

# Heuristic Optimization Techniques

## Exercise 3

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### 1 General Variable Neighborhood search

In this programming exercise, we build on our local search from the previous exercise and implement a generalized variable neighborhood search heuristic.

#### 1.1 Neighborhoods

We re-use the neighborhoods from assignment two and added a parameterized node move variant:

**X-node-move** Defined as all subsets where  $X$  vertices of the initial solution are moved to other positions.

- Size of neighborhood:  $(n - 1)^2 * X$
- Objective Function: Incremental, only crossings for moved edges are recalculated.

The neighborhoods from last assignment:

**1-node flip** This neighborhood is defined as all subsets where two vertices of the initial solution are flipped.

- Size of neighborhood:  $n(n - 1)/2$
- Objective Function: Incremental, crossings from flipped vertices are subtracted and recalculated.

**1-edge move** This neighborhood consists of all solutions where one edge is moved to a different page.

- Size of neighborhood:  $(pages - 1)edges$
- Objective Function: Incremental, only crossings for moved edges are recalculated.

**1-node edge move** This neighborhood consists of all solutions where the edges of one vertex on a specific page are moved to all different pages.

- Size of neighborhood:  $(pages - 1)edges$  as worst case.<sup>1</sup>
- Objective Function: Incremental, only crossings for moved edges are recalculated.

## 1.2 Neighborhoodstructures in General VNS

As shown in the algorithm in section 1 we define two non-overlapping sets of neighborhoods:

VNS: used for the shaking, we decided to take the complex neighborhoods here, which cannot be completely searched.

1-node-move, 2-node-move, 3-node-move

VND: used for variable neighborhood descent in the inner loop:

1-edge-move, 1-node-edge-move

## 2 Results

We executed the code on a desktop computer with a Core i7 Quad-Core CPU with 2.67Ghz and 24 GB of main memory. Table 1 shows the number of crossings for the GVNS. We report average and standard deviations for 30 executions as well as the minimum number of crossings. The run-time of our algorithm can be seen in 4

We have set a timeout of 5 minutes for the calculations.

**Order of the neighborhoods** Tables 2 and 3 show the results when we change the order of our neighborhoods. In Table /reftab:resultsDeterministic we report the results where we keep the order of the stochastic neighborhoods used for shaking the same and invert the order of the other neighborhoods. Table 3 shows the results where we flip the order of the neighborhoods used for shaking and keep the others the same. As expected, changing the order of our neighborhoods does influence the results of the GVNS.

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<sup>1</sup>depending on number of edges on respective node pages

|                  | var        | avg          | sd     | min       | min_run |
|------------------|------------|--------------|--------|-----------|---------|
| automatic-1.txt  | 1.51       | 9.42         | 1.23   | 9         | 2       |
| automatic-2.txt  | 35.90      | 41.68        | 5.99   | 36        | 0       |
| automatic-3.txt  | 20.88      | 67.53        | 4.57   | 59        | 10      |
| automatic-4.txt  | 234.26     | 97.05        | 15.31  | 84        | 0       |
| automatic-5.txt  | 0.95       | 40.68        | 0.98   | 39        | 0       |
| automatic-6.txt  | 0.00       | 6,153,059.00 | 0.00   | 6,153,059 | 0       |
| automatic-7.txt  | 714,058.88 | 136,518.53   | 845.02 | 135,111   | 1       |
| automatic-8.txt  | 581,888.04 | 541,495.47   | 762.82 | 539,317   | 11      |
| automatic-9.txt  | 687,002.37 | 1,266,526.95 | 828.86 | 1,265,326 | 7       |
| automatic-10.txt | 24,946.19  | 55,289.26    | 157.94 | 55,035    | 16      |

Table 1: This table shows the results of our algorithm.

|                  | var            | avg          | sd        | min       | min_run |
|------------------|----------------|--------------|-----------|-----------|---------|
| automatic-1.txt  | 2.35           | 16.47        | 1.53      | 12        | 0       |
| automatic-2.txt  | 19.15          | 37.89        | 4.38      | 36        | 0       |
| automatic-3.txt  | 27.64          | 64.79        | 5.26      | 56        | 6       |
| automatic-4.txt  | 116.13         | 137.37       | 10.78     | 108       | 6       |
| automatic-5.txt  | 30.85          | 47.32        | 5.55      | 40        | 0       |
| automatic-6.txt  | 954,027,528.83 | 6,011,850.26 | 30,887.34 | 5,962,740 | 16      |
| automatic-7.txt  | 52,922.03      | 137,904.84   | 230.05    | 137,553   | 4       |
| automatic-8.txt  | 5,544,075.63   | 504,782.95   | 2,354.59  | 501,545   | 12      |
| automatic-9.txt  | 13,573,379.20  | 1,188,287.47 | 3,684.21  | 1,185,819 | 4       |
| automatic-10.txt | 28,478.53      | 55,428.00    | 168.76    | 55,178    | 18      |

