1.4 300

Andre Monteiro

Term 3, 2025

Problem Summary

We have a classroom of 300 students arranged in a 15×20 grid. Define:

- A = tallest among the shortest students in each column
- B = shortest among the tallest students in each row

We aim to compare A and B, and illustrate the pattern using smaller sample matrices.

Sample Matrix 1

$$A = \begin{bmatrix} 2 & 5 & 3 \\ 4 & 1 & 6 \\ 1 & 3 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 5 & 3 \\ 4 & 1 & 6 \\ 1 & 3 & 2 \end{bmatrix}$$

Solution

- In the first matrix, $A = 2$.
- In the second matrix, $B = 3$.

$$\boxed{A < B}$$

Sample Matrix 2

$$A = \begin{bmatrix} 7 & 4 & 5 \\ 3 & 6 & 2 \\ 1 & 5 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 7 & 4 & 5 \\ 3 & 6 & 2 \\ 1 & 5 & 6 \end{bmatrix}$$

Solution

- In the first matrix, $A = 4$.
- In the second matrix, $B = 6$.

$$\boxed{A < B}$$

Proof

- For each column, we found the shortest student. Then A is the tallest among these. So A is bigger than or equal to any of the column minima.
- For each row, we found the tallest student. Then B is the shortest among these. So B is smaller than or equal to any of the row maxima.

Now, look at any student in the grid. Each student is **at least as tall as the shortest in their column** and **at most as tall as the tallest in their row**.

This means the tallest of all column minima (A) can never be taller than the shortest of all row maxima (B), and since we also apply the restriction:

$$\boxed{A \neq B}$$

We can therefore state that no matter how the students are arranged:

$$\boxed{A < B, (A \neq B)}$$

The previous examples make this clear in practice: green (A) is always less than red (B).