Using Geometric Brownian Motion to Model Options Strategies

Jake Freedle, Pierre Visconti

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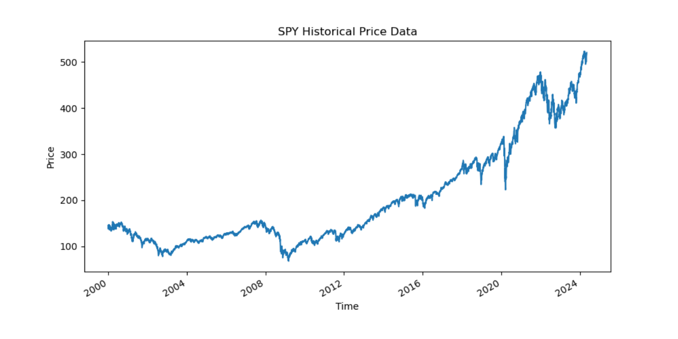
Outline

Mathematical Groundwork

What is Geometric Brownian Motion

Geometric Brownian Motion (GBM) is a continuous-time stochastic process in which the logarithm of a randomly varying value follows a Brownian motion with drift.



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When does a Stochastic Process follow GBM

A stochastic process S_t is said to follow a GBM if it satisfies the following stochastic differential equation (SDE):

$$dS_t = \mu S_t \, dt + \sigma S_t \, dW_t.$$

We can apply Itô's formula and find the following solution

$$S_t = S_0 \left(e^{\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t} \right).$$

Standard Brownian Motion W_t

1. Initial Value: $W_0 = 0$.
2. Independent Increments: The increments of the process $W_t - W_s$ for $0 \leq s < t$ are independent of the past values $W(u)$ for $u \leq s$.
3. Normal Distribution of Increments: The increments $W_t - W_s$ are normally distributed with mean 0 and variance $t - s$. Specifically, $W_t - W_s \sim N(0, t - s)$.
4. Continuous Paths: The paths of W_t are continuous functions of t, W_t is continuous in t.

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Application to Financial Assets

Financial Interpretation of the Solution

$$S_t = S_0 \left(e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t} \right)$$

S_t: Asset Price

t: Time

$\mu - \frac{\sigma^2}{2}$: Expected Return (Drift)

σ: Standard Deviation (Volatility)

W_t: Standard Brownian Process

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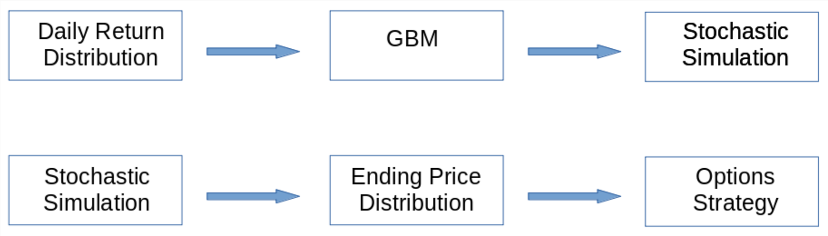
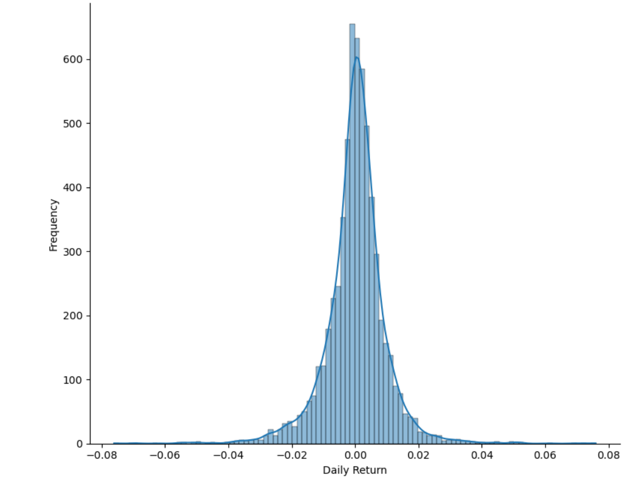
Stock Price Modeling

