Mutlivariat Assignment 1

February 4, 2021

[1]:
```
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
import random
from scipy.spatial import distance
```

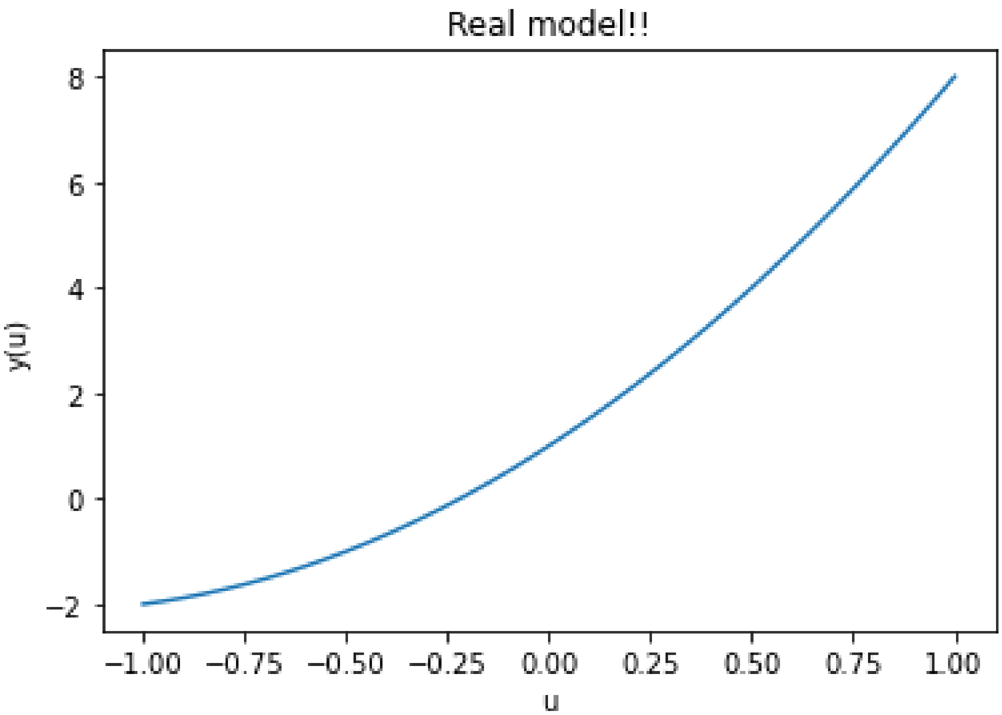
1 Multivariat Assignment 1

This assignment is brought to you by: Petter Skau-Nilsen, Siri S. Nysted, Ivar Tesdal Galtung

Task 1.1 - Create a true modell

[2]:
```
def arbitrary_poly(params):
    poly_model = lambda x: sum([p*(x**i) for i, p in enumerate(params)])
    return poly_model

theta = [1,5,2] # The real parameters of the true model

ymod = arbitrary_poly(theta)

N = 200 # Number of measurements

u = np.linspace(-1,1,N) # "Measured" inputs.

yreal = ymod(u) # Create the modell


# Plot
plt.figure()
plt.plot(u,yreal)
plt.xlabel('u')
plt.ylabel('y(u)')
plt.title('Real model!!')
```

[2]:
```
Text(0.5, 1.0, 'Real model!!')
```



Task 1.2 - Generate noise signal

[3]:
```
def chooseDist(dist1,dist2,alpha): # Choose distribution given alpha to be␣
 ↪probability of distribution 1.
    if(random.uniform(0,1)>=(1-alpha)):
        return dist1
    return dist2

from scipy.stats import norm, laplace

def genNoise(alpha, N, mean, sigma, beta, magnitude): # Generate the noise of␣
 ↪the signal
    e = np.zeros(N)
    for i in range(0,N):
        dist = chooseDist("Gauss","Laplace",alpha)
        if dist == "Laplace":
            #pdf = laplace_pdf
            pdf = laplace.pdf
            e[i] = magnitude * np.random.laplace(mean, beta)
        elif dist == "Gauss":
            #pdf = gauss_pdf
            pdf = norm.pdf
            e[i] = magnitude * np.random.normal(mean, sigma)
```

```
        else:
            raise Exception("Distribution not implemented, choose \"laplace\" or␣
↪\"gauss\"")

    return e
```

[4]:
```
mean = 0

magnitude = 1
```

[73]:
```
alpha = 0.5

sigma = 1

beta = 1

e = genNoise(alpha, N, mean, sigma, beta, magnitude)

y = yreal + e
```

