AI1110
Assignment-5

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Outline

Question

Papoullis EX 7-1:

Given independent random variables x_i with respective densities $f_i(x_i)$, we form random variables $y_k = x_1 + x_2 + x_3 + ... + x_k$. Then show that random variables y_i are also independent

Solution

The system

$y_1 = x_1$

$y_2 = x_1 + x_2$

$y_3 = x_1 + x_2 + x_3$

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$y_n = x_1 + x_2 + x_3 + ... + x_n$

has a unique solution

$x_k = y_k - y_{k-1})$

its jacobian transformation is

$$J(x_1 x_2 ... x_n) = \begin{vmatrix} \frac{\partial g_1}{\partial x_1} & .. & \frac{\partial g_1}{\partial x_n} \\ . & .... & . \\ . & .... & . \\ \frac{\partial g_n}{\partial x_1} & .. & \frac{\partial g_n}{\partial x_n} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & .. & 0 & 0 \\ 0 & 1 & .. & 0 & 0 \\ . & . & .. & . & . \\ 0 & 0 & .. & 1 & 0 \\ 0 & 0 & .. & 0 & 1 \end{vmatrix} = 1$$

then the joint density of y_k *is*

$$
\begin{aligned}
f(y_1 y_2..y_n) &= \frac{f(x_1 x_2..x_n)}{J} && (1)\\
&= f(x_1 x_2..x_n) && (2)\\
&= f(x_1)f(x_2)...f(x_n) && (3)\\
&= f(y_2 - y_1)f(y_3 - y_2)...f(y_n - y_{n-1}) && (4)
\end{aligned}
$$

It follows that any subset of the set x_i is a set of independent random variables.
$\{y_1 \leq y_1\} = \{x_1 \in A_1\}, .., \{y_n \leq y_n\} = \{x_n \in A_n\}$

$$
\begin{aligned}
f(y_1 \leq y_1 \cap y_2 \leq y_2.. \cap y_n \leq y_n) &= f(x_1 \in A_1 \cap x_2 \in A_2.. \cap x_n \in A_N) && (5)\\
&= f(x_1 \in A_1)f(x_2 \in A_2)...f(x_n \in A_n) && (6)\\
&= f(y_1 \leq y_1)f(y_2 \leq y_2)...f(y_n \leq y_n) && (7)
\end{aligned}
$$

$\therefore$ If the random variables x_i are independent, then the random variables
$y_1 = g(x_1), y_2 = g(x_2), .., y_n = g(x_n)$
are also independent