosophy.

The articles we cite "Unified Analysis of Discontinuous Galerkin Methods for Elliptic Problems" (Douglas et. al.), the habilitation thesis "Discontinuous Galerkin Methods for Viscous Incompressible Flow" by Kanschat and a SIAM book on DG methods that we have added in the revised manuscript "Discontinuous Galerkin Methods for Solving Elliptic and Parabolic Equations: Theory and

Implementation" by Rivereére are resources that provide an excellent description of the SIP method for elliptic problems (along with many DG variants).

The DG technique for the radiation diffusion equation is the standard Symmetric Interior Penalty technique. We chose it because it yields a symmetric linear system that can be efficiently solved using a preconditioned conjugate gradient technique. The novelty is the use SIP on polygonal meshes using PWLD basis functions (and to perform AMR with PWLD).

The DG bilinear form, Eq.(8), may look a bit complex. However, it only contains volume (surface in 2D) integrals and face (edge in 2D) integrals. The volume integrals are split on each element (thus we loop over the elements). Each integral in the volume of a given element only contribute entries in the global matrix for the unknowns present in that element. The face integrals are split over the faces (thus we loop over the set of interior faces). Each face integral contributes entries in the global matrix for the unknowns present in the two elements separating by that face. Thus, the manner in which cell/face information is accessed and used is different. It felt like coding would be clearer if our implementation would mimic the various terms found in the bilinear form of Eq.(8).

One can, of course, use linear basis functions on quadrilaterals. However, the ultimate goal is to employ the diffusion equation as a preconditioner to a radiation transport solver. In the thick diffusion limit, employing linear basis functions on quadrilaterals leads to a locking of the transport solution to the wrong answer. In reactor problems, where the thick diffusion limit is not a relevant regime, researcher may prefer to use linear basis functions ($a+bx+cy+dz$)

on hexadredal cells to reduce the number of unknowns without drastically reducing accuracy.

What code was this method implemented in? Was it just a test code? Was it MATLAB? Were there any parallel runs? What linear solver package was used, if any? The test bed code is MATLAB. We use both the built-in optimized LU package but also a conjugate gradient solve preconditioned with an algebraic multigrid method. The PWLD SIP technique for diffusion is now being implement in a massively C++ radiation transport to be used either as a standalone solver or a synthetic preconditioner.

I think a potential weakness of the paper is that the author does not compare the new method to any of the existing methods. If this is not hard to do, these results would be very interesting to include in the paper. Additionally, these results could be included in a future paper or conference proceedings.
Thank you

For the linear test problem results, what are the iteration counts for the linear solve as the mesh is distorted? Was the iteration count a lot different for the different mesh types? For quads vs. polygons?
Thank you

A reference for computing bounded Voronoi diagrams is a good idea.
Thank you

For the convergence plots, I think the terminology is a bit inconsistent. In the text the author writes in terms of dof. In the plots, he uses number of unknowns. This could be a bit more consistent, but is a minor detail.
Thank you

In general, an understanding of how resistant this method is to negative solutions would be another interesting thing to study, especially in a time dependent case, with a delta-function like source. I would anticipate that this method would perform a lot better than the CFEMs.
Thank you

Does the author anticipate a straight-forward application to RZ geometry and 3D geometry? Will the method recover spherical symmetry in RZ geometry. Brunner, et. Al. have observed an issue with this for RZ geometry. See T.A. Brunner, "Perserving Spherical Symmetry in Axisymmetric Coordinates for Diffusion," in Proc. International Conference on Mathematics and Computational Methods Applied to Nuclear Science and Engineering, May 5-9, 2013, Sun Valley, ID (2013), CD-ROM
Thank you