Nash Equilibrium

Definition

- $u^i(s^i, s^{-i}) \geq u^i(t^i, s^{-i})$ for all $t^i \in S^i$
- A combination of strategies such that every player is best-responding to every other player
- Can be pure or mixed, unique or not
- A pure strategy NE is a in A such that $u^i(a^i, a^{-i}) \geq u^i(b^i, a^{-i})$ for all $a^i \in A^i$

Some properties

- **Exists in any finite normal form game** (John Nash Nobel Pprize, 1994)

- If a^i is played with positive probability in some Nash Equilibrium, then it is rationalizable. Therefore, it survives IESDA

- Ex: if a unique action profile survives IESDA (like in the beauty contest game), then it is NE, since we know a NE exists

- This means, we can always make it easier to find NE actions by first reducing the set of available actions using IESDA

NE in experiments

- Why should we expect NE to emerge in experiments?
  - Introspection (thinking about the game)
  - Learning, especially in games with unique pure strategy Nash (beauty contest, Prisoner’s dilemma)

- Experiments typically conducted using one of the following protocols:
  - One-shot (game played once and only once, as in Agranov, et al (2012)
  - Strangers (random re-matching, as in the beginning of class today)
    - If no feedback, learning based on introspection
  - Partners (same partner held fixed over multiple rounds)
    - Dynamic game, will discuss strategic properties later

Mixed strategies

- Consider a two player game
- In a mixed strategy NE, the mixing probabilities of player i are determined by the **other** player's payoffs
- E.g., 1 mixes to make player 2 indifferent, which means 2's payoffs affect 1's mixing probabilities
- (Why must players be indifferent between their actions? If not indifferent, mixing is not a best response!)
- Lettting $u^1(T, L)=x$, the mixing probabilities are (½ , ½) for player 1 and (40/x, 1-40/x) for player 2

Learning doesn't help!

- Ochs (1995):

Player Two

		A	B
Player One	A	(1,0)	(0,1)
	B	(0,1)	(1,0)

Game One

	A	B
A	(9,0)	(0,1)
B	(0,1)	(1,0)

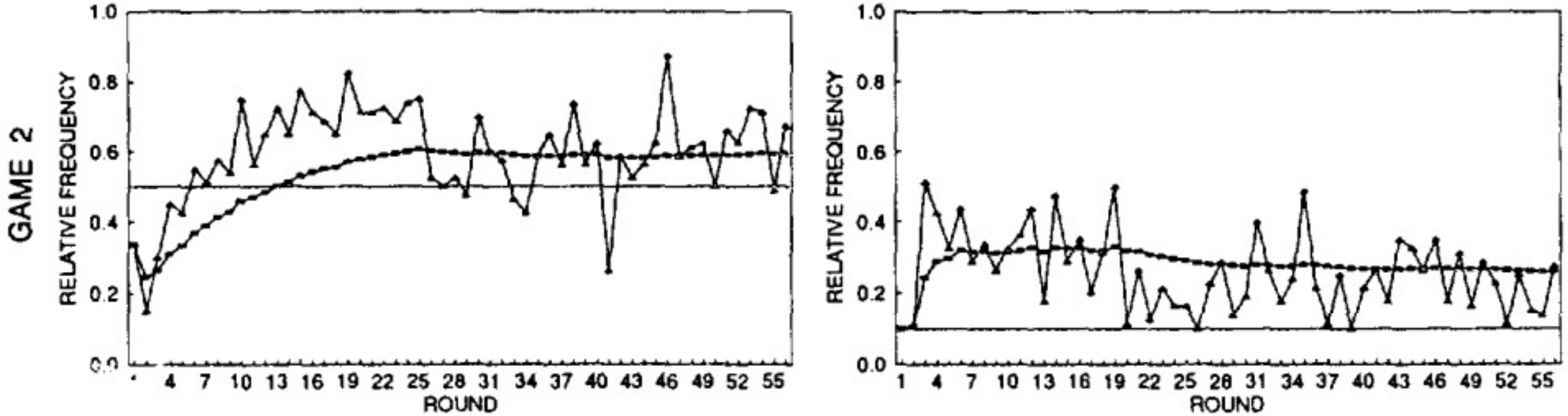
Game Two

	A	B
A	(4,0)	(0,1)
B	(0,1)	(1,0)

