Introduction to Colour Science

NPGR025

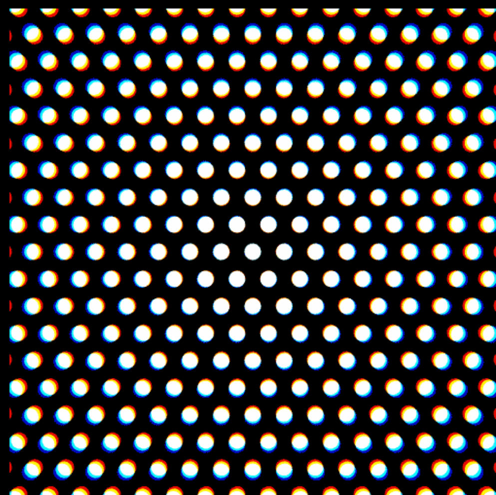
Unit 3: Introduction to Colorimetry

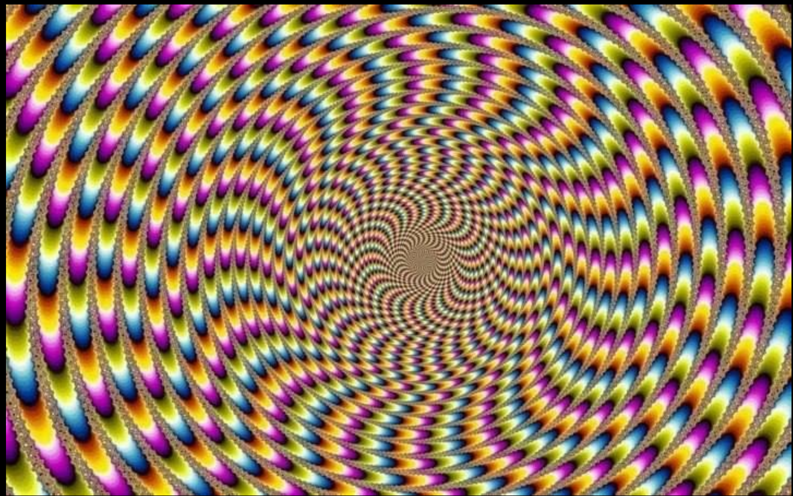
Computer Graphics Charles University

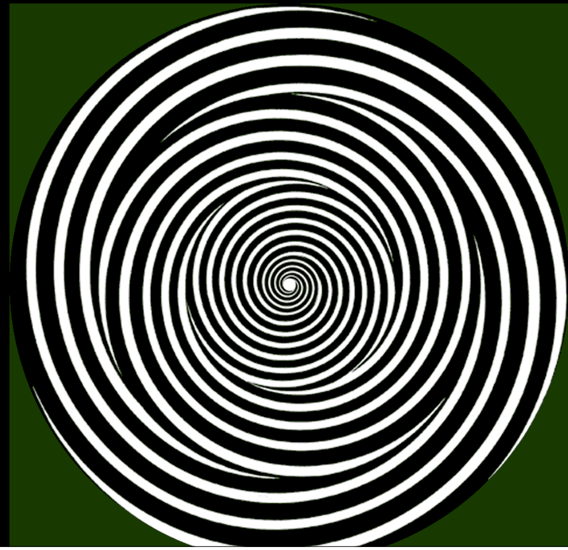
Sources:
K. Nassau, Color for Science, Art and Technology
R. Boynton, Human Color Vision
Wyszecki & Stiles

Overview

- Terminology
- Colour Matching Experiments
- CIE Colour Spaces
  - RGB
  - XYZ
  - L*u*v*
  - L*a*b*







What is our goal?

To be able to quantify colour in a **meaningful**, **expressive**, **consistent** and **reproducible** way.

Colour - A Visual Sensation

Object → Light stimulus → Eye → Nerve signal → Brain

Electromagnetic rays → Colour sensation

Realm of direct observables

Realm of psychology

Colorimetry (CM)

- CM is the branch of colour science concerned with numerically specifying the colour of a physically defined visual stimulus in such manner that
  - Stimuli with the same specification look alike under the same viewing conditions
  - Stimuli that look alike have the same specification
  - The numbers used are continuous functions of the physical parameters

Colorimetry Properties

- Colorimetry only considers the visual discriminability of physical beams of radiation
- For the purposes of CM „colours" are an equivalence class of mutually indiscriminable beams
- Colours in this sense cannot be said to be "red", "green" or any other "colour name"
- Discriminability is decided before the brain comes into action - CM is not psychology
- That is why CM can be a science, and so-called "Colour Science" often is not

CIE

- The Commission International de l' Eclairage (CIE) was founded in 1913
- It succeeded the Commission International de Photometrie
- The recognised international standards body for questions of human perception and colour
- The CIE colorimetric system is the standard from which all others are derived
- Published the Basic Colorimetric Terms (BCT)

CIE BCT: Colour

- Colour in the psychophysical sense is that characteristic of a visible radiant power by which an observer may distinguish differences between two structure-free fields of view of the same size and shape, such as may be caused by differences in the spectral composition of the radiant power concerned in the observation.
  Psychophysical colour is specified by the tristimulus values of the radiant power entering the eye.

CIE BCT: Colour Stimulus

- A colour stimulus is radiant power of given magnitude and spectral composition, entering the eye and producing a sensation of colour.

CIE BCT: Stimuli

- A monochromatic stimulus is monochromatic radiant power of given magnitude and wavelength (or frequency), entering the eye and producing a sensation of light of colour.
- An achromatic stimulus is the stimulus chosen because it usually yields a colour perception which is devoid of hue under the desired viewing conditions.

Colour Matching Experiments (generic)

Trichromatic Generalisation

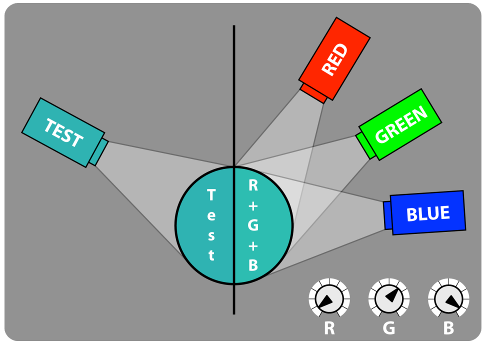
- Key to colorimetry!
  - Based on the physiology of the eye - 3 cone types!
- Over a wide range of conditions, many colour stimuli can be matched in colour completely by additive mixtures of three fixed primary stimuli whose radiant powers have been suitably adjusted
- These three primaries can be chosen (almost) arbitrarily – they just have to be mutually independent
- The three lights are usually chosen from the long, medium and short wavebands (RGB)

Omitted Problems

- Dependence of the match on the observational conditions under which the match is made
- The possible effects of previous exposures to colour stimuli on the eyes of the observer
- (Differences between observers)
  → averaging

