Chores

This is a **regular task**. You must submit a PDF, which can be produced using the LaTeX template on Moodle, exported from a word processor, hand-written or any other method.

You are given n chores to complete over the next n days. Each chore takes one day to complete, and only one chore can be done each day. Your parents have kindly agreed to pay you $p_i > 0$ cents if you finish the ith chore at or before the end of the corresponding due date $t_i \geq 1$. You do not receive payment if you complete the ith chore after day t_i.

For example, suppose that we have the chores specified in the table below. Doing chore X on either of the first two days earns you 20 cents, but doing chore X on day 3 earns you no money. The best solution is to do chores Z and Y on days 1 and 2 respectively, for a total of 80 cents.

i	X	Y	Z
p_i	20	50	30
t_i	2	2	1

Suppose we have a greedy algorithm that finds the optimal solution, that is, a way of scheduling the chores that maximises the total profit.

Consider the following input.

i	U	V	W	X	Y	Z
p_i	40	20	30	50	60	40
t_i	1	2	3	3	6	6

On this input, our algorithm produces the following schedule of chores.

Day	1	2	3	4	5	6
Chore (i)	U	W	X	·	Z	Y

In this case we have

$$\text{Total Profit} = 40 + 30 + 50 + 40 + 60 = 220.$$

(a) Determine the greedy heuristic used by the algorithm applied above.

(b) **(Optional)** Design and analyse an $O(n^2)$ algorithm[a] which uses the greedy heuristic from part **??** to schedule your chores to maximise your total income.

[a] This can be improved further to $O(n \log n)$.

Advice.

(a) There are multiple correct answers.

Expected length: Up to a few sentences.

Solution.

Attribution.