Game Three

FIGURE 1

- Relative freq. of choosing A over B for Game 2 (16 subjects with “stranger" matching, P1 on the left, P2 on the right):



Games with multiple equilibria

- Coordination games
- Classic example: stag hunt

	Stag	Hare
Stag	13, 13	-7, 10
Hare	10, -7	3, 3

- One equilibrium payoff-dominant, the other risk-dominant
  - A **payoff-dominant** equilibrium is Pareto superior to all other equilibria (A is Pareto-superior to B if everyone weakly prefers it and at least one player strictly prefers it)
  - In a symmetric 2x2 game, a **risk-dominant** equilibrium is an equilibrium where each player best responds to a uniform prior
    - Equivalently, an equilibrium that has the largest basin of attraction

Basin of attraction

- In a symmetric 2x2 game, basin of attraction of B is the largest probability of the other player playing A that makes the player in different between A and B

- Stag hunt example

- Dal Bó, P., Fréchette, G. R., & Kim, J. (2021). The determinants of efficient behavior in coordination games. Games and Economic Behavior, 130, 352-368.

Van Huyck, et al (1990)

- This paper studies games with payoffs as follows (sometimes called minimum effort or weakest link games):

$$U(e^i,e^{-i})=a*\min(e^1, e^2, \ldots, e^N)-b*e^i,\ a>b>0$$

- Any combination of equal effort levels is a pure straregy Nash equilibrium (why?)

- **Payoff-dominant** equilibrium has everyone choosing the highest effort level
- Secure (maxmin) equilibrium has everyone choosing the lowest effort level

Novelty of the paper

- One of the first to study equilibrium selection using experimental methods
    - If theory is silent on what equilibrium should be played, experiments can be used to give us an idea

- The first paper to do this using the weakest link game

- Most closely related paper: Cooper, et al (1987) use experiments to study equilibrium selection in matrix games

Strategic uncertainty

- What if players don't know what other players are doing?
  - Strategic uncertainty = uncertainty about the behavior of others

- Easy to show that the cdf of the minimum satisfies $F(x) – (1-G(x))^{n-1}$, where $G(x)$ is the cdf of of each player (see class notes)

- Therefore, even the probability of each player choosing 1 is small ($G(1)$ is close to zero), $F(1)$ converges to 1 as n increases

Payoffs

PAYOFF TABLE *A*

		Smallest Value of X Chosen						
		7	6	5	4	3	2	1
Your	7	1.30	1.10	0.90	0.70	0.50	0.30	0.10
Choice	6	–	1.20	1.00	0.80	0.60	0.40	0.20
of	5	–	–	1.10	0.90	0.70	0.50	0.30
X	4	–	–	–	1.00	0.80	0.60	0.40
	3	–	–	–	–	0.90	0.70	0.50
	2	–	–	–	–	–	0.80	0.60
	1	–	–	–	–	–	–	0.70

PAYOFF TABLE *B*

		Smallest Value of X Chosen						
		7	6	5	4	3	2	1
Your	7	1.30	1.20	1.10	1.00	0.90	0.80	0.70
Choice	6	–	1.20	1.10	1.00	0.90	0.80	0.70
of	5	–	–	1.10	1.00	0.90	0.80	0.70
X	4	–	–	–	1.00	0.90	0.80	0.70
X	3	–	–	–	–	0.90	0.80	0.70
	2	–	–	–	–	–	0.80	0.70
	1	–	–	–	–	–	–	0.70