CME Principle

- Observes had to match a test light by combining three fixed primaries



- Goal: find the unique RGB coordinates for each stimulus

Tristimulus Values

- The values RQ, GQ and BQ of a stimulus Q that fulfil

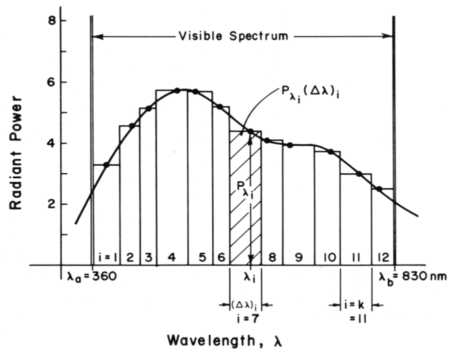
$$Q = R_Q \cdot R + G_Q \cdot G + B_Q \cdot B$$

are called the tristimulus values of Q

- In the case of a monochromatic stimulus Qλ, the values Rλ, Gλ and Bλ are called the spectral tristimulus values

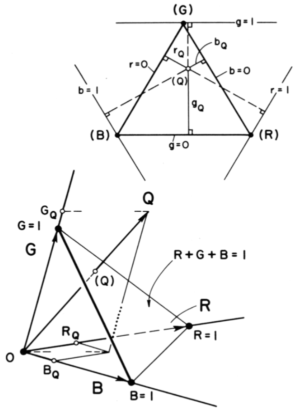
Monochromatic Stimulus

- A monochromatic stimulus of wavelength λ is defined as the radiant power in the wavelength interval dλ centred at λ with the intensity Pλ
- dλ is assumed to go towards zero



(R,G,B) Coordinates

- Primary stimuli are represented by unit vectors R, G, B with common origin 0
- A colour stimulus Q is rep. by RQ, GQ, BQ
- Q intersects the unit plane in point (Q)
- Plane contains a chromaticity diagram



Tristimulus vs. Chromaticity

- Tristimulus values are informative, but not convenient
- A two-dimensional representation of the colour space would be desirable
- One possible solution is to use the intersection of Q with the unit plane R+G+B=1
- Shows “pure” chroma information
- Intensity information is lost

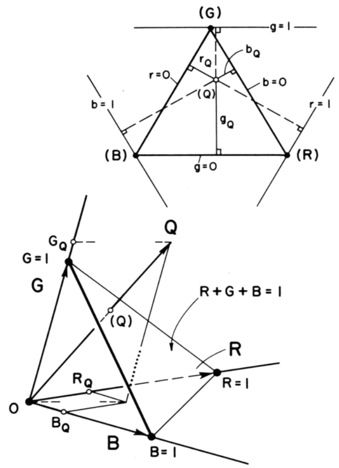
(R,G,B) → (r,g,b) Conversion

- Yields colour coordinate in a barycentric (i.e. over-defined) system

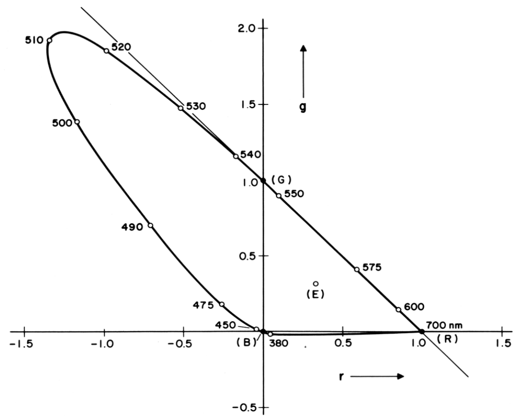
$$r_Q = \frac{R_Q}{R_Q + G_Q + B_Q}$$

$$g_Q = \frac{G_Q}{R_Q + G_Q + B_Q}$$

$$b_Q = \frac{B_Q}{R_Q + G_Q + B_Q}$$

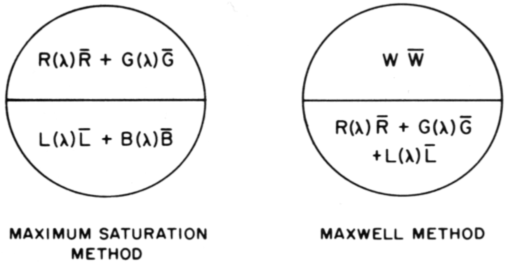


(r,g) Chromaticity Coordinates



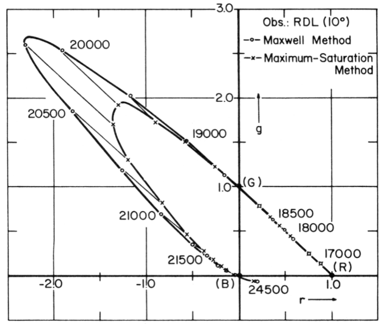
Maxwell vs. MS Colour Matching

- Introduced by Maxwell (1860)
- A test light L is matched against white W
- Two primaries and the test light L are used to match W



Maxwell Matching

- Large differences to MSM for pure colours
- More accurate near the pre-defined white point
- Better if linearity of trichromatic matching is not quite sound
  - → Constant luminance matching!



Colour Matching Experiments (CIE)

Absolute Trichromatic Values

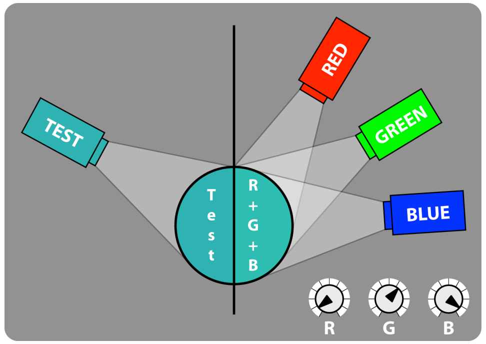
- After the idea of tristimulus values had been developed, the next step was to attempt the definition of an absolute reference system for human colour vision
  - For fixed reference values of R, G, B
  - Extensive experiments to determine average human response to these stimuli

CIE Colour Matching Experiments

- A sequence of experiments first conducted by the CIE from 1920-29 led to the CIE 1931 XYZ colour system
- A particular set of R, G and B lights and viewing conditions was chosen
- Experiments were conducted with observers that were given a two degree field of vision
- Only monochromatic stimuli were matched
- Data is only valid for photopic vision (cones)

CME Principle

- Observes had to match a test light by combining three fixed primaries



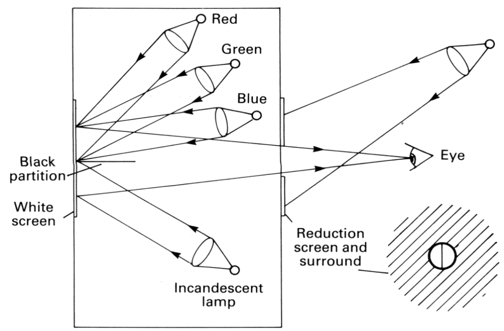
- Goal: find the unique RGB coordinates for each stimulus

