Objective: Learn to do clustering and noise reduction in data using PCA

In [1]:
```
import matplotlib.pyplot as plt
import numpy as np
from numpy.linalg import svd
from sklearn.datasets import load_digits
digits = load_digits()
digits.data.shape
```

Out[1]:
```
(1797, 64)
```

PCA using SVD

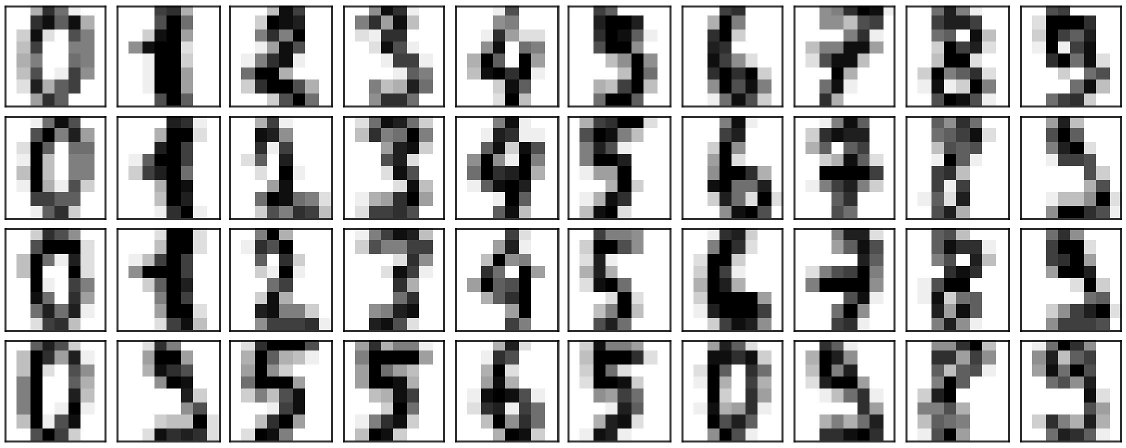
In [2]:
```
def pca(X):
    U, S, PTrans = svd(X, full_matrices=False)
    Sigma = np.diag(S)
    T=np.dot(U,Sigma)
    P=PTrans.T
    return T, Sigma, P #Score, Variace, Loadings
```

In [3]:
```
def plot_digits(data):
    fig, axes = plt.subplots(4, 10, figsize=(10, 4),
                             subplot_kw={'xticks':[], 'yticks':[]},
                             gridspec_kw=dict(hspace=0.1, wspace=0.1))
    for i, ax in enumerate(axes.flat):
        ax.imshow(data[i].reshape(8, 8),
                  cmap='binary', interpolation='nearest',
                  clim=(0, 16))
```

In [4]:
```
# Find out the original dimension of the data
X=digits.data
y=digits.target
print("Shape of X",X.shape)
print("Shape of y", y.shape)
```

```
Shape of X (1797, 64)
Shape of y (1797,)
```

