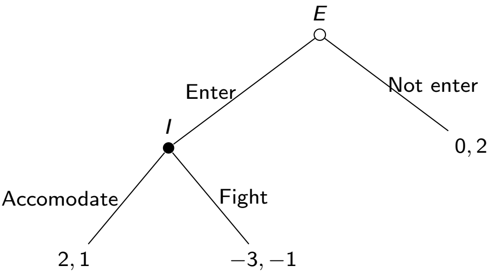
Dynamic Games

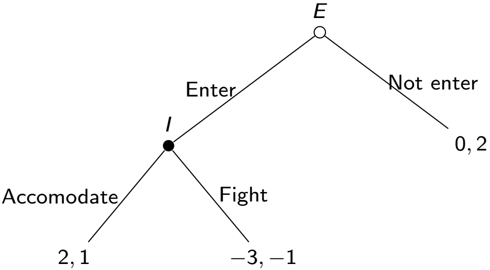
January 26, 2022

- **Dynamic (extensive form) games** can be represented using game trees



- **Dynamic (extensive form) games** can be represented using game trees

- How do we look for a Nash Equilibrium of this game?



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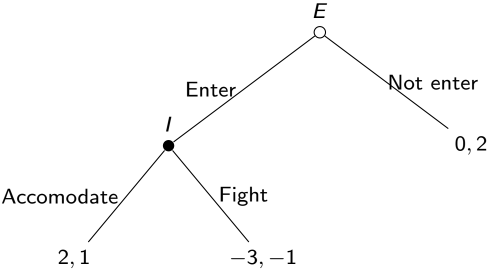


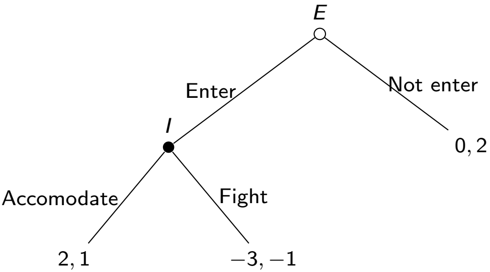
- How do we look for a Nash Equilibrium of this game?

- First, turn it into a normal form game:

	Accomodate	Fight
Enter	2, 1	-3, -1
Not enter	0, 2	0, 2

IMPORTANT: A **pure strategy** is a complete contingent plan

- Two pure-strategy Nash equilibria (blue)



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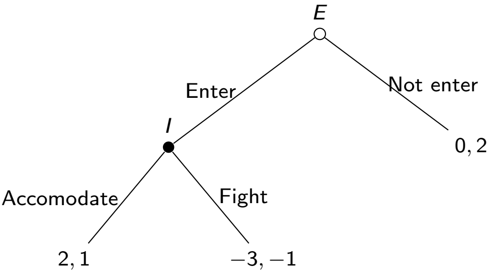
- Two pure-strategy Nash equilibria (blue)
- What's the problem with the (Not Enter, Fight) equilibrium?



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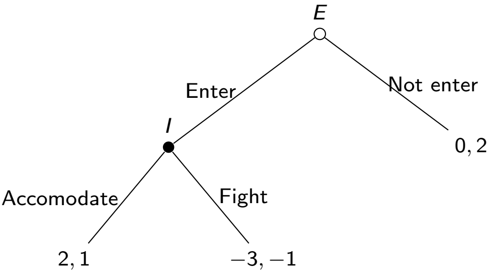
- Two pure-strategy Nash equilibria (blue)
- What's the problem with the (Not Enter, Fight) equilibrium?
- It involves a threat that is **not credible**
- If entrant opts out, incumbent does not need to make a decision. But if he *did* have to make one, he would never choose Fight



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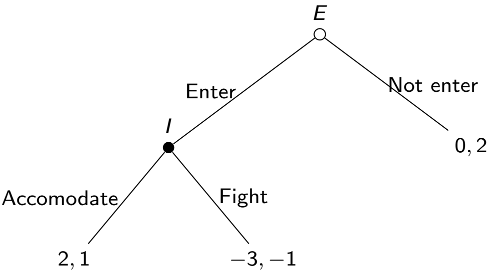
- As game theorists, we want to rule out the equilibrium that uses a non-credible threat



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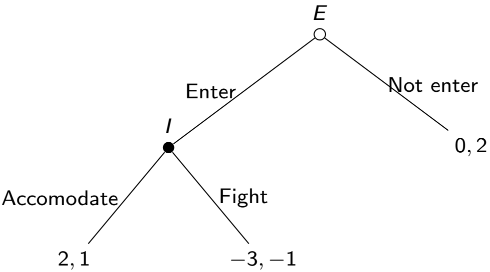
- As game theorists, we want to rule out the equilibrium that uses a non-credible threat
- This motivates the idea of **subgame perfection**