Laws of Colour Matching

- Symmetry law: if A matches B, then B matches A
- Transitivity: if A matches B and B matches C, then A matches C
  (in practice, this turns out to be questionable!)
- Proportionality: if A matches B, then x*A matches x*B
- Additivity: if any two of A m B, C m D and (A+C) m (B+D) hold true, then so does (A+D) m (B+C)
  with + denoting additive colour mixture

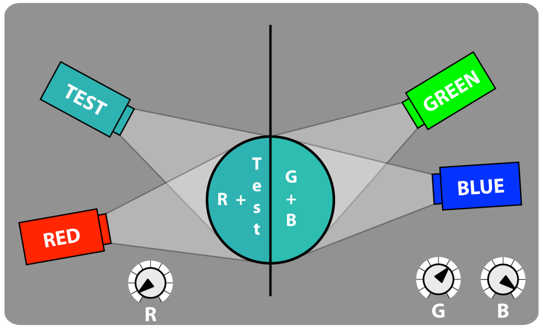
CME Details

- Diameter of the aperture originally was 2 degrees, later 10
- RGB were fixed at 700.0, 546.1 and 435.8nm



"Negative" Light in a CME

- If a match using only positive RGB values proved impossible, observers could simulate a subtraction of red from the match side by adding it to the test side



Equal Energy Stimulus

- Obtained when all the monochromatic stimuli Qλ contained in a spectrum Q have unit power
- Denoted by E, and its components are also sometimes referred to as Eλ
- A colour match for a given Eλ can always be obtained through

$$E_\lambda = r(\lambda)\cdot R + g(\lambda)\cdot G + b(\lambda)\cdot B$$

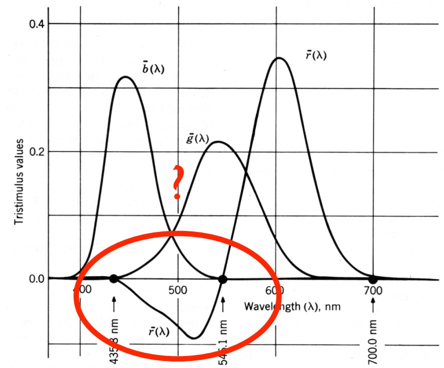
CIE RGB Colour Matching Functions

- Aim of the CME: to obtain values for the

$$r(\lambda), g(\lambda), b(\lambda)$$

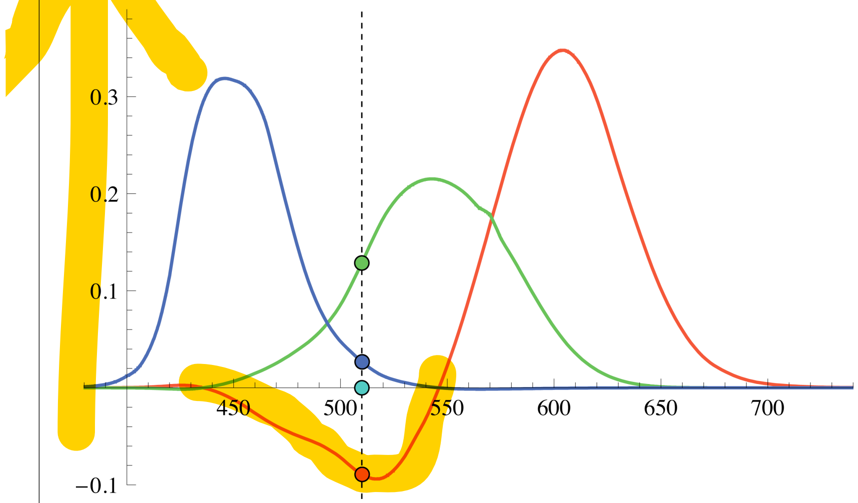
  colour matching functions just mentioned
- These r, g, b functions have no underlying mathematical law – they are empirical data which has to be measured
- They also differ slightly for individuals and strongly for other viewing conditions

CIE RGB Colour Matching Functions



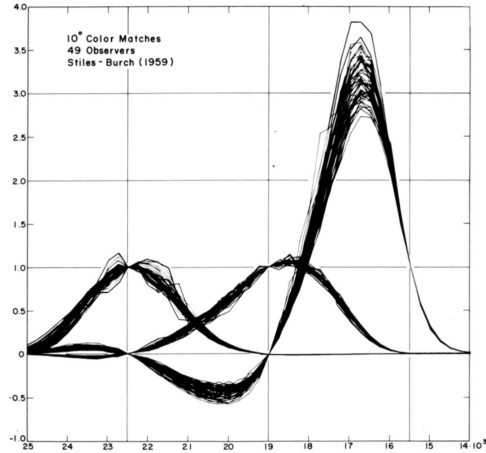
Computer Graphics Charles University

CIE RGB Matching Function Utility



Computer Graphics Charles University

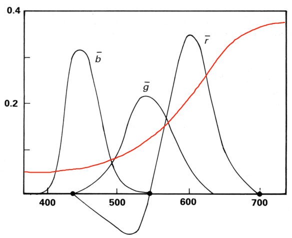
CIE CMF Accuracy



Computer Graphics Charles University

Tristimulus Values of Complex Stimuli

- A complex stimulus is a non-monochromatic wavelength distribution in the visible spectrum between λa and λb
- For a given power distribution Pλ, the corresponding CIE RGB values are given by

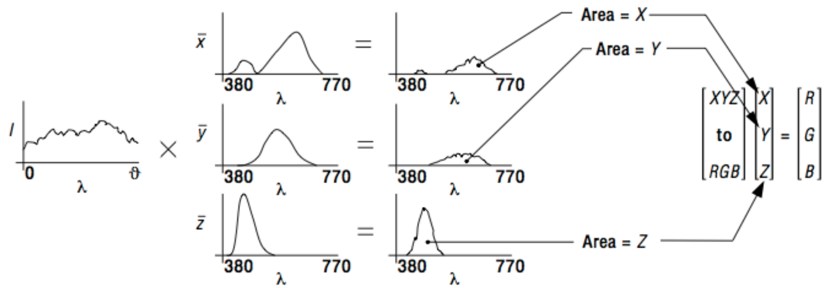


$$R = \int_{\lambda_a}^{\lambda_b} P_\lambda \, r(\lambda) \, d\lambda$$

Similar forms for G and B

Computer Graphics Charles University

XYZ → RGB



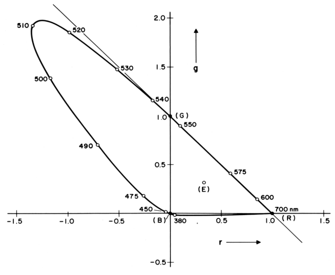
Computer Graphics Charles University

CIE XYZ

Computer Graphics Charles University

Discussion of CIE RGB

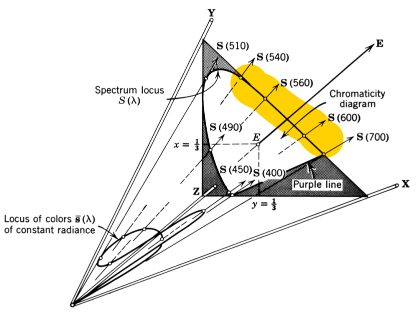
- Pro:
  - Maps all human colour sensations to tristimulus values
- Con:
  - Negative components can occur
  - (r,g) chromaticity coordinates are unintuitive



