Assymetric rooted bicolored binary trees $R_o := (Z_U + R_\bullet)^2$ $R_\bullet := (Z_U + R_\bullet)^2 \times Z_L$ $\hat{R}_\circ := Z_U + R_\bullet \times Z_U + Z_U \times R_\bullet + R_\bullet^2$ $\hat{R}_\circ := \hat{R}_\circ \times Z_L \times Z_U^2 + Z_U^2 \times Z_L \times \hat{R}_\circ + \hat{R}_\circ \times Z_L \times \hat{R}_\circ$ $R_\circ^{(ab)} := \hat{R}_\circ \times Z_L \times Z_U + Z_U \times Z_L \times R_\circ + \hat{R}_\circ \times Z_L \times R_\circ^2$ $R_\circ^{(ab)} := \hat{R}_\circ \times Z_L \times Z_U + Z_U \times Z_L \times R_\circ + Z_L \times R_\circ^2$ $R_\circ^{(ab)} := R_\circ \times Z_L \times Z_U + Z_U \times Z_L \times R_\circ + Z_L \times R_\circ^2$ $R_\circ^{(ab)} := R_\circ^{(ab)} + R_\circ^{(ab)}$ Assymetric (unrooted) bicolored binary trees $R_\circ^{(ab)} := R_\circ^{(ab)} \times R_\circ^{(ab)}$ Two possible techniques: 1. Sample from $\Gamma_L^{(ab)}$ until Bern $\left(\frac{2}{ \Gamma^{(ab)} }\right)$ 2. Apply rejection already during the sampling of $R_\circ^{(ab)}$ (Lemma 12) 2. is more efficient. Assymetric 1-derived bicolored binary trees $R_\circ^{(ab)} := R_\circ^{(ab)} \times R_\circ^{(ab)}$ $R_\circ^{(ab)} := R_\circ^{$
$\mathcal{R}_{\circ} := (\mathcal{Z}_{U} + \mathcal{R}_{\bullet})^{2}$ $\mathcal{R}_{\bullet} := (\mathcal{Z}_{U} + \mathcal{R}_{\circ})^{2} * \mathcal{Z}_{L}$ $\hat{\mathcal{R}}_{\circ} := \mathcal{Z}_{U} + \mathcal{R}_{\bullet} * \mathcal{Z}_{U} * \mathcal{R}_{\bullet} + \mathcal{R}_{\bullet}^{2}$ $\hat{\mathcal{R}}_{\bullet} := \hat{\mathcal{R}}_{\circ} * \mathcal{Z}_{L} * \mathcal{Z}_{U}^{2} * \mathcal{Z}_{L} * \hat{\mathcal{R}}_{\circ} + \hat{\mathcal{R}}_{\circ} * \mathcal{Z}_{L} * \hat{\mathcal{R}}_{\circ}$ $\hat{\mathcal{R}}_{\circ}^{(as)} := \hat{\mathcal{R}}_{\bullet} * \mathcal{Z}_{L} * \mathcal{Z}_{U}^{2} * \mathcal{Z}_{L} * \hat{\mathcal{R}}_{\circ} + \hat{\mathcal{R}}_{\circ} * \mathcal{Z}_{L} * \hat{\mathcal{R}}_{\circ}$ $\mathcal{R}_{\bullet}^{(as)} := \mathcal{R}_{\bullet} * \mathcal{Z}_{L} * \mathcal{Z}_{U} + \mathcal{Z}_{U} * \mathcal{Z}_{L} * \mathcal{R}_{\circ} + \mathcal{Z}_{L} * \mathcal{R}_{\circ}^{2}$ $\mathcal{R}_{\bullet}^{(as)} := \mathcal{R}_{\bullet} * \mathcal{Z}_{L} * \mathcal{Z}_{U} + \mathcal{Z}_{U} * \mathcal{Z}_{L} * \mathcal{R}_{\circ} + \mathcal{Z}_{L} * \mathcal{R}_{\circ}^{2}$ $\mathcal{R}_{\bullet}^{(as)} := \mathcal{R}_{\bullet} * \mathcal{Z}_{L} * \mathcal{Z}_{U} + \mathcal{Z}_{U} * \mathcal{Z}_{L} * \mathcal{R}_{\circ} + \mathcal{Z}_{L} * \mathcal{R}_{\circ}^{2}$ $\mathcal{R}_{\bullet}^{(as)} := \mathcal{R}_{\bullet} * \mathcal{Z}_{L} * \mathcal{Z}_{U} + \mathcal{Z}_{U} * \mathcal{Z}_{L} * \mathcal{R}_{\circ} + \mathcal{Z}_{L} * \mathcal{R}_{\circ}^{2}$ $\mathcal{R}_{\bullet}^{(as)} := \mathcal{R}_{\bullet} * \mathcal{Z}_{L} * \mathcal{Z}_{U} + \mathcal{Z}_{U} * \mathcal{Z}_{L} * \mathcal{R}_{\circ} + \mathcal{Z}_{L} * \mathcal{R}_{\circ}^{2}$ $\mathcal{R}_{\bullet}^{(as)} := \mathcal{R}_{\bullet} * \mathcal{Z}_{L} * \mathcal{Z}_{U} * \mathcal{Z}$