Task 1.3 - Algorithm for Least Square Estimation

[6]:
```
# Matrix form
u_tensor_0 = np.reshape(u,(N,1))

ones_vec = np.ones((N,1))
u_tensor = np.append(ones_vec, u_tensor_0, axis=1)

for i in range(2,len(theta)):
    u_tensor = np.append(u_tensor, np.power(u_tensor_0, i) ,axis=1)
```

Want to find $(u^{\mathrm{T}} \cdot u)^{-1}$

[17]:
```
u_transpose_dot_u = np.dot(u_tensor.T,u_tensor)

u_transpose_dot_u_inv = np.linalg.inv(u_transpose_dot_u)
```

Next step is to find $(u^T \cdot y)$

[74]:
```
u_transpose_dot_y = np.dot(u_tensor.T,y)
```

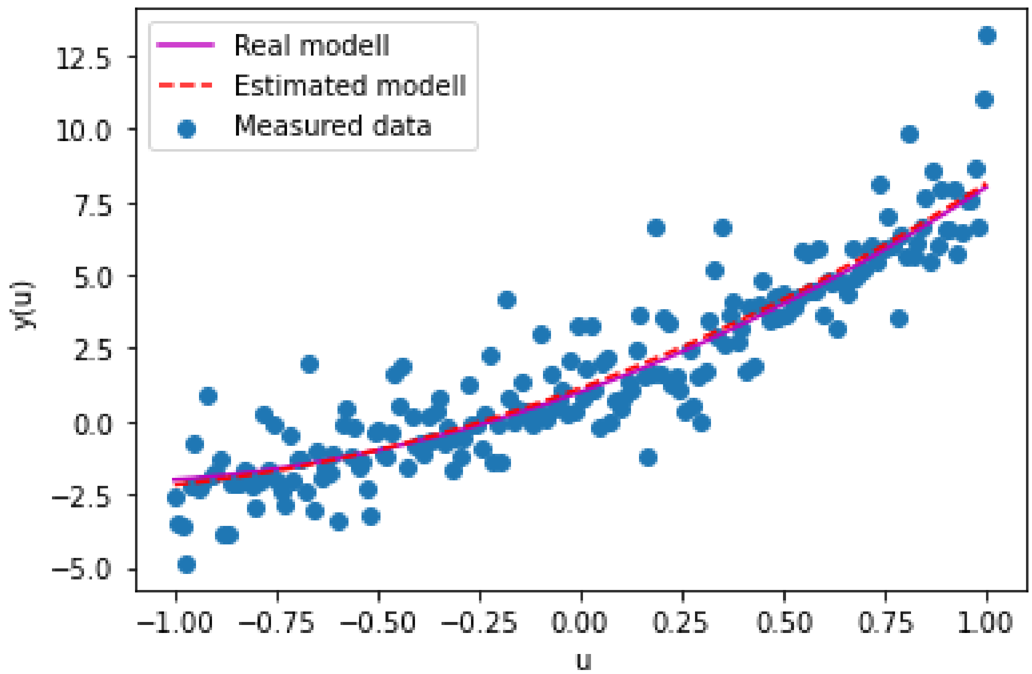
Now to the last, final, shit show of a step!!! $\hat{\theta} = (u^{\mathrm{T}} \cdot u)^{-1} \cdot (u^T \cdot y)$

[75]:
```
est_params = np.dot(u_transpose_dot_u_inv,u_transpose_dot_y) # Calculate equation

est_params = est_params.tolist() # Format parameters to list

d = distance.euclidean(theta, est_params)
```

```
print(d)

y_est = arbitrary_poly(est_params) # Create estimated response

plt.figure()
plt.plot(u,yreal,"m",label = "Real modell")
plt.scatter(u,y, label = "Measured data")
plt.plot(u,y_est(u),"r--",label = "Estimated modell")
plt.ylabel('y(u)')
plt.xlabel('u')
plt.legend()
```

```
0.24772178911737636
```