Computer Graphics Charles University

CIE XYZ

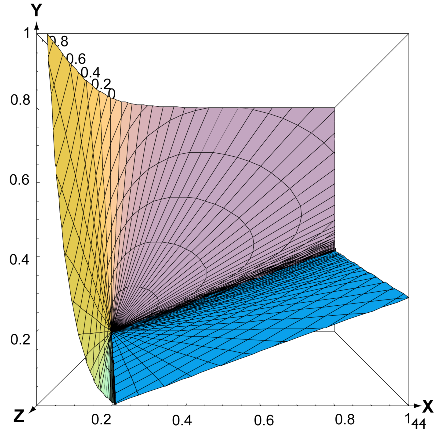
- To solve these problems, a tristimulus system derived from RGB and based on imaginary primaries referred to as XYZ was defined in 1931
- All three are outside the human visual gamut
- Hence only positive XYZ values can occur!

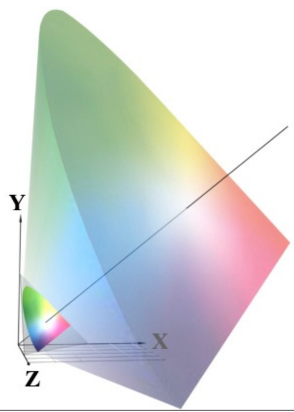


Computer Graphics Charles University

CIE XYZ Space

- Valid colour values only a subspace of the first octant





Transformation CIE RGB → XYZ

- Projective transformation specifically designed so that Y = V(λ)
  (luminous efficiency function)
- XYZ → CIE RGB uses inverse matrix
- XYZ → any other RGB: matrix is device dependent!

$$X = 0.723R + 0.273G + 0.166B$$
$$Y = 0.265R + 0.717G + 0.008B$$
$$Z = 0.000R + 0.008G + 0.824B$$

RGB Matrix Example: sRGB

- sRGB is a standardised set of RGB phosphors which in turn define a RGB colour space

$$r_{sRGB} = 3.240X - 0.969Y + 0.55Z$$
$$g_{sRGB} = -1.537X + 1.875Y - 0.204Z$$
$$b_{sRGB} = -0.498X + 0.041Y + 1.057Z$$

- Note that negative (r,g,b) values are possible!
  → gamut mapping can be necessary!

CIE RGB vs. XYZ

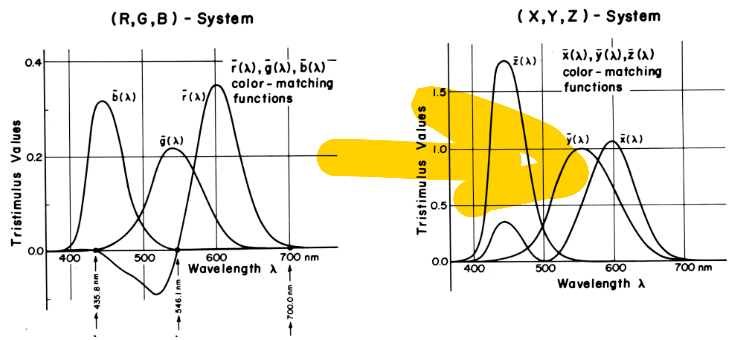
	R, G, B System (Chromaticity Coordinates)			**X, Y, Z** System (Chromaticity Coordinates)		
Stimulus	r	g	b	x	y	z
(R) 700.0 nm	1	0	0	0.73467	0.26533	0.00000
(G) 546.1 nm	0	1	0	0.27376	0.71741	0.00883
(B) 435.8 nm	0	0	1	0.16658	0.00886	0.82456
Illuminant E	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
Illuminant A	0.55255	0.32126	0.12619	0.44757	0.40745	0.14498
Illuminant B	0.36230	0.34305	0.29465	0.34842	0.35161	0.29997
Illuminant C	0.28226	0.33326	0.38448	0.31006	0.31616	0.37378

- XYZ values can be obtained from xyz through

$$X = \frac{x}{y}V, \quad Y = V, \quad Z = \frac{z}{y}V$$

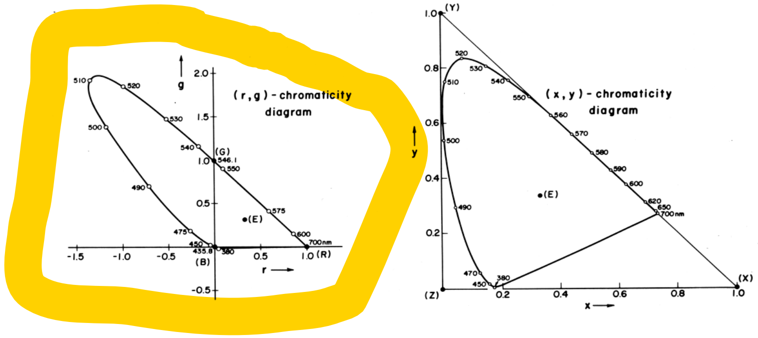
RGB vs. XYZ #1

- Negative component disappears
- y(λ) is the achromatic luminance sensitivity



RGB vs. XYZ #2

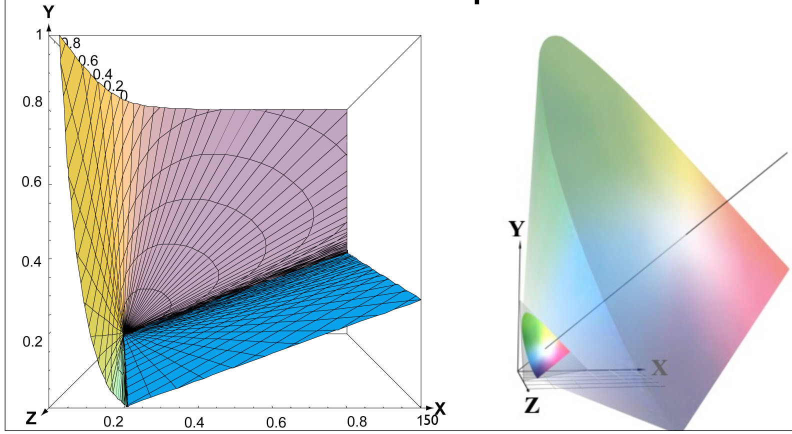
- E remains constant
- All colours are now in the positive quadrant
- Y = achromatic – no colour information is lost!