$\begin{array}{lll} \hat{\mathcal{R}}_{\circ} := \mathcal{Z}_{U} + \mathcal{R}_{\bullet} \star \mathcal{Z}_{U} + \mathcal{Z}_{U} \star \mathcal{R}_{\bullet} + \mathcal{R}_{\bullet}^{2} \\ \hat{\mathcal{R}}_{\bullet} := \hat{\mathcal{R}}_{\circ} \star \mathcal{Z}_{L} \star \mathcal{Z}_{U}^{2} + \mathcal{Z}_{U}^{2} \star \mathcal{Z}_{L} \star \hat{\mathcal{R}}_{\circ} + \hat{\mathcal{R}}_{\circ} \star \mathcal{Z}_{L} \star \hat{\mathcal{R}}_{\circ} \\ \hat{\mathcal{R}}_{\circ}^{(as)} := \hat{\mathcal{R}}_{\circ} \star \mathcal{Z}_{L} \star \mathcal{Z}_{U} + \mathcal{Z}_{U} \star \hat{\mathcal{R}}_{\bullet} + \mathcal{R}_{\bullet}^{2} \\ \mathcal{R}_{\bullet}^{(as)} := \mathcal{R}_{\circ}^{(as)} \star \mathcal{Z}_{L} \star \mathcal{Z}_{U} + \mathcal{Z}_{U} \star \mathcal{Z}_{L} \star \mathcal{R}_{\circ} + \mathcal{Z}_{L} \star \mathcal{R}_{\circ}^{2} \\ \mathcal{K} := \mathcal{R}_{\bullet}^{(as)} + \mathcal{R}_{\circ}^{(as)} \\ \text{Assymetric (unrooted) bicolored binary trees} & \text{Get rid of the rooting} = \text{get rid of the u-derivation} \\ \mathcal{K} \leftarrow rejection(\underline{\mathcal{K}}) & \text{Two possible techniques:} \\ 1. \text{ Sample from } \Gamma \underline{\mathcal{K}} \text{ until Bern}(\frac{2}{ \gamma }) \\ 2. \text{ Apply rejection already during the sampling of } \\ \mathcal{K} \text{ (Lemma 12)} \\ 2. \text{ is more efficient.} \\ \text{Assymetric 1-derived bicolored binary trees} \\ \mathcal{K}' \leftarrow dx_from_dy(\underline{\mathcal{K}}) & \alpha_{L/U} = 2/3, \text{ see 5.3.1} \\ \text{Irreducible dissections (of the hexagon)} \\ \mathcal{I} \leftarrow bijection(\mathcal{K}) & \text{Closure} \\ \mathcal{I}' \leftarrow bijection(\mathcal{K}') & \text{Closure} \\ \mathcal{I}' = 3 \star \mathcal{Z}_{L} \star \mathcal{Z}_{U} \star \mathcal{I} \\ \mathcal{I}' := 3 \star \mathcal{Z}_{L} \star \mathcal{Z}_{U} \star \mathcal{I} \\ \mathcal{I}' := 3 \star \mathcal{Z}_{U} \star \mathcal{I} + 3 \star \mathcal{Z}_{L} \star \mathcal{Z}_{U} \star \mathcal{I}' \\ \mathcal{I}_{\sigma} \leftarrow rejection(\mathcal{I}) & admissible \text{ rooted irreducible dissections; sample } \delta \\ \end{array}$
