1 Normal Form Games

A normal form game is $(I, (A^i)_{i=1,...,n}, (u^i)_{i=1,...,n})$, where $\forall i$ A_i is an action set, $A = \times_{i=1}^n A^i$, and $u_i : A \to \mathbb{R}$ $\forall i$.

$I = \{1, ..., n\}$ is the set of players.

Assume A^i is finite for all i.

Examples: Coordination, Matching pennies, Prisoner's dilemma, Battle of the Sexes.

1.1 Dominance

Let $S^i = \Delta(A^i) = \{(s(a_1^i), ..., s(a_{k_i}^i)) : \forall i,\ s(a_i) \geq 0, \sum_{A^i} s(a^i) = 1\}$.

A mixed extension of a normal form game is $(I, (S^i)_{i=1,...,n}, (u^i)_{i=1,...,n})$, where $\forall$ $S^i = \Delta(A^i)$, $S = \prod_{i=1}^n S^i$ and $u^i : S \to \mathbb{R}$ is defined by

$$u^i(s^1, ..., s^n) = \sum_{a \in A} u^i(a) \prod_{i=1}^n s^i(a^i).$$

We write $Pr_s(a) = \prod_{i=1}^n s^i(a^i) \in \Delta A$.

Example: Show that in the game below, the player can get a better payoff by mixing T and M than by playing B, no matter what his belief is about what his partner is doing.

	L	R
T	3	0
M	0	3
B	1	1

We say $s^i \in S^i$ strictly dominates $a^i \in A^i$ iff for all a^{-i}

$$u^i(s^i, a^{-i}) > u^i(a^i, a^{-i}).$$

Alternatively,

$$s^i D_2 a^i \Leftrightarrow \forall s^{-i} \in S^{-i} \quad u^i(s^i, s^{-i}) > u_i(a^i, s^{-i})$$

or

$$s^i D_3 a^i \Leftrightarrow \forall \mu \in \Delta(A^{-i}) \quad u^i(s^i, \mu) > u_i(a^i, \mu)$$

Exercise: $s^i D_3 a^i \Leftrightarrow s^i D_2 a^i \Leftrightarrow s^i D_1 a^i$.

Example: Note that T and L are both dominated in the game below.

	L	R
T	-2,-2	-10,-1
B	-1,-10	-5,-5

This leads to the counter-intuitive prediction of playing (B,R). Of course this doesn't happen in real life.

Example:

	L	R
T	3	0
M	0	3
B	x	x

Consider a belief p for Player 1 that Player 2 chooses L. Note that if $x < \frac{3}{2}$, B is never a best response. For every belief, Player 1 is better of playing T or M. Dually, $\exists s^1 \in S^1$ that dominates B.

If $x = \frac{3}{2}$, there exists a belief ($p = 0.5$) for which B is a best response. Dually, B is not strictly dominated.

This example suggests that an action is never a best response if and only if it is strictly dominated by a strategy.

Definition: An action $a^i \in A^i$ is never a best response if there is no $\mu \in \Delta(A^{-i})$ such that $u^i(a^i, \mu) \geq u^i(b^i, \mu)$ for all b^i.

Theorem: An action $a^i \in A^i$ is strictly dominated if and only if it is never a best response.

One direction is easy to prove (see your class notes). The proof for the other direction can be found in Osborne and Rubinstein.

1.2 IESDA

We illustrate this with examples:

	L	R
T	0,-2	-10,-1
B	-1,-10	-5,-5

	L	R
T	3,0	0,1
M	0,0	3,1
B	1,1	1,0

Example (Cournot Duopoly):

Consider a two player game with two firms $i = 1, 2$. Each firm faces the demand curve $p = a - b(q_1 + q_2)$ and per-unit costs of production c. Show that iterated elimination of strictly dominated actions yields a unique outcome in which each firm produces $\frac{a-c}{3b}$.