In [5]:
```
#Visualize the original data
plot_digits(X)
```

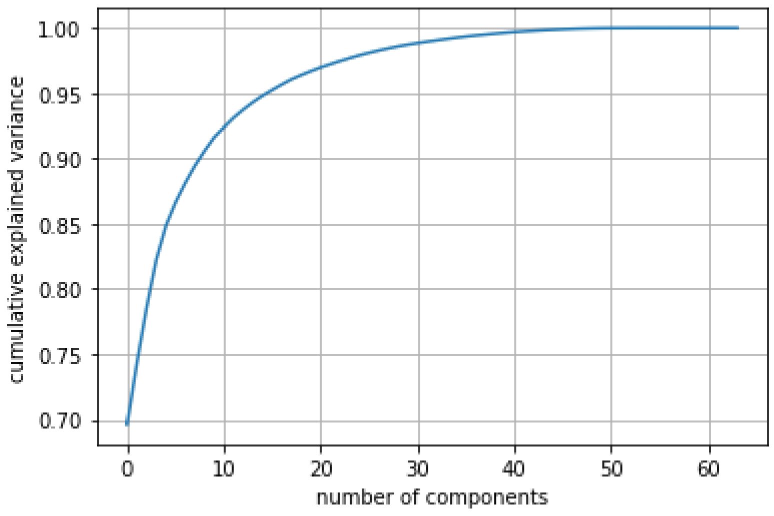


Task 1: Dimensionality reduction: Conduct PCA on the the matrix

X to find out the dimension required to capture 80% of the variance

In [6]:
```
T, sigma, P = pca(X)
print(T.shape,P.shape)
def variance_keeper(T, P, sigma, variance_limit):
    sigmas = []
    sum_sigma = np.trace(sigma)
    for i in range(len(np.diag(sigma))):
        if (sum(sigmas)/sum_sigma > variance_limit):
            break
        else:
            sigmas.append(sigma[i,i])


    return(T[:,0:len(sigmas)], P[:,0:len(sigmas)], np.array(sigmas), len(sigmas))


SS = np.diag(sigma)

explained_variance = (SS ** 2)/4
explained_variance_ratio = (explained_variance / explained_variance.sum())
plt.plot(np.cumsum(explained_variance_ratio))
plt.grid()
plt.xlabel('number of components')
plt.ylabel('cumulative explained variance');
```