$\begin{array}{lll} \hat{\mathcal{R}}_{\bullet} := \hat{\mathcal{R}}_{\circ} \star \mathcal{Z}_{L} \star \mathcal{Z}_{U}^{2} + \mathcal{Z}_{U}^{2} \star \mathcal{Z}_{L} \star \hat{\mathcal{R}}_{\circ} + \hat{\mathcal{R}}_{\circ} \star \mathcal{Z}_{L} \star \hat{\mathcal{R}}_{\circ} \\ \mathcal{R}_{\circ}^{(as)} := \hat{\mathcal{R}}_{\bullet} \star \mathcal{Z}_{U} + \mathcal{Z}_{U} \star \hat{\mathcal{R}}_{\bullet} + \mathcal{R}_{\bullet}^{2} \\ \mathcal{R}_{\bullet}^{(as)} := \mathcal{R}_{\bullet} \star \mathcal{Z}_{L} \star \mathcal{Z}_{U} + \mathcal{Z}_{U} \star \mathcal{Z}_{L} \star \mathcal{R}_{\circ} + \mathcal{Z}_{L} \star \mathcal{R}_{\circ}^{2} \\ \mathcal{K} := \mathcal{R}_{\bullet}^{(as)} := \mathcal{R}_{\bullet} \star \mathcal{Z}_{L} \star \mathcal{Z}_{U} + \mathcal{Z}_{U} \star \mathcal{Z}_{L} \star \mathcal{R}_{\circ} + \mathcal{Z}_{L} \star \mathcal{R}_{\circ}^{2} \\ \mathcal{K} := \mathcal{R}_{\bullet}^{(as)} := \mathcal{R}_{\bullet} \star \mathcal{Z}_{L} \star \mathcal{Z}_{U} \star \mathcal{Z}_{U} \star \mathcal{Z}_{L} \star \mathcal{R}_{\circ} + \mathcal{Z}_{L} \star \mathcal{R}_{\circ}^{2} \\ \mathcal{K} := \mathcal{R}_{\bullet}^{(as)} := \mathcal{R}_{\bullet} \star \mathcal{Z}_{U} \star \mathcal{Z}_{U} \star \mathcal{Z}_{U} \star \mathcal{R}_{\circ} + \mathcal{Z}_{L} \star \mathcal{R}_{\circ}^{2} \\ \mathcal{K} := \mathcal{R}_{\bullet}^{(as)} := \mathcal{R}_{\bullet} \star \mathcal{Z}_{U} \star \mathcal{Z}_{U} \star \mathcal{Z}_{U} \star \mathcal{Z}_{U} \star \mathcal{R}_{\circ} + \mathcal{Z}_{L} \star \mathcal{R}_{\circ}^{2} \\ \mathcal{K} := \mathcal{R}_{\bullet}^{(as)} := \mathcal{R}_{\bullet} \star \mathcal{Z}_{U} \star \mathcal{Z}_{$
$\mathcal{R}_{\diamond}^{(as)} := \hat{\mathcal{R}}_{\bullet} \star \mathcal{Z}_{U} + \mathcal{Z}_{U} \star \hat{\mathcal{R}}_{\bullet} + \mathcal{R}_{\bullet}^{2}$ $\mathcal{R}_{\bullet}^{(as)} := \mathcal{R}_{\diamond} \star \mathcal{Z}_{L} \star \mathcal{Z}_{U} + \mathcal{Z}_{U} \star \mathcal{R}_{\bullet} + \mathcal{R}_{\bullet}^{2}$ $\mathcal{K} := \mathcal{R}_{\bullet}^{(as)} + \mathcal{R}_{\diamond}^{(as)}$ Assymetric (unrooted) bicolored binary trees $\mathcal{K} \leftarrow rejection(\mathcal{K})$ $\mathcal{K} \leftarrow rejection(\mathcal{K})$ Two possible techniques: $1. \text{ Sample from } \Gamma \underline{\mathcal{K}} \text{ until Bern}(\frac{2}{ \gamma })$ $2. \text{ Apply rejection already during the sampling of } \underline{\mathcal{K}} \text{ (Lemma 12)}$ $2. \text{ is, more. efficient.}$ Assymetric 1-derived bicolored binary trees $\mathcal{K}' \leftarrow dx \text{-} from \text{-} dy(\underline{\mathcal{K}})$ $\mathcal{I} \leftarrow bijection(\mathcal{K})$ $\mathcal{I} \leftarrow bijection(\mathcal{K})$ $\mathcal{I}' \leftarrow bijection(\mathcal{K}')$ $\mathcal{I}' = 3 \star \mathcal{Z}_{L} \star \mathcal{Z}_{U} \star \mathcal{I}$ $\mathcal{I}' := 3 \star \mathcal{Z}_{U} \star \mathcal{I} + 3 \star \mathcal{Z}_{L} \star \mathcal{Z}_{U} \star \mathcal{I}'$ $\mathcal{I}_{a} \leftarrow rejection(\mathcal{I})$ $\mathcal{I} \rightarrow dmissible \text{ rooted irreducible dissections; sample } \delta$