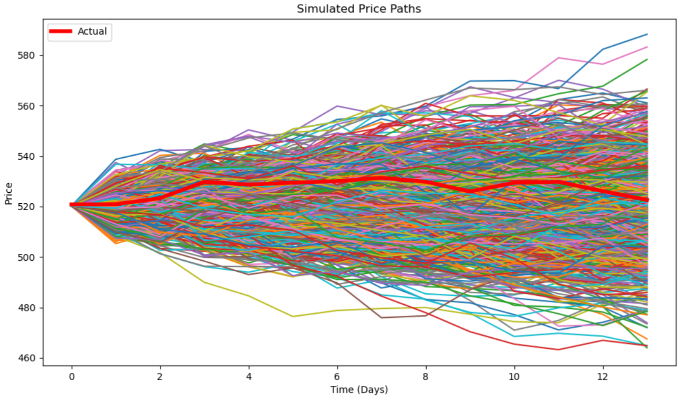
Modeling Process



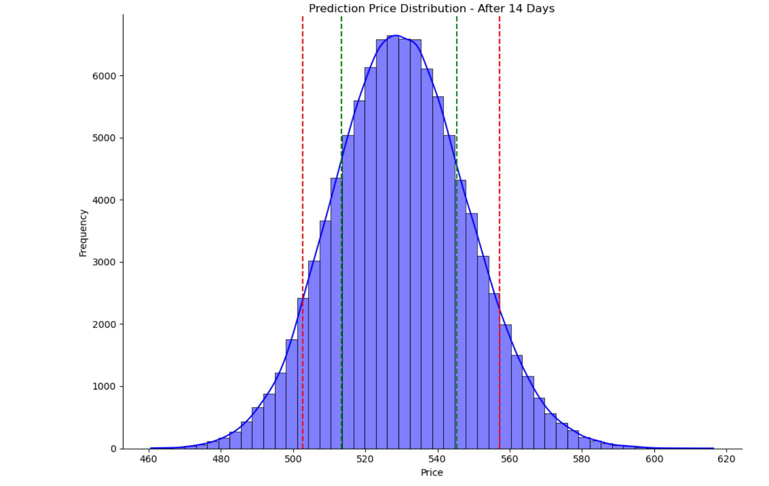
Historical Daily Percent Change Distribution



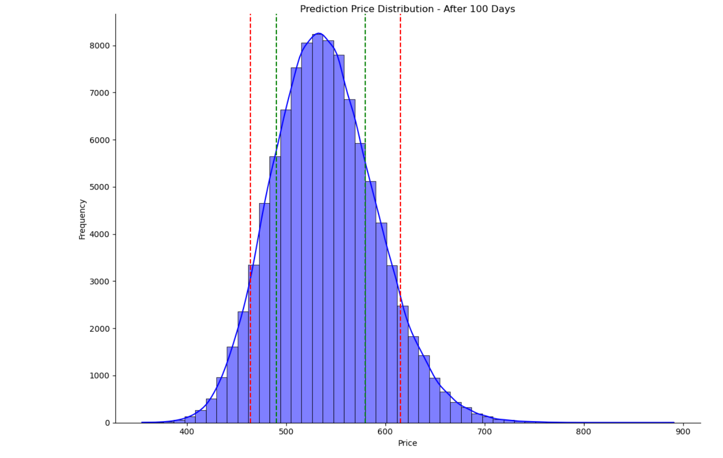
Simulated Price Paths



Ending Price Probability Distribution

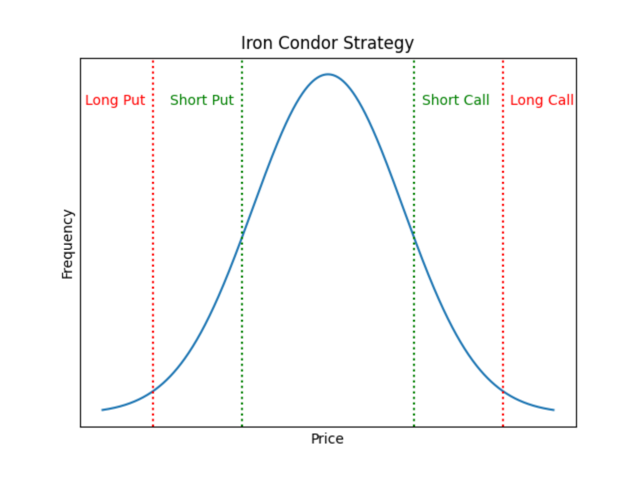


Ending Price Probability Distribution (Large T)