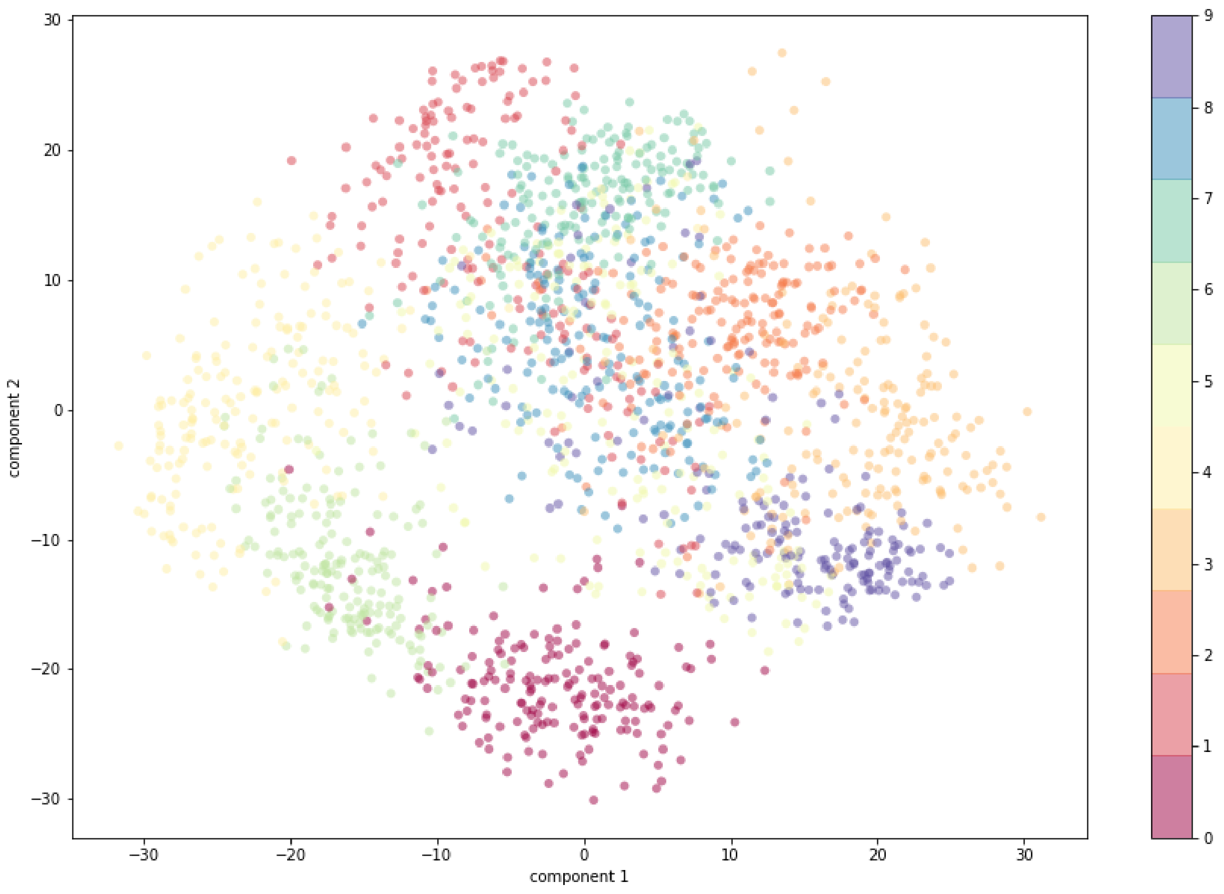
```
(1797, 64) (64, 64)
```



We need about 3/4 components to cover 80% of the variance

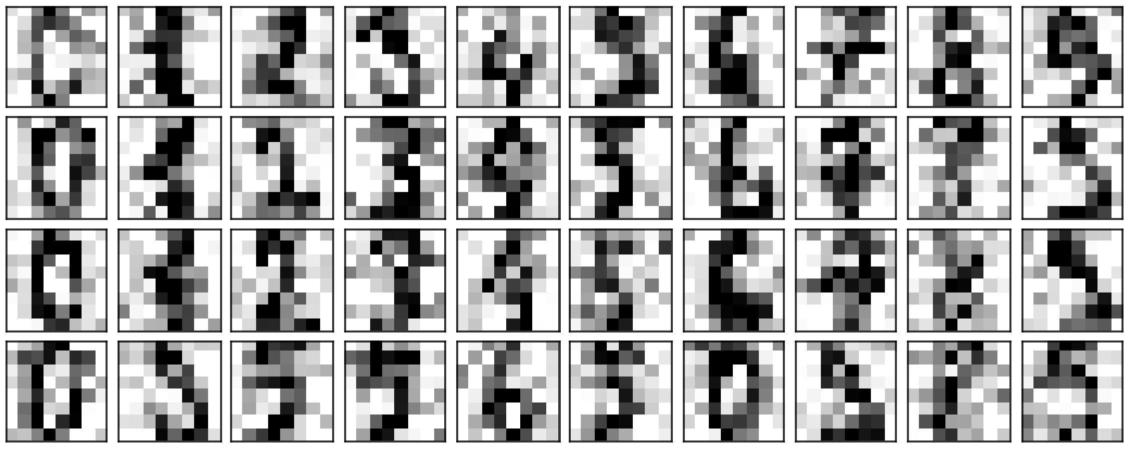
Task 2: Clustering: Project the original data matrix X on the first two PCs and draw the scalar plot

In [90]:
```
t1= T[:,0]
t2= T[:,1]

plt.figure(figsize=(15,10))
plt.scatter(t1, t2,
            c=digits.target, edgecolor='none', alpha=0.5,
            cmap=plt.cm.get_cmap('Spectral', 10))
plt.xlabel('component 1')
plt.ylabel('component 2')
plt.colorbar();
```



Task 3: Denoising: Remove noise from the noisy data

In [33]:
```
# Adding noise to the original data
X=digits.data
y=digits.target
np.random.seed(42)
noisy = np.random.normal(X, 4)
plot_digits(noisy)
```



Tips:

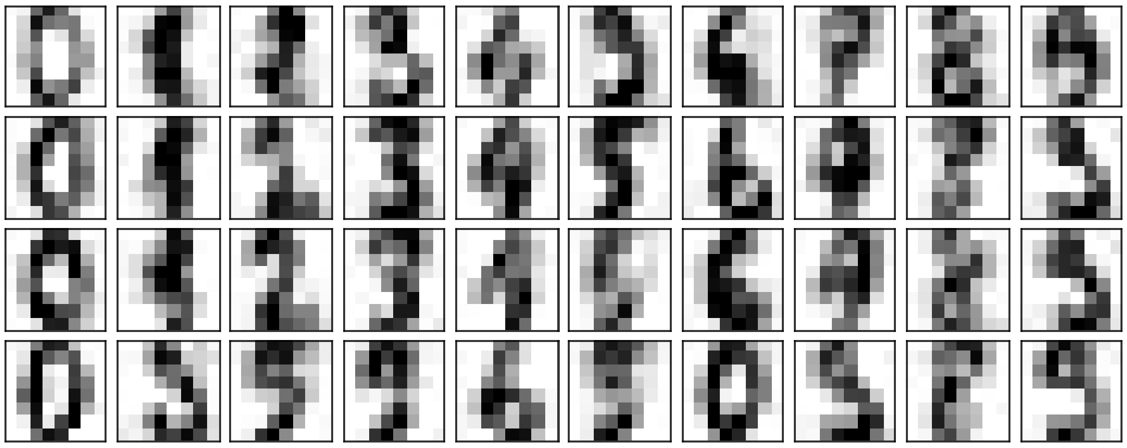
- Decompose the noisy data using PCA
- Reconstruct the data using just a few dominant components.For eg. check the variance plot

Since the nature of the noise is more or less similar across all the digits, they are not the fearues with enough variance to discriminate between the digits.

In [79]:
```
T, sigma, P = pca(noisy)
```

```
print(T.shape)

n_comp = 12

T_new = T[:,0:n_comp]
P_new = P[:,0:n_comp]

print(T_new.shape)

#T_new, P_new, sigmas, n = variance_keeper(T, P, sigma, 0.4)
#print(n)


X_denoised = T_new.dot(P_new.T)
plot_digits(X_denoised)
```

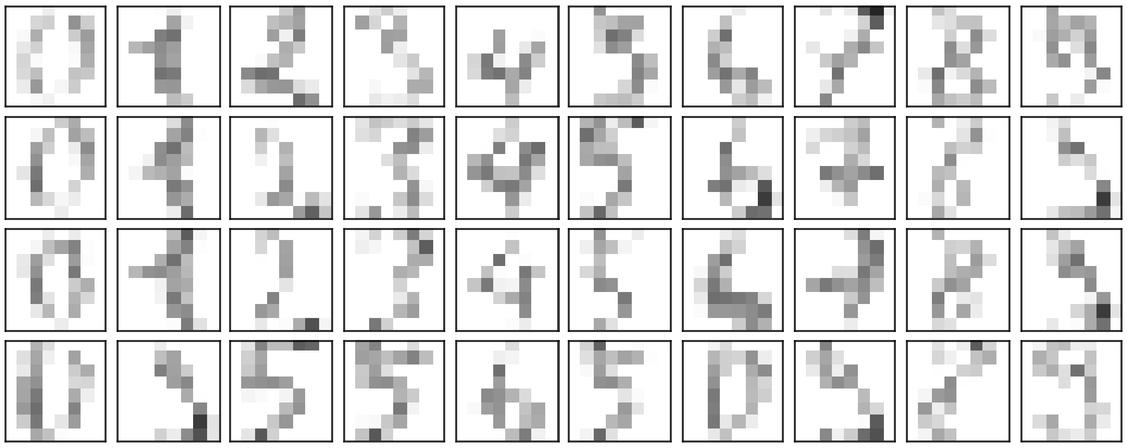
```
(1797, 64)
(1797, 12)
```



Task 4: Study the impact of normalization of the dataset before conducting PCA. Discuss if it is critical to normalize this particular data compared to the dataset in other notebooks