Computer Graphics Charles University

Usefulness of XYZ in 3D Graphics

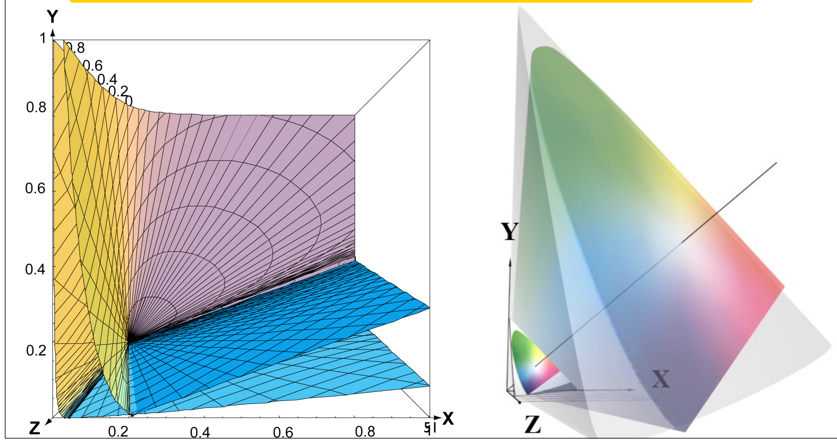
- Valid colours a subspace of the first octant
- XYZ not closed under multiplication!



Computer Graphics Charles University

XYZ: Problem of Multiplication

- Component-wise XYZ multiplications can yield invalid results (positive, but outside subspace)



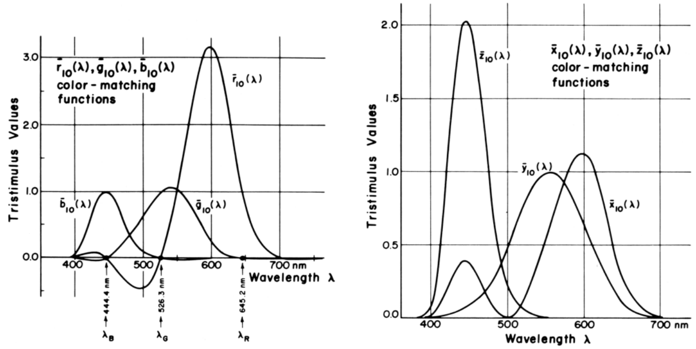
Computer Graphics Charles University

CIE XYZ vs.RGB

- Problem does not exist for RGB spaces:
  - They occupy the entire first octant, are closed under multiplication
- RGB cannot represent all visible colours
  - XYZ is no alternative due to the discussed problems!
  - Older literature still recommends using XYZ!
- For image synthesis spectral rendering or „extreme“ RGB spaces (Sharp RGB) are an alternative

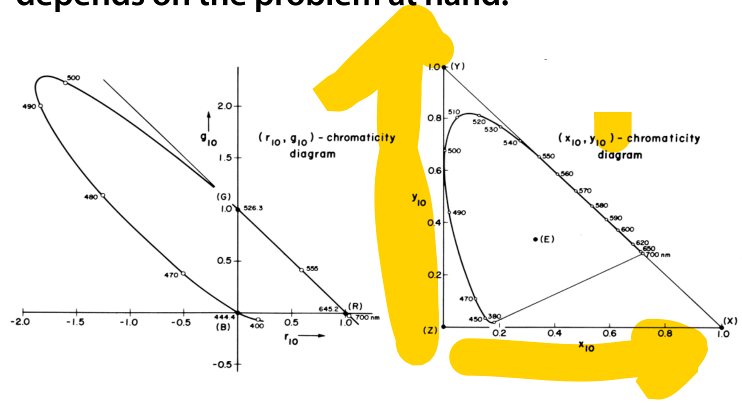
CIE 1964 Observer CMF

- Qualitatively similar to the 2 degree observer
- Minor differences



CIE 1964 Observer CD

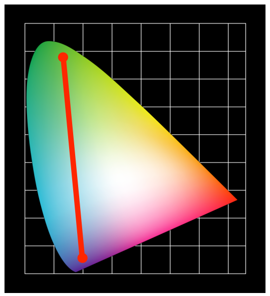
- Which of the two standard observers is used depends on the problem at hand!



Chromaticity Diagram Properties

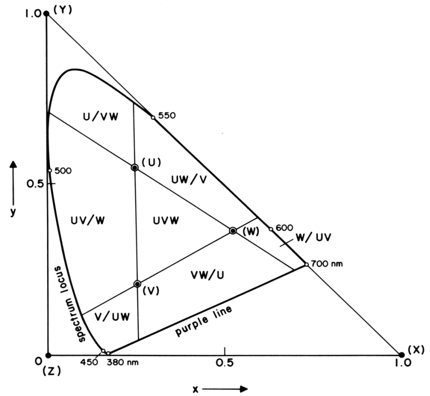
Chromaticity Diagram Properties

- Intensity information is not represented
- All colours that can be realised by an additive mixture of two stimuli lie on a straight line between these two points
- The sum of R+G+B > 0 for all real stimuli
- All real chromaticities plot in a finite domain bounded by the spectrum locus and the purple line



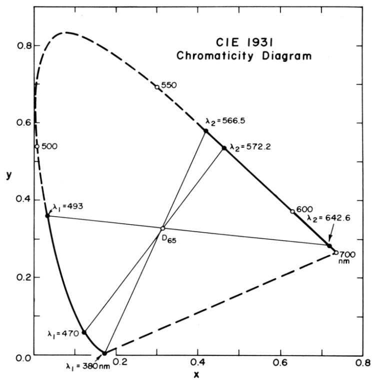
Tristimulus Colour Mixing

- Separation of x,y space into areas that can be described by particular mix-tures of UVW
- Left of dash = positive
- Right of dash = negative



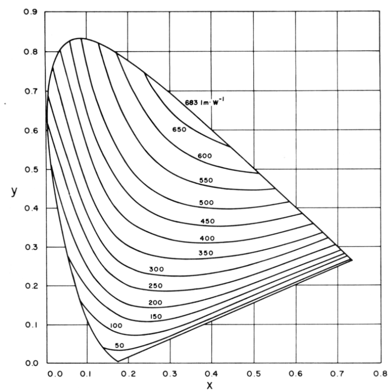
Complementary Colours

- Monochromatic complementary stimuli are only possible for colours that are not oppo-site the, or on the magenta line
- Otherwise also dependent on the given white point!