Table 2: This table shows the results of our algorithm using when we flipped the order of the deterministic neighborhoods.

|                  | var          | avg          | sd       | min       | min_run |
|------------------|--------------|--------------|----------|-----------|---------|
| automatic-1.txt  | 2.53         | 9.68         | 1.59     | 9         | 3       |
| automatic-2.txt  | 41.53        | 39.21        | 6.44     | 25        | 12      |
| automatic-3.txt  | 39.41        | 64.53        | 6.28     | 50        | 14      |
| automatic-4.txt  | 13.62        | 127.53       | 3.69     | 112       | 7       |
| automatic-5.txt  | 107.82       | 45.84        | 10.38    | 30        | 18      |
| automatic-6.txt  | 0.00         | 6,153,059.00 | 0.00     | 6,153,059 | 0       |
| automatic-7.txt  | 1,033,834.45 | 136,568.16   | 1,016.78 | 134,450   | 11      |
| automatic-8.txt  | 0.00         | 541,969.00   | 0.00     | 541,969   | 0       |
| automatic-9.txt  | 1,096,844.32 | 1,267,338.32 | 1,047.30 | 1,265,185 | 18      |
| automatic-10.txt | 76,894.93    | 55,855.74    | 277.30   | 55,425    | 5       |

Table 3: This table shows the results of our algorithm where we flipped the order of the stochastic neighborhoods used for shaking.

|                  | var    | avg    | sd    | min | min_run |
|------------------|--------|--------|-------|-----|---------|
| automatic-1.txt  | 0.00   | 0.03   | 0.00  | 0   | 1       |
| automatic-2.txt  | 0.00   | 0.02   | 0.01  | 0   | 10      |
| automatic-3.txt  | 0.05   | 0.61   | 0.22  | 0   | 5       |
| automatic-4.txt  | 0.00   | 0.08   | 0.01  | 0   | 11      |
| automatic-5.txt  | 0.00   | 0.06   | 0.01  | 0   | 4       |
| automatic-6.txt  | 0.26   | 309.22 | 0.51  | 309 | 13      |
| automatic-7.txt  | 712.56 | 277.61 | 26.69 | 228 | 9       |
| automatic-8.txt  | 0.01   | 302.82 | 0.09  | 303 | 16      |
| automatic-9.txt  | 0.00   | 303.16 | 0.06  | 303 | 0       |
| automatic-10.txt | 0.00   | 302.35 | 0.04  | 302 | 7       |

Table 4: This table shows the runtime of our algorithm.

|                  | var  | avg    | sd   | min | min_run |
|------------------|------|--------|------|-----|---------|
| automatic-1.txt  | 0.00 | 0.01   | 0.00 | 0   | 2       |
| automatic-2.txt  | 0.00 | 0.04   | 0.01 | 0   | 16      |
| automatic-3.txt  | 0.12 | 0.74   | 0.35 | 0   | 13      |
| automatic-4.txt  | 0.00 | 0.06   | 0.02 | 0   | 7       |
| automatic-5.txt  | 0.00 | 0.08   | 0.01 | 0   | 4       |
| automatic-6.txt  | 0.90 | 309.33 | 0.95 | 309 | 16      |
| automatic-7.txt  | 0.00 | 301.15 | 0.01 | 301 | 0       |
| automatic-8.txt  | 0.00 | 302.81 | 0.02 | 303 | 6       |
| automatic-9.txt  | 0.06 | 303.22 | 0.24 | 303 | 16      |
| automatic-10.txt | 0.00 | 302.34 | 0.02 | 302 | 18      |

Table 5: This table shows the runtime of our algorithm (det-order).

|                  | var    | avg    | sd    | min | min_run |
|------------------|--------|--------|-------|-----|---------|
| automatic-1.txt  | 0.00   | 0.02   | 0.00  | 0   | 0       |
| automatic-2.txt  | 0.00   | 0.02   | 0.01  | 0   | 13      |
| automatic-3.txt  | 0.22   | 0.74   | 0.47  | 0   | 2       |
| automatic-4.txt  | 0.00   | 0.05   | 0.02  | 0   | 8       |
| automatic-5.txt  | 0.00   | 0.12   | 0.04  | 0   | 5       |
| automatic-6.txt  | 0.01   | 309.07 | 0.11  | 309 | 12      |
| automatic-7.txt  | 894.86 | 275.49 | 29.91 | 216 | 6       |
| automatic-8.txt  | 0.00   | 302.81 | 0.03  | 303 | 0       |
| automatic-9.txt  | 0.06   | 303.23 | 0.24  | 303 | 6       |
| automatic-10.txt | 0.00   | 302.34 | 0.01  | 302 | 6       |

Table 6: This table shows the runtime of our algorithm (stoch-order).