In [95]:
```
X_norm = X - np.mean(X,axis = 0)
#X_norm = noisy - np.mean(noisy,axis = 0)
T, sigma, P = pca(X_norm)
print(X_norm.shape)

print(np.max(X))

n_comp = 64

T_new = T[:,0:n_comp]
P_new = P[:,0:n_comp]

print(T_new.shape)



X_denoised = T_new.dot(P_new.T)
plot_digits(X_denoised)
```

```
(1797, 64)
16.0
(1797, 64)
```



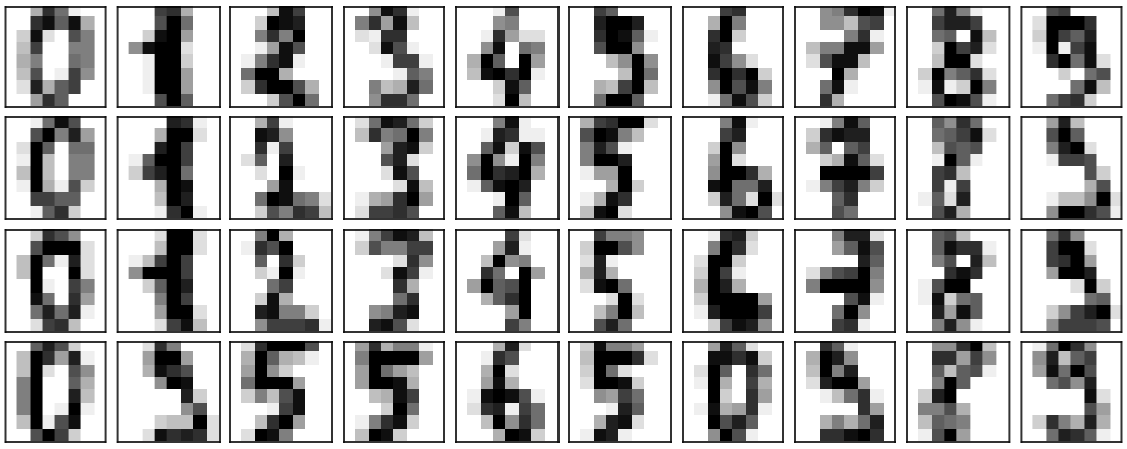
It is not necessary to normalize the data in this task. Here, the data lives in the area $\in [0, 16]$, thus shifting the data values so that the mean is about 0, makes no sense. This will only result in the loss of much data. However, it may be usefull to normalize the postion of the digits in each image. I.e. shifting the pixel position so that the center position of the digit in each image lay in the center of the image.