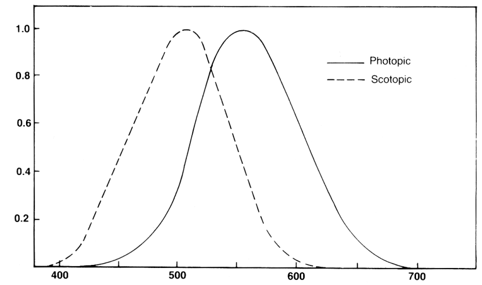
Maximum Luminous Efficacy

- Shape of isolines directly dependent on luminous efficacy function V(λ)
- Highest LE in the yellow-green area
- Lowest in the blue region



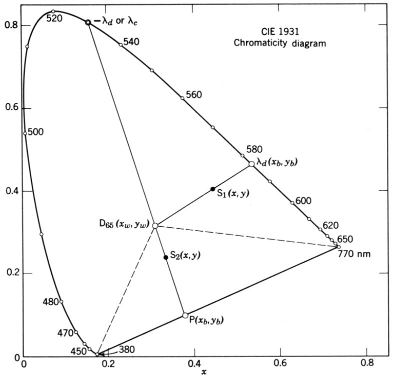
Relative Luminous Efficiency

- Photopic – adapted to bright light (cones)
- Scotopic – adapted to the dark (rods)
- Mesopic – intermediate



Dominant Wavelength

- The dominant w. λd is the wavelength of a monochromatic stimulus that when mixed with the achromatic stimulus can match a given stimulus
- Loosely corresponds to hue

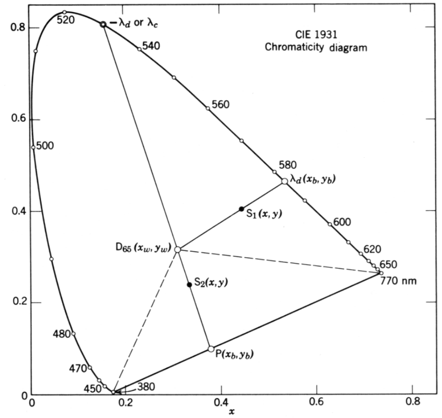


Excitation Purity

- Exactly defined ratio of distances in the CD

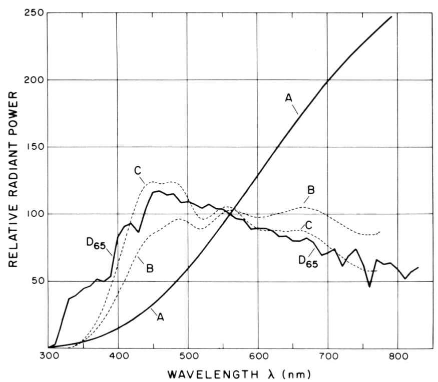
$$p_e = \frac{x - x_w}{x_b - x_w}$$

- y values can be used alternatively
- Loosely corresponds to saturation

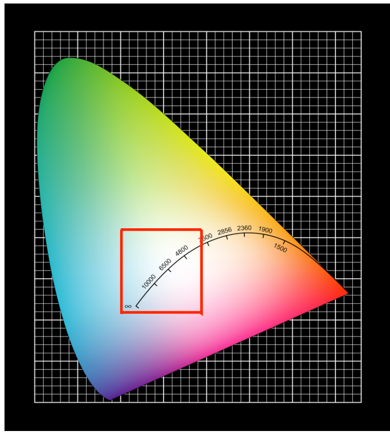


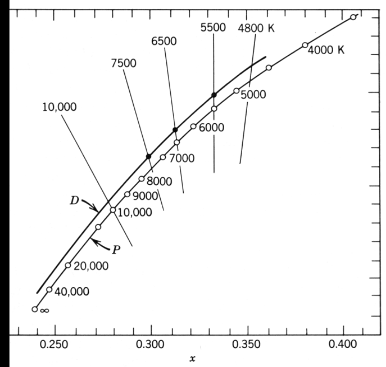
CIE Standard Illuminants

- Purpose: standardised illuminants for colour comparisons
- A – incandescent
- D65 – daylight
- B, C – daylight (no longer used due to missing UV energy)



Blackbody vs. Dx Daylight





CIE XYZ: Perceptual Issues

Computer Graphics Charles University

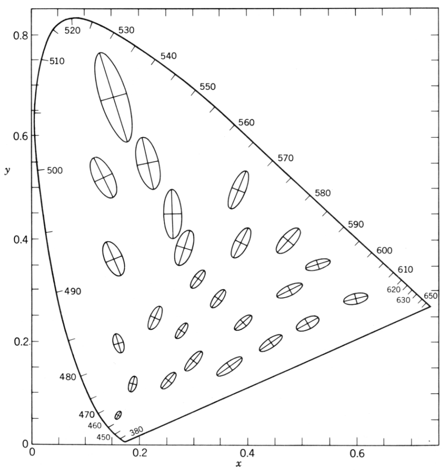
CIE Perceptual Issues

- Desirable feature of a chromaticity diagram: geometric distance between chromaticities corresponds to perceptual difference (difference measure)
  - → the just noticeably different chromaticities should be situated in a circle around a given point x, y
- Use of difference measures: colour picking, design choice evaluation

Computer Graphics Charles University

MacAdam's Ellipses

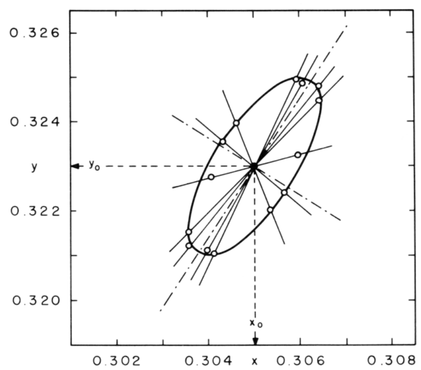
- Demonstrates the perceptual non-uniformity of the CIE x,y chromaticity diagram
- First compiled by MacAdam in 1942



Computer Graphics Charles University

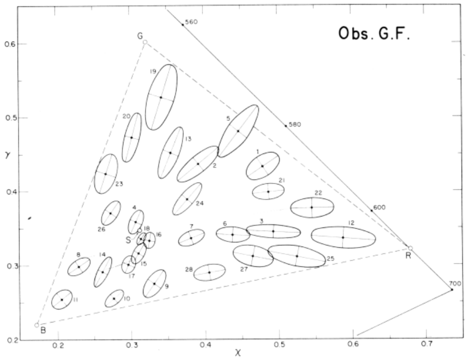
MacAdam's Experiment

- For a fixed colour x0, y0 the observer only had to vary one variable at a time – a shift of colour along a straight line in x, y space that went through x0, y0
- Apparatus held luminance constant
- Ellipses were fitted through data points



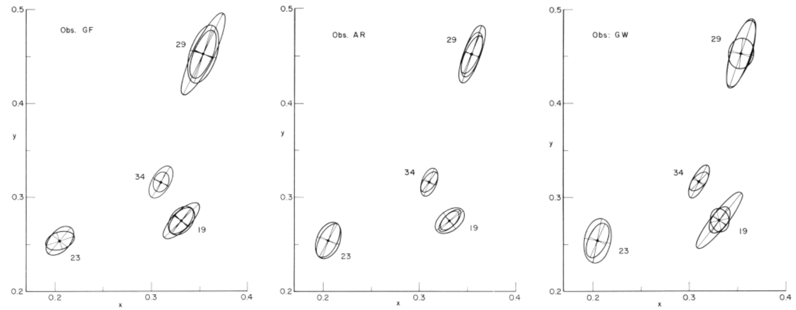
Variability Between Observers

- Experiments by Wyszecki (1971) suggest a marked dependency on the observer



Variability For a Given Observer

- Same observer, identical viewing conditions
- Poor repeatability demonstrates qualitative aspect of MacAdam's diagram



