# Simulation of Dynamic Behavior in an Extended Public Goods Game

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# Outline

- Introduction
- Public Goods Game
- Model
- Results
- Conclusions

# Aim and purpose

The classical assumption of rational behavior implies that all individuals behave rationally and selfishly.

Inconsistent with observations and laboratory experiments.

This is often explained by the existence of social norms and preferences for equality, altruism, etc.

Aims of our study:

- ◇ Study establishment and upholding of norms in a society.
- ◇ Investigate ways to reach states of cooperation.
- ◇ Analyze the effect of cost-sharing institutional structure.

# Approach

Construct and simulate an agent-based model for the public goods game with punishment.

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# (Single round) Public Goods Game

Players have the option to contribute credits to a common pool.

The common pool grows and its contents are distributed equally between all players.

Unique Nash equilibrium where all players contribute zero credits.

# (Repeated) Public Goods Game

Introduce multiple rounds.

For a finite number of rounds there is a unique Nash equilibrium where all players contribute zero credits.

Reciprocity can be introduced, e.g. in the form of punishment.

# Excerpts from previous research

In laboratory experiments contributions are commonly observed, but they decline as the game progresses (Burlando and Guala, 2005).

Large variations across cultures (Henrich et al., 2005).

Punishment may help to enforce cooperation (Ellingsen et al., 2012; Fehr and Gächter, 2000).

More surprising behavior also observed, e.g. punishing contributors (Herrmann et al., 2008).

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# Description of the model

Game with $n$ players, each employing one of three strategies:

- *Defectors* – Players who neither contribute nor punish.
- *Enforcers* – Players who both contribute and punish defectors.
- *Cooperators* – Players who contribute but who do not punish.

Played for $T$ rounds, each consisting of three stages:

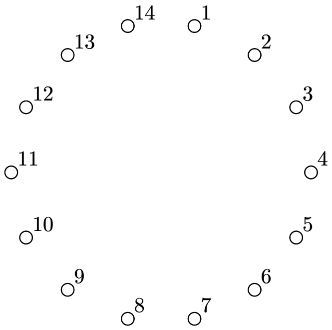
- *Contribution stage* – Contribution to common pool.
- *Punishment stage* – Option to punish other players.
- *Strategy revision stage* – Introduction of game dynamics.

# Basic set-up

Generation of $n$ players with strategies from some underlying strategy distribution.

Players are placed on a circle.

Punishments are limited to nearest neighbors to model a lack of information.




Circle with $14$ sites.

# Game dynamics

Game dynamics is introduced based on evolutionary game theory.

A number of players are selected for possible strategy revision.

The probability that a selected player revises her strategy depends on her relative payoff:

$$\text{Poor performance} \quad \longrightarrow \quad \text{High revision probability}$$

Three possibilities for strategy revision:

- ◇ Local imitation – Imitation of a nearest neighbor
- ◇ Global imitation – Imitation of a random player
- ◇ Mutation – Mutation to a random strategy

# Model summary

Important parameters:

- Growth factor $\kappa$ : $\kappa = 1.6$
- Cost of punishing $c$ : $c = 1$
- Effectiveness of punishment $\theta$ : $\theta > 0$
- Local-global imitation delimiter $\xi_1$ : $0 \leq \xi_1 \leq 1$
- Imitation-mutation delimiter $\xi_2$ : $\xi_1 \leq \xi_2 \leq 1$

$\Longrightarrow$ Cost of being punished is $\theta c$.