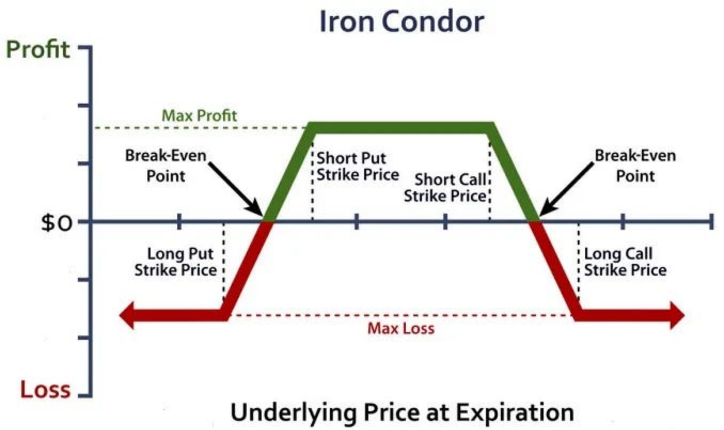


Iron Condor Options Strategy

An Introduction to Options and the Iron Condor Strategy



Developing an Options Strategy



Determining the Optimal Trade For a Given Interval

Calculating statistics:

- maxProfit $= (SP_p + SC_p - LP_p - LC_p) * 100.0$
- maxLoss$_p$ = maxProfit- (SP$_s - LP_s) * 100.0$
- maxLoss$_c$ = maxProfit - (LC$_s - SC_s) * 100.0$
- breakEven$_p$ =SP$_s - (SP_p + LP_p)$
- breakEven$_c$ =SC$_s + (SC_p + LC_p)$
- RR = maxProfit / max{maxLoss$_p$,maxLoss$_c$}

Optimizing the outer contracts:

- Define RR as a reward to risk ratio for the trade, where reward=maxProfit and risk=max{maxLoss$_p$,maxLoss$_c$}.
- Iterate over all the possible strikes for the long put and long call contracts to find the highest RR.

Expected Value of a Trade

x : profit (loss) per trade.

W : amount won if trade is a win (reward).

L : amount lost if trade is a loss.

$$E[X] = W \cdot P_w - L \cdot P_\ell$$

Determining Minimum RR For a Given Interval

We can represent W as a ratio of reward/loss: RR.

$$\frac{E[X]}{L} = \frac{W}{L} \cdot P_w - \frac{L}{L} \cdot P_\ell$$

$$E_r[X] = RR \cdot P_w - 1 \cdot P_\ell$$

Can solve for minimum RR by setting $E_r[X] = 0$:

$$\mathrm{RR}_{\mathrm{min}} = \frac{P_\ell}{P_w}$$

Ex: Assuming an interval of 90%

$$\mathrm{RR}_{\mathrm{min}} = \frac{0.1}{0.9} = 0.111$$

Determining if a Trade Should Be Taken

Ex: Assume an interval of 90%, a max profit of \$50, and a max loss of \$120.

$$\text{RR}_{\text{min}} = \frac{0.1}{0.9} = 0.111$$

$$\text{RR}_{\text{trade}} = \frac{50}{120} = 0.417$$

$$E[X] = 50 \cdot 0.9 - 120 \cdot 0.1 = \$33$$

$$E_r[X] = 0.417 \cdot 0.9 - 1 \cdot 0.1 = 0.275$$

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Finding The Optimal Trade

Algorithm:

Iterate over all assets with high options trading volume: {SPY, QQQ, IWM, SLV,...}.

- For each asset, iterate over a list of pre-defined intervals: {0.9, 0.85, 0.8,...}.
  - For each interval, determine the optimal iron condor strategy.

Output:

A list of the optimal iron condor setups for every interval and asset. The optimal trade is the one with the highest $E_r[X]$.

Results

Live coding demo.

Conclusion

Notes and Future Work

- Geometric Brownian Motion is likely too simple.
- Back-testing with historical data to test different assets.
- Building a more complex options model.

Disclaimer: This is not investment advice!

References

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Iron Condor: How This Options Strategy Works, With Examples, April 2024.

Elias Melul.
Monte Carlo Simulations for Stock Price Predictions [Python], May 2020.

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Geometric brownian motion — Wikipedia, the free encyclopedia, 2024.