Rules	**Remarks**
Assymetric rooted bicolored binary trees	Only 4 such trees are *not* assymetric. The rooting corresponds to a u-derivation.
$\mathcal{R}_\circ := (\mathcal{Z}_U + \mathcal{R}_\bullet)^2$ $\mathcal{R}_\bullet := (\mathcal{Z}_U + \mathcal{R}_\circ)^2 \star \mathcal{Z}_L$ $\hat{\mathcal{R}}_\circ := \mathcal{Z}_U + \mathcal{R}_\bullet \star \mathcal{Z}_U + \mathcal{Z}_U \star \mathcal{R}_\bullet + \mathcal{R}_\bullet^2$ $\hat{\mathcal{R}}_\bullet := \hat{\mathcal{R}}_\circ \star \mathcal{Z}_L \star \mathcal{Z}_U^2 + \mathcal{Z}_U^2 \star \mathcal{Z}_L \star \hat{\mathcal{R}}_\circ + \hat{\mathcal{R}}_\circ \star \mathcal{Z}_L \star \hat{\mathcal{R}}_\circ$ $\mathcal{R}_\circ^{(\mathrm{as})} := \hat{\mathcal{R}}_\bullet \star \mathcal{Z}_U + \mathcal{Z}_U \star \hat{\mathcal{R}}_\bullet + \mathcal{R}_\bullet^2$ $\mathcal{R}_\bullet^{(\mathrm{as})} := \mathcal{R}_\circ \star \mathcal{Z}_L \star \mathcal{Z}_U + \mathcal{Z}_U \star \mathcal{Z}_L \star \mathcal{R}_\circ + \mathcal{Z}_L \star \mathcal{R}_\circ^2$ $\underline{\mathcal{K}} := \mathcal{R}_\bullet^{(\mathrm{as})} + \mathcal{R}_\circ^{(\mathrm{as})}$	
Assymetric (unrooted) bicolored binary trees	Get rid of the rooting = get rid of the u-derivation
$\mathcal{K} \leftarrow rejection(\underline{\mathcal{K}})$	Two possible techniques: 1. Sample from $\Gamma\underline{\mathcal{K}}$ until Bern($\frac{2}{\lvert\lvert\gamma\rvert\rvert}$) 2. Apply rejection already during the sampling of $\underline{\mathcal{K}}$ (Lemma 12) 2. is more efficient.
Assymetric l-derived bicolored binary trees	
$\mathcal{K}' \leftarrow dx\_from\_dy(\underline{\mathcal{K}})$	$\alpha_{L/U} = 2/3$, see 5.3.1
Irreducible dissections (of the hexagon)	
$\mathcal{I} \leftarrow bijection(\mathcal{K})$ $\mathcal{I}' \leftarrow bijection(\mathcal{K}')$ $\mathcal{J} := 3 \star \mathcal{Z}_L \star \mathcal{Z}_U \star \mathcal{I}$ $\mathcal{J}' := 3 \star \mathcal{Z}_U \star \mathcal{I} + 3 \star \mathcal{Z}_L \star \mathcal{Z}_U \star \mathcal{I}'$ $\mathcal{J}_a \leftarrow rejection(\mathcal{J})$ $\mathcal{J}_a' \leftarrow rejection(\mathcal{J}')$	Closure Closure $3 \star \mathcal{Z}_L = \mathcal{Z}_L + \mathcal{Z}_L + \mathcal{Z}_L$ *admissible* rooted irreducible dissections; sample δ from $\Gamma\mathcal{J}$ until $\delta \in \mathcal{J}_a$
3-connected edge rooted planar graphs	
$\overrightarrow{\mathcal{M}_3} \leftarrow bijection(\mathcal{J}_a)$ $\overrightarrow{\mathcal{M}_3}' \leftarrow bijection(\mathcal{J}_a')$ $\overrightarrow{\mathcal{G}}_3 \leftarrow bijection(\overrightarrow{\mathcal{M}_3})$ $\overrightarrow{\mathcal{G}}'_3 \leftarrow bijection(\overrightarrow{\mathcal{M}_3}')$ $\underline{\overrightarrow{\mathcal{G}}}_3 \leftarrow dy\_from\_dx(\overrightarrow{\mathcal{G}}'_3)$	Primal map Primal map Just forget the planar embedding $\alpha_{U/L} = 3$, see 5.3.3