CIE Uniform Colour Spaces

CIE Uniform Colour Spaces

- Chromaticity diagrams only show proportions of tristimulus values
- For comparisons of stimuli luminance has to be taken into account
- Goal: a device-independent colour space in which the euclidian distance between stimuli (incl. Y) is proportional to their difference
- 1960: first attempt
- 1976 the CIE proposed two such colour spaces: CIE L*u*v* and CIE L*a*b*

CIE 1960 UCS

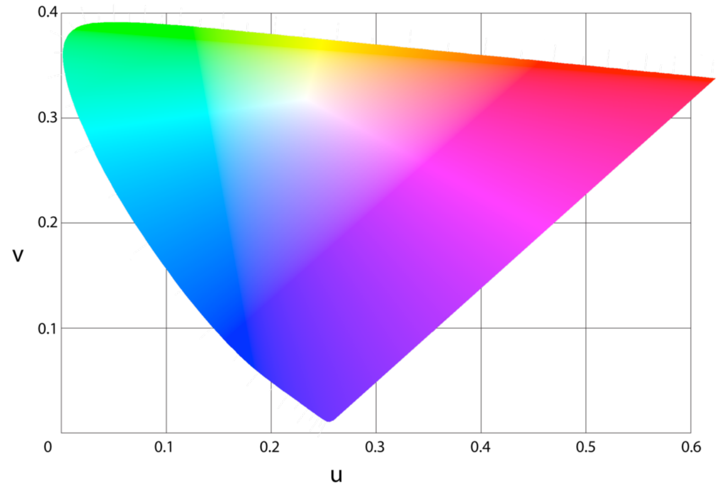
- First attempt at a CIE UCS
  - Developed by Judd
- Simple linear transform of XYZ
- Still used for CRI computations

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 3.1956 & 2.4478 & -0.1434 \\ -2.5455 & 7.0492 & 0.9963 \\ 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$u = \frac{0.4661x + 0.1593y}{y - 0.15735x + 0.2424}$$

$$v = \frac{0.6581y}{y - 0.15735x + 0.2424}$$

CIE 1960 (u,v)



CIE 1976 L*u*v*

- Derived from earlier u', v' colour space
- Rectangular coordinates
- un, vn are coordinates of reference white
- Yn is set to 100
- Originally mainly used for additive colour systems, e.g. for monitors

$$L^* = 116 \sqrt[3]{\frac{Y}{Y_n}} - 16$$

$$u^* = 13L^*(u' - u'_n)$$

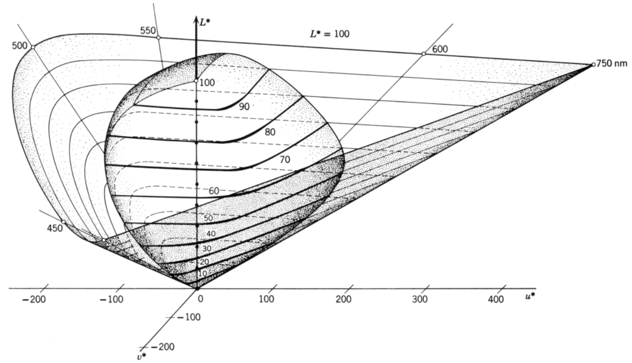
$$v^* = 13L^*(v' - v'_n)$$

$$u' = \frac{4X}{X + 15Y + 3Z}$$

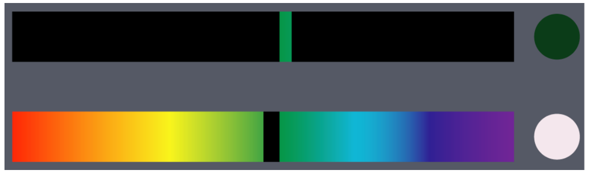
$$v' = \frac{9Y}{X + 15Y + 3Z}$$

CIE L*u*v* Visualisation

- Outer surface: monochromatic colours
- Inner surface: D65 / optimal object colours

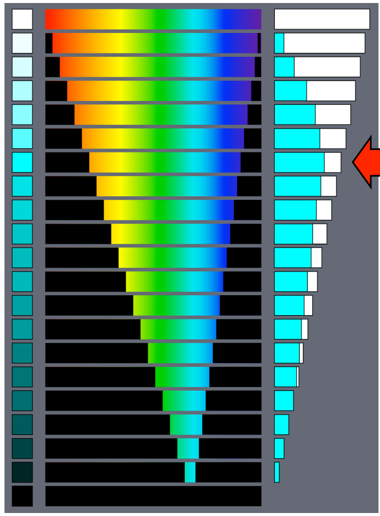


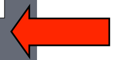
Object Colours: Reflection Spectra



- Monochromatic (or band pass) paints are necessarily almost black
- Narrow band gap paints are necessarily almost white
- Highly colourful paints must be "full" colours, or "semichromes"

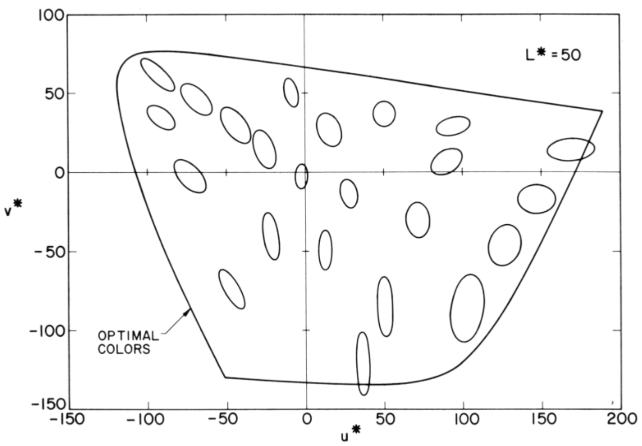
Solid Colour Ramp



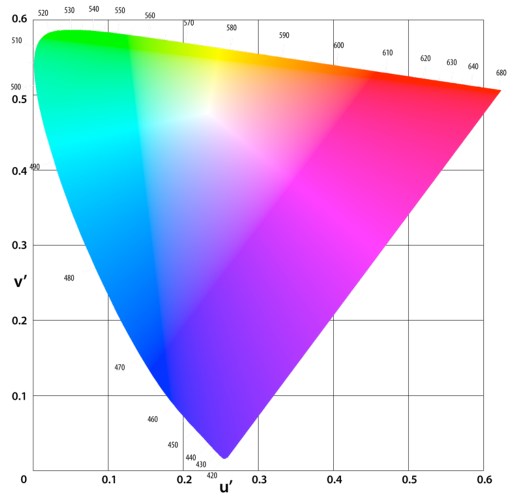


- Full colours have the maximum colour content for a given hue
- Band limits have to be at complementary wavelengths for maximum hue