$\mathcal{R}^{(as)}_{\bullet} := \mathcal{R}_{\circ} \star \mathcal{Z}_{L} \star \mathcal{Z}_{U} + \mathcal{Z}_{U} \star \mathcal{Z}_{L} \star \mathcal{R}_{\circ} + \mathcal{Z}_{L} \star \mathcal{R}_{\circ}^{2}$ $\mathcal{K} := \mathcal{R}^{(as)}_{\bullet} + \mathcal{R}^{(as)}_{\circ}$ Assymetric (unrooted) bicolored binary trees $\mathcal{K} \leftarrow rejection(\mathcal{K})$ $\mathcal{K} \leftarrow rejection(\mathcal{K})$ Two possible techniques: $1. \text{ Sample from } \Gamma \underline{\mathcal{K}} \text{ until Bern}(\frac{2}{ \gamma })$ $2. \text{ Apply rejection already during the sampling of } \underline{\mathcal{K}} \text{ (Lemma 12)}$ $2. \text{ is more efficient.}$ Assymetric 1-derived bicolored binary trees $\mathcal{K}' \leftarrow dx \text{-} from \text{-} dy(\underline{\mathcal{K}})$ $\mathcal{L} \leftarrow bijection(\mathcal{K})$ $\mathcal{L} \leftarrow bijection(\mathcal{K})$ $\mathcal{L}' \leftarrow bijection(\mathcal{K}')$ $\mathcal{L}' \leftarrow bijection(\mathcal{K}')$ $\mathcal{L}' = 3 \star \mathcal{L}_{L} \star \mathcal{L}_{U} \star \mathcal{L}$ $\mathcal{L}' := 3 \star \mathcal{L}_{L} \star \mathcal{L}_{U} \star \mathcal{L}$ $\mathcal{L}' := 3 \star \mathcal{L}_{L} \star \mathcal{L}_{L} \star \mathcal{L}_{U} \star \mathcal{L}$ $\mathcal{L}' \leftarrow bijection(\mathcal{L}')$ $\mathcal{L}' \leftarrow $
$\underline{\mathcal{K}} := \mathcal{R}^{(as)}_{\bullet} + \mathcal{R}^{(as)}_{\circ}$ Assymetric (unrooted) bicolored binary trees $\mathcal{K} \leftarrow rejection(\underline{\mathcal{K}})$ Two possible techniques: $1. \text{ Sample from } \Gamma\underline{\mathcal{K}} \text{ until Bern}(\frac{2}{ \gamma })$ $2. \text{ Apply rejection already during the sampling of } \underline{\mathcal{K}} \text{ (Lemma 12)}$ $2. \text{ is more efficient.}$ Assymetric l-derived bicolored binary trees $\mathcal{K}' \leftarrow dx_from_dy(\underline{\mathcal{K}})$ $\mathcal{L} \leftarrow bijection(\mathcal{K})$ $\mathcal{L} \leftarrow bijection(\mathcal{K})$ $\mathcal{L}' \leftarrow bijection(\mathcal{K}')$ $\mathcal{L}' \leftarrow bijection(\mathcal{K}')$ $\mathcal{L}' \leftarrow bijection(\mathcal{K}')$ $\mathcal{L}' = 3*\mathcal{L}_L*\mathcal{L}_U*\mathcal{L}$ $\mathcal{L}' = 3*\mathcal{L}_L*\mathcal{L}_L*\mathcal{L}_L*\mathcal{L}$ $\mathcal{L}' = 3*\mathcal{L}_L*\mathcal{L}_L*\mathcal{L}_L*\mathcal{L}_L*\mathcal{L}$ $\mathcal{L}' = 3*\mathcal{L}_L*L$
