

### Tema 3

$$\textcircled{3} \quad \begin{array}{l} n - \text{prim} \\ \hline 2^n - 1 \text{ prim} \end{array}$$

$$\begin{array}{l} 2^{n^2} - 1 - \text{prim} \\ \hline n - \text{prim} \\ \hline \end{array}$$

verificare:

$$\begin{array}{l} 2^2 - 1 = 3 \\ 2^3 - 1 = 7 \end{array} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{prim}$$

p.p. R. A.  $n$  - compus  $\Rightarrow$

$$\Rightarrow \exists a, b \in \mathbb{N}^* \text{ a.i. } n = a \cdot b.$$

$$\Rightarrow 2^n - 1 = 2^{a \cdot b} - 1 = (2^a)^b - 1 \Rightarrow$$

$$\Rightarrow (2^a - 1) \cdot ((2^a)^{b-1} + (2^a)^{b-2} + (2^a)^{b-3} + \dots + 2^a + 1) =$$

$$= (2^a)^b - 1.$$

$b$  - termenii

$$1 < a < n \Leftrightarrow 2 < 2^a < 2^n \Leftrightarrow 1 < 2^a - 1 < 2^n - 1$$

$$\Rightarrow (2^a - 1) \mid 2^n - 1, \Rightarrow 2^n - 1 - \text{compus (F)} \Rightarrow$$

$$\neq 2. \Rightarrow \boxed{n - \text{prim}}$$