Networks	
$\mathcal{D} := \mathcal{Z}_U + \mathcal{S} + \mathcal{P} + \mathcal{H}$ $\mathcal{S} := (\mathcal{Z}_U + \mathcal{P} + \mathcal{H}) \star \mathcal{Z}_L \star \mathcal{D}$ $\mathcal{P} := \mathcal{Z}_U \star SET_{\geq 1}(\mathcal{S} + \mathcal{H}) + SET_{\geq 2}(\mathcal{S} + \mathcal{H})$ $\mathcal{H} := \overrightarrow{\mathcal{G}}_3 \circ_U \mathcal{D}$	 series-network parallel-network polyhedral-network, this is the only time we need u-substitution
L-derived networks	
$\mathcal{D}' := \mathcal{S}' + \mathcal{P}' + \mathcal{H}'$ $\mathcal{S}' := (\mathcal{P}' + \mathcal{H}') \star \mathcal{Z}_L \star \mathcal{D} + (\mathcal{Z}_U + \mathcal{P} + \mathcal{H}) \star (\mathcal{D} + \mathcal{Z}_L \star \mathcal{D}')$ $\mathcal{P}' := \mathcal{Z}_U \star (\mathcal{S}' + \mathcal{H}') \star SET(\mathcal{S} + \mathcal{H}) + (\mathcal{S}' + \mathcal{H}') \star SET_{\geq 1}(\mathcal{S} + \mathcal{H})$ $\mathcal{H}' := \overrightarrow{\mathcal{G}}'_3 \circ_U \mathcal{D} + \mathcal{D}' \star (\underline{\overrightarrow{\mathcal{G}}}_3 \circ_U \mathcal{D})$	
2-connected planar graphs	

$(1+\mathcal{Z}_U)\star\overrightarrow{\mathcal{G}}_2=(1+\mathcal{D})$	Here we need a special technique to 'solve' this class equation: Sample network from $\Gamma(1+\mathcal{D})$ and add an edge between the poles (Lemma 14).
$\mathcal{F}:=\mathcal{Z}_L^2\star\overrightarrow{\mathcal{G}}_2$	$\mathcal{F}$ ist just an intermediate auxiliary class
$2\star\underline{\mathcal{G}}_2=\mathcal{F}$	Once again we need a special sampler: Sample from $\Gamma\mathcal{F}$ and forget direction of the root
$G_2'\leftarrow dx\_from\_dy(\underline{\mathcal{G}_2})$	Obtain l-derived class from u-derived class, see Lemma 6. Here, $\alpha_{L/U}=2.0$, see 4.2
L-derived 2-connected planar graphs	
$(1+\mathcal{Z}_U)\star\overrightarrow{\mathcal{G}}'_2=\mathcal{D}'$	Same special samples techniques as before, see 5.5
$\mathcal{F}':=\mathcal{Z}_L^2\star\overrightarrow{\mathcal{G}}'_2+2\star\mathcal{Z}_L\star\overrightarrow{G}_2$	
$2\star\underline{\mathcal{G}}'_2=\mathcal{F}'$	
$G_2''\leftarrow dx\_from\_dy(\underline{\mathcal{G}_2'})$	$\alpha_{L/U}=1.0$, see 5.5
1-connected planar graphs	
$\mathcal{G}_1':=SET(\mathcal{G}_2'\circ_L(\mathcal{Z}_L\star\mathcal{G}_1'))$	Block decomposition. Only time we need l-substitution
$\mathcal{G}_1'':=(\mathcal{G}_1'+\mathcal{Z}_L\star\mathcal{G}_1'')\star(\mathcal{G}_2''\circ_L(\mathcal{Z}_L\star\mathcal{G}_1'))\star\mathcal{G}_1'$	
$\mathcal{G}_1\leftarrow rejection(\mathcal{G}_1')$	see Lemma 15
Planar graphs	
$\mathcal{G}:=SET(G_1)$	
$\mathcal{G}':=\mathcal{G}_1'\star\mathcal{G}$	
$\mathcal{G}'':=\mathcal{G}_1''\star\mathcal{G}+\mathcal{G}_1'\star\mathcal{G}'$	Our final sampler will sample from this rule and then forget the two marked vertices.