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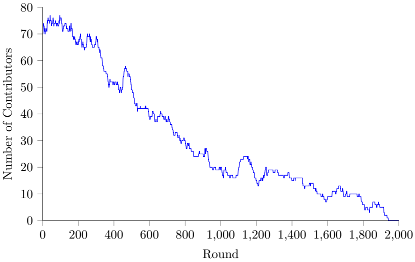
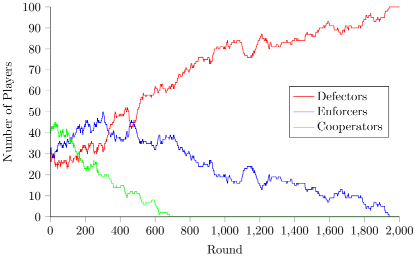
# Example simulation: $\theta = 2$, 100 players



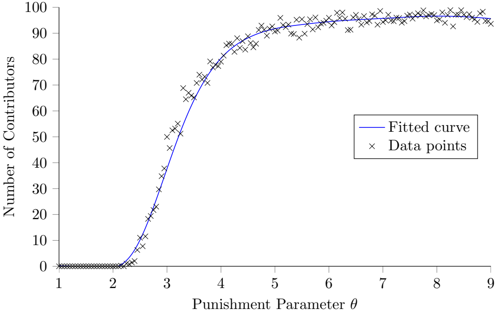

Number of contributors




Number of players for each strategy

Contributions decline over time, as often observed in laboratory experiments (Burlando and Guala, 2005).

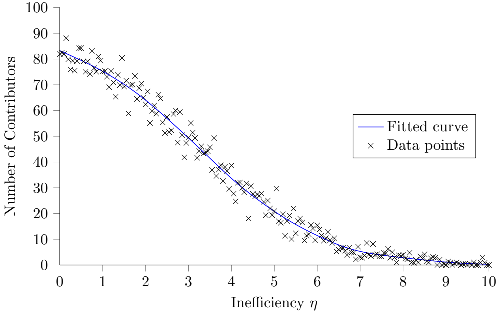
# Parameter study of $\theta$




(Averages of 100 simulations with 100 players of 10,000 rounds each.)

# Institutional cost-sharing

Enforcers share costs of punishing equally, but with an overhead cost $\eta$.




For $\theta = 2.5$ and 100 players, institutional cost sharing adds value for an inefficiency up to 600%.

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# A selection of conclusions

Results imply that:

- Punishment has an important effect on establishment and sustainment of norms.
- Favorable to break up local structures.
- Sharing the costs of punishment through an institution can be valuable, even for high degrees of inefficiency.

Suggestions for future work:

- Generalizations (different network structures, range of interactions, budget constraints).
- Empirical studies.

Thank you for your attention!

Questions?

# Mean field approximation

Ordinary differential equations for strategy ratios ($\zeta^1, \zeta^2, \zeta^3$):

$$\dot{\zeta}^k(t) = -p^k(t)\zeta^k(t) + \sum_{h=1}^{3} \left( \xi_2 \zeta^k(t) + \frac{1}{3}(1 - \xi_2) \right) p^h(t)\zeta^h(t) \tag{1}$$

Revision probability:

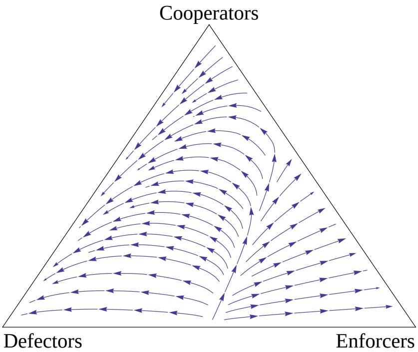
$$p^k(t) = \lambda e^{-\lambda y^k(t)} \tag{2}$$

Relative payoffs:

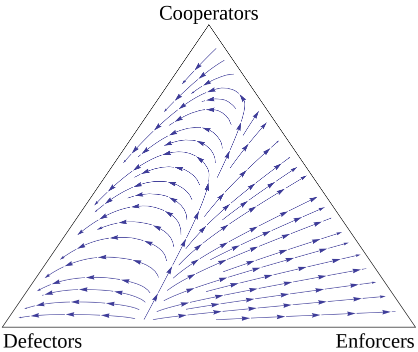
$$y^k(t) = 2 \cdot \frac{\pi^k(t) - \pi_{\min}(t)}{\pi_{\max}(t) - \pi_{\min}(t)} \tag{3}$$

where the payoffs $\left\{\pi^k\right\}_{k=1,2,3}$ depend explicitly on $\left\{\zeta^k\right\}_{k=1,2,3}$.