Experimental design

PAYOFF TABLE *A*

		Smallest Value of X Chosen						
		7	6	5	4	3	2	1
Your	7	1.30	1.10	0.90	0.70	0.50	0.30	0.10
Choice	6	–	1.20	1.00	0.80	0.60	0.40	0.20
of	5	–	–	1.10	0.90	0.70	0.50	0.30
X	4	–	–	–	1.00	0.80	0.60	0.40
	3	–	–	–	–	0.90	0.70	0.50
	2	–	–	–	–	–	0.80	0.60
	1	–	–	–	–	–	–	0.70

PAYOFF TABLE *B*

		Smallest Value of X Chosen						
		7	6	5	4	3	2	1
Your	7	1.30	1.20	1.10	1.00	0.90	0.80	0.70
Choice	6	–	1.20	1.10	1.00	0.90	0.80	0.70
of	5	–	–	1.10	1.00	0.90	0.80	0.70
X	4	–	–	–	1.00	0.90	0.80	0.70
X	3	–	–	–	–	0.90	0.80	0.70
	2	–	–	–	–	–	0.80	0.70
	1	–	–	–	–	–	–	0.70

Results, Treatment A

- Period 1: neither payoff-dominant nor secure action predict behavior well , lack of coordination
- Convergence to 1

TABLE 2—EXPERIMENTAL RESULTS FOR TREATMENT *A*, Continued

	Period									
	1	2	3	4	5	6	7	8	9	10
Experiment 5										
No. of 7's	2	2	3	1	1	1	1	0	0	0
No. of 6's	1	3	1	0	0	0	0	0	0	0
No. of 5's	9	3	0	4	1	0	2	0	0	0
No. of 4's	3	4	6	2	1	2	0	2	1	1
No. of 3's	1	2	2	4	6	0	0	0	0	1
No. of 2's	0	2	2	3	4	6	5	2	5	3
No. of 1's	0	0	2	2	3	7	8	12	10	11
Minimum	3	2	1	1	1	1	1	1	1	1
Experiment 6										
No. of 7's	5	3	1	1	1	1	2	2	2	3
No. of 6's	2	0	0	0	1	0	0	0	0	0
No. of 5's	5	1	0	0	0	1	0	0	0	0
No. of 4's	2	3	4	0	0	0	0	0	0	0
No. of 3's	1	5	4	2	2	2	1	0	2	0
No. of 2's	0	2	4	5	3	3	6	4	5	5
No. of 1's	1	2	3	8	9	9	7	10	7	8
Minimum	1	1	1	1	1	1	1	1	1	1
Experiment 7										
No. of 7's	4	3	1	1	1	1	1	1	1	1
No. of 6's	1	0	0	0	0	0	0	0	0	0
No. of 5's	2	3	0	0	0	0	0	0	0	0
No. of 4's	4	0	1	2	1	0	0	0	0	0
No. of 3's	1	3	2	1	1	0	0	0	0	0
No. of 2's	1	3	2	2	4	4	4	4	5	3
No. of 1's	1	2	8	8	7	9	9	9	8	10
Minimum	1	1	1	1	1	1	1	1	1	1

Results, Treatments B and A’

- Subjects coordinate on the efficient action in Treatment B but return to the minimum in Treatment A’
- By period 20, 84% choose the secure action in Treatment A’