CIE 1976 u*v* Ellipses



CIE 1976 (u′,v′)



L*u*v* Colour Difference

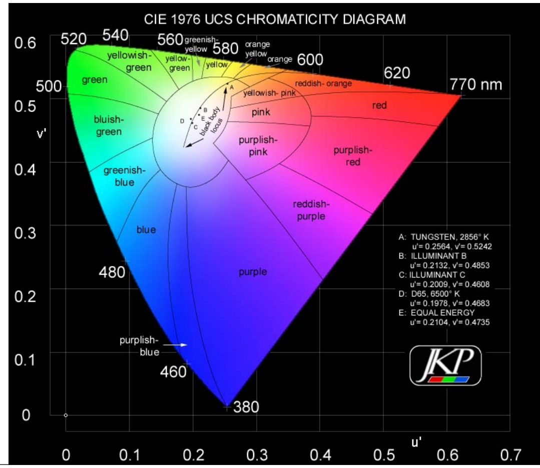
- Euclidian distance between sample points

$$E_{uv}^{*} = \sqrt{(\Delta L^{*})^{2} + (\Delta u^{*})^{2} + (\Delta v^{*})^{2}}$$

- Also known as total colour difference
- For constant lightness L* the u*, v* plane is a transformed x, y chroma diagram
- Straight lines in the x, y diagram are also straight in the u*, v* diagram (colour mixing!)

CIE 1976 UCS Diagram

- Chromaticity diagram for the 1976 L*u*v* space



CIE 1976 L*a*b*

- Based on Hunter's earlier Lab colour space
- No associated chromaticity diagram due to cube root in a* and b*
- Originally devised for use in the paint industry

$$L^{*} = 116\sqrt[3]{\frac{Y}{Y_n}} - 16$$

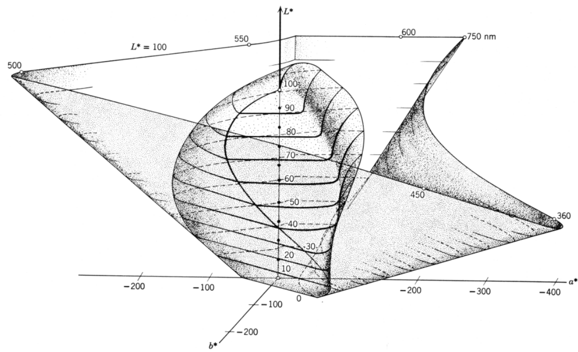
$$a^{*} = 500\left[\sqrt[3]{\frac{X}{X_n}} - \sqrt[3]{\frac{Y}{Y_n}}\right]$$

$$b^{*} = 200\left[\sqrt[3]{\frac{Y}{Y_n}} - \sqrt[3]{\frac{Z}{Z_n}}\right]$$

- Values of a*, b* are associated with complementary colours and hence intuitive

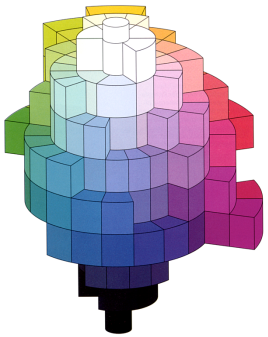
CIE L*a*b* Visualisation

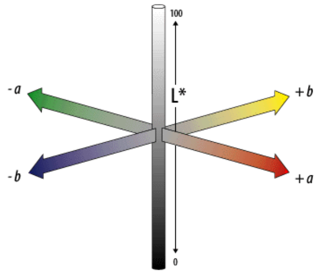
- Outer surface: monochromatic colours
- Inner surface: D65 / optimal object colours



CIE L*a*b*

- The a* and b* axes correspond to r/g and b/y opponent colour stimuli





L*a*b* Total Colour Difference

- Euclidian distance between sample points

$$E^*_{ab} = \sqrt{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2}$$

- Formula similar to L*u*v* distance metric
- Also known as the 1976 (L*a*b*) colour difference formula
- Sometimes abbreviated as CIELAB
- Well suited for most viewing conditions

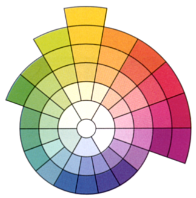
Chroma

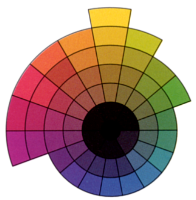
- For both L*a*b* and L*u*v* the chroma value C* of a stimulus can be determined through

$$C^*_{uv} = \sqrt{(u^*)^2 + (v^*)^2}$$

$$C^*_{ab} = \sqrt{(a^*)^2 + (b^*)^2}$$

respectively





Saturation

- Only in the case of L*u*v* the quantity s*uv can be defined as a correlate of saturation:

$$s^*_{uv} = \frac{C^*_{uv}}{L^*}$$

- For stimuli of constant chromaticity but changing luminance it remains constant

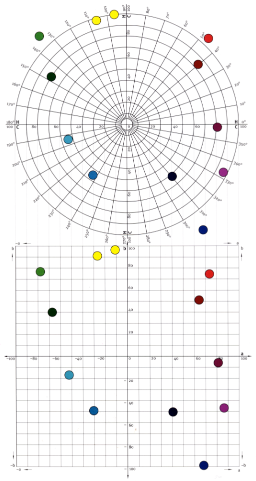
Hue Angle

- For both L*a*b* and L*u*v* the hue angle of a stimulus can be determined through

$$h_{ab} = \arctan\left(\frac{b^*}{a^*}\right)$$

$$h_{uv} = \arctan\left(\frac{v^*}{u^*}\right)$$

respectively



Hue Differences

- For both L*u*v* and L*a*b* the hue difference H of a pair of stimuli can be determined through

$$\Delta H^*_{uv} = \left[(\Delta E^*_{uv})^2 - (\Delta L^*)^2 - (\Delta C^*_{uv})^2\right]$$

$$\Delta H^*_{ab} = \left[(\Delta E^*_{ab})^2 - (\Delta L^*)^2 - (\Delta C^*_{ab})^2\right]$$

respectively

Alternative Total Colour Difference

- The total colour difference E* can also be defined as consisting of three components:
  - Luminosity difference ΔL*
  - Chromaticity difference ΔC*
  - Hue difference ΔH*

CIELUV vs. CIELAB Summary

- Both are basically equivalent colour spaces
- The existence of two similar colour spaces has historical and practical reasons
  - L*u*v* with its uniform chroma diagram is used for luminous colours
  - L*a*b* with its intuitive opponent coordinates is more suited for object colours
- There is no perfect colour space suited for all tasks