

Partial exam PMP (21.11.2025)
work time: 60 min.

The answers to the test must be written on this sheet or uploaded, by the end of the test, to your personal GitHub repository in a directory named “Partial”. In addition to the Python code, you must also upload screenshots of the executions as well as any relevant graphs.

Subject 1. SMART HOME ENERGY AND COMFORT SYSTEM

A smart home system monitors and controls heating, cooling, and energy usage based on environmental and behavioral factors. The Bayesian network models the relationships between **outside temperature, heating system status, window status, room temperature, energy consumption, and occupant comfort**.

Variables and Domains:

- OutsideTemp (O): {cold, mild}
- HeatingOn (H): {yes, no}
- WindowOpen (W): {yes, no}
- RoomTemp (R): {warm, cool}
- EnergyUse (E): {high, low}
- Comfort (C): {comfortable, uncomfortable}

Conditional Probability Tables (CPTs):

$$P(O = \text{cold}) = 0.3, P(O = \text{mild}) = 0.7$$

$$P(H = \text{yes} \mid O = \text{cold}) = 0.9, P(H = \text{yes} \mid O = \text{mild}) = 0.2$$

$$P(W = \text{yes} \mid O = \text{cold}) = 0.1, P(W = \text{yes} \mid O = \text{mild}) = 0.6$$

$$P(R = \text{warm} \mid H = \text{yes}, W = \text{yes}) = 0.6, P(R = \text{warm} \mid H = \text{yes}, W = \text{no}) = 0.9$$

$$P(R = \text{warm} \mid H = \text{no}, W = \text{yes}) = 0.3, P(R = \text{warm} \mid H = \text{no}, W = \text{no}) = 0.5$$

$$P(E = \text{high} \mid H = \text{yes}) = 0.8, P(E = \text{high} \mid H = \text{no}) = 0.2$$

$$P(C = \text{comfortable} \mid R = \text{warm}) = 0.85, P(C = \text{comfortable} \mid R = \text{cool}) = 0.40$$

Tasks:

- a) Build the Bayesian network (including graph) in pgmpy.
 - b) Perform inference using VariableElimination:
 - Compute $P(H = \text{yes} \mid C = \text{comfortable})$.
 - Compute $P(E = \text{high} \mid C = \text{comfortable})$.
 - Find the MAP estimate (greatest probability) for (H, W) given $C = \text{comfortable}$.
 - c) Independence reasoning:
 - Is $W \perp E \mid H$? Explain using d-separation.
 - Are O and C independent given R ? Justify.
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Subject 2. WEATHER AND ACTIVITY PREDICTION USING HMM

A fitness app predicts a user's *activity* (hidden state) based on *sensor readings* (observations). The user's activity can be either Walking (W), Running (R) or Resting (S). The app receives sensor observations about *heart rate* and *step count*, which are simplified into three categories: Low (L), Medium (M) and High (H).

The system is modeled as a *Hidden Markov Model (HMM)* with the following model parameters:

- *Initial state probabilities*:

$$P(W) = 0.4, P(R) = 0.3, P(S) = 0.3$$

- *Transition probabilities*:

$$P(W|W) = 0.6, P(R|W) = 0.3, P(S|W) = 0.1$$

$$P(W|R) = 0.2, P(R|R) = 0.7, P(S|R) = 0.1$$

$$P(W|S) = 0.3, P(R|S) = 0.2, P(S|S) = 0.5$$

- *Emission probabilities*:

$$P(L|W) = 0.1, P(M|W) = 0.7, P(H|W) = 0.2$$

$$P(L|R) = 0.05, P(M|R) = 0.25, P(H|R) = 0.7$$

$$P(L|S) = 0.8, P(M|S) = 0.15, P(H|S) = 0.05$$

Tasks:

a) *Model Construction*: Define the HMM using `hmmlearn` (or any suitable library). Specify the initial, transition, and emission probabilities.

b) *Forward Algorithm*: Compute the probability of observing the sequence:

$$\text{Observations} = [\text{Medium}, \text{High}, \text{Low}]$$

c) *Viterbi Algorithm*: Find the most likely sequence of hidden states for the same observation sequence. If the observation sequence were longer, why might the Viterbi algorithm be preferred over brute-force enumeration?

d) Generate 10,000 sequences using the HMM and estimate the empirical probability of the observation sequence [M, H, L]. Compare with the exact probability from the Forward algorithm.

Score table:

subject:	1.a	1.b	2.c	2.a	2.b	2.c	2.d	ex officio	Total
max. score:	4	5	4	4	4	4	4	1	30