All the above excercise can be done using the SKLEAR library as follows

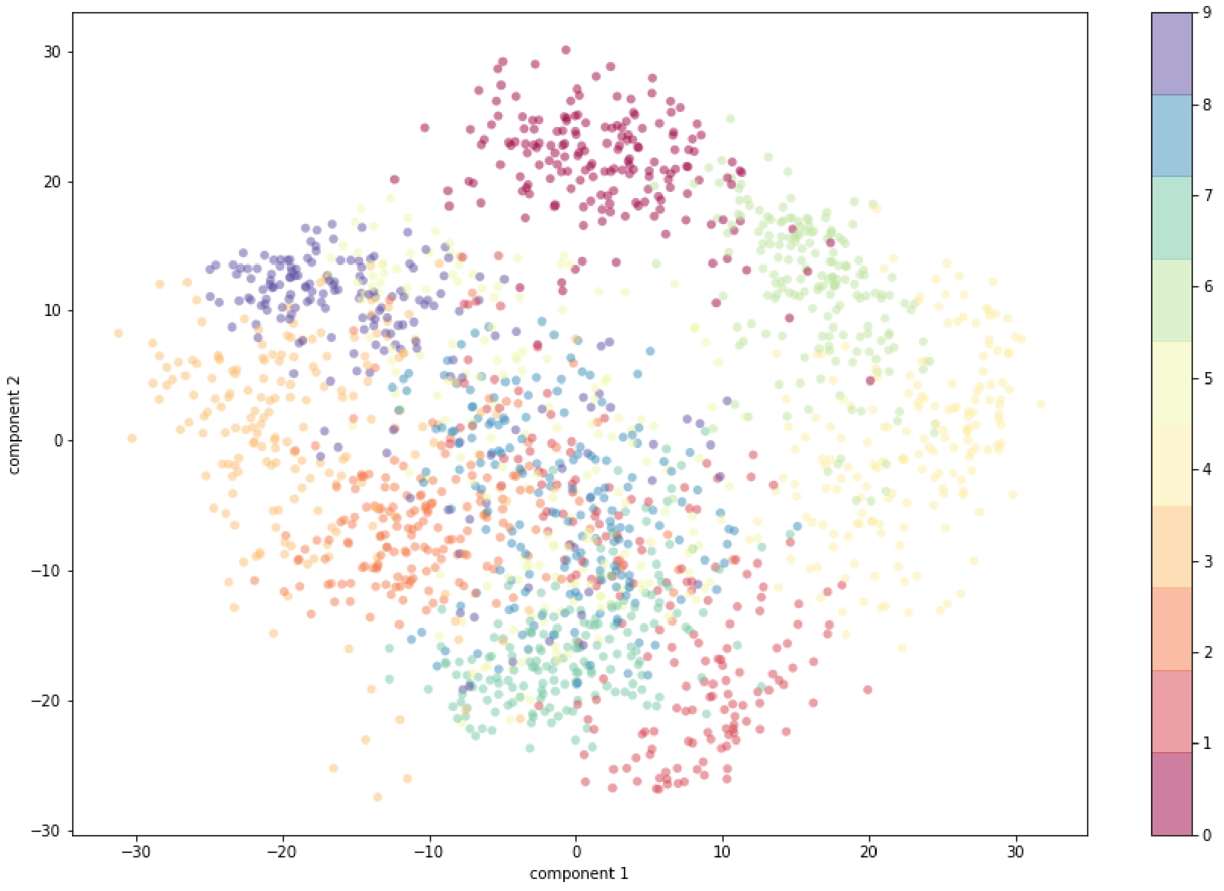
In [6]:
```
from sklearn.decomposition import PCA
X=digits.data
y=digits.target
```

In [7]:
```
pca = PCA(2)  # project from 64 to 2 dimensions
projected = pca.fit_transform(digits.data)
print(digits.data.shape)
print(projected.shape)
plot_digits(digits.data)
```