Assymetric (unrooted) bicolored binary trees $\mathcal{K} \leftarrow rejection(\underline{\mathcal{K}})$ Two possible techniques: $1. \text{ Sample from } \underline{\Gamma}\underline{\mathcal{K}} \text{ until Bern}(\frac{2}{ \gamma })$ $2. \text{ Apply rejection already during the sampling of } \underline{\mathcal{K}} \text{ (Lemma 12)}$ $2. \text{ is more efficient.}$ Assymetric l-derived bicolored binary trees $\mathcal{K}' \leftarrow dx_from_dy(\underline{\mathcal{K}})$ $\mathcal{A}_{L/U} = 2/3, \text{ see } 5.3.1$ Irreducible dissections (of the hexagon) $\mathcal{I} \leftarrow bijection(\mathcal{K})$ Closure $\mathcal{I}' \leftarrow bijection(\mathcal{K}')$ Closure $\mathcal{I}' \leftarrow bijection(\mathcal{K}')$ Closure $\mathcal{I}' = 3*\mathcal{Z}_L*\mathcal{Z}_U*\mathcal{I}$ $\mathcal{I}' := 3*\mathcal{Z}_L*\mathcal{Z}_U*\mathcal{I}$ $\mathcal{I}' := 3*\mathcal{Z}_L*\mathcal{Z}_L*\mathcal{Z}_U*\mathcal{I}'$ $\mathcal{I}' \leftarrow admissible \text{ rooted irreducible dissections; sample } \delta$
$\mathcal{K} \leftarrow rejection(\underline{\mathcal{K}}) \hspace{1cm} \text{Two possible techniques:} \\ 1. \hspace{1cm} \text{Sample from } \Gamma \underline{\mathcal{K}} \text{ until Bern}(\frac{2}{ \gamma }) \\ 2. \hspace{1cm} \text{Apply rejection already during the sampling of} \\ \underline{\mathcal{K}} \hspace{1cm} \text{(Lemma 12)} \\ 2. \hspace{1cm} \text{is more efficient.} \\ \text{Assymetric 1-derived bicolored binary trees} \\ \mathcal{K}' \leftarrow dx_from_dy(\underline{\mathcal{K}}) \hspace{1cm} \alpha_{L/U} = 2/3, \hspace{1cm} \text{see } 5.3.1 \\ \text{Irreducible dissections (of the hexagon)} \\ \mathcal{I} \leftarrow bijection(\mathcal{K}) \hspace{1cm} \text{Closure} \\ \mathcal{I}' \leftarrow bijection(\mathcal{K}') \hspace{1cm} \text{Closure} \\ \mathcal{I} := 3 * \mathcal{Z}_L * \mathcal{Z}_U * \mathcal{I} \\ \mathcal{I}' := 3 * \mathcal{Z}_U * \mathcal{I} + 3 * \mathcal{Z}_L * \mathcal{Z}_U * \mathcal{I}' \\ \mathcal{I}_a \leftarrow rejection(\mathcal{I}) \hspace{1cm} admissible \hspace{1cm} \text{rooted irreducible dissections; sample } \delta$
2. Apply rejection already during the sampling of $\underline{\mathcal{K}}$ (Lemma 12) 2. is more efficient. Assymetric 1-derived bicolored binary trees $\mathcal{K}' \leftarrow dx_from_dy(\underline{\mathcal{K}})$ $\mathcal{L} \leftarrow bijection(\mathcal{K})$ $\mathcal{I}' \leftarrow bijection(\mathcal{K}')$ $\mathcal{I}' \leftarrow bijection(\mathcal{K}')$ $\mathcal{I}' = 3 * \mathcal{Z}_L * \mathcal{Z}_U * \mathcal{I}$ $\mathcal{I}' := 3 * \mathcal{Z}_U * \mathcal{I} + 3 * \mathcal{Z}_L * \mathcal{Z}_U * \mathcal{I}'$ $\mathcal{I}_a \leftarrow rejection(\mathcal{I})$ 2. is more efficient. $\alpha_{L/U} = 2/3, \text{ see 5.3.1}$ Closure $\mathcal{I} \leftarrow bijection(\mathcal{K})$ $\mathcal{I}' = 2/3, \text{ see 5.3.1}$ Closure $\mathcal{I}' \leftarrow bijection(\mathcal{K}')$ $\mathcal{I}' = 2/3, \text{ see 5.3.1}$ Closure $\mathcal{I}' \leftarrow bijection(\mathcal{K}')$ $\mathcal{I}' = 3 * \mathcal{I}_L * $
