1.4 RGB

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Algorithm

Reduce the problem to an ordinary shortest path instance. First filter the graph so that it only contains edges that join vertices of *different* colours, then a plain BFS gives the required path.

(a) Build a new graph $G' = (V, E')$ with

$$E' = \{\{x, y\} \in E : \text{colour}(x) \neq \text{colour}(y)\}.$$

Checking each original edge once costs $O(|E|)$.

(b) Run a standard BFS from u to v on G' and output the path it finds.

$$\text{Time complexity of BFS: } O(|V| + |E|)$$

Complexity

Total time is

$$O(|E|) + O(|V| + |E'|) = O(|V| + |E| + |E'|).$$

Since $E' \subseteq E$, we have $|E'| \leq |E|$, so

$$O(|V| + |E| + |E'|) = O(|V| + 2|E|) = O(|V| + |E|).$$

Conclusion

By reducing the graph first and then calling BFS, the shortest colour alternating path is found in:

$$\boxed{O(|V| + |E|)}$$