

# Distances and similarity

## Distances and similarity

- Distance and similarity are more or less opposite concepts.
- Distance is a **numerical measure** describing how are two objects (defined by certain variables) different (*pairwise distance*).
- Different distance measures exist for different data types.

## Distance

- Scale  $0 - \infty$
- 0 – Two objects with 0 distance between them.
- $\infty$  – Two objects with infinite distance.
- In practice, maximum distance is often 1.
- Denoted by  $D$  (for distance, or dissimilarity).
- $D = 1 - S$

## Similarity

- Scale  $0 - 1$
- 0 – Two objects completely dissimilar (0%).
- 1 – Two objects completely similar (100%).
- Denoted by  $S$  (for similarity).
- $S = 1 - D$

## Different distance measures

- Dichotomous variables
  - Symmetrical – Simple matching distance
  - Asymmetrical – Jaccard index (binary distance)

- Categorical variables
  - Hamming distance
- Numeric continuous variables
  - Euclidean distance
  - Mahalanobis distance
- Mixed data sets
  - Gower's distance

## Binary distances

- For TRUE/FALSE, 1/0, presence/absence (etc.) data

### Symmetrical

- Two **presences** as **match**.
- Two **absences** as **match**.

If a trait is present, two objects are more similar. If a trait is absent, two objects are more similar. For example if *biological sex* is encoded in one variable with 0 for *male* and 1 for *female*, it is symmetrical.

- **Simple matching distance**

### Asymmetrical

- Two **presences** as **match**.
- Two **absences** as **mismatch**.

If a trait is present, two objects are more similar. If a trait is **absent** in both cases, e.g. **undetermined**, **missing** etc., this does not affect similarity. This is more practical in archaeology.

- **Jaccard index**, i.e. **binary distance**

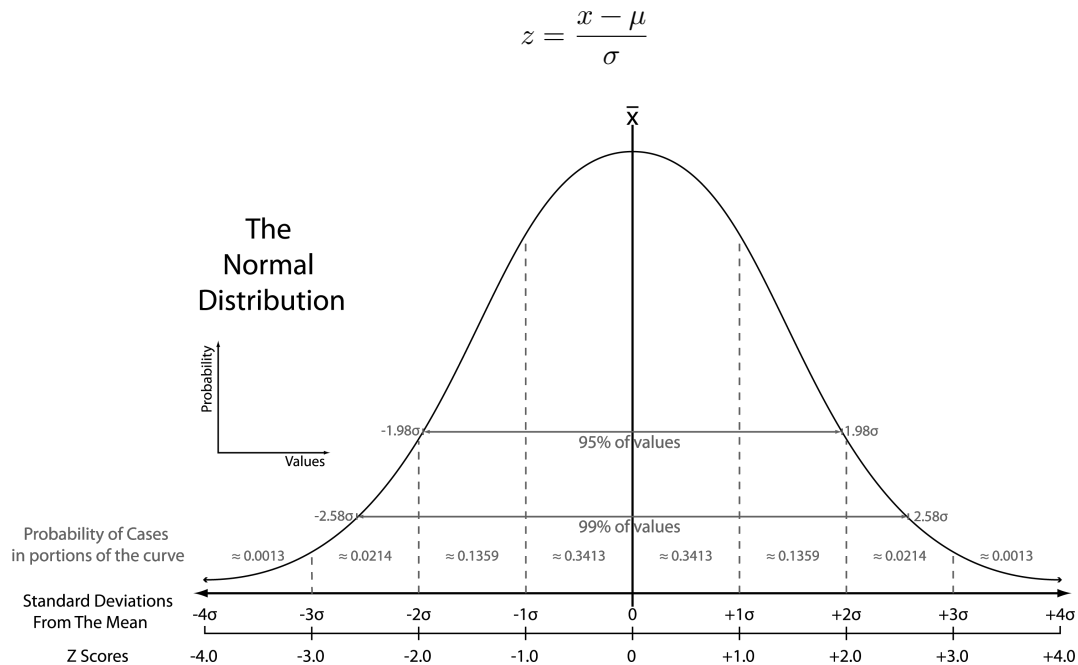
```
dist(x, method = "binary")
```

## Distance between (continuous) numeric data

- To **remove effects of scale** (different units etc.), variables should be scaled (normalized).

## Normalization

- *z-score* or *z-transformation*



## Euclidean distance

- Defined for a **Cartesian coordinate space**.
- Uses **Pythagorean theorem**.

$$d(p, q) = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2}$$

## In R...

Normalization:

```
scale(x, center = TRUE, scale = TRUE)
```

Euclidean distance:

```
dist(x, method = "euclidean")
```

## Code along

```
library(dplyr)
darts <- read.csv("https://petrpajdla.github.io/stat4arch/lect/w09/data/dartpoints_numeric.csv")
```

```
# summary of values
summary(select(darts, Length, Width, Thickness, Weight))
```

