Tema 3

(3) n - prim $\not\Rightarrow$ $2^n - 1$ prim

$2^{n^4} - 1$ - prim $\Rightarrow$ n - prim

verificare:

$2^2 - 1 = 3$
$2^3 - 1 = 7$ } prim

p.p. R.A. n - compus $\Rightarrow$

$\Rightarrow \exists a, b \in \mathbb{N}^* \setminus \{1\}$ a.î. $n = a \cdot b$.

$\Rightarrow 2^n - 1 = 2^{a \cdot b} - 1 = (2^a)^b - 1 \Rightarrow$

$\Rightarrow (2^a - 1) \cdot \underbrace{((2^a)^{b-1} + (2^a)^{b-2} + (2^a)^{b-3} + \dots + 2^a + 1)}_{b \text{ - termeni}} =$

$= b^{ab} - 1.$

$1 < a < n \Leftrightarrow 2 < 2^a < 2^n \Leftrightarrow 1 < 2^a - 1 < 2^n - 1$

$\Rightarrow (2^a - 1) \mid (2^n - 1) \Rightarrow 2^n - 1$ - compus ($\lightning$) $\Rightarrow$

$\underbrace{2^a - 1}_{\geq 2} \Rightarrow$ $\boxed{n \text{ - prim}}$