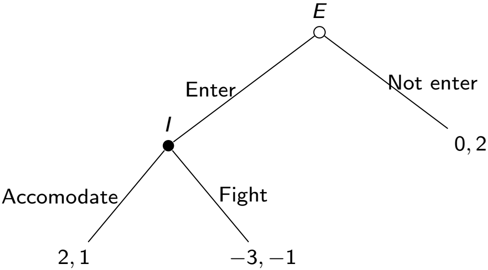
Dynamic Games

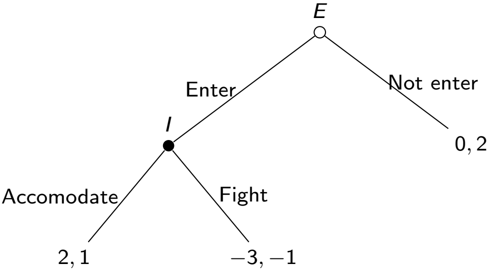
February 2, 2022

- **Dynamic (extensive form) games** can be represented using game trees

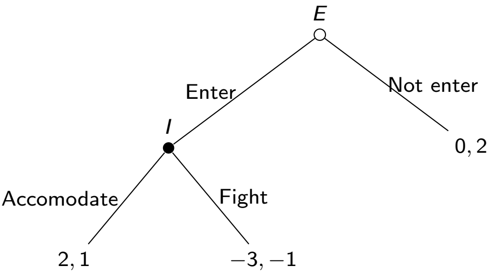


- **Dynamic (extensive form) games** can be represented using game trees

- How do we look for a Nash Equilibrium of this game?



- **Dynamic (extensive form) games** can be represented using game trees



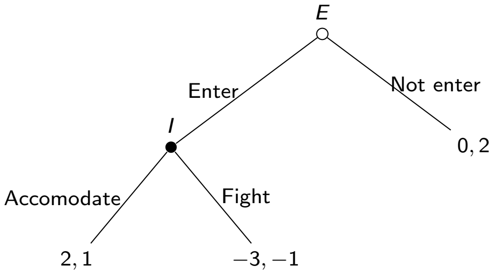
- How do we look for a Nash Equilibrium of this game?

- First, turn it into a normal form game:

	Accomodate	Fight
Enter	2, 1	-3, -1
Not enter	0, 2	0, 2

IMPORTANT: A **pure strategy** is a complete contingent plan

- Two pure-strategy Nash equilibria (blue)



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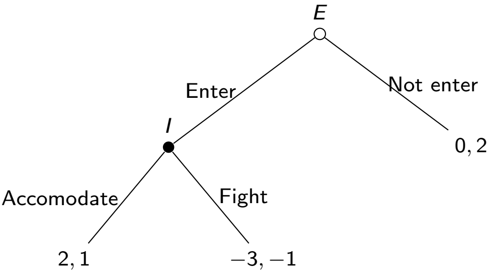
- Two pure-strategy Nash equilibria (blue)
- What's the problem with the (Not Enter, Fight) equilibrium?



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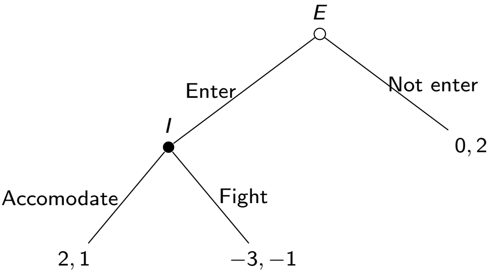
- Two pure-strategy Nash equilibria (blue)
- What's the problem with the (Not Enter, Fight) equilibrium?
- It involves a threat that is **not credible**
- If entrant opts out, incumbent does not need to make a decision. But if he *did* have to make one, he would never choose Fight



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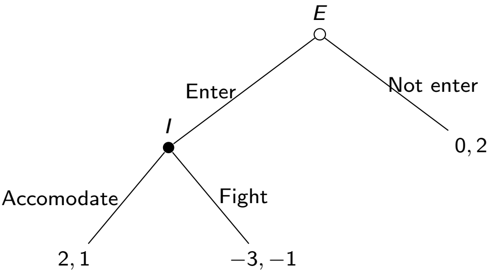
- As game theorists, we want to rule out the equilibrium that uses a non-credible threat



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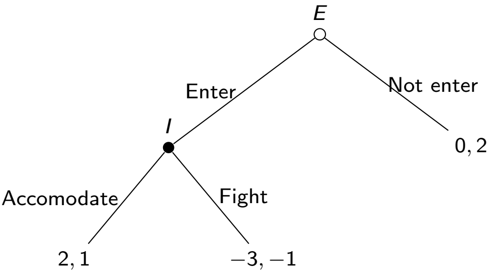
- As game theorists, we want to rule out the equilibrium that uses a non-credible threat
- This motivates the idea of **subgame perfection**



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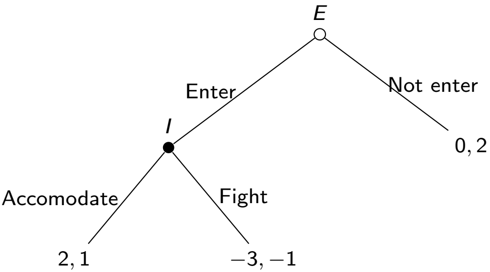
- As game theorists, we want to rule out the equilibrium that uses a non-credible threat
- This motivates the idea of **subgame perfection**
- A Nash Equilibrium is **subgame perfect** if it induces a Nash equilibrium for any **subgame**
- In a complete information game, a **subgame** is a game beginning at any non-terminal node



