<u>**Logic Coursework**</u>

1) i)
Formalising the argument, we get:

1. $P \leftrightarrow \neg(O \wedge P)$	given	
2. $(\neg O \rightarrow P) \rightarrow DDI$	given	premises
3. $(O \wedge R \rightarrow \neg P) \rightarrow DAI$	given	

$DDI \wedge DAI$		conclusion

ii)
We want to show that the argument is valid.

Let us first prove the following lemma : $\neg P \leftrightarrow (O \wedge R)$

We have:

1. $P \leftrightarrow \neg(O \wedge P)$	given
2. $P \rightarrow \neg(O \wedge P)$	$\leftrightarrow$ E ,1
3. $(O \wedge R) \rightarrow \neg P$	contraposition, 2
4. $\neg(O \wedge R) \rightarrow P$	$\leftrightarrow$ E ,1
5. $\neg P \rightarrow O \wedge R$	contraposition, 4
6. $\neg P \leftrightarrow O \wedge R$	$\leftrightarrow$ I,5,3

So we get : lemma1 : $\neg P \leftrightarrow O\ R$
So now, using the premises and lemma1, we can derive the conclusion :

1. $P \leftrightarrow \neg(O \wedge P)$	given
2. $\neg P \leftrightarrow O \wedge R$	lemma1
3. $(\neg O \rightarrow P) \rightarrow DDI$	given
4. $(O \wedge R \rightarrow \neg P) \rightarrow DAI$	given
5. $O \wedge R \rightarrow \neg P$	$\leftrightarrow$ E ,2
6. DAI	$\rightarrow$ E,4,5
~~7.~~ $\neg O$	**Assume**
~~8.~~ $\neg P$	**Assume**
9. $\neg P \rightarrow O \wedge R$	$\leftrightarrow$ E ,2
10. $O \wedge R$	$\rightarrow$ E,8,9
11. O	$\wedge$E,10
12. P	RAA,11,7
13. $\neg O \rightarrow P$	$\rightarrow$ I,7,12
14. DDI	$\rightarrow$ E,13,3
15. $DDI \wedge DAI$	$\wedge$ I,6,14

The argument is valid.

2)i)
In terms of the standard connectives, the definition of @(P,Q,R) is:

$(P \rightarrow Q) \wedge (\neg P \rightarrow R)$

ii) We want to show : @(A, B, C), @(A, ¬B, C) |- C

We have that @(A, B, C) and @(A, ¬B, C) are given, so replacing by their definition we get:

1. $(A \rightarrow B) \wedge (\neg A \rightarrow C)$	given
2. $(A \rightarrow \neg B) \wedge (\neg A \rightarrow C)$	given
3. $\neg A \rightarrow C$	$\wedge$E,1
~~4.~~ A	**Assume**
5. $A \rightarrow B$	$\wedge$E,1
6. $A \rightarrow \neg B$	$\wedge$E,2
7. B	$\rightarrow$ E,4,5
8. $\neg B$	$\rightarrow$ E,4,6
9. $\neg A$	RAA,7,8
10. C	$\rightarrow$ E,3,9

Hence : @(A, B, C), @(A, ¬B, C) |- C .
Q.E.D

3)i)

a. $\forall X(\text{length}(X,0) \leftrightarrow \forall Y\ \neg\text{member}(Y,X))$

b. $\forall$ L1, L2, L3 (append(L1,L2,L3) $\rightarrow$
$\forall$X(member(X,L3) $\rightarrow$ (member(X,L1) $\vee$ member(X,L2))))

c. $\forall$ L,X (length(L,X) $\rightarrow$ (X=0 $\vee$ X>0))

d. $\forall$ L1, L2 (same_elts(L1,L2) $\leftrightarrow$
$\forall$ X (member(X,L1) $\leftrightarrow$ member(X,L2)))

e. $\exists$L length(L,0)

ii)
Take :
X= [1,2]
Y=[3,4]
Z=[1,1,2,3,4]

Then, for the above example :
The left hand of the principal implication holds while the right hand does not .

Hence the statement is false.