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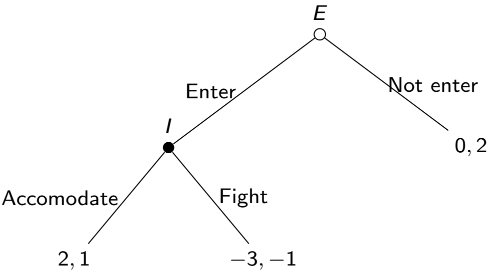
- As game theorists, we want to rule out the equilibrium that uses a non-credible threat

- This motivates the idea of **subgame perfection**

- A Nash Equilibrium is **subgame perfect** if it induces a Nash equilibrium for any **subgame**

- In a complete information game, a **subgame** is a game beginning at any non-terminal node



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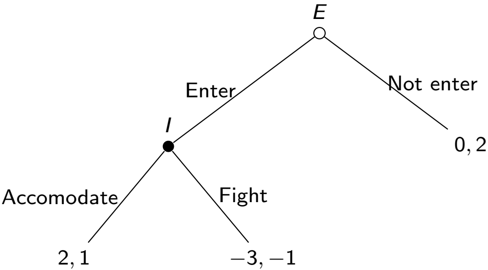
- In our example, the game has two subgames:



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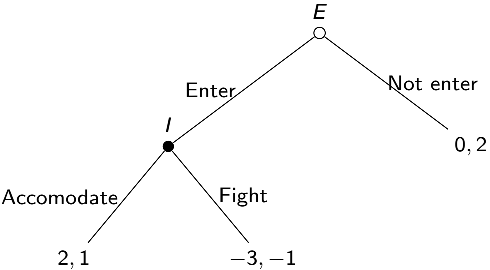
- In our example, the game has two subgames:

- The game beginning at the initial node (EFG is always a subgame of itself)



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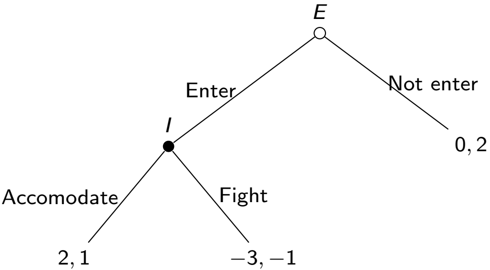
- In our example, the game has two subgames:
- The game beginning at the initial node (EFG is always a subgame of itself)
- The game beginning at the incumbent's decision node



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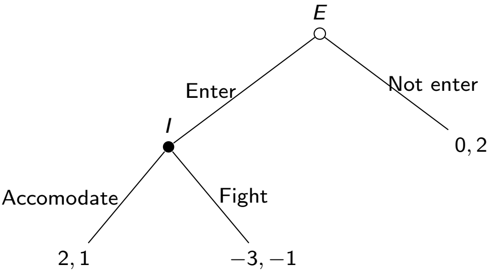
- (In, Accomodate) induces a Nash Equilibrium for both subgames
- (Out, Fight) does not
- Therefore, (Out, Fight) is not **subgame perfect**
- We succeeded in introducing an equilibrium concept that rules out non-credible threats being played



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Backward induction

The **backward induction** procedure can be used to find the subgame perfect Nash equilibrium:

1. Look at immediate predecessors of the final nodes.
2. Each such node has a player controlling it. Choose the action that gives him the largest payoff (break ties arbitrarily).
3. Replace the node with a final node having utility for each player equal to the utility induced by the action chosen in Step 2.
4. Repeated the procedure in steps 1-3 in the new game until only one node is left.

Race game

Gneezy, Rustichini, and Vostroknutov (2010):

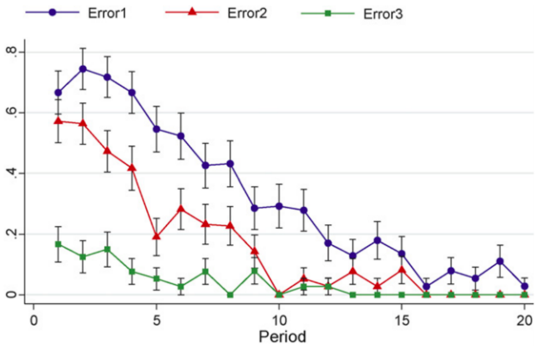

Fig. 1. Average error per round, $G(15, 3)$.

Ultimatum game

- Game played between a proposer and a responder
- Proposer endowed with some amount, e.g. \$10
- Decides amount $x \in \{0.01, 0.02, ..., 9.98, 9.99, 10\}$ to offer to the responder
- If responder accepts, proposer gets $10 - x$ and responder gets x
- If responder rejects, no one gets anything