

AI1110

Assignment-3

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May 31, 2022

Outline

- 1 Question
- 2 Definitions
- 3 Solution

Question

Papoullis 4-10:

If x is $N(0,2)$ find (a) $P\{1 \leq x \leq 2\}$ and (b) $P\{1 \leq X \leq 2 | x \geq 1\}$

Definitions

x is $N(\mu, \sigma^2)$ will be used to represent the Gaussian p.d.f

Normal(Gaussian) Distribution Function

If $f_X(x)$ is probability density function of a random variable X , then its corresponding distribution function $F_X(x)$ is

$$F_X(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-\mu)^2}{2\sigma^2}} dy = G\left(\frac{x-\mu}{\sigma}\right) \quad (1)$$

Solution

From the given information; $N(0,2)$

so,

$$\mu = 0$$

,

$$\sigma = \sqrt{2}$$

(a)

$$P\{1 \leq x \leq 2\} = P\{x \leq 2\} - P\{x \leq 1\} \quad (2)$$

$$= F_x(2) - F_x(1) \quad (3)$$

$$= G\left(\frac{2-\mu}{\sigma}\right) - G\left(\frac{1-\mu}{\sigma}\right) \quad (4)$$

$$= G\left(\frac{2-0}{\sqrt{2}}\right) - G\left(\frac{1-0}{\sqrt{2}}\right) \quad (5)$$

$$= \int_{-\infty}^{\sqrt{2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy - \int_{-\infty}^{\frac{1}{\sqrt{2}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \quad (6)$$

$$= 0.41924 - 0.25804 \quad (7)$$

$$= 0.1612 \quad (8)$$

$$(9)$$

(b)

$$P\{1 \leq X \leq 2 | x \geq 1\} = \frac{P\{1 \leq x \leq 2\}}{P\{x \geq 1\}} \quad (10)$$

$$= \frac{P\{x \leq 2\} - P\{x \leq 1\}}{P\{x \geq 1\}} \quad (11)$$

$$= \frac{F_x(2) - F_x(1)}{1 - F_x(1)} \quad (12)$$

$$= \frac{G\left(\frac{2-0}{\sqrt{2}}\right) - G\left(\frac{1-0}{\sqrt{2}}\right)}{1 - G\left(\frac{1-0}{\sqrt{2}}\right)} \quad (13)$$

$$= \frac{0.41924 - 0.25804}{1 - 0.75804} \quad (14)$$

$$= 0.67 \quad (15)$$