[75]:
```
<matplotlib.legend.Legend at 0x1428bf6ad30>
```



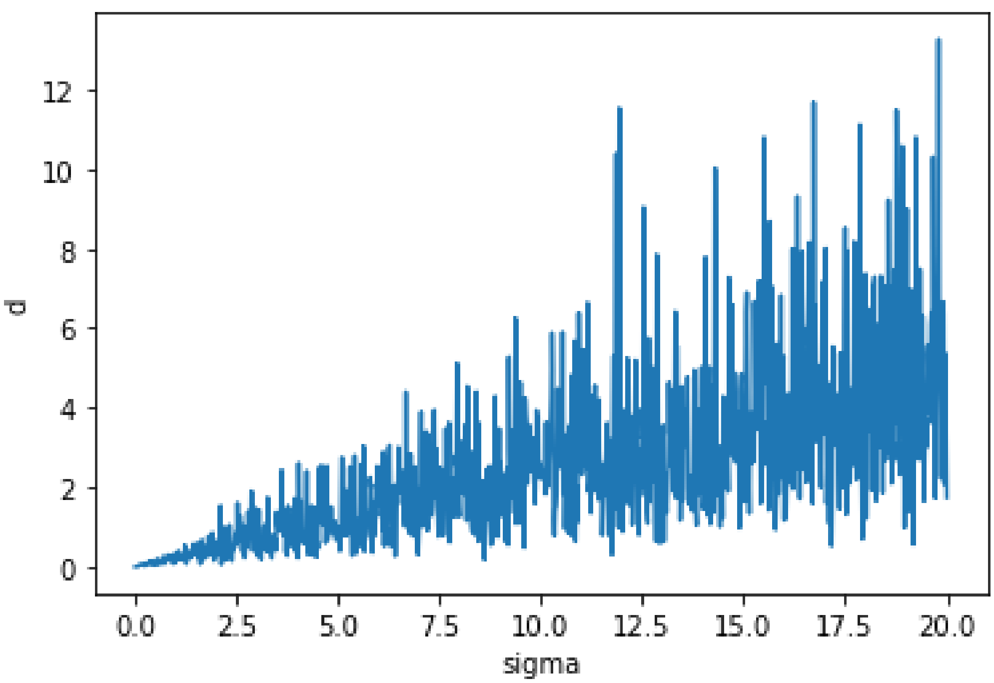
Task 1.4 - Analysis of parameters

Analysis of d when $\alpha = 1$ and σ is varying.

[10]:
```
alpha = 1

sigma_space = np.linspace(0,20,1000)

beta = 1
```

```
e = np.array([genNoise(alpha, N, mean, sigma, beta, magnitude) for sigma in
 ↪sigma_space])

y = yreal + e

u_transpose_dot_y = np.array([np.dot(u_tensor.T,y_i) for y_i in y])

est_params = np.array([np.dot(u_transpose_dot_u_inv,elt) for elt in
 ↪u_transpose_dot_y]) # Calculate equation

est_params = est_params.tolist() # Format parameters to list

d = np.array([distance.euclidean(theta, elt) for elt in est_params])

plt.figure()
plt.plot(sigma_space,d)
plt.xlabel('sigma')
plt.ylabel('d')
```

[10]: Text(0, 0.5, 'd')

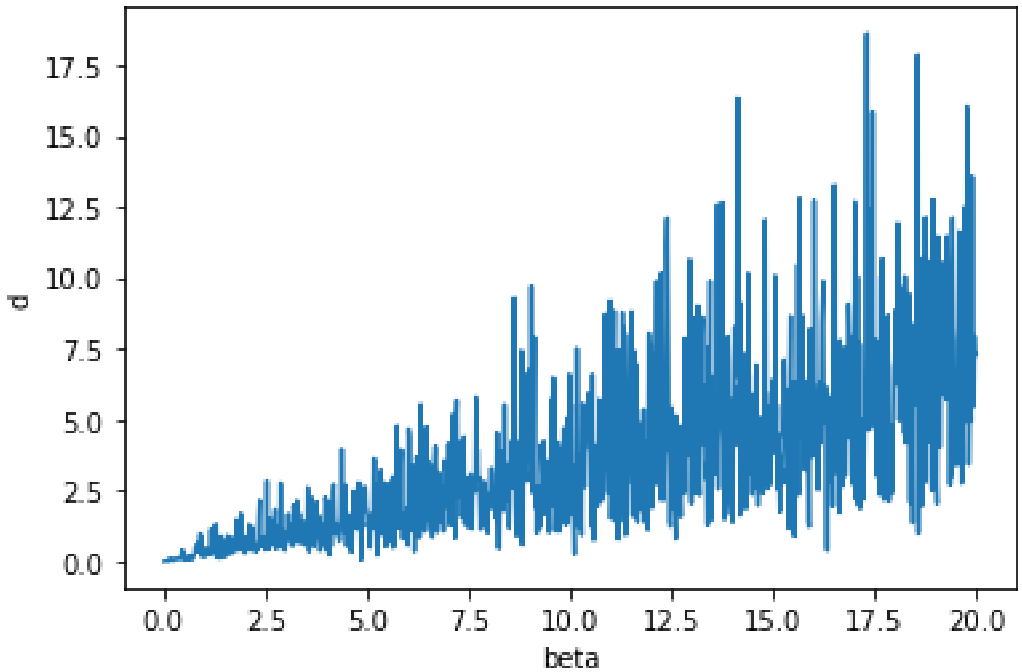


Comment: We observe that increasing σ will result in increasing variance of the distance d. An increase in σ leads to a greater variance in the distance between the LS solution and the real values.