# Solutions of approximate ODE

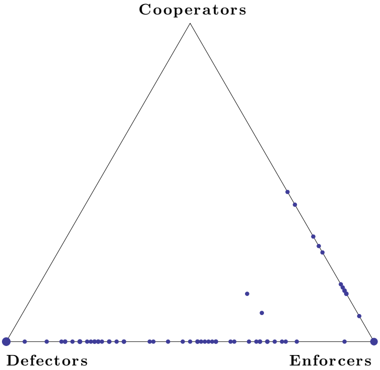



$\theta = 3,\ \xi_2 = 1$

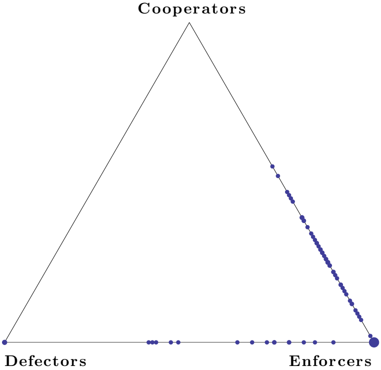



$\theta = 5,\ \xi_2 = 1$

# Simulation outcomes on a simplex

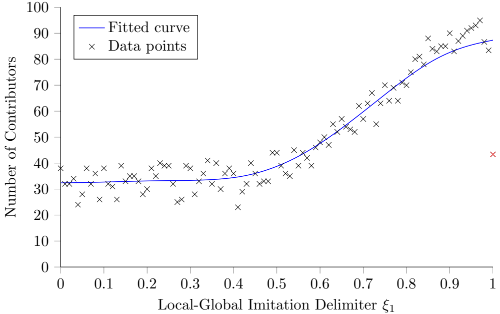


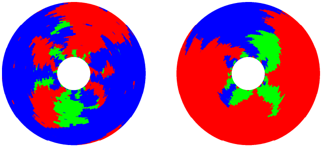

$\theta = 3.0$




$\theta = 5$

# Parameter study of $\xi_1$



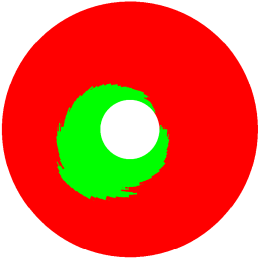



$\xi_1 = 0.97$ $\xi_1 = 1$

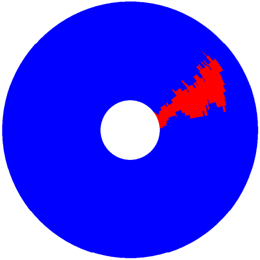
Two qualitative examples.

A very small amount of global imitation ($\xi_1 < 1$) breaks local structures and benefits enforcers.

# Stability



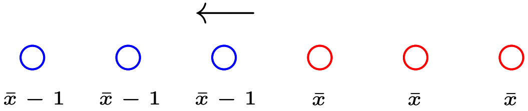
Red: enforcers, green: cooperators



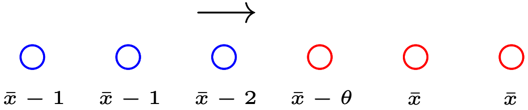
Blue: enforcers, red: defectors

# Punishment situation

Defector not detected:




Defector detected and punished:



# Research question – forgotten or intentionally excluded?

Structure of the report reflects the line of work.

Use model to study interesting phenomena *in the model*, not to answer specific questions about reality.

A predefined hypothesis could bias the construction of the model.

# References I

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Fehr, E. and Gächter, S. (2000), “Fairness and retaliation: The economics of reciprocity”, *Journal of Economic Perspectives* , Vol. 14, pp. 159–181.

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