$ \underbrace{\mathcal{K}} \text{ (Lemma 12)} $ $ 2. \text{ is more efficient.} $ Assymetric l-derived bicolored binary trees $ \mathcal{K}' \leftarrow dx_from_dy(\underline{\mathcal{K}}) $ $ \alpha_{L/U} = 2/3, \text{ see } 5.3.1 $ Irreducible dissections (of the hexagon) $ \mathcal{I} \leftarrow bijection(\mathcal{K}) $ $ \mathcal{I}' \leftarrow bijection(\mathcal{K}') $ $ \mathcal{I} := 3 \star \mathcal{Z}_L \star \mathcal{Z}_U \star \mathcal{I} $ $ \mathcal{I}' := 3 \star \mathcal{Z}_L \star \mathcal{Z}_U \star \mathcal{I} $ $ \mathcal{I}' := 3 \star \mathcal{Z}_U \star \mathcal{I} + 3 \star \mathcal{Z}_L \star \mathcal{Z}_U \star \mathcal{I}' $ $ \mathcal{I}_{a} \leftarrow rejection(\mathcal{I}) $ $ admissible \text{ rooted irreducible dissections; sample } \delta $
Assymetric 1-derived bicolored binary trees $\mathcal{K}' \leftarrow dx_from_dy(\underline{\mathcal{K}}) \qquad \qquad \alpha_{L/U} = 2/3, \text{ see } 5.3.1$ Irreducible dissections (of the hexagon) $\mathcal{I} \leftarrow bijection(\mathcal{K}) \qquad \qquad \text{Closure}$ $\mathcal{I}' \leftarrow bijection(\mathcal{K}') \qquad \qquad \text{Closure}$ $\mathcal{I} := 3 \star \mathcal{Z}_L \star \mathcal{Z}_U \star \mathcal{I} \qquad \qquad 3 \star \mathcal{Z}_L = \mathcal{Z}_L + \mathcal{Z}_L + \mathcal{Z}_L$ $\mathcal{J}' := 3 \star \mathcal{Z}_U \star \mathcal{I} + 3 \star \mathcal{Z}_L \star \mathcal{Z}_U \star \mathcal{I}'$ $\mathcal{J}_a \leftarrow rejection(\mathcal{J}) \qquad \qquad admissible \text{ rooted irreducible dissections; sample } \delta$
$\mathcal{K}' \leftarrow dx_from_dy(\underline{\mathcal{K}}) \qquad \qquad \alpha_{L/U} = 2/3, \text{ see 5.3.1}$ $\mathcal{I} \leftarrow bijection(\mathcal{K}) \qquad \qquad \text{Closure}$ $\mathcal{I}' \leftarrow bijection(\mathcal{K}') \qquad \qquad \text{Closure}$ $\mathcal{I} := 3 \star \mathcal{Z}_L \star \mathcal{Z}_U \star \mathcal{I} \qquad \qquad 3 \star \mathcal{Z}_L = \mathcal{Z}_L + \mathcal{Z}_L + \mathcal{Z}_L$ $\mathcal{J}' := 3 \star \mathcal{Z}_U \star \mathcal{I} + 3 \star \mathcal{Z}_L \star \mathcal{Z}_U \star \mathcal{I}'$ $\mathcal{J}_a \leftarrow rejection(\mathcal{J}) \qquad \qquad admissible \text{ rooted irreducible dissections; sample } \delta$
Irreducible dissections (of the hexagon) $\mathcal{I} \leftarrow bijection(\mathcal{K}) \qquad \qquad \text{Closure}$ $\mathcal{I}' \leftarrow bijection(\mathcal{K}') \qquad \qquad \text{Closure}$ $\mathcal{J} := 3 \star \mathcal{Z}_L \star \mathcal{Z}_U \star \mathcal{I} \qquad \qquad 3 \star \mathcal{Z}_L = \mathcal{Z}_L + \mathcal{Z}_L + \mathcal{Z}_L$ $\mathcal{J}' := 3 \star \mathcal{Z}_U \star \mathcal{I} + 3 \star \mathcal{Z}_L \star \mathcal{Z}_U \star \mathcal{I}'$ $\mathcal{J}_a \leftarrow rejection(\mathcal{J}) \qquad \qquad admissible \text{ rooted irreducible dissections; sample } \delta$