Length	Width	Thickness	Weight
Min. : 30.60	Min. :14.50	Min. : 4.000	Min. : 2.300
1st Qu.: 40.85	1st Qu.:18.55	1st Qu.: 6.250	1st Qu.: 4.550
Median : 47.10	Median :21.10	Median : 7.200	Median : 6.800
Mean : 49.33	Mean :22.08	Mean : 7.271	Mean : 7.643
3rd Qu.: 55.80	3rd Qu.:25.15	3rd Qu.: 8.250	3rd Qu.:10.050
Max. :109.50	Max. :49.30	Max. :10.700	Max. :28.800

```
# normalization
darts_norm <- darts %>%
  mutate(Length = scale(Length), Width = scale(Width),
         Thickness = scale(Thickness), Weight = scale(Weight))

# or this shorthand can be used
darts_norm <- darts %>%
  mutate(across(all_of(c("Length", "Width", "Thickness", "Weight")), scale))
```

```
# summary of normalized values
summary(select(darts_norm, Length, Width, Thickness, Weight))
```

Length.V1	Width.V1	Thickness.V1
Min. :-1.470673	Min. :-1.469440	Min. :-2.1363403
1st Qu.: -0.665879	1st Qu.: -0.683998	1st Qu.: -0.6670233

Median :-0.175152	Median :-0.189461	Median :-0.0466450
Mean : 0.000000	Mean : 0.000000	Mean : 0.0000000
3rd Qu.: 0.507941	3rd Qu.: 0.595981	3rd Qu.: 0.6390363
Max. : 4.724271	Max. : 5.279540	Max. : 2.2389592

Weight.V1

Min. :-1.269966
1st Qu.: -0.735154
Median :-0.200342
Mean : 0.000000
3rd Qu.: 0.572164
Max. : 5.028928

## Code along

```
# subset of Travis and Darl types of dart points
darts_subset <- filter(darts_norm, Name %in% c("Travis", "Darl"))

# matrix with numerical variables only
darts_mx <- darts_subset %>%
  select(Length, Width, Thickness, Weight) %>%
  as.matrix()

# add row names to the matrix
rownames(darts_mx) <- darts_subset$Name

# count Euclidean distance
darts_d <- dist(darts_mx, method = "euclidean", diag = TRUE)

round(as.matrix(darts_d)[1:6, 1:6], 2)
```

```
      Darl Darl Darl Darl Darl Darl
Darl 0.00 0.42 0.47 0.40 1.57 1.14
Darl 0.42 0.00 0.43 0.43 1.50 1.18
Darl 0.47 0.43 0.00 0.28 1.51 1.36
Darl 0.40 0.43 0.28 0.00 1.74 1.47
Darl 1.57 1.50 1.51 1.74 0.00 0.90
Darl 1.14 1.18 1.36 1.47 0.90 0.00
```

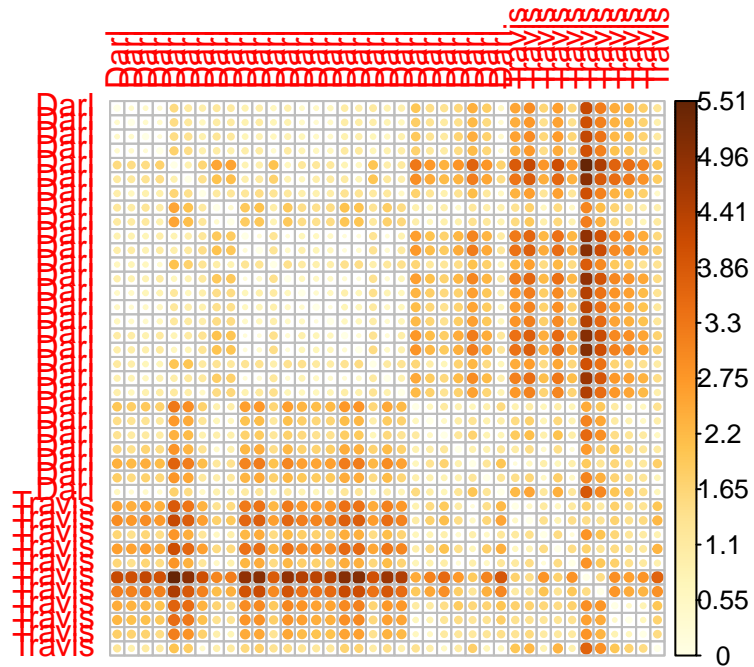
- Result is a **distance matrix**.
- It is **symmetrical**. **Lower triangular** is the same as **upper triangular**.
- On the **diagonal**, there is distance of the given object to itself, i.e. 0.

## Visualizing distance matrix

- Package `corrplot` has a nice way of plotting heatmaps.

```
library(corrplot)

# arg. is.corr set to FALSE, because we are not visualizing correlation matrix
corrplot(as.matrix(darts_d), is.corr = FALSE)
```



## Resources

For a much more detailed overview of distance methods, see the tutorial on classification by Schmidt, S. C. et al. DOI: [10.5281/zenodo.6325372](https://doi.org/10.5281/zenodo.6325372) (direct link to a HTML file is [here](#)).