Analysis when $\alpha = 0$ and β varying.

[11]:
```
alpha = 0

beta_space = np.linspace(0,20,1000)

sigma = 1

e = np.array([genNoise(alpha, N, mean, sigma, beta, magnitude) for beta in
 ↪beta_space])

y = yreal + e

u_transpose_dot_y = np.array([np.dot(u_tensor.T,y_i) for y_i in y])

est_params = np.array([np.dot(u_transpose_dot_u_inv,elt) for elt in
 ↪u_transpose_dot_y]) # Calculate equation

est_params = est_params.tolist() # Format parameters to list

d = np.array([distance.euclidean(theta, elt) for elt in est_params])

plt.figure()
plt.plot(beta_space,d)
plt.xlabel('beta')
plt.ylabel('d')
```

[11]:
```
Text(0, 0.5, 'd')
```

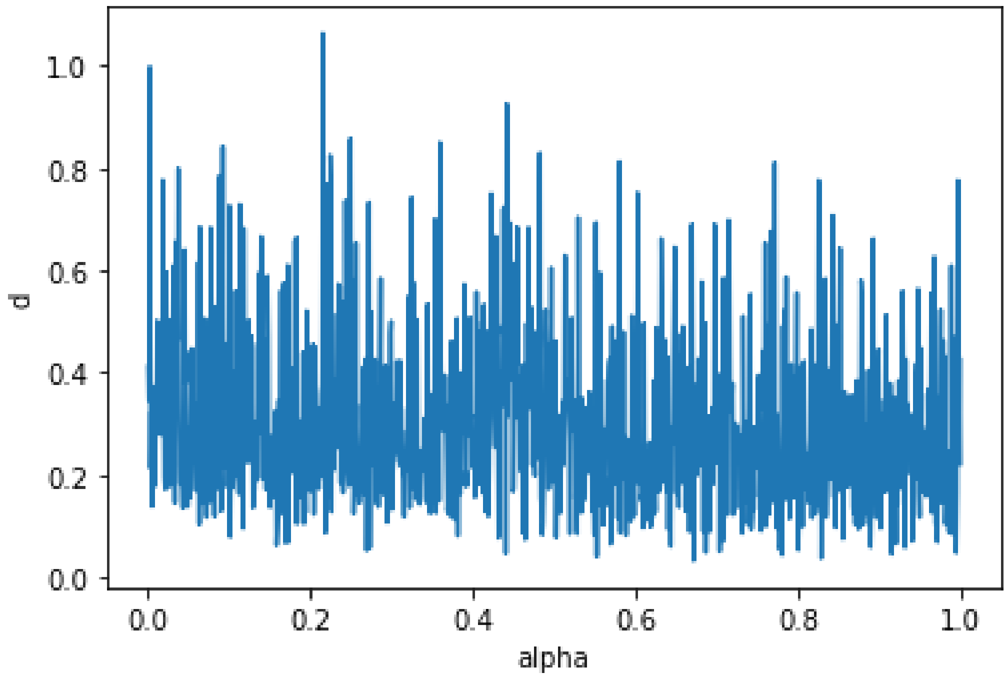


Comment: We observe somewhat the same result as for varying σ, but with more increasing variance of the value d. It also looks like the mean of d. The LS solution is further from the real values as β increases.

[66]:
```
alpha_space = np.linspace(0,1,1000)

beta = 1

sigma = 1

e = np.array([genNoise(alpha, N, mean, sigma, beta, magnitude) for alpha in␣
 ↪alpha_space])

y = yreal + e

u_transpose_dot_y = np.array([np.dot(u_tensor.T,y_i) for y_i in y])

est_params = np.array([np.dot(u_transpose_dot_u_inv,elt) for elt in␣
 ↪u_transpose_dot_y]) # Calculate equation

est_params = est_params.tolist() # Format parameters to list

d = np.array([distance.euclidean(theta, elt) for elt in est_params])

plt.figure()
```

```
plt.plot(alpha_space,d)
plt.xlabel('alpha')
plt.ylabel('d')
```

[66]: Text(0, 0.5, 'd')



Observation: When the two distributions have equal variance, the weighting of their contribution has little effect on the distance d.