$\mathcal{I} \leftarrow bijection(\mathcal{K})$ $\mathcal{I}' \leftarrow bijection(\mathcal{K}')$ $\mathcal{I} := 3 \star \mathcal{Z}_L \star \mathcal{Z}_U \star \mathcal{I}$ $\mathcal{I}' := 3 \star \mathcal{Z}_L \star \mathcal{Z}_U \star \mathcal{I}$ $\mathcal{I}' := 3 \star \mathcal{Z}_U \star \mathcal{I} + 3 \star \mathcal{Z}_L \star \mathcal{Z}_U \star \mathcal{I}'$ $\mathcal{I}_a \leftarrow rejection(\mathcal{I})$ Closure $3 \star \mathcal{Z}_L = \mathcal{Z}_L + \mathcal{Z}_L + \mathcal{Z}_L$ $admissible rooted irreducible dissections; sample \delta$
$\mathcal{I}' \leftarrow bijection(\mathcal{K}')$ $\mathcal{I} := 3 \star \mathcal{Z}_L \star \mathcal{Z}_U \star \mathcal{I}$ $\mathcal{I}' := 3 \star \mathcal{Z}_L \star \mathcal{Z}_U \star \mathcal{I} + 3 \star \mathcal{Z}_L \star \mathcal{Z}_U \star \mathcal{I}'$ $\mathcal{I}_a \leftarrow rejection(\mathcal{I})$ $\mathcal{I}_a \leftarrow rejection(\mathcal{I})$ Closure $3 \star \mathcal{Z}_L = \mathcal{Z}_L + \mathcal{Z}_L + \mathcal{Z}_L$ $admissible \text{ rooted irreducible dissections; sample } \delta$
$\mathcal{J} := 3 \star \mathcal{Z}_L \star \mathcal{Z}_U \star \mathcal{I}$ $\mathcal{J}' := 3 \star \mathcal{Z}_U \star \mathcal{I} + 3 \star \mathcal{Z}_L \star \mathcal{Z}_U \star \mathcal{I}'$ $\mathcal{J}_a \leftarrow rejection(\mathcal{J})$ $3 \star \mathcal{Z}_L = \mathcal{Z}_L + \mathcal{Z}_L + \mathcal{Z}_L$ $admissible \text{ rooted irreducible dissections; sample } \delta$
$ \mathcal{J}' := 3 \star \mathcal{Z}_U \star \mathcal{I} + 3 \star \mathcal{Z}_L \star \mathcal{Z}_U \star \mathcal{I}' $ $ \mathcal{J}_a \leftarrow rejection(\mathcal{J}) $ $ admissible \text{ rooted irreducible dissections; sample } \delta $
$\mathcal{J}_a \leftarrow rejection(\mathcal{J})$ admissible rooted irreducible dissections; sample δ
from $\Gamma \mathcal{J}$ until $\delta \in \mathcal{J}_a$
$\mathcal{J}_a' \leftarrow rejection(\mathcal{J}')$
3-connected edge rooted planar graphs
$\overrightarrow{\mathcal{M}_3} \leftarrow bijection(\mathcal{J}_a)$ Primal map
$\overrightarrow{\mathcal{M}_3}' \leftarrow bijection(\mathcal{J}_a')$ Primal map
$\overrightarrow{\mathcal{G}}_3 \leftarrow bijection(\overrightarrow{\mathcal{M}}_3)$ \longrightarrow Just forget the planar embedding
$\overrightarrow{\mathcal{G}}_3' \leftarrow bijection(\overrightarrow{\mathcal{M}}_3'') \rightarrow$
$\overrightarrow{\mathcal{G}}_3 \leftarrow dy_from_dx(\overrightarrow{\mathcal{G}}_3')$ $\alpha_{U/L} = 3, \text{ see } 5.3.3$
Networks
$\mathcal{D} := \mathcal{Z}_U + \mathcal{S} + \mathcal{P} + \mathcal{H}$
$\mathcal{S} := (\mathcal{Z}_U + \mathcal{P} + \mathcal{H}) \star \mathcal{Z}_L \star \mathcal{D}$ series-network
