AI1110
Assignment-4

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Outline

1. Question
2. Solution

Question

Papoullis 4-11:

The space S contains of all points t_i in the interval (0,1) and $P\{0 \leq t_i \leq y\} = y$ for every$y \leq 1$. The function G(x) is increasing from $G(-\infty) = 0$ to $G(\infty) = 1$; hence it has an inverse $G^{-1}(y) = H(y)$. The random variable$X(t_i) = H(t_i)$. Show that $F_X(x) = G(x)$

Solution

From the given information;
$t_i(0, 1)$
$P\{0 \leq t_i \leq y\} = y$
$X(t_i) = H(t_i)$
$H(x) = G^{-1}(x)$

$$F_X(x) = P\{X(t_i) \leq x\} \quad (1)$$
$$= P\{H(t_i) \leq x\} \quad (2)$$
$$(3)$$

$$X(t_i) \leq x \tag{4}$$

$$H(t_i) \leq x \tag{5}$$

$$t_i \leq H^{-1}(x) \tag{6}$$

$$t_i \leq G(x) \tag{7}$$

$$\tag{8}$$

$$P\{X(t_i) \leq x\} = P\{t_i \leq G(x)\} \tag{9}$$

$$\tag{10}$$

As $0 \leq G(x) \leq 1$

$$P\{t_i \leq G(x)\} = P\{0 \leq t_i \leq G(x)\} \tag{11}$$
$$= G(x) \tag{12}$$
$$\tag{13}$$

$P\{X(t_i) \leq x\} = G(x) \because eq(12)$
$\therefore \mathrm{F}_X(x) = G(x)$