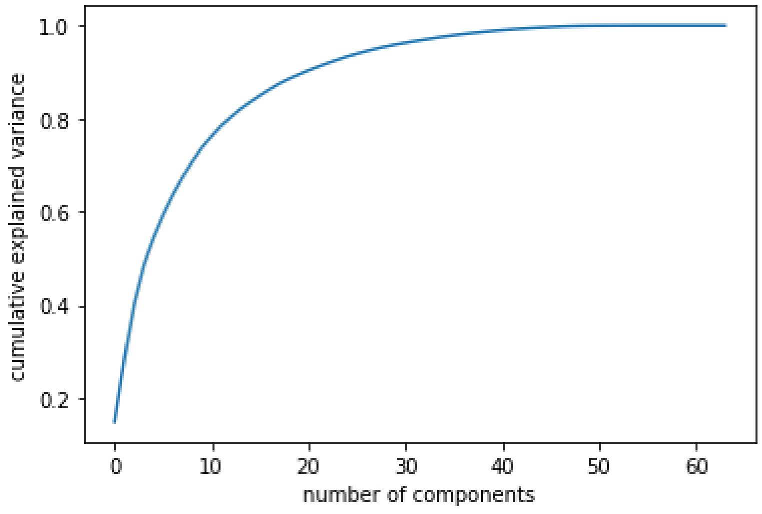
```
(1797, 64)
(1797, 2)
```



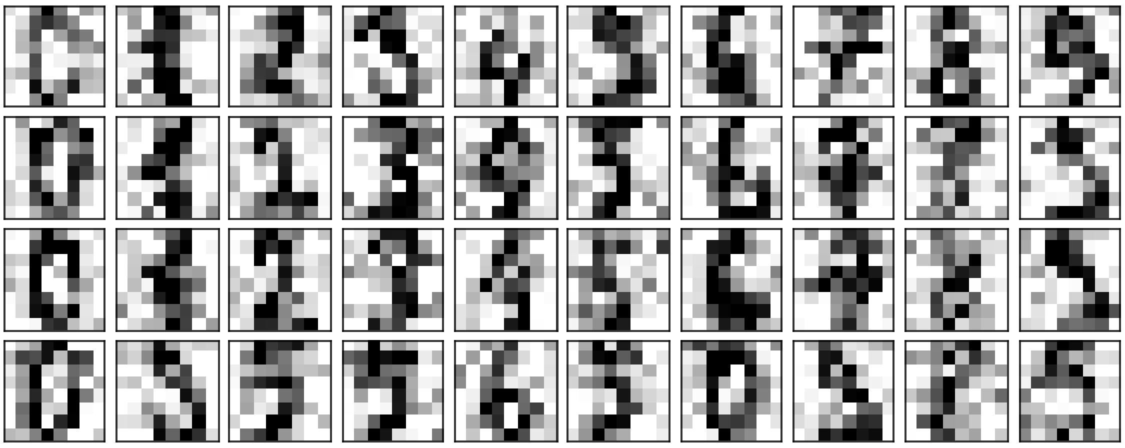
In [8]:
```
plt.figure(figsize=(15,10))
plt.scatter(projected[:, 0], projected[:, 1],
            c=digits.target, edgecolor='none', alpha=0.5,
            cmap=plt.cm.get_cmap('Spectral', 10))
plt.xlabel('component 1')
plt.ylabel('component 2')
plt.colorbar();
```



In [9]:
```
pca = PCA().fit(digits.data)
plt.plot(np.cumsum(pca.explained_variance_ratio_))
plt.xlabel('number of components')
plt.ylabel('cumulative explained variance');
```

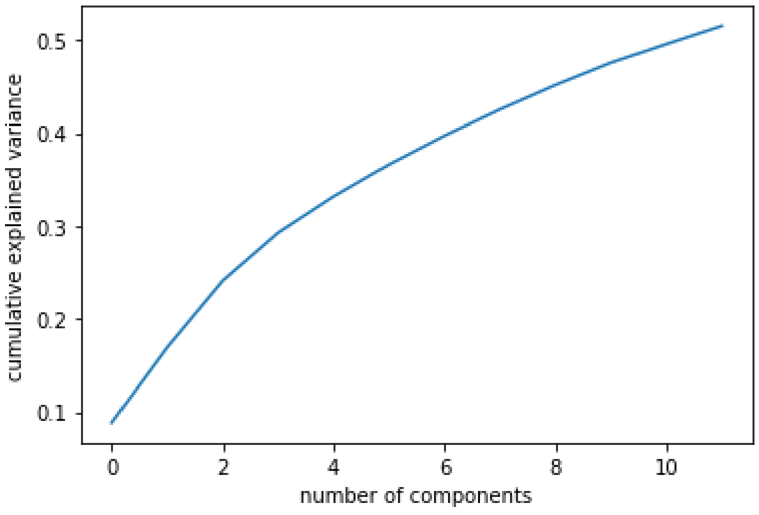


In [10]:
```
np.random.seed(42)
noisy = np.random.normal(digits.data, 4)
plot_digits(noisy)
```



In [11]:

```python
pca = PCA(0.50).fit(noisy) # 50% of the variance amounts to 12 principal components.
pca.n_components_
plt.plot(np.cumsum(pca.explained_variance_ratio_))
plt.xlabel('number of components')
plt.ylabel('cumulative explained variance');
```



In [12]:

```python
components = pca.transform(noisy)
filtered = pca.inverse_transform(components)
plot_digits(filtered)
```