$\mathcal{P} := \mathcal{Z}_U \star SET_{\geq 1}(\mathcal{S} + \mathcal{H}) + SET_{\geq 2}(\mathcal{S} + \mathcal{H})$ parallel-network
$\mathcal{H}:=\overrightarrow{\mathcal{G}}_3\circ_U\mathcal{D}$ polyhedral-network, this is the only time we need usubstitution
L-derived networks
$\mathcal{D}' := \mathcal{S}' + \mathcal{P}' + \mathcal{H}'$
$\mathcal{S}' := (\mathcal{P}' + \mathcal{H}') \star \mathcal{Z}_L \star \mathcal{D} + (\mathcal{Z}_U + \mathcal{P} + \mathcal{H}) \star (\mathcal{D} + \mathcal{Z}_L \star \mathcal{D}')$
$\mathcal{P}' := \mathcal{Z}_U \star (\mathcal{S}' + \mathcal{H}') \star SET(\mathcal{S} + \mathcal{H}) + (\mathcal{S}' + \mathcal{H}') \star SET_{\geq 1}(\mathcal{S} + \mathcal{H})$
$\mathcal{H}' := \overrightarrow{\mathcal{G}}_3' \circ_U \mathcal{D} + \mathcal{D}' \star (\overrightarrow{\underline{\mathcal{G}}}_3 \circ_U \mathcal{D})$
2-connected planar graphs

$(1 + \mathcal{Z}_U) \star \overrightarrow{\mathcal{G}}_2 = (1 + \mathcal{D})$ $\mathcal{F} := \mathcal{Z}_L^2 \star \overrightarrow{\mathcal{G}}_2$	Here we need a special technique to 'solve' this class equation: Sample network from $\Gamma(1+\mathcal{D})$ and add an edge between the poles (Lemma 14). \mathcal{F} ist just an intermediate auxiliary class
$2 \star \underline{\mathcal{G}}_{2} = \mathcal{F}$ $G'_{2} \leftarrow dx_from_dy(\underline{\mathcal{G}}_{2})$	Once again we need a special sampler: Sample from $\Gamma \mathcal{F}$ and forget direction of the root Obtain 1-derived class from u-derived class, see Lemma 6. Here, $\alpha_{L/U}=2.0$, see 4.2
L-derived 2-connected planar graphs	
$(1 + \mathcal{Z}_U) \star \overrightarrow{\mathcal{G}}_2' = \mathcal{D}'$	Same special samples techniques as before, see 5.5
$\mathcal{F}' := \mathcal{Z}_L^2 \star \overrightarrow{\mathcal{G}}_2' + 2 \star \mathcal{Z}_L \star \overrightarrow{G}_2$	
$2\star \underline{\mathcal{G}}_2' = \mathcal{F}'$	
$G_2'' \leftarrow dx_from_dy(\underline{\mathcal{G}_2'})$	$\alpha_{L/U} = 1.0$, see 5.5
1-connected planar graphs	
$\mathcal{G}_1' := SET(\mathcal{G}_2' \circ_L (\mathcal{Z}_L \star \mathcal{G}_1'))$	Block decomposition. Only time we need l-
$\mathcal{G}_1'' := \left(\mathcal{G}_1' + \mathcal{Z}_L \star \mathcal{G}_1''\right) \star \left(\mathcal{G}_2'' \circ_L \left(\mathcal{Z}_L \star \mathcal{G}_1'\right)\right) \star \mathcal{G}_1'$	substitution
$\mathcal{G}_1 \leftarrow rejection(\mathcal{G}_1')$	see Lemma 15
Planar graphs	
$\mathcal{G} := SET(G_1)$	
$ig \mathcal{G}' := \mathcal{G}_1' \star \mathcal{G}$	
$\mathcal{G}'' := \mathcal{G}_1'' \star \mathcal{G} + \mathcal{G}_1' \star \mathcal{G}'$	Our final sampler will sample from this rule and then forget the two marked vertices.