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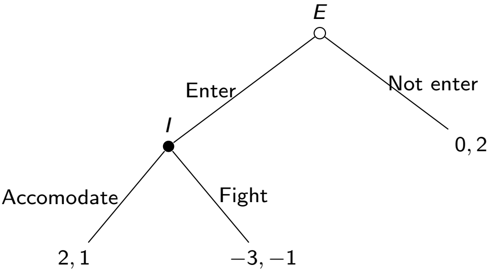
- In our example, the game has two subgames:



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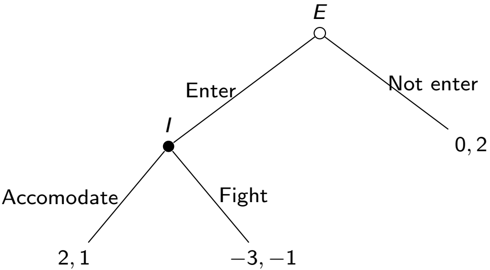
- In our example, the game has two subgames:
- The game beginning at the initial node (EFG is always a subgame of itself)



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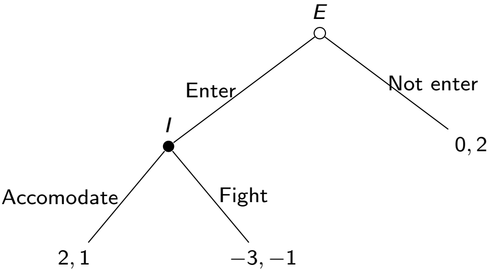
- In our example, the game has two subgames:
- The game beginning at the initial node (EFG is always a subgame of itself)
- The game beginning at the incumbent's decision node



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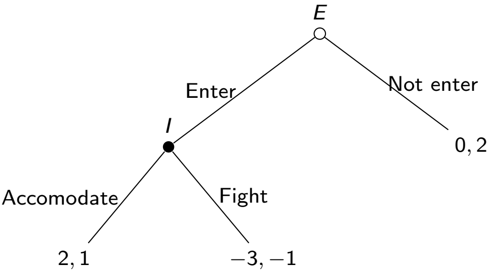
- (In, Accomodate) induces a Nash Equilibrium for both subgames
- (Out, Fight) does not
- Therefore, (Out, Fight) is not **subgame perfect**
- We succeeded in introducing an equilibrium concept that rules out non-credible threats being played



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Backward induction

The **backward induction** procedure can be used to find the subgame perfect Nash equilibrium:

1. Look at immediate predecessors of the final nodes.
2. Each such node has a player controlling it. Choose the action that gives him the largest payoff (break ties arbitrarily).
3. Replace the node with a final node having utility for each player equal to the utility induced by the action chosen in Step 2.
4. Repeated the procedure in steps 1-3 in the new game until only one node is left.

Empirical evidence?

- Not great (see Goeree and Holt (2001), experiments on ultimatum games)

Evidence of people learning backward induction

Gneezy, Rustichini, and Vostroknutov (2010):

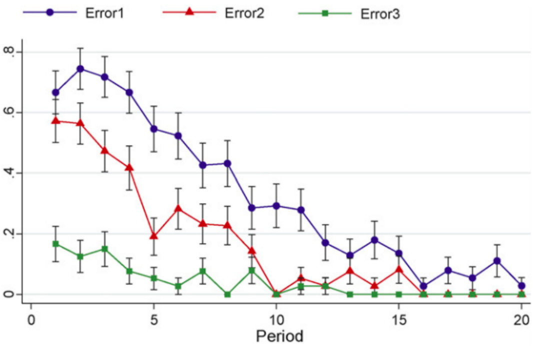

Fig. 1. Average error per round, $G(15, 3)$.

Repeated games

- A repeated game consists of a stage game played many times by the same players
- The stage game is a normal form game (e.g., prisoner's dilemma)
- Can be finitely or infinitely repeated
- **The main message of the theory of repeated games is that repetition can be used to help players sustain more cooperation**
- But there are some caveats...

Finitely repeated games

- Unraveling in stage games with **unique** NE
  - If a game has a unique NE, the only SPNE is for that NE to be played every period (just apply backward induction!)
  - I.e., repetition does not add anything
  - E.g., prisoner's dilemma
- In practice, though, subjects cooperate in finitely repeated PD
  - Embrey, M., Fréchette, G. R., & Yuksel, S. (2018). Cooperation in the finitely repeated prisoner's dilemma. The Quarterly Journal of Economics, 133(1), 509-551.
- If the stage game has **multiple** NE, different equilibria can be used to provide dynamic incentives

Example

		Player 2		
		b_1	b_2	b_3
	a_1	10, 10	2, 12	0, 13
Player 1	a_2	12, 2	5, 5	0, 0
	a_3	13, 0	0, 0	1, 1

Example

		Player 2		
		b_1	b_2	b_3
Player 1	a_1	10, 10	2, 12	0, 13
	a_2	12, 2	5, 5	0, 0
	a_3	13, 0	0, 0	1, 1

- This shows that outcomes that are non-Nash in one-shot games can be played in SPNE of the finitely repeated game
- HW: Find all pure strategy subgame perfect Nash Equilibria in this game

Infinitely Repeated Games

- Classic example: infinitely repeated PD

	C	D
C	2, 2	0, 3
D	3, 0	1, 1

- Assume discount factor $\delta \in (0, 1)$
- Consider grim trigger strategy:
  - Play C in period 1
  - In period $t > 1$, play C as long as (C,C) was played in period $t - 1$. Otherwise, play D forever
- Expected payoff $2/(1 - \delta)$ converges to the fully cooperative payoff as $\delta \to 1$
- See class notes for why GT is a SPNE for high enough δ