TABLE 3—EXPERIMENTAL RESULTS FOR TREATMENT *B* AND TREATMENT *A′*

	Treatment *B*					Treatment *A′*				
	11	12	13	14	15	16	17	18	19	20
Experiment 2										
No. of 7’s	13	15	16	16	16	8	2	0	0	0
No. of 6’s	1	0	0	0	0	0	0	0	0	0
No. of 5’s	0	1	0	0	0	1	0	0	0	0
No. of 4’s	1	0	0	0	0	1	2	0	0	0
No. of 3’s	1	0	0	0	0	1	1	1	1	0
No. of 2’s	0	0	0	0	0	3	3	4	2	0
No. of 1’s	0	0	0	0	0	2	8	11	13	16
Minimum	3	5	7*	7*	7*	1	1	1	1	1*
Experiment 3										
No. of 7’s	13	13	12	13	14	6	2	2	1	1
No. of 6’s	0	0	1	1	0	1	0	0	0	0
No. of 5’s	0	0	1	0	0	0	2	1	0	0
No. of 4’s	1	0	0	0	0	1	0	0	0	1
No. of 3’s	0	1	0	0	0	0	0	0	0	0
No. of 2’s	0	0	0	0	0	2	4	2	3	0
No. of 1’s	0	0	0	0	0	4	6	9	10	12
Minimum	4	3	5	6	7*	1	1	1	1	1

Treatment A with monitoring

TABLE 3—EXPERIMENTAL RESULTS FOR TREATMENT *B* AND TREATMENT *A′*

	Treatment *B*					Treatment *A′*				
	11	12	13	14	15	16	17	18	19	20
Experiment 2										
No. of 7's	13	15	16	16	16	8	2	0	0	0
No. of 6's	1	0	0	0	0	0	0	0	0	0
No. of 5's	0	1	0	0	0	1	0	0	0	0
No. of 4's	1	0	0	0	0	1	2	0	0	0
No. of 3's	1	0	0	0	0	1	1	1	1	0
No. of 2's	0	0	0	0	0	3	3	4	2	0
No. of 1's	0	0	0	0	0	2	8	11	13	16
Minimum	3	5	7*	7*	7*	1	1	1	1	1*
Experiment 3										
No. of 7's	13	13	12	13	14	6	2	2	1	1
No. of 6's	0	0	1	1	0	1	0	0	0	0
No. of 5's	0	0	1	0	0	0	2	1	0	0
No. of 4's	1	0	0	0	0	1	0	0	0	1
No. of 3's	0	1	0	0	0	0	0	0	0	0
No. of 2's	0	0	0	0	0	2	4	2	3	0
No. of 1's	0	0	0	0	0	4	6	9	10	12
Minimum	4	3	5	6	7*	1	1	1	1	1

Treatment C

TABLE 4—FIXED PAIRINGS, Continued

	Period						
	21	22	23	24	25	26	27
Pair 5							
Subject 6	4	5	7	7	7	7	7
Subject 11	4	5	7	7	7	7	7
Minimum	4*	5*	7*	7*	7*	7*	7*
Pair 6							
Subject 7	5	7	7	7	7	7	7
Subject 10	5	7	7	7	7	7	7
Minimum	5*	7*	7*	7*	7*	7*	7*

* ~ Denotes a mutual best-response outcome.

TABLE 5—DISTRIBUTION OF ACTIONS FOR TREATMENT C: RANDOM PAIRINGS

	Period				
	21	22	23	24	25
Experiment 6					
No. of 7's	5	5	4	10	8
No. of 6's	0	1	3	0	0
No. of 5's	2	5	3	3	4
No. of 4's	3	1	1	1	1
No. of 3's	1	1	1	0	0
No. of 2's	1	1	2	2	2
No. of 1's	4	2	2	0	1
Experiment 7					
No. of 7's	–	–	6	5	5
No. of 6's	–	–	1	0	1
No. of 5's	–	–	0	3	0
No. of 4's	–	–	2	1	4
No. of 3's	–	–	2	0	0
No. of 2's	–	–	0	0	1
No. of 1's	–	–	3	5	3

Summary

- Payoff-dominance does not seem to predict behaviror (consistent with other behavior)

- Results consistent with strategic uncertainty and security

Overcoming coordination failure

- Many experiments show coordination failure (stag hunt, weakest link)
- How can we influence subjects to converge to efficient equilibrium?
- One common intervention: **communication**
- Example: Blume and Ortmann (2007)

Results without (left) and with (right) communication